Quantum Theory of Many-Body systems in Condensed Matter (4302112) 2020

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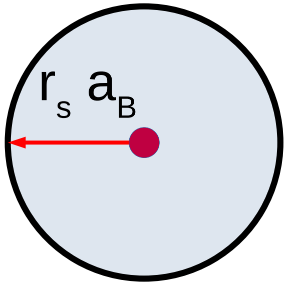
Today's class: *Interacting electron gas – RPA*

- Ground-state energy vs density
- First-order correction: $1/r_s$
- Higher-order diagrams: log-divergency .
- Re-summation of Feynman diagrams.
- "Random Phase Approximation": RPA.

The r_s parameter in the electron gas



"single electron volume"

$$V_{1\mathrm{el}} = \frac{4}{3}\pi(r_s a_B)^3$$

Density:

$$n = \frac{1}{V_{1\mathrm{el}}} = \frac{3}{4\pi(a_B)^3}\frac{1}{(r_s)^3}$$

Bohr radius:

$$\frac{\hbar^2}{2m_e(a_B)^2} = \frac{e^2}{4\pi\epsilon_0(2a_B)} = 1\ \mathrm{Ry} = 13.6\ \mathrm{eV}$$

"Free" electron gas (3D):

$$k_F^3 = 3\pi^2 n$$

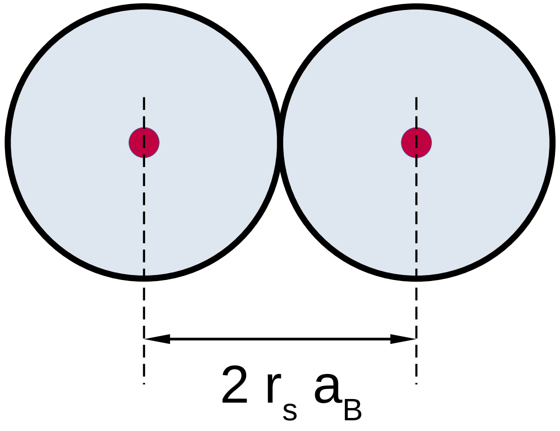
$$\varepsilon_F = \left(\frac{\hbar^2(3\pi^2)^{\frac{2}{3}}}{2m_e}\right) n^{\frac{2}{3}}$$

Kinetic energy per particle:

$$\frac{E^{(0)}}{N} = \frac{3}{5}\varepsilon_F = \frac{2.21}{r_s^2}\ \mathrm{Ry}$$

The r_s parameter in the electron gas

"Typical" electron-electron interaction :



$$\frac{E_{\mathrm{int}}}{N/2} \sim \frac{e^2}{4\pi\epsilon_0(2r_s a_B)} = \frac{1}{r_s}\ \mathrm{Ry}$$

$$\frac{E_{\mathrm{int}}}{E^{(0)}} \sim \frac{(r_s)^2}{r_s} \sim r_s \sim \frac{1}{n^{1/3}}$$

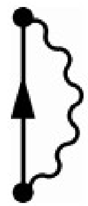
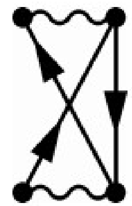
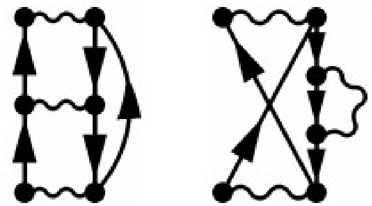
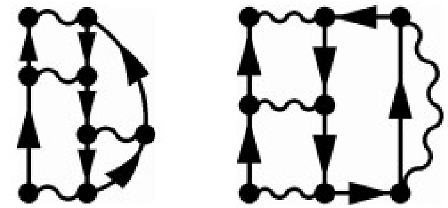
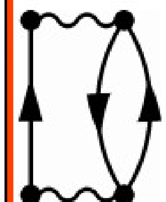
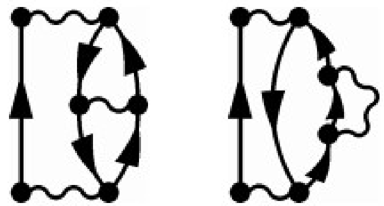
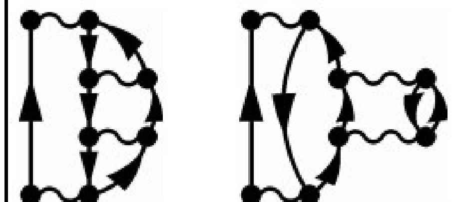
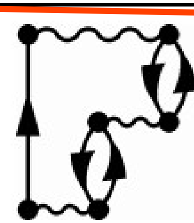
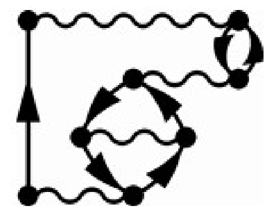
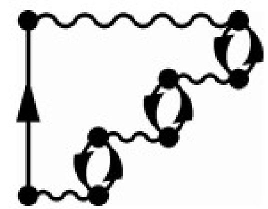
high density	⟺	weakly interacting
low density	⟺	strongly interacting

The “most divergent” diagrams

n interaction lines

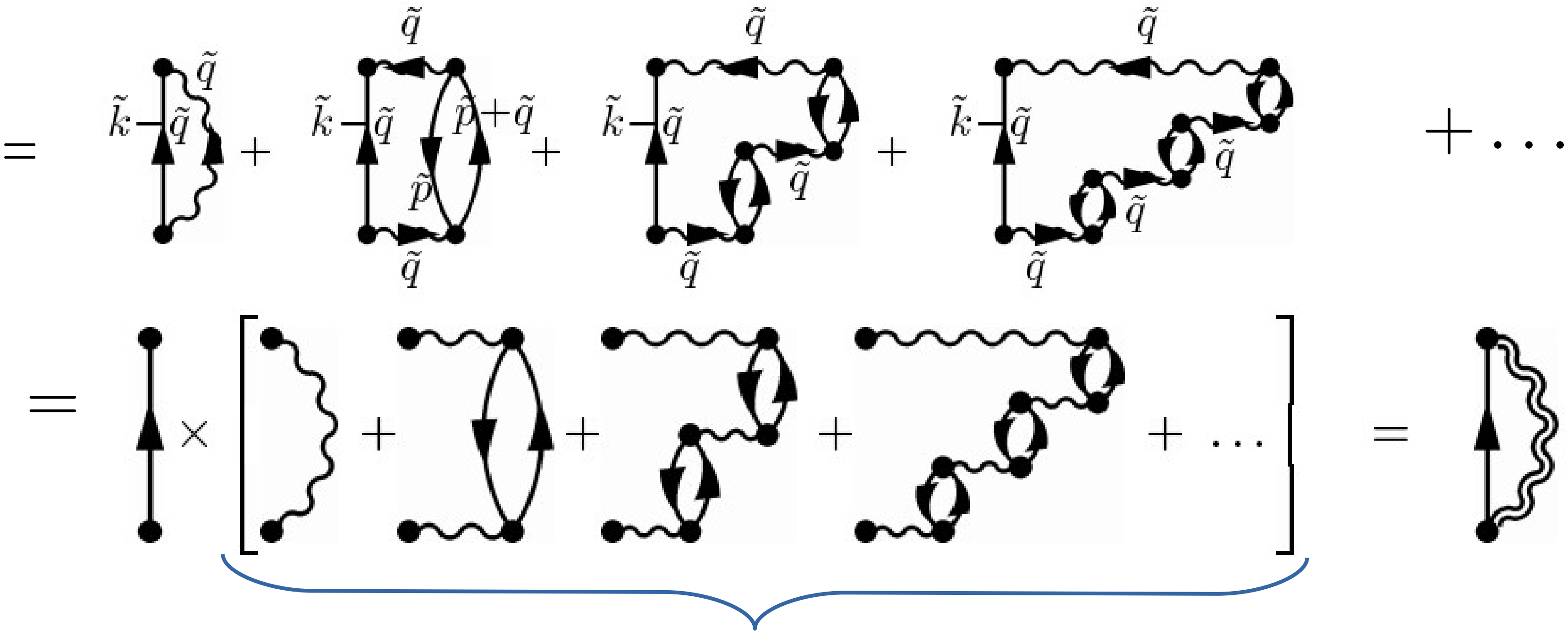
δ interaction lines with same momentum.

δ=n → divergent diagrams.

$\Sigma_\sigma(\tilde{k})$	$n=1$	$n=2$	$n=3$	$n=4$
$\delta=1$				
$\delta=2$	–			
$\delta=3$	–	–		
$\delta=4$	–	–	–	

RPA approximation for the self-energy

$$-i\Sigma^{\mathrm{RPA}}(\vec{k},\omega) =$$



$$\equiv (-i)V^{\mathrm{RPA}}(\vec{q})$$

Why "random phase"? The name persists for historical reasons.
See, e.g., D. Bohm and D. Pines, *Phys. Rev.* **92** 609 (1953).

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A Collective Description of Electron Interactions: III. Coulomb Interactions in a Degenerate Electron Gas

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AND
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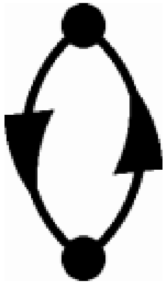
(Received May 21, 1953)

Loop diagram: Polarizability

$$i\Pi^{(0)}(\vec{q},\omega_q) = \quad \vec{p},\omega_p$$



$$\vec{p}+\vec{q},\omega_q+\omega_p$$

$$\Pi^{(0)}(\vec{q},\omega_q) = (-i)\sum_\sigma \int \frac{d\vec{p}}{(2\pi)^3}\frac{d\omega_p}{2\pi}\mathcal{G}^{(0)}_{\sigma\sigma}(\vec{p},\omega_p)\mathcal{G}^{(0)}_{\sigma\sigma}(\vec{p}+\vec{q},\omega_p+\omega_q)$$

Free propagator:

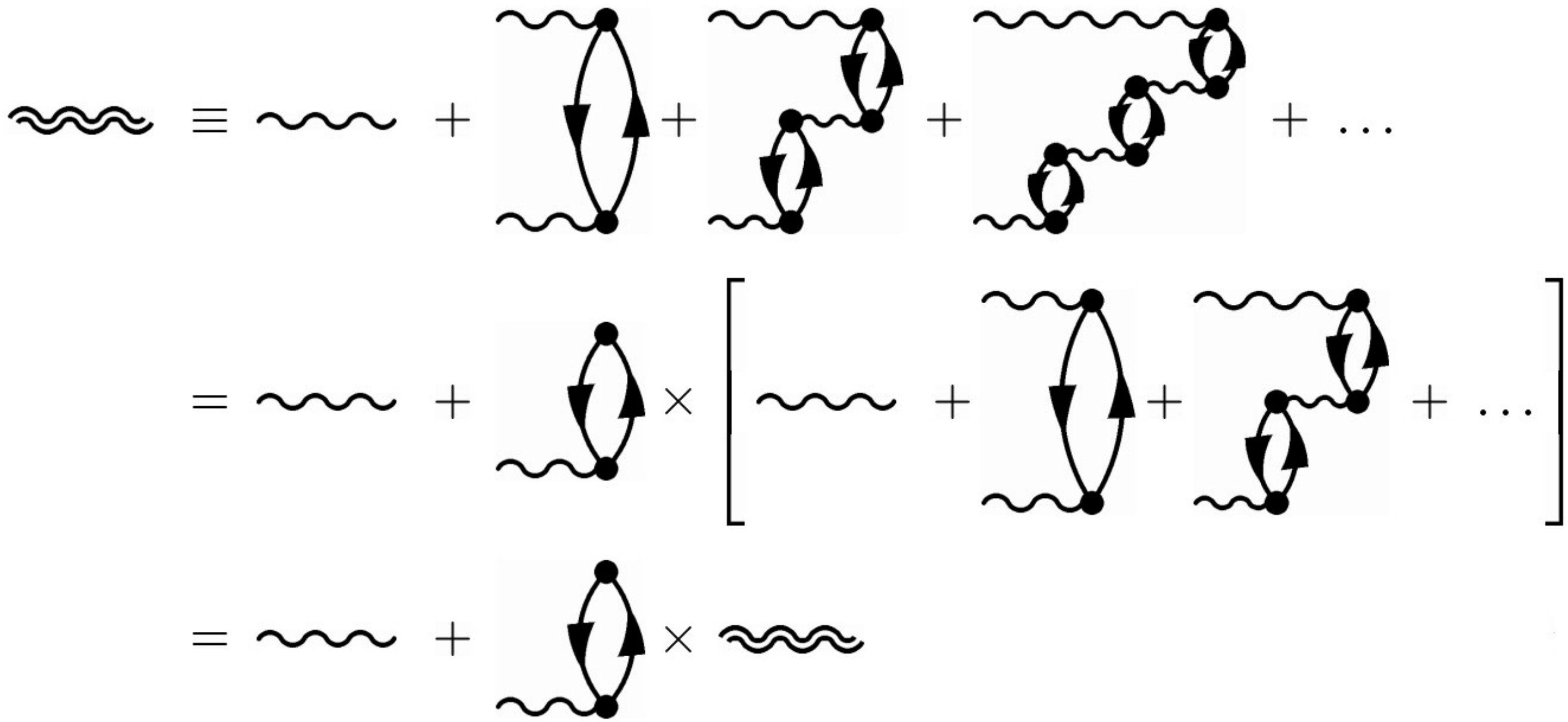
$$\mathcal{G}^{(0)}_{\sigma\sigma}(\vec{p},\omega_p) = \frac{\theta(|\vec{p}|-k_F)}{\omega_p-\epsilon_{|\vec{p}|}+i\eta}+\frac{\theta(k_F-|\vec{p}|)}{\omega_p-\epsilon_{|\vec{p}|}-i\eta}$$

Assignment:

$$\Pi^{(0)}(\vec{q},\omega_q) = -2\int\frac{d\vec{p}}{(2\pi)^3}\left(\frac{\theta(k_F-|\vec{p}|)\theta(|\vec{p}+\vec{q}|-k_F)}{\omega_q+\epsilon_{|\vec{p}|}-\epsilon_{|\vec{p}+\vec{q}|}+i\eta}-\frac{\theta(|\vec{p}|-k_F)\theta(k_F-|\vec{p}+\vec{q}|)}{\omega_q+\epsilon_{|\vec{p}|}-\epsilon_{|\vec{p}+\vec{q}|}-i\eta}\right)$$

RPA potential

$$(-i)V^{\mathrm{RPA}}(\vec{q},\omega_q) =$$

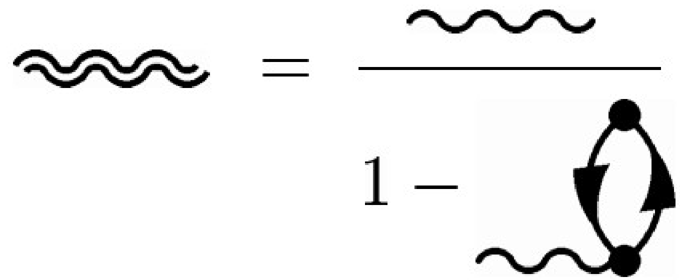


$$= (-i)V(\vec{q}) + (-i)V(\vec{q})i\Pi^{(0)}(\vec{q},\omega_q)(-i)V^{\mathrm{RPA}}(\vec{q},\omega_q)$$

$$\boxed{V^{\mathrm{RPA}}(\vec{q},\omega_q) = V(\vec{q}) + V(\vec{q})\Pi^{(0)}(\vec{q},\omega_q)V^{\mathrm{RPA}}(\vec{q},\omega_q)}$$

RPA: some comments

$$V^{\mathrm{RPA}}(\vec{q},\omega_q) = \frac{V(\vec{q})}{1-\Pi^{(0)}(\vec{q},\omega_q)V(\vec{q})}$$



RPA contribution to GS energy ("correlation energy"):

(see Fetter & Walecka, sec. 12)

$$\frac{E^{(2)}_{\mathrm{RPA}}}{N} = 0.0622\ln r_s - 0.094 + \mathcal{O}(r_s \ln r_s)\ \mathrm{Ry}$$

Thomas-Fermi appoximation
(screening):

[redacted] $$V^{\mathrm{RPA}}(\vec{q},0) \underset{q\to 0}{=} \frac{4\pi e^2}{q^2+q_{\mathrm{TF}}^2}$$

$$\begin{cases} q_{\mathrm{TF}}^2 = 0.66 r_s k_F^2 \\ V_{\mathrm{TF}}(r) = \dfrac{e^2}{4\pi\epsilon_0}\dfrac{e^{-q_{\mathrm{TF}}r}}{r} \end{cases}$$

Screening: Yukawa-like potential