$$\langle u, v \rangle = \int_{-\pi}^{\pi} u(x)\, v(x)\, dx$$

VAMOS GERAR OS POL. ORT.
MÔNICOS

$$\boxed{P_0(x) = 1}$$

$$P_1(x) = (x - \alpha_0) P_0(x)$$

$$\alpha_0 = \frac{\langle x P_0, P_0 \rangle}{\langle P_0, P_0 \rangle}$$

$$\langle x P_0, P_0 \rangle = \int_{-\pi}^{\pi} x\, dx = 0$$

$$P_1(x) = x$$

$$P_2(x) = (x - \alpha_1) P_1(x) - \beta_1 P_0(x)$$

$$\alpha_1 = \frac{\langle x P_1, P_1 \rangle}{\langle P_1, P_1 \rangle}$$

$$\langle x P_1, P_1 \rangle = \int_{-\pi}^{\pi} x^3 \, dx = 0$$

$$\Longrightarrow \alpha_1 = 0$$

$$\beta_1 = \frac{\langle P_1, P_1 \rangle}{\langle P_0, P_0 \rangle}$$

$$\langle P_1, P_1 \rangle = \int_{-\pi}^{\pi} P_1(x)^2\,dx$$

$$= \int_{-\pi}^{\pi} x^2\,dx = 2\int_0^{\pi} x^2\,dx$$

$$= 2\,\frac{x^3}{3}\Big|_0^{\pi} = 2\,\frac{\pi^3}{3}$$

$$\langle P_0, P_0 \rangle = \int_{-\pi}^{\pi} P_0(x)^2\,dx = \int_{-\pi}^{\pi} dx$$

$$= 2\pi$$

$$\beta_1 = \frac{\langle P_1, P_1 \rangle}{\langle P_0, P_0 \rangle} = \frac{\pi^2}{3}$$

$$P_2(x) = x P_1(x) - \frac{\pi^2}{3} P_0(x)$$

$$= x^2 - \frac{\pi^2}{3}$$

—— / ——

$P_0(x) = 1$, $P_1(x) = x$, $P_2(x) = x^2 - \frac{\pi^2}{3}$

VAMOS APROXIMAR

$f(x) = \text{SEN}(x)$ POR UM

POLINÔMIO $g(x)$ DE

GRAU ≤ 2 QUE MINI-

MIZA A

$$\int_{-\pi}^{\pi} [\operatorname{sen}(x) - g(x)]^2 \, dx$$

PODEMOS BUSCAR A APROX NA FORMA

$$g(x) = c_0 P_0(x) + c_1 P_1(x) + c_2 P_2(x)$$

COMO ESTES POLINÔMIOS SÃO ORT., USANDO ESTA BASE O SIST. NORMAL É DIAGONAL E TEMOS

$$c_K = \frac{\langle P_K, f \rangle}{\langle P_K, P_K \rangle},$$

$K = 0, 1$ E 2.

$$C_0 = \frac{\langle P_0, f\rangle}{\langle P_0, P_0\rangle} = \frac{1}{2\pi}\int_{-\pi}^{\pi} \underbrace{\text{sen}(x)\,dx}_{\text{IMPAR}} = 0$$

$$C_1 = \frac{\langle P_1, f\rangle}{\langle P_1, P_1\rangle} = \frac{3}{2\pi^3}\int_{-\pi}^{\pi} x\,\text{sen}(x)\,dx$$

$$= \frac{3}{2\pi^3}\,2\pi = \frac{3}{\pi^2}$$

$$C_2 = \frac{\langle P_2, f\rangle}{\langle P_2, P_2\rangle} = \frac{1}{\langle P_2, P_2\rangle}\int_{-\pi}^{\pi} \underbrace{\left(x^2 - \frac{\pi^2}{3}\right)\text{sen}(x)}_{\text{IMPAR}}$$

$$= 0$$

$$g(x) = 0 \cdot 1 + \frac{3}{\pi^2} \cdot x + 0 \cdot \left(x^2 - \frac{\pi^2}{3}\right)$$

$$= \frac{3}{\pi^2} \cdot x$$

$$= a + bx + cx^2$$

$$\left.\begin{array}{l} a = 0 \\ b = \frac{3}{\pi^2} \\ c = 0 \end{array}\right\}$$

OBS

QUALQUER POLINÔMIO q DE GRAU MENOR OU IGUAL A 2 PODE SER ESCRITO COMO

$$q(x) = a_0 + a_1 x + a_2 x^2$$

$$= \gamma_0 + \gamma_1 \cdot x + \gamma_2 \left(x^2 - \frac{3}{\pi^2} \right)$$

EXERCÍCIO

$f\ [0,1] \longrightarrow \mathbb{R}$

$$f(t) = \begin{cases} 1-2t, & 0 \leq t \leq \frac{1}{2} \\ 2t-1, & \frac{1}{2} \leq t \leq 1 \end{cases}$$

DETERMINE a, b E c

QUE MINIMIZAM

$$\int_0^1 [f(t)-a-bt-ct^2]^2 dt$$

NOTE QUE A SOLUÇÃO NOS DÁ O ÚNICO POL. DE GRAU ≤ 2 QUE MINIMIZA O ERRO QUAD SEGUNDO O PROD INTERNO

$$(u,v) = \int_0^1 u(t)\,v(t)\,dt$$

RESOLVA USANDO A ORTOGONALIDADE DOS POL DE LEGENDRE

$$\langle u,v\rangle = \int_{-1}^{1} u(x)\,v(x)\,dx$$

$$P_0(x) = 1, \quad P_1(x) = x$$

$$P_2(x) = \frac{3}{2}x^2 - \frac{1}{2}$$

$$\langle P_k, P_l\rangle = \begin{cases} 0, & k \neq l \\ \dfrac{2}{2k+1}, & k = l \end{cases}$$

MUDANÇA DE VARIÁVEL P/ FICARMOS COM UMA FUNÇÃO DEF. EM [-1,1]

$$\varphi : [-1, 1] \longrightarrow [0, 1]$$

$$\varphi(x) = \alpha x + \beta$$

$$F(x) = f(\varphi(x)),$$

$$x \in [-1, 1]$$

$$\varphi(-1) = 0 \qquad -\alpha + \beta = 0$$

$$\varphi(1) = 1 \qquad \alpha + \beta = 1$$

$$\alpha = \beta = \frac{1}{2}$$

$$\boxed{\varphi(x) = \frac{x+1}{2}}$$

$$F(x) = f\left(\frac{x+1}{2}\right)$$

$$f(x) = \frac{x+1}{2}$$

$$-1 \leq x \leq 0 \Longrightarrow 0 \leq f(x) \leq \frac{1}{2}$$

$$\Longrightarrow F(x) = 1 - 2f(x)$$

$$= 1 - 2\frac{x+1}{2} = -x$$

$$0 \leq x \leq 1 \Longrightarrow \frac{1}{2} \leq f(x) \leq 1$$

$$\Longrightarrow F(x) = 2f(x) - 1$$

$$= 2\frac{x+1}{2} - 1 = x$$

$$\boxed{F(x) = |x|, \quad x \in [-1, 1]}$$

$$G(x) = c_0 P_0(x) + c_1 P_1(x)$$
$$+ c_2 P_2(x)$$

$$c_0 = \frac{\langle P_0, F \rangle}{\langle P_0, P_0 \rangle} = \frac{1}{2} \int_{-1}^{1} \underbrace{|x|}_{\text{PAR}} \, dx$$

$$= \frac{1}{2} \cdot 2 \int_0^1 |x| \, dx = \int_0^1 x \, dx$$

$$= \frac{1}{2} \quad //$$

$$c_1 = \frac{\langle P_1, F \rangle}{\langle P_1, P_1 \rangle} = \cancelto{0}{\frac{1}{2/3} \int_{-1}^{1} \underbrace{x|x|}_{\text{IMPAR}} \, dx}$$

$$C_2 = \frac{\langle P_2, F\rangle}{\langle P_2, P_2\rangle} = \frac{5}{2}\int_{-1}^{1} \underbrace{\left(\frac{3}{2}x^2 - \frac{1}{2}\right)|x|}_{\text{PAR}}\,dx$$

$$= 5\int_{0}^{1} \left(\frac{3}{2}x^2 - \frac{1}{2}\right)x\,dx$$

$$= 5\int_{0}^{1} \left(\frac{3}{2}x^3 - \frac{1}{2}x\right)dx$$

$$= 5\left(\frac{3}{2}\,\frac{1}{4} - \frac{1}{2}\,\frac{1}{2}\right) = \frac{5}{8}$$

$$G(x) = \frac{1}{2} + \frac{5}{8}\left(\frac{3}{2}x^2 - \frac{1}{2}\right)$$

$$= \frac{15}{16}x^2 + \frac{3}{16}$$

SOLUÇÃO

$$g(t) = G(\psi(t))$$

$$\psi : [0,1] \longrightarrow [-1,1]$$

$$\psi(t) = \gamma t + \delta$$

$$\psi(0) = -1 \qquad \delta = -1$$

$$\psi(1) = 1 \qquad \gamma + \delta = 1$$

$$\psi(t) = 2t - 1$$

$$g(t) = \frac{15}{16}(2t-1)^2 + \frac{3}{16}$$

$$= \frac{15}{16}(4t^2 - 4t + 1) + \frac{3}{16}$$

$$= \frac{15}{4}t^2 - \frac{15}{4}t + \frac{9}{8}$$

$a = \frac{9}{8}$; $b = -\frac{15}{16}$

$c = \frac{15}{16}$