Expansão da Matriz S

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_I \,, \qquad \mathcal{L}_0 = :\bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi: - \frac{1}{2} :\partial_\nu A_\mu \partial^\nu A^\mu:$$

$$H = H_0 + H_I \qquad \mathcal{L}_I = e :\bar{\psi} \cancel{A} \psi \qquad \pi = \frac{\delta \mathcal{L}}{\delta \dot{\psi}} = \frac{\delta \mathcal{L}_0}{\delta \dot{\psi}}$$

Rep Interação

$$\frac{d\psi_I}{dt} = [\psi^I, H_0] \qquad i\frac{d}{dt}|\phi\rangle = H_I^I |\phi\rangle$$

Matriz S

$$|\phi(\infty)\rangle = S\,|\phi(-\infty)\rangle$$

$$S = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 \cdots \int_{-\infty}^{\infty} dt_n \, T\left(H_I(t_1) \cdots H_I(t_n)\right)$$

$$S = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int d^4x_1 \cdots \int d^4x_n \, \underline{T\left(\mathcal{H}_I(x_1) \cdots \mathcal{H}_I(x_n)\right)}$$

$$\mathcal{H}_I = -e :\bar{\psi} \cancel{A} \psi:$$

Teorema de Wick (1950)

A, B lineares em a e a^+ $\begin{pmatrix} \psi & \bar{\psi} \\ & A_\mu \end{pmatrix}$

$$:AB: = :(A^+ + A^-)(B^+ + B^-):$$

$$= :\underline{A^+B^+} + \underline{A^-B^-} + \underbrace{A^+B^-}_{(-1)^f B^- A^+} + \underline{A^-B^+}:$$

$$(-1)^f = \begin{cases} -1 & A \text{ e } B \text{ termi...} \\ 1 & \text{demais} \end{cases}$$

$$AB - :AB: = \begin{cases} \{A^+, B^-\} & A \text{ e } B \text{ são termiônicos} \\ [A^+, B^-] & \text{demais casos} \end{cases}$$

c-numbers

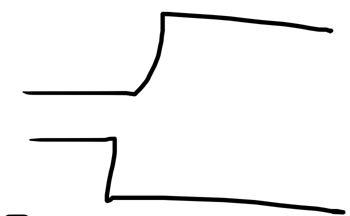
$$\underline{\langle 0|AB|0\rangle} = \begin{cases} \{A^+, B^-\} \\ [A^+, B^-] \end{cases}$$

$$\boxed{AB = :AB: + \langle 0|AB|0\rangle}$$

$$x_1^0 \neq x_2^0$$

$$T\left(A(x_1)B(x_2)\right) = A(x_1)B(x_2)\,\theta(x_1^0 - x_2^0)$$

$$+ (-1)^f B(x_2)A(x_1)\,\theta(x_2^0 - x_1^0)$$



$$= \left(:A(x_1)B(x_2): + \langle 0|A(x_1)B(x_2)|0\rangle\right)\theta(x_1^0 - x_2^0)$$

$$+ (-1)^f\left(:B(x_2)A(x_1): + \langle 0|B(x_2)A(x_1)|0\rangle\right)\theta(x_2^0 - x_1^0)$$

$$= :A(x_1)B(x_2): + \langle 0|T\left(A(x_1)B(x_2)\right)|0\rangle$$

$$T(A(x_1)B(x_2)) = :A(x_1)B(x_2): + \underbrace{\langle 0|T(A(x_1)B(x_2))|0\rangle}_{\underbracket{A(x_1)\,B}(x_2)}$$

$$\cancel{A}$$

$$\underbracket{\phi(x_1)\,\phi}(x_2) = \langle 0|T(\phi(x_1)\phi(x_2))|0\rangle$$
$$= \underline{i\Delta_F(x_1 - x_2)}$$

$$\underbracket{\phi(x_1)\,\phi^\dagger}(x_2)$$

$$\underbracket{\psi_\alpha(x_1)\,\bar{\psi}}(x_2) = i(S_F)_{\alpha\beta}(x_1 - x_2)$$

$$\underbracket{A_\mu(x_1)\,A}_\nu(x_2) = iD_F^{\mu\nu}(x_1 - x_2)$$

$a =$ grau de $A = \begin{cases} 0 & \text{se } A \text{ é boson} \\ 1 & \text{se } A \text{ é férmion} \end{cases}$

$$[A,B]_s = AB - (-1)^{ab} BA$$

$a+b$

$$C[A,B]_s = (-1)^{c(a+b)} [A,B]_s C$$

$$\langle 0 | AB | 0 \rangle = [A^+, B^-]_s$$

$$ABC = (A^+ + A^-)(B^+ + B^-)(C^+ + C^-)$$

$$A^+ \underbrace{B^+ C^-}$$

$$(-1)^{ac} C^- B^+ + [B^+, C^-]_s$$

$$ABC = :ABC: + (-1)^{a(b+c)} [B^+, C^-]_s A + [A^+, B^-]_s C$$

$$+ (-1)^{bc} [A^+, C^-] B^+ + (-1)^{ab} (-1)^{b(a+c)} [A^+, C^-] B^-$$

$$ABC = :ABC: + (-1)^{a(b+c)} \langle 0|BC|0\rangle A + (-1)^{bc} \langle 0|AC|0\rangle B + \langle 0|AB|0\rangle C$$

$$T(A(x_1)B(x_2)C(x_3)) = A(x_1)B(x_2)C(x_3)\,\theta(x_1^0 - x_2^0)\,\theta(x_2^0 - x_3^0)$$
$$+ (-1)^{bc} A(x_1)C(x_3)B(x_2)\,\theta(x_1^0 - x_3^0)\,\theta(x_3^0 - x_2^0)$$

$$\vdots$$

$$T(A(x_1)B(x_2)C(x_3)) = :A(x_1)B(x_2)C(x_3): + (-1)^{a(b+c)} \langle 0|T(B(x_2)C(x_3))|0\rangle A(x_1)$$
$$+ (-1)^{bc} \langle 0|T(A(x_1)C(x_3))|0\rangle B(x_2) + \langle 0|T(A(x_1)B(x_2))|0\rangle C(x_3)$$

$$T\left(A(x_1)B(x_2)C(x_3)\right) = :ABC: + :A\underbrace{BC}: + :\underbrace{ABC}: + :\underbrace{AB}C:$$

$$T(ABCD \quad XYZ) = :ABC \ldots XYZ: + :\underbrace{AB}\ldots XYZ: + :\underbrace{ABC}\ldots XYZ: + \ldots$$

$$+ :\underbrace{AB}\,\underbrace{CD}\ldots: + :\underbrace{A\underbrace{BC}D}\ldots: + \ldots$$

$$\langle 0|AB|0\rangle :CD \ldots XYZ:$$

$$T\left(\mathcal{H}_I(x_1)\, \mathcal{H}_I(x_2) \cdots \mathcal{H}_I(x_n) \right)$$

$:ABC:(x_1)$

FAY

$$A = \underline{A}^{+} + \underline{A}^{-}$$

$$x_r = (x_r^0, \vec{x}_r) \Rightarrow \xi_r = (x_r^0 \pm \varepsilon, \vec{x}_r)$$

$+\varepsilon$ op criação

$-\varepsilon$ op destruição

$$A(x_1) B(x_2)\, \theta(x_1^0 - x_2^0)$$

$$T\left(\mathcal{H}_I(x_1) \cdots \mathcal{H}_I(x_n) \right) = \lim_{\varepsilon \to 0} T\left((ABC)_{\xi_1} \cdots (ABC)_{\xi_n} \right)$$

$$T\left(\mathcal{H}_I(x_1)\,\mathcal{H}_I(x_2)\right) = T\left(\bar{\psi}\not{A}\psi_{x_1}\,\bar{\psi}\not{A}\psi_{x_2}\right)$$

$$= :\bar{\psi}\not{A}\psi(x_1)\,\bar{\psi}\not{A}\psi(x_2):$$

$$+ :\bar{\psi}\not{A}\psi(x_1)\,\bar{\psi}\not{A}\psi(x_2): + :\bar{\psi}\not{A}\psi(x_1)\,\bar{\psi}\not{A}\psi(x_2):$$

$$+ :\bar{\psi}\not{A}\psi(x_1)\,\bar{\psi}\not{A}\psi(x_2):$$

$$+$$

$$\bar{\psi}\not{A}\psi(x_1)\,\bar{\psi}\not{A}\psi(x_2)$$

$$[A^+, B^-],$$
$$\|$$
$$\langle 0|AB|0\rangle$$

$$\langle f | S | i \rangle$$

e^+e^- $\qquad\qquad\qquad\qquad$ e^+e^-