

# Cyclic Code Redundancy (CCR Separável)

## CRC - Código de Redundância Cíclica Separável

Para a transmissão de longos blocos de informação é mais eficiente empregar a codificação separável, que irá permitir o dado recebido já ser utilizado na recepção.

Seja  $D(x) = d_{k-1} \cdot x^{k-1} + d_{k-2} \cdot x^{k-2} + \dots + d_0$  (grau  $k-1$ )

Seja  $G(x)$  de grau  $(n-k)$ .

Seja  $\bar{D}(x) = d_{k-1} \cdot x^{n-1} + d_{k-2} \cdot x^{n-2} + \dots + d_0 \cdot x^{n-k}$

(desloca  $D(x)$  para a esquerda deixando  $(n-k)$  zeros na posição menos significativa)

Em seguida divide-se  $\bar{D}(x)$  por  $G(x)$ , obtendo um resto  $R(x)$  com grau  $< (n-k)$ .

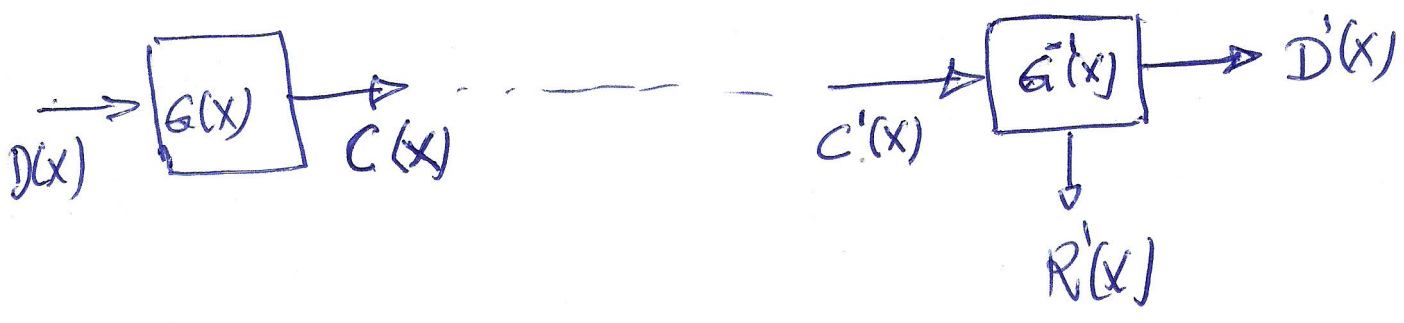
$$\Rightarrow \bar{D}(x) = Q(x) \cdot G(x) + R(x)$$

$$C(x) = \bar{D}(x) + R(x) \equiv Q(x) \cdot G(x) \pmod{2}$$

que será o polinômio a ser transmitido.

$\bar{D}(x)$  e  $R(x)$  não tem termo (informação) em  $n$  posições.

sendo que os primeiros  $k$  bits de  $C(x)$  correspondem à informação original.



Se  $R'(x)$   $\left\{ \begin{array}{l} = \emptyset; \text{ mensagem correta} \\ \neq \emptyset; \text{ mensagem incorreta.} \end{array} \right.$

# Exercícios

3

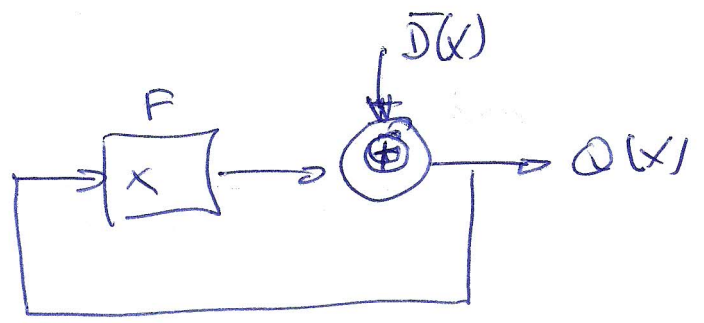
4)  $G(x) = x + 1$  Qual dado deve ser transmitido?  $C(x)$ .  
 $D(x) = x^2 + x$  (0110)

$$\bar{D}(x) = x^3 + x^2 \quad ( \underline{1} \quad \underline{1} \quad \underline{0} \quad \underline{0} )$$

$$\bar{D}(x) = Q(x) \cdot G(x)$$

$$\bar{D}(x) = Q(x)(x+1) = Q(x) \cdot x + Q(x)$$

$$Q(x) = \bar{D}(x) + Q(x) \cdot x$$



Clock	$\bar{D}(x)$	F	$Q(x)$
0	0	0	0
1	0	0	0
2	1	0	1
3	1	1	0
4	0	0	0

At clock 4, the remainder  $R(x) = 0$  is shown in a box.

$$C(x) = \bar{D}(x) + R(x) = x^3 + x^2 \iff \boxed{0110|0}$$

2)  $G(x) = x + 1$

$D(x) = x^3 + x^2 + x \rightarrow$  1110

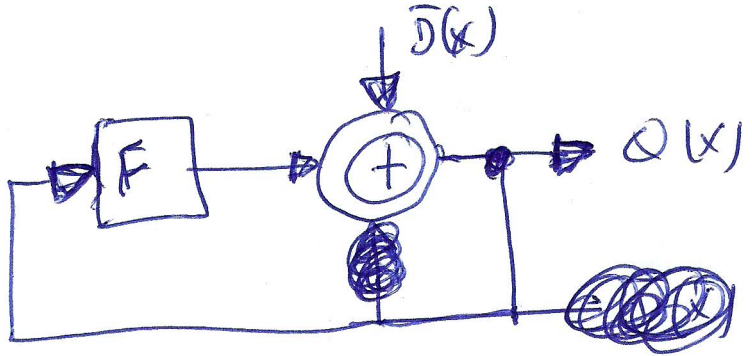
$\bar{D}(x) = x^4 + x^3 + x^2 \rightarrow$  11100

$\bar{D}(x) = Q(x) \cdot G(x)$

$\bar{D}(x) = Q(x) \cdot (x + 1)$

$\bar{D}(x) = Q(x) \cdot x + Q(x)$

$Q(x) = \bar{D}(x) + Q(x) \cdot x$



Clock	$\bar{D}(x)$	F	$Q(x)$
0	0	0	0
1	0	0	0
2	1	0	1
3	1	1	0
4	1	0	1
5	0	1	1

$\boxed{1} - R(x) = 1$

$\Rightarrow C(x) = \bar{D}(x) + R(x)$   
 $C(x) = x^4 + x^3 + x^2 + 1$

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11101



③  $G(x) = x + x^2$   ~~$x^3 + x^2 + 1$~~  ⑤

$D(x) = x^3 + x^2 + 1 \rightarrow \overline{D(x)} = x^5 + x^4 + x^2$

$\overline{D(x)} = \begin{pmatrix} 1 & 1 & 0 & 1 \\ Q(x) & Q(x) \end{pmatrix}$   $\boxed{\begin{matrix} 1 & 1 & 0 & 1 & 0 & 0 \\ \hline 1 & 1 & 0 & 1 & 0 & 0 \end{matrix}}$

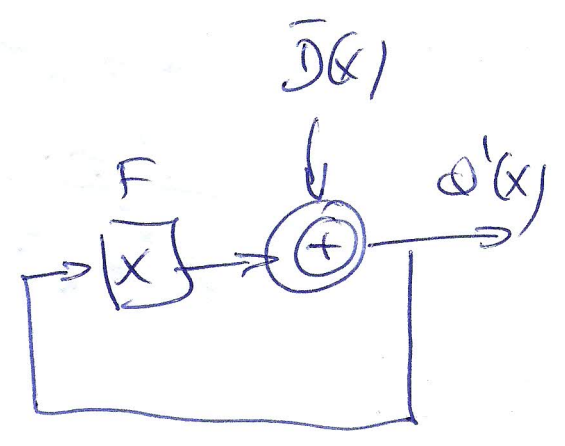
$\overline{D(x)} = (x + x^2) \cdot Q(x)$

$\overline{D(x)} = x \cdot Q(x) + x^2 \cdot Q(x)$

$x \cdot Q(x) = Q'(x)$

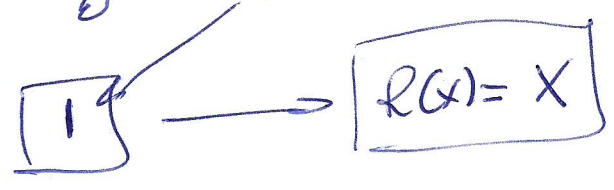
$\overline{D(x)} = Q'(x) + x \cdot Q'(x)$

$Q'(x) = \overline{D(x)} + x \cdot Q'(x)$



clock	$\overline{D(x)}$	F	$Q'(x)$
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0	0	0	0
1	0	0	0
2	1	0	1
3	0	1	1
4	1	1	0
5	1	0	1
6	0	1	1



(devido ao deslocamento anterior de  $d \rightarrow Q'$ )

$C(x) = \overline{D(x)} + R(x) = x^5 + x^4 + x^2 + x$



④  $D(x) = x^3$   $G(x) = x^2 + 1$  1000

⑥

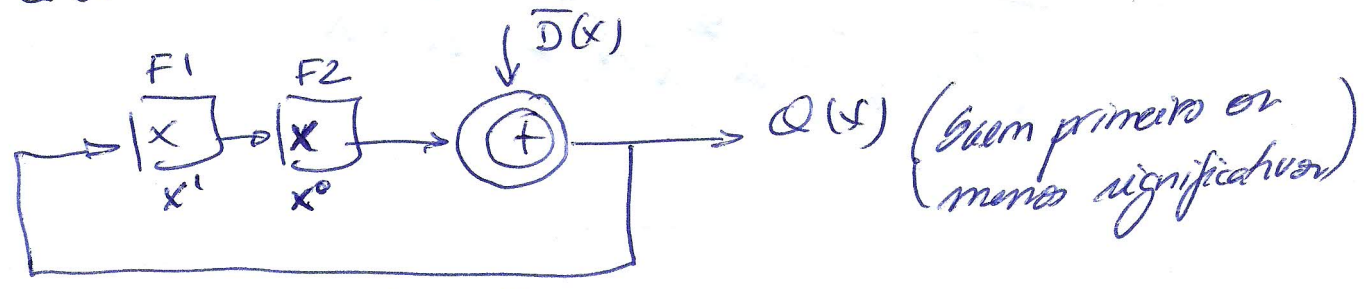
$\bar{D}(x) = x^5$  1000|00

$\bar{D}(x) = Q(x) \cdot G(x)$

$\bar{D}(x) = Q(x)(x^2 + 1)$

$\bar{D}(x) = Q(x) \cdot x^2 + Q(x)$

$Q(x) = \bar{D}(x) + Q(x) \cdot x^2$



clock	$\bar{D}(x)$	$(x^1)$ F1	$(x^0)$ F2	Q(x)
0	0	0	0	0
1	0	0	0	0
2	0	0	0	0
3	0	0	0	0
4	0	0	0	0
5	1	0	0	1
6	0	1	0	0

$R(x) = x$

$C(x) = \bar{D}(x) + R(x) = x^5 + x$  1000|10

5  $C(x) = x^7 + x^5 + x^3 + x^2 + x + 1 \rightarrow$  1 0 1 0 1 1 1 1 7

$G(x) = x^2 + 1$

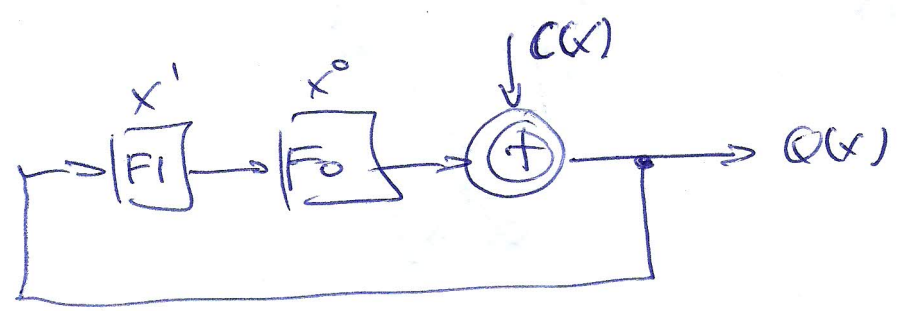
A mensagem está correta?  
 E sem qual o dado original?

$C(x) = Q(x) \cdot G(x)$

$C(x) = Q(x) \cdot (x^2 + 1)$

$C(x) = Q(x)x^2 + Q(x) \cdot 1$

$Q(x) = C(x) + Q(x) \cdot x^2$

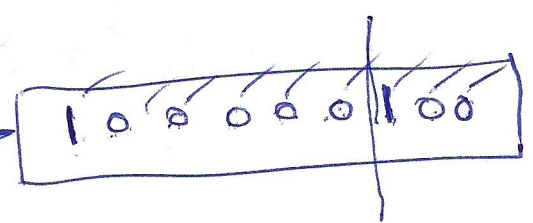


clock	C(x)	F1(x)	F0(x <sup>0</sup> )	Q(x)
0	1	0	0	1
1	1	1	0	1
2	1	1	1	0
3	1	0	1	0
4	0	0	0	0
5	1	0	0	1
6	0	1	0	0
7	1	0	1	0
8	0	0 0		0

$\Rightarrow$  1 0 1 0 1 1  
 $D(x) = x^5 + x^3 + x + 1$

$R'(x) = 0$

⑥  $E(x) = X^8 + X^2$   
 $G(x) = X^3 + X$  } A mensagem está correta?  
 Se não, qual o dado original? ⑧



$C(x) = Q(x) \cdot G(x)$

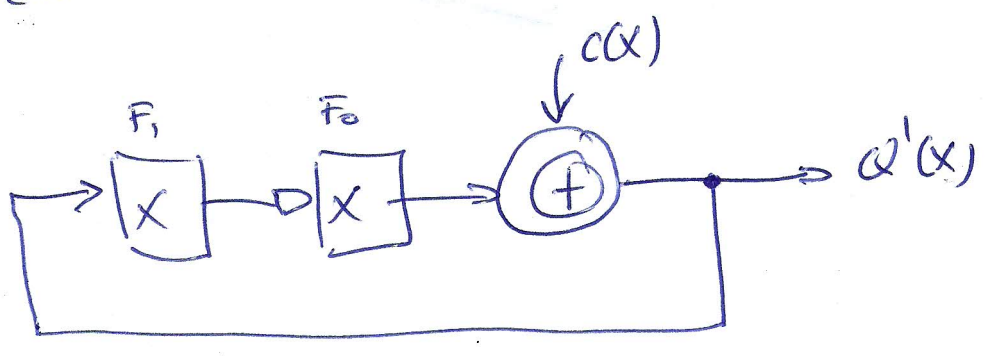
$C(x) = Q(x) (X^3 + X)$

$C(x) = Q(x) \cdot X^3 + Q(x) \cdot X$

$Q(x) \cdot X = Q'(x)$

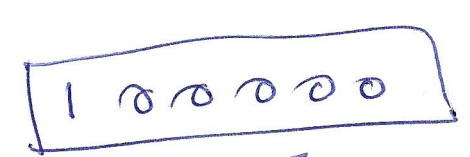
$C(x) = Q'(x) \cdot X^2 + Q'(x)$

$Q'(x) = C(x) + Q'(x) \cdot X^2$



clock    C(x)    F1    F0    Q'(x)

0	0	0	0	0
1	0	0	0	0
2	1	0	0	1
3	0	1	0	0
4	0	0	1	1
5	0	1	0	0
6	0	0	1	1
7	0	1	0	0
8	1	0	1	0
9	0	0	0	0



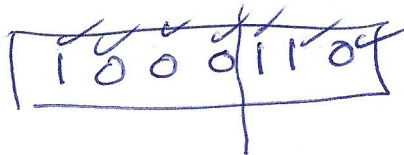
$Q(x) = X^5$

Mensagem



9

(7)  $C(x) = x^6 + x^2 + x \rightarrow$



$G(x) = x^3 + x^2 + 1$

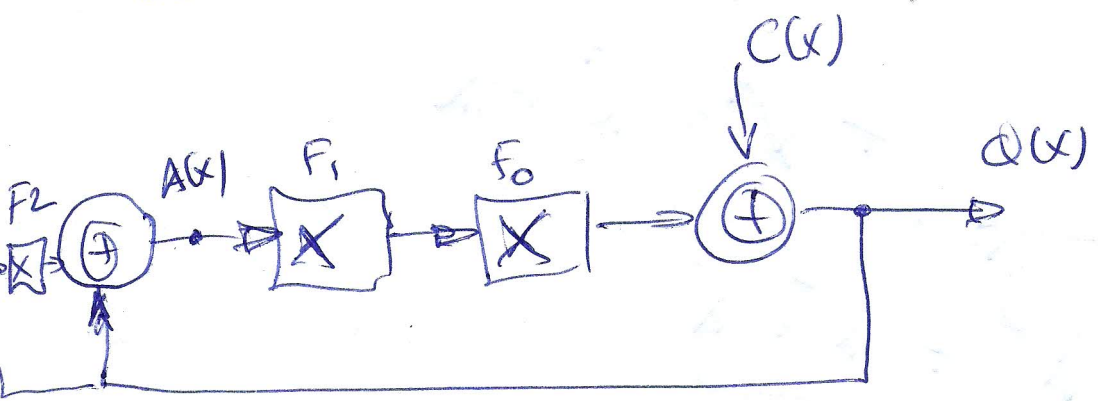
A mensagem está correta? Se sim, qual o dado original?

$C(x) = Q(x) \cdot G(x)$

$C(x) = Q(x) (x^3 + x^2 + 1)$

$C(x) = Q(x) (x^3 + x^2) + Q(x)$

$Q(x) = C(x) + x^3(Q(x) \cdot x + Q(x))$



clock	C(x)	F2	F1	F0	AG(x)	Q(x)
0	0	0	0	0	0	0
1	1	0	0	0	1	1
2	1	1	1	0	0	1
3	0	1	0	1	0	1
4	0	1	0	0	1	0
5	0	0	1	0	0	0
6	1	0	0	1	0	0
7	0	0	0	0	0	0

Dado  
1000  
 $D(x) = x^3$

msg ok

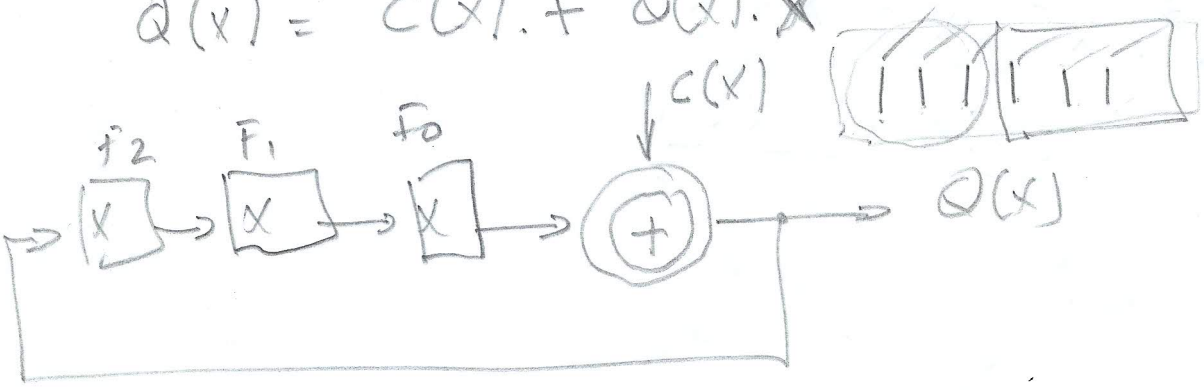
8) 
$$\begin{cases} G(x) = x^3 + 1 \\ C(x) = x^5 + x^4 + x^3 + x^2 + x + 1 \end{cases} \quad D(x) = ?$$

$$C(x) = Q(x) \cdot G(x)$$

$$C(x) = Q(x) (x^3 + 1)$$

$$C(x) = Q(x)x^3 + Q(x)$$

$$Q(x) = C(x) \div (x^3 + 1)$$



clock	C(x)	f2	f1	f0	Q(x)
0	1	0	0	0	1
1	1	1	0	0	1
2	1	1	1	0	1
3	1	1	1	1	0
4	1	0	1	1	0
5	1	0	0	1	0
6	0	0	0	0	0

$$\bar{D}(x) = x^5 + x^4 + x^3 \rightarrow D(x) = x^2 + x + 1$$

9

$C(x) = ?$

11

$$G(x) = x^3 + 1$$

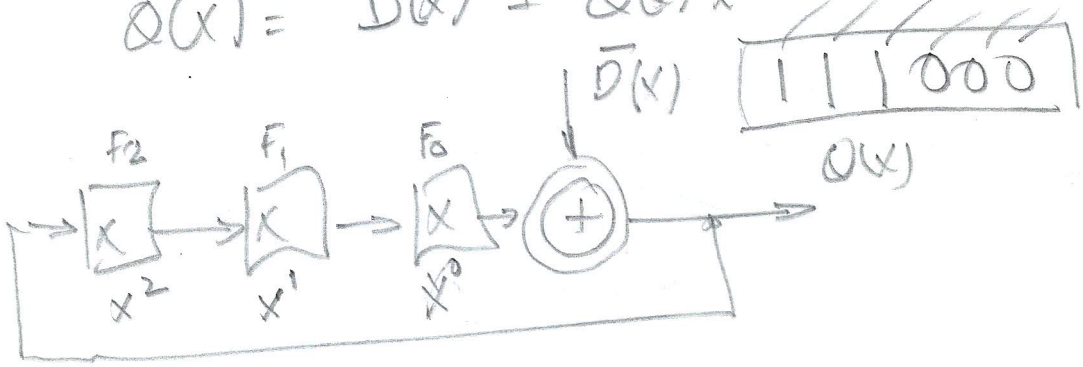
$$D(x) = x^2 + x + 1 \rightarrow \bar{D}(x) = x^5 + x^4 + x^3$$

$$\bar{D}(x) = Q(x) \cdot G(x) + R(x)$$

$$\bar{D}(x) = Q(x) (x^3 + 1)$$

$$\bar{D}(x) = Q(x) \cdot x^3 + Q(x)$$

$$Q(x) = \bar{D}(x) + Q(x)x^3$$



clock	$\bar{D}(x)$	F2	F1	F0	Q(x)
0	0	0	0	0	0
1	0	0	0	0	0
2	0	0	0	0	0
3	1	0	0	0	1
4	1	1	0	0	1
5	1	1	1	0	1
6	0	1	1	1	1

$$R(x) = x^2 + x + 1 \Rightarrow C(x) = x^5 + x^4 + x^3 + x^2 + x + 1$$

10) 
$$\begin{cases} G(x) = x^3 + x + 1 \\ C(x) = x^5 + x^3 + x^2 \end{cases} \quad D(x) = ?$$

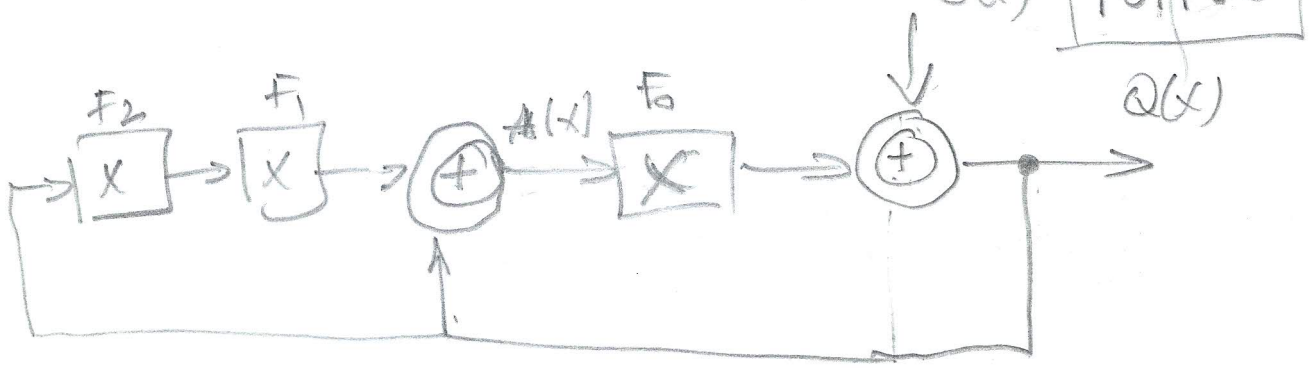
$C(x) = Q(x) \cdot G(x)$

$C(x) = Q(x)(x^3 + x + 1)$

$C(x) = Q(x) + Q(x)(x^3 + x)$

$Q(x) = C(x) + Q(x)(x^3 + x)$

$Q(x) = C(x) + \underbrace{Q(x)x^2 + Q(x)}_{A(x)}$



clock	C(x)	F2	F1	F0	Q(x)	A(x)
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	1	0	0	0	1	1
3	1	1	0	1	0	0
4	0	0	1	0	0	1
5	1	0	0	1	0	0
6	0	0	0	0	0	0

$\Rightarrow D(x) = x^2 + 1$



1)  $G(x) = x^3 + x^2 + 1$

$D(x) = x^3 \rightsquigarrow \overline{D}(x) = x^6$

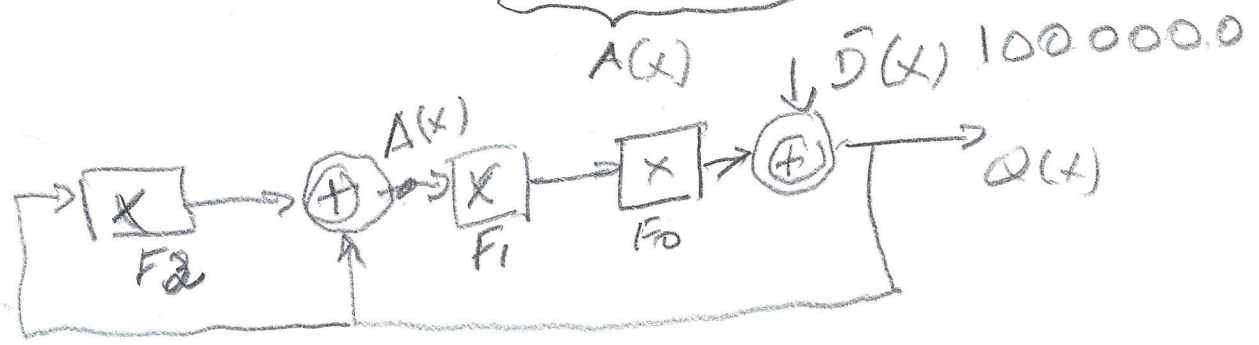
$\overline{D}(x) = Q(x) \cdot G(x)$

$\overline{D}(x) = Q(x) (1 + x^2 + x^3)$

$\overline{D}(x) = Q(x) + Q(x)x^2 + Q(x)x^3$

$Q(x) = \overline{D}(x) + Q(x)x^2 + Q(x)x^3$

$Q(x) = \overline{D}(x) + x^2(Q(x) + Q(x)x)$



Clock	$\overline{D}(x)$	$F_2$	$F_1$	$F_0$	$Q(x)$	$A(x)$
0	0	0	0	0	0	0
1	1	0	0	0	1	1
7	0	1	1	0	0	1

$R(x) = x^2 + x$

⇓

$C(x) = x^6 + x^2 + x$

$$12) G(x) = x^2 + 1$$

(14)

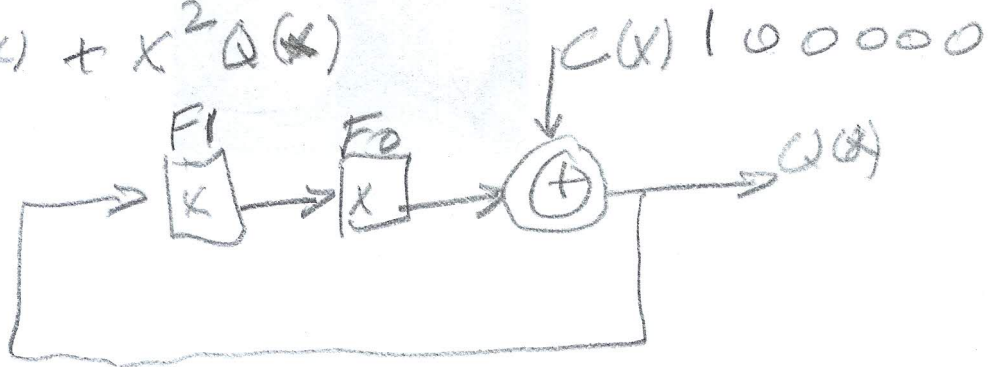
$$D(x) = x^3 \longrightarrow \overline{D}(x) = x^5 \quad 1000000$$

$$C(x) = Q(x)G(x)$$

$$C(x) = Q(x)(1+x^2)$$

$$C(x) = Q(x) + x^2Q(x)$$

$$Q(x) = C(x) + x^2Q(x)$$



clock	C(x)	F1	F0	Q(x)
0	0	0	0	0
6	1	0	0	1
7	0	1	0	0

$$R(x) = x$$

⇓

$$\underline{\underline{C(x) = x^5 + x}}$$

13)  $G(x) = x^5 + x^3$

$C(x) = x^7 + x^6 + x^4 + x^3 \rightarrow 110 | 1000$

Se: mensagem correta  $\rightarrow D(x) = ?$

$C(x) = Q(x) \cdot G(x)$

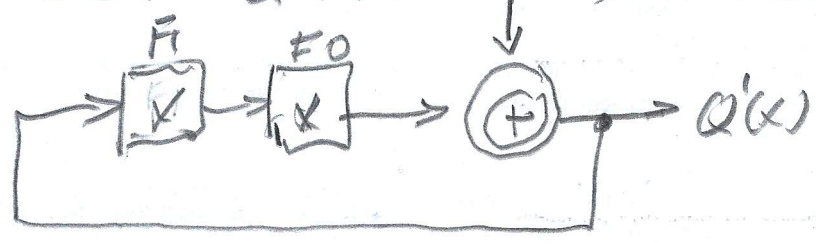
$C(x) = Q(x) (x^5 + x^3)$

$C(x) = Q(x) \cdot x^5 + Q(x) \cdot x^3$

$Q'(x)$  (desloca tres para a esquerda)

$C(x) = Q'(x)x^2 + Q'(x)$

$Q'(x) = C(x) + Q'(x)x^2$



Clock	C(x)	FL	FO	Q'(x)
0	0	0	0	0
1	0	0	0	0
3	1	0	0	1
4	1	1	0	1
5	0	1	1	1
6	1	1	1	0
7	1	0	1	0
8	0	0	0	0

$\Rightarrow D(x) = x^2 + x$

14)  $G(x) = x^5 + x^3$

$D(x) = x^2 + x \rightarrow \overline{D}(x) = x^7 + x^6 \quad 110|00000$

$C(x) = ?$

$\overline{D}(x) = Q(x) G(x)$

$\overline{D}(x) = Q(x) (x^5 + x^3)$

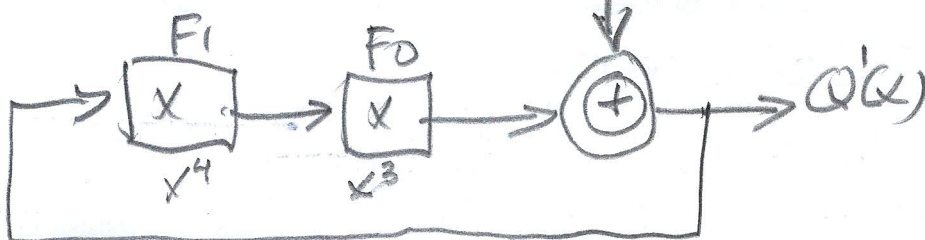
$\overline{D}(x) = Q(x) x^5 + Q(x) x^3$

$Q(x)$  desbaca 3 para a esquerda

$\overline{D}(x) = Q'(x) x^2 + Q'(x)$

$Q'(x) = \overline{D}(x) + Q'(x) x^2$

$\overline{D}(x) \quad 11000000$



Clock	$\overline{D}(x)$	FI	FO	$Q'(x)$
0	0	0	0	0
1	0	0	0	0
6	1	0	0	1
7	1	1	0	1
8	0	1	1	1

$\boxed{\begin{matrix} 1 & 1 \\ 1 & 1 \end{matrix}} \rightarrow x^4 + x^3$

$C(x) = x^7 + x^6 + x^4 + x^3 \rightarrow 110|11000$



15)  $G(x) = X^3 + X^2 + X + 1$

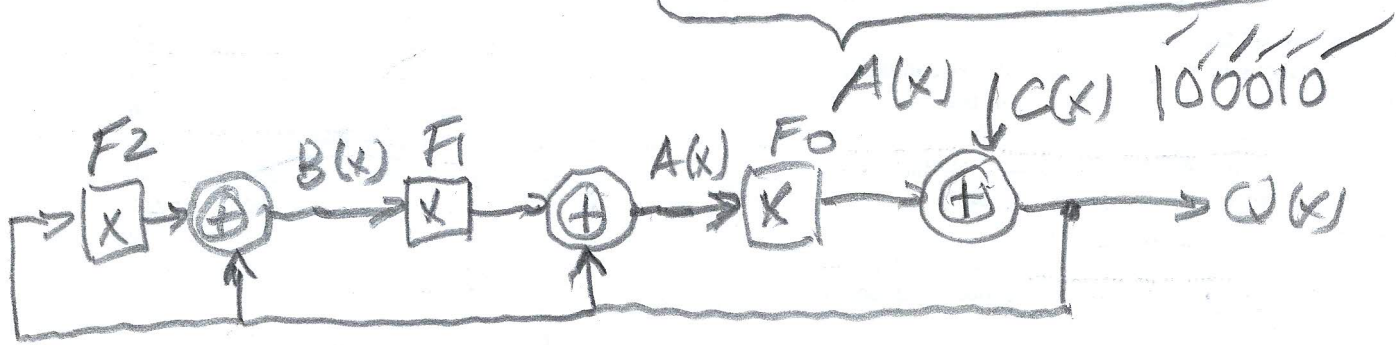
$C(x) = X^5 + X$  Qual  $D(x)$  se msg estiver correta?

$C(x) = Q(x) \cdot G(x)$

$C(x) = Q(x) (1 + X + X^2 + X^3)$

$C(x) = Q(x) + Q(x)X + Q(x)X^2 + Q(x)X^3$

$Q(x) = C(x) + X(Q(x) + X(Q(x) + XQ(x)))$



Clock	$C(x)$	$F_2$	$F_1$	$F_0$	$B(x)$	$A(x)$	$Q(x)$
0	0	0	0	0	0	0	0
1	1	0	0	0	1	1	1
2	0	1	1	1	0	0	1
3	0	1	0	0	1	0	0
4	0	0	1	0	0	1	0
5	1	0	0	1	0	0	0
6	0	0	0	0	0	0	0

msg OK

$\Rightarrow \underline{100} \mid \underline{010} \Rightarrow \underline{D(x) = X^2}$