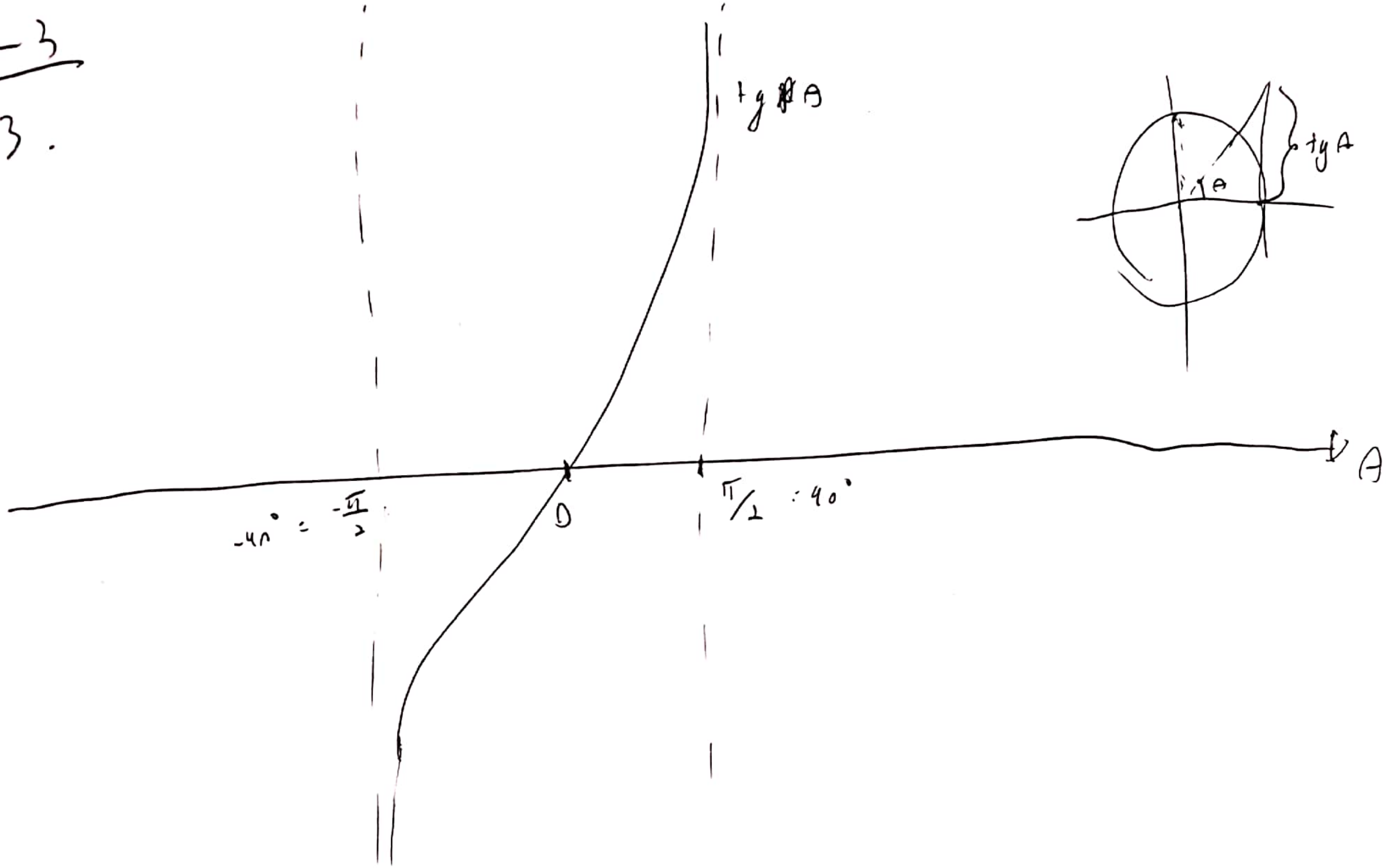
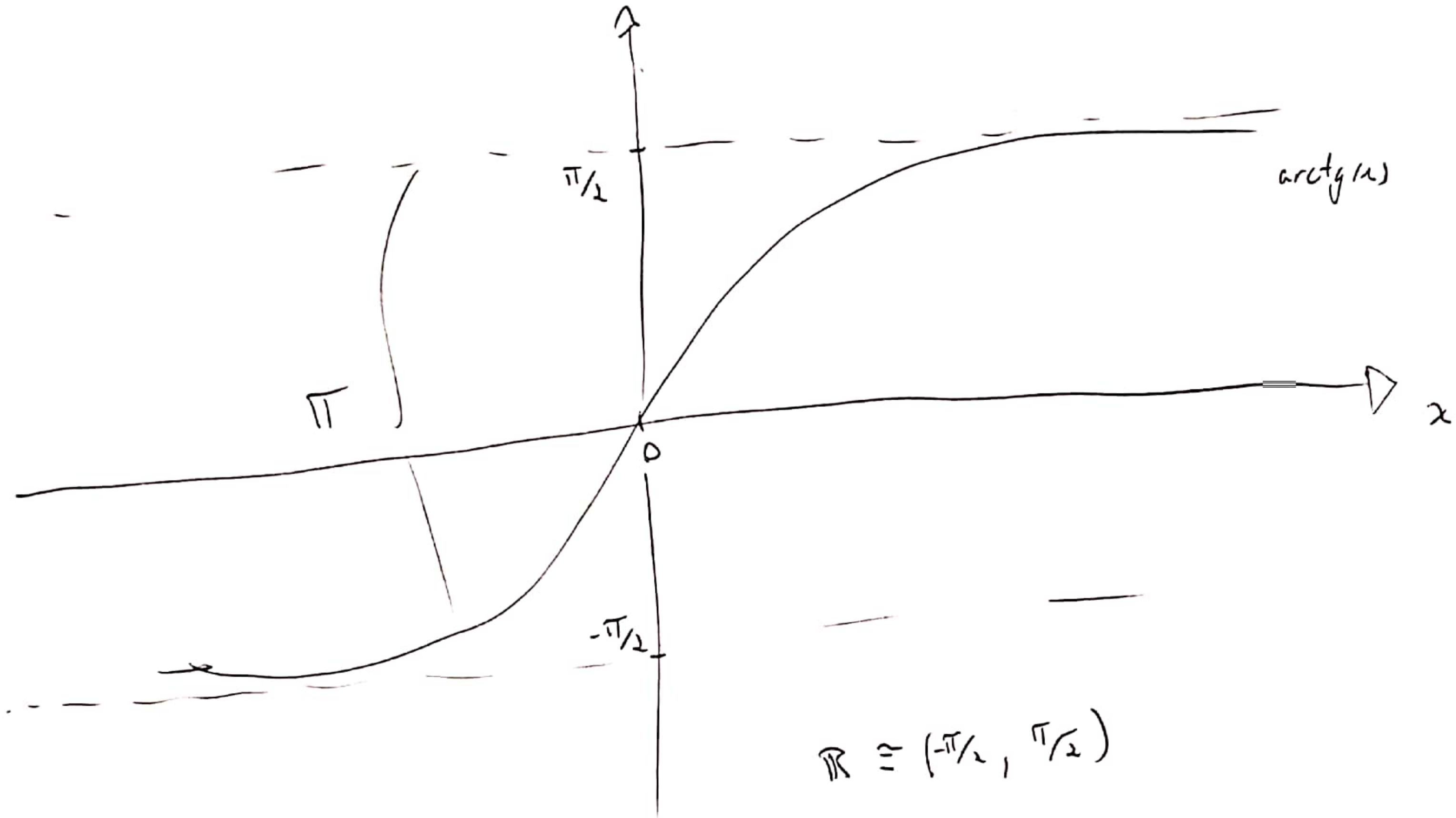
L3

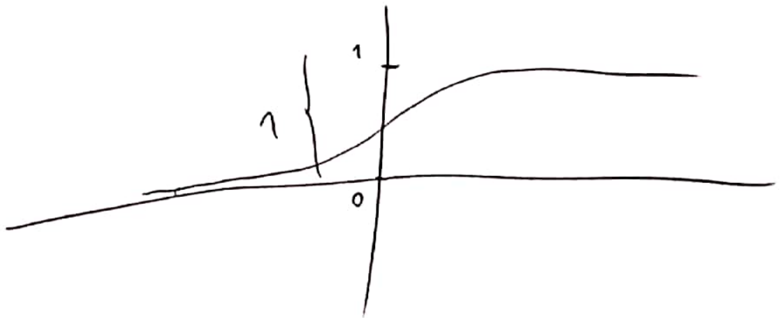
3.





$\mathbb{R} \cong (-\pi/2, \pi/2)$

~~arctg x~~ $\quad \operatorname{arctg} : \mathbb{R} \to \underbrace{(-\pi/2, \pi/2)}_{\simeq (0,1)}$ é injetora e sobrejetora!



$f(x)$ $\qquad$ $a f(x)$ $\qquad$ $\operatorname{arctg}(x) = f(x)$

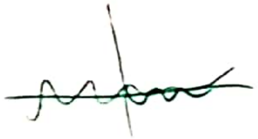
$2\,\text{sen}$

$$-\frac{\pi}{2} < f(x) < \frac{\pi}{2}$$

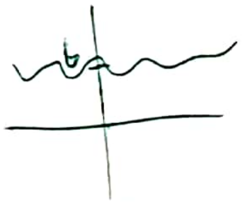
$$\Downarrow$$

$$\cancel{0 < f(x) < 1}$$

$f(x)$



$f(x) + b$



$$0 < a f(x) + b < 1$$

$$-b < a f(x) < 1 - b$$

$$\frac{-b}{a} < f(x) < \frac{1-b}{a}$$

$-\frac{\pi}{2}$ $\qquad$ $\frac{\pi}{2}$

$$\Rightarrow 0 < \frac{1}{\pi} f(x) + \frac{1}{2} < 1$$

$$\Downarrow$$

$$-\frac{\pi}{2} < f(x) < \frac{\pi}{2}$$

$$-\frac{b}{a} = -\frac{\pi}{2} \qquad \frac{1-b}{a} = \frac{\pi}{2}$$

$$\Rightarrow \begin{cases} \frac{\pi}{2} a - b = 0 \quad (*) \\ \frac{\pi}{2} a + b = 1 \end{cases}$$

$$\pi a = 1 \quad \Rightarrow a = \frac{1}{\pi}$$

$$\frac{\pi}{2} \cdot \frac{1}{\pi} - b = 0 \quad \Rightarrow \frac{1}{2} - b = 0 \quad \Rightarrow b = \frac{1}{2}$$

$f: \mathbb{R} \to (0,1)$ bijetora.

$$f(x) = \frac{1}{\pi}\,atg(x) + \frac{1}{2}$$

$f(x) = (atg)^2$

I. injetora:

$f(x) = f(y)$, $x, y \in \mathbb{R}$

$$\Updownarrow$$

$$\cancel{\frac{1}{\pi}}\,atg(x) + \cancel{\frac{1}{2}} = \cancel{\frac{1}{\pi}}\,atg(y) + \cancel{\frac{1}{2}} \Rightarrow atg(x) = atg(y) \overset{atg}{\underset{inj}{\Rightarrow}} x = y$$

II. sobre.

$\exists x$ t.q. $f(x) = z$

- $z \in (0,1)$

$$0 < z < 1 \Rightarrow -\frac{1}{2} < z - \frac{1}{2} < \frac{1}{2}$$

$atg: \mathbb{R} \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

E-genharia reversa:

$$z = \frac{1}{\pi}\,atg(x) + \frac{1}{2} \Leftrightarrow z - \frac{1}{2} = \frac{1}{\pi}\,atg(x) \Leftrightarrow \underbrace{\pi\left(z - \frac{1}{2}\right)}_{\in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)} = atg(x)$$

$\overset{\exists x}{\Rightarrow} atg(x)$

$$0 < z < 1 \overset{-\frac{1}{2}}{\Rightarrow} -\frac{1}{2} < z - \frac{1}{2} < \frac{1}{2} \overset{\cdot\pi}{\Rightarrow} -\frac{\pi}{2} < \pi\left(z - \frac{1}{2}\right) < \frac{\pi}{2}$$

$$\Downarrow$$

$$\pi\left(z - \frac{1}{2}\right) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$\therefore$ Pela sobrejetividade do atg, $\exists\, x \in \mathbb{R}$ t.q. $\pi\left(z - \frac{1}{2}\right) = \operatorname{atg} x$

$$\overset{\times\frac{1}{\pi}}{\Rightarrow} z - \frac{1}{2} = \frac{1}{\pi}\operatorname{atg} x \overset{+\frac{1}{2}}{\Rightarrow} z = \frac{1}{\pi}\operatorname{atg}(x) + \frac{1}{2} = f(x) \;//$$

✓ Sobre.

$\Rightarrow f$ é bijetora

3

Provinha

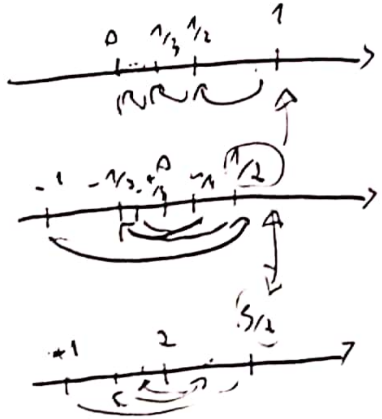
$$A = \left\{ 2 + \frac{(-1)^n}{n} \text{ tal que } n \in \mathbb{N} \right\}$$

$\sup A = \frac{5}{2}$

I. $\frac{1}{n}$:

II. $(-1)^n \cdot \frac{1}{n}$

III. $2 + (-1)^n \cdot \frac{1}{n}$



$$a_1 = 2 + \frac{(-1)^1}{1} = 2 - 1 = 1$$

$$a_2 = 2 + \frac{(-1)^2}{2} = 2 + \frac{1}{2} = \frac{5}{2} = 2{,}5$$

$$a_3 = 2 + \frac{-1}{3} = \frac{6}{3} - \frac{1}{3} = \frac{5}{3} = 1{,}\overline{6}$$

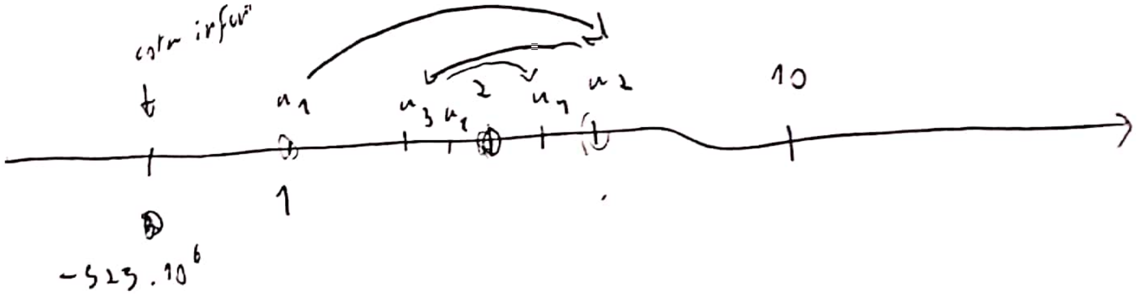
$$a_4 = 2 + \frac{(-1)^4}{4} = 2 + \frac{1}{4} = \frac{9}{4} = 2{,}25$$

$$a_5 = 2 + \frac{(-1)^5}{5} = \frac{9}{5} = 1{,}8$$

$a \leq b \quad \forall a \in A$

$a + 2 \leq b + 2$

$\frac{1}{2} + \frac{1}{3} =$



2.2

4.e)

$I = (a, +\infty) := \{x \in \mathbb{R} : x > a\}$

I. $a = \inf I$

$x \in I \overset{def}{\Rightarrow} x > a \Rightarrow x \geq a$ (usando a definição)

$\forall x \in I,\ x \geq a \Rightarrow a$ é cota inferior

II. Seja $M > a$. Vou mostrar que M não é cota inferior.

Tome $x = \frac{a+M}{2}$. Sabemos, como $a < M$, $a < x < M$

$a < M \Rightarrow a + a < M + a \Rightarrow 2a < a + M \Rightarrow a < \frac{a+M}{2} = x$

$a < M \overset{+M}{\Rightarrow} a + M < M + M \Rightarrow a + M < 2M \Rightarrow x = \frac{a+M}{2} < M$

$x \geq r$ (def)

$x > r \overset{def}{\Rightarrow} x \in I$ ✓

então, achamos $x \in I$ t.q. $x < m \Rightarrow M$ é não cota inferior ✓

$a = \inf I$ // $\square$

---

supõe p/ absurdo que K é cota superior

$K \geq x \quad \forall x \in I$

$K \geq x \overset{def}{>} r \Rightarrow K > r \Rightarrow K \in I$

$K+1 > K$
$\in I$

$K + 1 > K \Rightarrow$

$K > r \Rightarrow K+1 > r \Rightarrow K+1 \in I$