$$f(x) = \begin{cases} x^2, & x \in [0,1] \\ -x^2, & x \in [-1,0] \end{cases}$$

$$P(x) = \frac{3}{4}x \,; \quad \langle u, v \rangle = \int_{-1}^{1} u(x)\,v(x)\,dx$$

$$1, \quad x, \quad x^2 - \frac{1}{3} \qquad \underline{\text{P. O. MÔNICOS}}$$

$$P_0 \qquad P_1 \qquad P_2$$

APROX. P/ GRAU ≤ 2

$$C_0 P_0(x) + C_1 P_1(x) + C_2 P_2(x)$$

$$= C_0 + C_1 \cdot x + C_2 \cdot \left(x^2 - \frac{1}{3}\right)$$

$$= \frac{3}{4} x$$

$$C_0 = 0 \,; \quad C_1 = \frac{3}{4} \,; \quad C_2 = 0$$

$P_3(x)$: POLINÔMIO DE GRAU 3 DA FAMÍLIA DOS POLINÔMIOS ORTOGONAIS

DEVIDO À $\boxed{\text{ORTOGONALIDADE}}$ A APROXIMAÇÃO POR UM POLINÔMIO DE GRAU ≤ 3 PELO M.M.Q. SERÁ IGUAL A:

$$c_0 P_0(x) + c_1 P_1(x) + c_2 P_2(x) + c_3 P_3(x)$$

COM

$$c_K = \frac{\langle P_K, g \rangle}{\langle P_K, P_K \rangle}$$

$$\Longrightarrow c_0 = 0; \quad c_1 = \frac{3}{4}; \quad c_2 = 0$$

CÁLCULO DE $P_3(x)$ MÓNICO

$$P_{k+1}(x) = (x - \alpha_k) P_k(x) - \beta_k P_{k-1}(x)$$

$$P_3(x) = (x - \alpha_2) P_2(x) - \beta_2 P_1(x)$$

$$\alpha_2 = \langle x P_2,$$

JC

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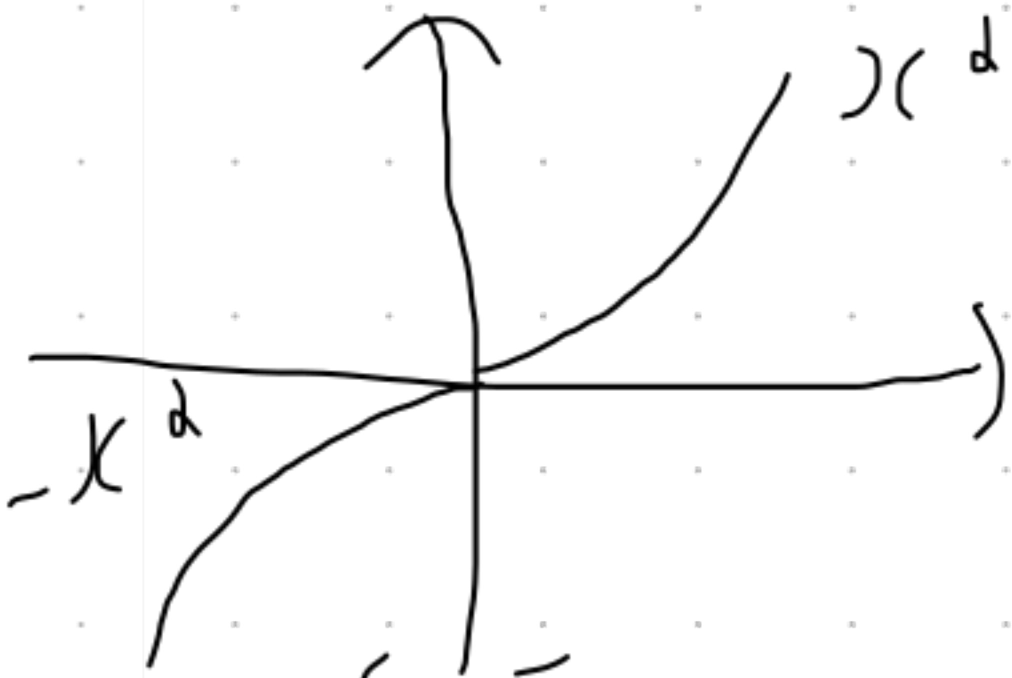
$$c_3 = \frac{\langle P_3, f\rangle}{\langle P_3, P_3\rangle}$$



f É IMPAR

$$\langle P_3, f\rangle = \int_{-1}^{1} \underbrace{\left(x^3 - \frac{3}{5}x\right) f(x)}_{\text{PAR}}\, dx$$

$$= 2\int_{0}^{1} \left(x^3 - \frac{3}{5}x\right) f(x)\, dx$$

$$= 2\int_{0}^{1} \left(x^3 - \frac{3}{5}x\right) x^2\, dx =$$

$$2\int_0^1 \left(x^5 - \frac{3}{5}x^3\right) dx = 2\left[\frac{1}{6} - \frac{3}{5}\cdot\frac{1}{4}\right]$$

$$= 2\left(\frac{1}{6} - \frac{3}{20}\right) = \frac{1}{3} - \frac{3}{10} = \frac{1}{30}$$

$\langle P_3, f\rangle$

$$\langle P_3, P_3\rangle = \int_{-1}^{1} \left(x^3 - \frac{3}{5}x\right)^2 dx$$

$$= \int_{-1}^{1} \left(x^6 - \frac{6}{5}x^4 + \frac{9}{25}x^2\right) dx$$

$$= 2\left(\frac{1}{7} - \frac{6}{5}\cdot\frac{1}{5} + \frac{9}{25}\cdot\frac{1}{3}\right)$$

$$= 2\left(\frac{1}{7} - \frac{3}{25}\right) = 2\cdot\frac{25-21}{175} = \frac{8}{175}$$

$$c_3 = \frac{1}{30} \cdot \frac{175}{8} = \frac{35}{48}$$

$$\underbrace{g_3(x) = \frac{3}{4}x + \frac{35}{48}\left(x^3 - \frac{3}{5}x\right)}$$

$$= \frac{3}{4}x + \frac{35}{48}x^3 - \frac{7}{16}x$$

$$= \frac{35}{48}x^3 + \frac{5}{16}x$$