Simple Uncertainty-Principle Experiment

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Recent discussions in *TPT* have considered the pros and cons of an early introduction of students to modern physics.[1-4] In this regard, appropriate experiments with classical waves can help students understand some important but counterintuitive aspects of modern physics. I recently taught a summer course that introduced high school seniors who had graduated with some background in physics and algebra to aspects of quantum physics and relativity. In this paper I'll describe a simple experiment I used to introduce these students to uncertainty principles in quantum physics.

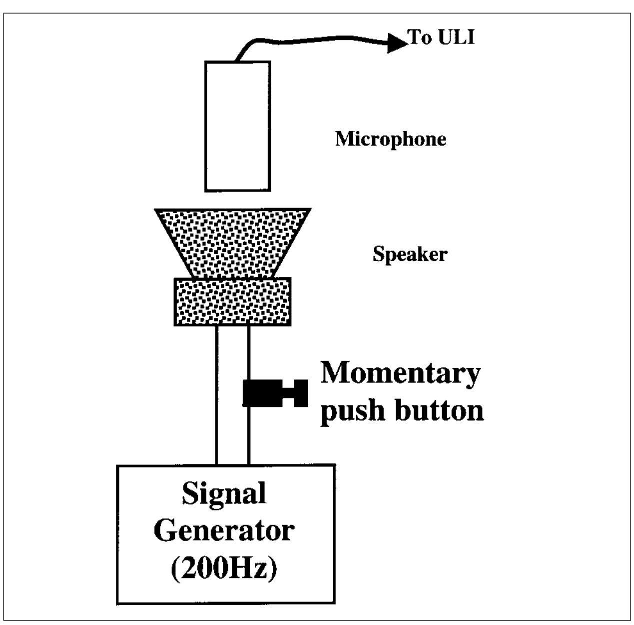

Fig. 1. Diagram of the apparatus discussed in this paper. While the momentary push button is held down, the signal generator set to 200 Hz is connected to the speaker. The ULI II interface collects sound data that is transferred to a computer for analysis.

The experimental setup shown in Fig. 1 allows demonstration of the uncertainty principle $\Delta f \Delta t \geq 1$. Here Δt is the length in time of a pulse of waves while Δf is the range of frequencies in the pulse. A simple derivation of this relation is given in Appendix I. The present experiment nicely complements an LED determination of Planck's constant h reported earlier.[5]

The described experiment provides students with a basis to understand the uncertainty principles $\Delta P_x \Delta x \geq h$ and $\Delta E \Delta t \geq h$ that can be easily derived from the time-frequency uncertainty relation. These two relations allowed my students to understand other experiments they performed as well as interesting applications of quantum physics.[6] Details of the preparation of my particular class for this experiment are included in Appendix II.

As seen in Fig. 1, this experiment uses a computer-interfaced microphone to record 200-Hz sound signals of various lengths produced by depressing a momentary push button. The electrical signal can be recorded directly but I wanted students to hear the length of the pulse. Table I presents data collected[7] in one experiment. In this case $\Delta f \Delta t \approx 1.2$. The Logger Pro software was set up to begin recording at the start of the sound signal and to display both the recorded signal and the amplitude of its Fourier transform. The "Examine" function in the Analyze menu of Logger Pro allowed students to accurately measure the time length of the signal as well as the full width at half maximum amplitude of the 200-Hz FFT peak.

Careful choice of the data collection parameters plays an important part in obtaining a useful Fourier

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Table I. Data collected using the apparatus shown in Fig. 1 with a 200-Hz sound signal. Each row represents a measurement in which 8.5 s of data were collected at 500 Hz. The time width Δt was measured from the data collected by the ULI interface. The frequency width Δf was measured as the full width at half maximum of the 200-Hz peak in the amplitude of the 4096-point Fourier transform of the recorded data. The smallest Δf value in the last row was determined by interpolation of points around half maximum amplitude.

Time Width Δt (s)	Frequency Width	$\Delta f \Delta t$
0.108	11.35	1.23
0.220	5.37	1.18
0.388	3.06	1.19
0.880	1.34	1.18
1.68	0.73	1.22
3.15	0.43	1.35

transform for analysis. I used a 200-Hz sound signal and collected data for 8.5 s at 500 points/s. These choices give a 4096-point (2^{12}) Fourier transform with small enough frequency steps so that Δf could be determined with reasonable accuracy. Discussion of signal processing principles that allow the intelligent choice of data collection parameters is not included in manuals supplied with student lab software and is almost never considered in physics classes. Some knowledge of signal processing is helpful in any experiment where Fourier transforms are used, so I include a short discussion of the relevant principles in Appendix III.

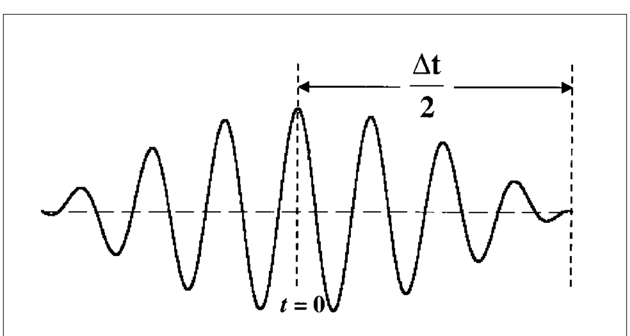

Fig. 2. One cycle of the beats produced by interference of the waves cos ($2\pi f_2 t$) and cos ($2\pi f_1 t$) is shown in an amplitude-vs-time plot.

Appendix I: Time-Frequency Uncertainty Relation

Figure 2 shows one cycle of the beats produced by interference of two waves cos $(2\pi f_1 t)$ and cos $(2\pi f_2 t)$ of frequency f_1 and f_2. I assume f_2 is greater than f_1. These waves are in phase at $t = 0$ and out of phase (by π) at $t = \Delta t/2$. This gives Eq. (1):

$$2\pi f_2 \frac{\Delta t}{2} - 2\pi f_1 \frac{\Delta t}{2} = \pi. \quad (1)$$

Taking $f_2 - f_1 = \Delta f$, Eq. (1) gives Eq. (2):

$$\Delta f \Delta t = 1. \quad (2)$$

The important aspect of Eq. (2) (that relates to the more familiar quantum uncertainty relations) is that the right-hand side is not zero. Making a pulse of waves narrower in time makes the range of frequencies in the pulse larger. A perceptive student might point out that only one of an infinite series of beats is considered here, so how can this derivation with only two frequencies relate to a real pulse of waves with time width Δt?

Wave peaks at times larger than those shown in Fig. 2 can be canceled by adding in more waves with frequencies between f_1 and f_2. Therefore, the difference $f_2 - f_1$ provides a reasonable estimate of Δf for a pulse with time length Δt. A more sophisticated analysis where an infinite number of waves are superimposed using integration and Δf and Δt are carefully defined gives $\Delta f \Delta t \geq 1$. The present derivation is not tied to a particular type of wave so considering the probability density waves of quantum physics (wavelength λ) and using the de Broglie relation $\lambda = h/p$ and $hf = E$ allows simple derivation of the useful relations $\Delta P_x \Delta x \geq h$ and $\Delta E \Delta t \geq h$.

Appendix II: Student Preparation for these Experiments

In the case discussed here, I had a small summer class of very good, recently graduated high school seniors selected from the entire state of North Dakota. They were not all chosen for particular preparation in mathematics and physics since my class was only one part of a program covering other disciplines. They all had some physics and at least high school algebra. The students were not familiar with either the equipment used to perform these experiments or most of the concepts required to understand their results. In the following paragraphs I present activities and discussions that were carried out to prepare this class for the uncertainty-principle experiment.

In particular, I first reviewed the properties of sinusoidal waves and we practiced use of the LabPro hardware and Logger Pro software. I wanted students to have some feel for the possibility and usefulness of representing time-dependent signals in frequency space (none of them had heard of Fourier transforms). Students performed an experiment in which they recorded single-frequency sounds produced by a digital function generator (with frequency display) connected to a speaker. They then compared the Logger Pro FFT display to the input frequency and examined the effect on the FFT of changing the frequency and loudness of the sound. They repeated this experiment recording several superimposed sounds with different frequencies (up to three). They changed the relative amplitudes of these sounds and described the changes they saw in the FFTs. We discussed the FFT as a tool that displays the strengths of the different frequencies present in a sound signal. I discussed the representation of sounds and other signals by sums of single frequency signals. As an illustration of this concept, they next used a Mathcad[8] simulation to sequentially add in higher frequency components to produce a good approximation to a square wave. I did not discuss the material in Appendix III because it was beyond the scope of the course, and all data collection used Logger Pro template files that I had set up for the experiments. I next discussed the material included in Appendix I, and they ran a Mathcad simulation in which two waves with different frequencies were added together to produce beats. They observed that as the frequencies moved closer together, the beat pulses became longer in time (as expected from the discussion of Appendix I). After this preparation they performed the uncertainty-principle experiment, which we discussed in terms of the activities detailed above.

Appendix III: Signal Processing

Assume that data is collected at f points/s. To represent a particular sine wave requires at least two data points per cycle so data collected at f points/s can only represent frequencies up to $f/2$. Student software used to process data normally uses a power-of-two-based Fourier transform algorithm so that if the number of data points is between 2^n and 2^{n+1}, the software calculates a 2^n-point Fourier transform of the data. Positive and negative frequencies between $-f/2$ and $+f/2$ are included in the calculation. The frequency steps in the Fourier transform then have a size given by $f/(2^n)$. This result shows that better frequency resolution in Fourier transforms can sometimes be obtained by using a lower data collection rate (f) and/or a longer recording time (larger n). The data collection rate must, of course, be at least twice the highest frequency component in the signal.

In the present experiment, Logger Pro only calculates Fourier transforms up to a maximum of 2^{12} = 4096 points. I always collected 8.5 s of data at 500 points/s for a total of 4250 points to ensure that 2^{12}-point Fourier transforms were calculated even though the actual time length of the 200-Hz signals was less than 8.5 s, as seen in Table I. In this case, the frequency steps in the Fourier transform are 500 Hz/4096 = 0.122 Hz. Collecting data for a time longer than the signal results in smaller frequency steps but does not improve frequency resolution. The effect of using a larger-than-needed Fourier transform is to interpolate the Fourier transform data. These smaller frequency steps make it easier to determine Δf accurately even if further interpolation is required. An engaging, readable introduction to signal processing (requiring only introductory calculus) has been written by Steiglitz.[9]

References

1. Ruth Howes, "Modern physics—Guest editorial," *Phys. Teach.* **38,** 73 (Feb. 2000).
2. Art Hobson, "Teaching 'modern' physics in introductory courses," *Phys. Teach.* **38,** 388 (Oct. 2000).
3. Ludwik Kowalski, "Teaching advanced physics: Who, when, and how?" *Phys. Teach.* **39,** 4 (Jan. 2001).

4. Michael George, "Response to teaching modern physics," *Phys. Teach.* **39,** 4–5 (Jan. 2001).

5. L. Nieves, G. Spavieri, B. Fernandez, and R.A. Guevara, "Measuring the Planck constant with LED's," *Phys. Teach*. **35,** 108 (Feb. 1997). Following this experiment with the one I discuss gives the students a feeling as to why the tiny (but nonzero) value found for h has profound consequences.

6. Experiments and discussion related to $\Delta P_x \Delta x \geq h$ include energy estimates of particles in bound systems such as electrons in atoms or nucleons in a nucleus, and divergence of laser beams and the effect of a beam expander as well as diffraction of light or particles passing through a slit. Applications of $\Delta E \Delta t \geq h$ include: (1) differences in the width of emission lines produced by high and low pressure sodium lamps related to lifetime (Δt) changes, and (2) discussion of Mossbauer-effect experiments such as gravitational red shift related to source height above the Earth's surface. (Here large Δt in nuclear decay gives small and very useful ΔE.)

7. I used a ULI II interface with Logger Pro software from Vernier Software to collect data in this experiment. 13979 SW Millikan Way, Beaverton, OR 97005-2886; 503-277-2299, http://www.vernier.com.

8. Mathcad software for Windows is available at academic discounts from MathSoft Engineering and Education Inc., Boston, MA; 800-628-4223, http://www.mathsoft.com.

9. K. Steiglitz, *A Digital Signal Processing Primer* (Addison-Wesley, New York, 1996).

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