Quantum Theory of Many-Body systems in Condensed Matter (4302112) 2020

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Today's class: *Feynman diagrams*

- Review: Time-ordered Green's functions.
- Perturbative expansion in the interaction picture.
- Feynman diagrams.
- Feynman diagrams in momentum/frequency space

Review: Time-ordered Green's functions (T=0)

$$i\mathcal{G}_{km}(t,t') = \left\langle \Psi_0 \left| \mathcal{T}\left[\hat{a}_k(t)\hat{a}^\dagger_m(t')\right]\right| \Psi_0 \right\rangle$$

$$\hat{a}_k(t) = e^{+i\hat{H}t}\hat{a}_k e^{-i\hat{H}t}$$

Heisenberg

$$\hat{H}|\Psi_0\rangle = E_0|\Psi_0\rangle \qquad \hat{H} = \hat{H}_0 + \hat{H}_1$$

- Time ordering operator: $\mathcal{T}\left[\hat{a}_k(t)\hat{a}^\dagger_{k'}(t')\right] = \begin{cases} \hat{a}_k(t)\hat{a}^\dagger_{k'}(t') \text{ if } t > t' \\ \pm\hat{a}^\dagger_{k'}(t')\hat{a}_k(t) \text{ if } t' > t \end{cases}$

+: Bosons -: Fermions

Non-interacting GFs:

$$i\mathcal{G}^{(0)}_{km}(t,t') = \left\langle \Phi_0 \left| \mathcal{T}\left[\hat{a}_k(t)_0\hat{a}^\dagger_m(t')_0\right]\right| \Phi_0 \right\rangle$$

$$\hat{H}_0|\Phi_0\rangle = E^{(0)}_0|\Phi_0\rangle$$

$$\hat{a}_k(t)_0 = e^{+i\hat{H}_0t}\hat{a}_k e^{-i\hat{H}_0t}$$

Review - Perturbative expansion

Perturbative expansion in the interaction (quartic) term:

$$i\mathcal{G}_{km}(t,t') = \frac{\langle\Phi_0|\sum_{n=0}^{\infty}\frac{(-i)^n}{n!}\int_{-\infty}^{+\infty}dt_1\dots dt_n\mathcal{T}\left[\hat{H}_1(t_1)\ \dots\hat{H}_1(t_n)(\hat{a}_k)_I(t)(\hat{a}_m^\dagger)_I(t')\right]|\Phi_0\rangle}{\langle\Phi_0|\hat{U}(+\infty,-\infty)|\Phi_0\rangle}$$

$$\begin{cases}\hat{H}_0 = \sum_k \epsilon_k \hat{a}_k^\dagger \hat{a}_k \\ \hat{H}_1 = \frac{1}{2}\sum_{\substack{km\\k'm'}} V_{\substack{km\\k'm'}} \hat{a}_k^\dagger \hat{a}_m^\dagger \hat{a}_{m'} \hat{a}_{k'}\end{cases}$$

$$\hat{H}_0|\Phi_0\rangle = E_0|\Phi_0\rangle$$

$$V_{\substack{km\\k'm'}} = \int d^3\vec{r}_1 d^3\vec{r}_2\ \varphi_k^*(\vec{r}_1)\varphi_m^*(\vec{r}_2)\ V(\vec{r}_1,\vec{r}_2)\ \varphi_{k'}(\vec{r}_1)\varphi_{m'}(\vec{r}_2)$$

Perturbation expansion (1st order)

n=0 n=1

$$i\mathcal{G}_{km}(t,t') \approx \frac{1}{\langle \Phi_0 | \hat{U}(+\infty,-\infty) | \Phi_0 \rangle} \Bigg(i\mathcal{G}^{(0)}_{km}(t,t') + \frac{(-i)}{2} \sum_{\substack{k'm' \\ k''m''}} \int_{-\infty}^{+\infty} dt_1$$

$$\Bigg[i\mathcal{G}^{(0)}_{km}(t,t') \left(i\mathcal{G}^{(0)}_{k''k'}(t_1,t_1) V_{\substack{k'm' \\ k''m''}} i\mathcal{G}^{(0)}_{m''m'}(t_1,t_1) - i\mathcal{G}^{(0)}_{m''k'}(t_1,t_1) V_{\substack{k'm' \\ k''m''}} i\mathcal{G}^{(0)}_{k''m'}(t_1,t_1) \right)$$

$$+ i\mathcal{G}^{(0)}_{kk'}(t,t_1) i\mathcal{G}^{(0)}_{k''m'}(t_1,t_1) V_{\substack{k'm' \\ k''m''}} i\mathcal{G}^{(0)}_{m''m}(t_1,t') - i\mathcal{G}^{(0)}_{km'}(t,t_1) i\mathcal{G}^{(0)}_{m''m'}(t_1,t_1) V_{\substack{k'm' \\ k''m''}} i\mathcal{G}^{(0)}_{k''m}(t_1,t')$$

$$+ i\mathcal{G}^{(0)}_{km'}(t,t_1) i\mathcal{G}^{(0)}_{m''k'}(t_1,t_1) V_{\substack{k'm' \\ k''m''}} i\mathcal{G}^{(0)}_{k''m}(t_1,t') - i\mathcal{G}^{(0)}_{kk'}(t,t_1) i\mathcal{G}^{(0)}_{k''k'}(t_1,t_1) V_{\substack{k'm' \\ k''m''}} i\mathcal{G}^{(0)}_{m''m}(t_1,t') \Bigg] \Bigg)$$

Rules for Feynman diagrams

$$i\mathcal{G}_{km}(t,t') = \frac{\langle\Phi_0|\sum_{n=0}^{\infty}\frac{(-i)^n}{n!}\int_{-\infty}^{+\infty}dt_1\ldots dt_n\mathcal{T}\left[\hat{H}_1(t_1)\;\ldots\hat{H}_1(t_n)(\hat{a}_k)_I(t)(\hat{a}_m^\dagger)_I(t')\right]|\Phi_0\rangle}{\langle\Phi_0|\hat{U}(+\infty,-\infty)|\Phi_0\rangle}$$

To evaluate the *n*-th order term in the expansion:

- Draw all connected, topologically distinct diagrams with *n* interaction lines beginning at (m,t') and ending at (k,t).
- Associate indices [(k',k''),(m',m''), etc.] to all interaction *2n* vertices.
- To every continuous line from (k_2, t_2) → (k_1, t_1) associate $i\mathcal{G}^{(0)}_{k_1k_2}(t_1,t_2)$
- To every interaction like, associate $(-i)V_{\substack{k'm'\\k''m''}}$
- Sum over internal indices and integrate over times t_n.
- Multiply each diagram by $(-1)^F$ where *F* =no. of fermionic closed loops.
- Multiply by $\frac{1}{n!}$

Rules for Feynman diagrams in (**k**,ω)

$$i\mathcal{G}(\vec{k},\omega)$$

To evaluate the *n*-th order term in the expansion:

- Draw all connected, topologically distinct diagrams with *n* interaction lines.
- Associate ($\mathbf{k}_i,\omega_i$) to propagating lines and ($\mathbf{q}$,0) to interaction lines.
- To every continuous line associate $i\mathcal{G}^{(0)}(\vec{k}_i,\omega_i)$
- To every interaction like, associate $(-i)V(\vec{q})$
- To every equal time propagator associate $e^{i\omega_i\eta}$ with $\eta\to 0^+$
- Apply conservation of momentum in each vertex.
- Integrate over internal momenta and frequencies .
- Multiply each diagram by $(-1)^F$ where *F* =no. of fermionic closed loops.
- Multiply by $\frac{1}{n!}$