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Perspectives on the Uncertainty Principle and Quantum Reality

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A student's first encounter with the uncertainty principle is likely to leave the impression that it is entirely *negative* in thrust. It rules out the possibility of simultaneously measuring or knowing two conjugate variables (e.g., position x and momentum p_x of a particle) with arbitrarily great precision. Seldom is the beginning student introduced to *positive* aspects of the principle. This is unfortunate because the uncertainty relations not only illustrate what can be known about atomic systems and how to rationalize counterintuitive quantum phenomena, they even provide a simple means for estimating fundamental quantities of chemical interest without having to solve Schroedinger's equation. Concrete examples of this approach not only make the uncertainty principle seem more alive and useful, they help convey the real meaning of the term "uncertainty."

Although a rigorous treatment of the subject requires a familiarity with quantum mechanics and its mathematical underpinnings, much of the flavor and substance can be communicated by a more qualitative approach. To this end we begin with a brief review of the significance of the wave function in quantum mechanics, an appreciation of which is essential before reasoned judgements about the uncertainty principle can be developed.

Probability Amplitudes

A working knowledge of the uncertainty principle requires an awareness of certain difficulties in visualizing elementary quantum phenomena. Quantum behavior is and must remain inexplicable in terms of everyday experience. The most graphic glimpse of quantum events available is that afforded by the "probability amplitude," or "wave function," commonly designated Ψ. From Schroedinger's equation of motion[1] can be deduced, in principle, the form of the wave function and its evolution with time corresponding to a particular way of preparing a system (*1*). Prescribed mathematical operations upon Ψ can extract information about any observable property of the system (*2*). For the present purposes let us consider a particularly simple system, namely a particle in motion with momentum $p = mv$. As predicted by de Broglie and strikingly confirmed by interference fringes produced by electrons, Ψ for our moving particle has an undulatory character with wavelength λ given by

$$\lambda = h/p \tag{1}$$

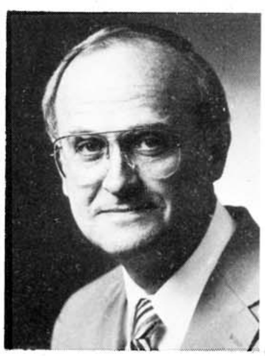
where h is Planck's constant. Because the predicted interference fringes are even generated by streams of electrons directed, one by one, at closely spaced slits cut to predetermined dimensions (*3*) it must not be supposed that the concept of wave function is too abstract to be practical. Notice that particle-wave duality is built into quantum mechanics at the outset in eqn. (*1*) and also in the Einstein relation

$$E = h\nu \tag{2}$$

inasmuch as both equations relate a property of an individual particle (p, or an energy, E) to a wave property (λ, or to the frequency, ν). Both equations apply equally well to photons and material particles when ν is suitably identified.

In double-slit interference experiments, for example, the *wave function* traverses *both* slits. What about individual

This feature is intended as a review of basic chemical principles and as a reappraisal of the state of the art. Comments, suggestions for topics, and contributions should be sent to the feature editor.



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Larsen has teamed with Dan Kallus to co-edit the *Chemical Principles Revisited* feature. They invite your contributions to this effort.

[1] Schroedinger's equation of motion (*1*) governing the unfolding of quantum processes with time is less familiar to chemists than his equation $H\psi = E\psi$ applying only to well-defined energy states.

particles? What can be said is this. In keeping with the particle-wave duality and at the root of the divergence of opinions about quantum interpretations of Ψ, is the fact that the interference fringes (and also the uncertainty spreads to be discussed) *can never be directly observed for individual particles.* In interference and other experiments, either a whole electron, or none, is ultimately detected at a given place, say, as a black dot on a photographic plate or scintillation from a fluorescent screen. A good electron interference pattern (or well-recorded uncertainty spread) requires the irreversible, indelible registration of a great many electrons, all initially prepared to be in identical states of motion by some quantum prescription. It is the probability distribution of the ensemble of electrons that is successfully predicted by Schroedinger's equation, being mapped by $|\Psi|^2$, the square of the absolute magnitude of the probability amplitude evaluated at the detector surface.

For the above reasons there are many different interpretations of the meaning of Ψ (*2–5*). Some theorists believe that Ψ applies to individual particles so that it is, as a rule, meaningless to ascribe to them either position *or* momentum, let alone position *and* momentum at some arbitrary time. Many sharing this opinion would say that it is the act of measurement itself that "collapses the wave function" and "brings into being" the observed position or momentum of the particle in question! Other theorists, and Einstein was among them, hold that Ψ is an attribute only of an ensemble of identically prepared particles and not of individual particle themselves. Einstein was also a member of a now dwindling group ascribing an actual position *and* momentum to a given particle at a given time (*6*), even though he fully agreed that restrictions intrinsic in quantum measurements exclude precise simultaneous determinations of both quantities. Still others have advocated the "many worlds" interpretation of wave functions in which the universe splits into a distribution of noninteracting universes at each measurement (*4*). Proposing several other boldly different interpretations of Ψ, Bohm (*7*) has even conceived of circumstances which, if achieved, might allow measurements to transcend the uncertainty principle.

Fortunately, it is unnecessary to choose between the conflicting points of view when applying quantum mechanics to chemical systems. Whatever the meaning of Ψ may be for individual elementary events, nobody contests the correspondence between probability amplitudes and observations made on large numbers of particles. This does not mean that quantum anomalies disappear when treating ensembles. The wave function continues to correlate, reciprocally, position and momentum in a way that would have been unimaginable to Newton. Arguments to lend plausibility to this connection are presented in the next section.

Rationale behind the Uncertainty Principle

Heisenberg[2] hit upon the uncertainty principle in 1927 while he was struggling to reconcile seeming contradictions. How can the readily observed paths of electrons in cloud chambers be accounted for by the quantum theory when the concept of path, or orbit, does not even arise in the theory? To be sure, Schroedinger[2] had already shown that some semblance of classical particle behavior could be recognized in quantum treatments if one constructed a "wave packet" by adding together judiciously enough probability amplitudes, each corresponding to a different de Broglie wavelength. Moreover, such a packet even tends to obey Newton's laws of motion if the potential energy varies only modestly over a packet's breadth. The trouble with this approach was that Schroedinger's packets could not naturally be made as small as an electron was imagined to be. Worse, the packet spread alarmingly as it progressed, spreading more violently, the smaller it was originally constructed to be. Escaping this dilemma seemed possible only if a description of quantum systems admitted just those quantities capable of being observed, in principle. Orbits in atoms had never been observed but neither had the electron trajectories in a cloud chamber to within atomic resolution.

Building upon this clue, Heisenberg proposed[3] a series of thought experiments purporting to demonstrate the impossibility, *in principle*, of watching an electron's undisturbed orbit. Among the most instructive of the thought experiments was Heisenberg's celebrated "γ-ray microscope," an object lesson appearing in a large fraction of today's textbooks on physical science. The central idea is that a microscope cannot resolve distances much smaller than the wavelength λ of the radiation used to view an object (an electron, in Heisenberg's example). If λ is decreased to reduce the uncertainty in position (Δx), the Compton recoil suffered by the illuminated electron increases and is comparable to the photon's momentum (h/λ). Because the trajectory of the illuminating photon through the objective lens of the microscope cannot be known (without spoiling the image) the magnitude and direction of the electron's change in momentum (Δp_x) cannot be known and is readily shown to be uncertain in conformity with the inequality

$$\Delta x \cdot \Delta p_x \gtrsim h \tag{3}$$

where p_x is the component of the momentum in the x direction. Analogous considerations relate ΔE, the sharpness with which the energy of a system can be known, to Δt, the lifetime of the system in the state being considered, by

$$\Delta E \cdot \Delta t \gtrsim h \tag{4}$$

As long as discussions of particle behavior presuppose no greater knowledge than compatible with eqns. (3) and (4), no logical contradictions arise.

A simple numerical example illustrates the consequences of eqn. (3). Consider a particle, mass m, orbiting a central body with velocity v. An easily appreciated indication of the precision with which we are allowed to know the velocity v_x when Δx is finite is, in view of eqn. (3),

$$\frac{\Delta v_x}{v} = \frac{\Delta p_x}{mv} \gtrsim \frac{h}{mv\Delta x} \tag{5}$$

It is convenient to work in MKS units, where $h = 6.6 \times 10^{-34}$ J s. Let us suppose that we wish to measure Δx to 1 Å(10^{-10} m). If the particle were an electron ($m \approx 9 \times 10^{-31}$ kg) in a $2p$ state of hydrogen ($v \approx 10^6$ m s^{-1}) this would scarcely tell us where in the atom the electron was because the atomic size is $\approx$1 Å. Yet this mild demand for precision implies, by eqn. (5), that $\Delta v_x/v \gtrsim 7$. Such a huge relative uncertainty confirms the impossibility of following the undisturbed orbital motion of an atomic electron. If, however, the particle were a 100-kg satellite orbiting the earth ($v \approx 7 \times 10^3$ m s^{-1}), the preposterous demand for 1 Å precision in observation of position would still allow us, according to eqn. (5), to establish v_x simultaneously with a delicacy $\Delta v_x/v \approx 9 \times 10^{-30}$. Obviously, the uncertainty principle imposes no practical limitation on the tracking of a satellite's orbit.

As discussed above, the first interpretations of the uncertainty relations attributed indeterminacy to the uncontrollable disturbance of a system by an act of measurement. This interpretation, adopted in most elementary textbooks, was also accepted by Bohr until later thought experiments[4] re-

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[2] For an excellent review of the history and philosophy of quantum mechanics see reference (*4*). For a sense of the lively controversy in the field, see the discussions in reference (*5*).

[3] For a translation of historic papers in the field of quantum measurements together with an up-to-date collection of selected reprints, see reference (*2*).

[4] In particular, the famous Einstein, Podolsky, Rosen paradox discussed at length in references (*2*) and (*4*).

vealed indeterminacies for particles not directly subjected to interactions with measuring instruments.

Heisenberg quickly recognized that the indeterminacy relations are even more fundamental than manifestations of disturbance during measurements. They are intrinsic in the quantum laws governing Ψ itself. Pursuit of this approach introduced a mathematical precision (if not a physical clarity) into the definitions of uncertainty. To show how a wave function *must* interconnect Δx and Δp_x, we return to the concept of "wave packet."

In wave mechanics, the wave function for any state can always be expressed mathematically as the "superposition" of (the sum of) other wave functions corresponding to alternative sets of states. This curious consequence of the form of Schroedinger's equation is used in Figure 1 to suggest how a sum of de Broglie waves (free particle states), each individually obeying eqn. (1), can reinforce in a given region of space [say, $x_0 \pm (\Delta x/2)$] and cancel outside this region where the probability of finding the particle vanishes. It can be seen at once that the existence of different wavelengths $\lambda_i = h/p_i$ contributing to Ψ necessarily means that different momenta contribute to the wave packet and, hence, that Δp_x cannot be zero if the packet is at all localized in space.[5] Consider two representative waves with wavelengths λ_1 and λ_2 that reinforce perfectly at x_0 but cancel at $x_0 \pm (\Delta x/2)$. Cancellation means that the waves are half a wavelength out of step, their amplitudes having opposite signs. In the distance $\Delta x/2$, one wave has n sinusoidal loops while the other has $n + 1/2$. Their *wave numbers* N_1 and N_2, or number of waves per length, then, are

$$N_1 = 1/\lambda_1 = n/(\Delta x/2) \tag{6}$$

and

$$N_2 = 1/\lambda_2 = (n + 1/2)/(\Delta x/2), \tag{7}$$

whence

$$\Delta N = (1\lambda_2) - (1/\lambda_1) \approx (1/2)/(\Delta x/2) \tag{8}$$

or

$$\Delta x \cdot \Delta N \approx 1 \tag{9}$$

Invoking de Broglie's relation, we recognize from eqn. (8) that $h\Delta N$ is none other than Δp_x. Therefore, eqn. (9) becomes

$$\Delta x \cdot \Delta p_x \approx h \tag{10}$$

Analogous reasoning leads to

$$\Delta \nu \cdot \Delta t \approx 1 \tag{11}$$

and

$$\Delta E \cdot \Delta t \approx h \tag{12}$$

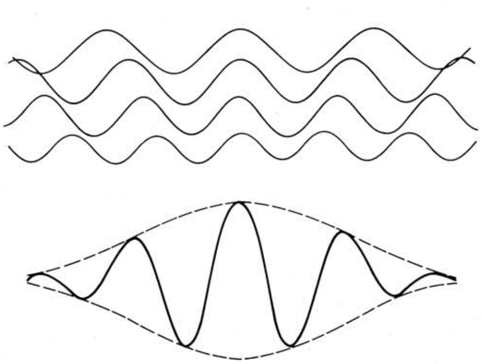

Figure 1. Sum of infinitely extended sine waves of different wavelength (upper curves) adding up to a localized "packet" (lower heavy curve). Abscissa can be distance, or time, implying eqn. (9) or (11).

The most natural way of giving a precise definition to the uncertainty (quantum spread, indeterminacy) in an observable is to relate it to the probability distribution which, as we have seen, is implied by the wave function for the system. The connection is made through the quantum recipe for "expectation value," or average value to be expected from a great many precise measurements upon identically prepared systems. Let ξ represent any dynamical variable (e.g., x, y, p_x, p_y, angular momentum, kinetic energy, etc.). Then the expectation values $\langle \xi \rangle$ and $\langle \xi^2 \rangle$ corresponding to ξ and ξ^2 can be derived from the wave function Ψ as shown in footnote 6. This allows us to identify the uncertainty $\Delta\xi$ as a *standard deviation*, meaning that $(\Delta\xi)^2$ is the statistical *variance* or mean-square deviation from the mean of a great many suitable measurements. Accordingly, $(\Delta\xi)^2$ is given formally[6] by $\langle (\xi) - \langle \xi \rangle)^2 \rangle$ or, equivalently, by $\langle \xi^2 \rangle - \langle \xi \rangle^2$. If observable ξ_1 is identified with position and ξ_2, with momentum, then it is possible to compute, say, values of Δx and Δp_x corresponding to a given allowed Ψ. Straightforward mathematical manipulations (*8*) can be shown to yield

$$\Delta x \cdot \Delta p_x \geqslant h/4\pi \tag{13}$$

and

$$\Delta y \cdot \Delta p_y \geqslant h/4\pi, \tag{14}$$

for example, where the 4π absent in the previous expressions has entered merely because of the precise definition of uncertainty introduced which was not used in the earlier, qualitative treatments. When variables are not "conjugate,"[7] as for example in $\Delta x \cdot \Delta p_y$ or $\Delta x \cdot \Delta y$, there may not be any restrictions on the smallness of the product. An analog of eqns. (4) and (12) can be derived, and is useful when properly interpreted. Many theorists refrain from calling it an "uncertainty relation" because time, in quantum theory, is not on the same footing as observables such as position and momentum. Uncertainty relations can be written for angular momentum (*8*) and other observables, as well.

An analysis of the observational consequences of the uncertainty relations is a central subject of "quantum mechanical measurement theory" (*2*), a field formulated in as many ways as there are interpretations of the wave function Ψ. Despite unresolved philosophic questions, it is possible and desirable to acquire a sense of the practical consequences of the uncertainty principle. In the following section are presented several examples to aid in this acquisition and to illustrate modes of thinking which, when mastered, can lead to quick and quite handy insights of wide applicability. Unfortunately, mastery is not won easily. Casual readers may prefer to skip to the final section.

Applications of Uncertainty Relations

We have seen that the quantum equations of motion imply, by their very form, the uncertainty relations. So intimate is the connection that we can regard the uncertainty relations, themselves, as it were, as simplified wave equations. With their aid it is possible to estimate, quite easily, magnitudes of

[5] This, of course, is responsible for some components of a wave packet racing ahead and some, lagging, causing the packet to spread as it travels.

[6] By the symbol $\langle \xi^n \rangle$, or "expectation value" of ξ^n is meant the integral $\int \Psi^* \hat{\xi}^n \Psi d\tau$ over all of configuration space, where $\hat{\xi}$ is the quantum operator (*1*) associated with ξ.

[7] Generalized coordinates and momenta are said to be conjugate when they satisfy certain relations discussed in reference (*1*), p. 15. For our purpose it is enough to consider them as components of the position and momentum vectors in a common direction, say along the x axis. At the heart of the uncertainty relations are the so-called commutation relations adopted as axioms in quantum mechanics. These relations between operators associated with conjugate variables were recognized by Heisenberg in his first paper on the topic (translated in reference (*2*)) to imply the uncertainty principle.

a variety of quantities of concern in chemistry and physics, a few of which appear below. Limitations of space restrict illustrations to rough and ready treatments. We can do a little better, as a rule, if we remember to apply eqn. (3) or its equivalent when uncertainties are intended to encompass more or less the full of the spread involved (e.g., a span perhaps threefold larger than the standard deviation as, for example, when the uncertainty in position of a photon traversing a lens of diameter D is assigned the value D). Alternatively, if we mean for uncertainties to represent something like standard deviations, we apply eqn. (13) or its equivalent. For order of magnitude estimates it scarcely matters which is used.

Resolving Power of Optical Instruments

A good optical instrument (telescope, microscope) forms an image whose sharpness is limited almost entirely by diffraction of light (or electrons, radar waves, etc.) by the objective lens. The idea is that, because photons (or electrons) were *restricted* to $\Delta y = D$ in their passage through the lens, they experience a characteristic sidewise kick Δp_y chaotically interfering with their trajectory and causing a point object to be imaged as a blur. The *angular* magnitude of this blur (see Fig. 2) figured from lens center to image and, therefore, from lens center to object, is

$$\Delta\theta_R \approx \Delta p_y/p \approx (h/\Delta y)/p$$

$$= \lambda/\Delta y = \lambda/D \qquad (15)$$

by virtue of the uncertainty relation, eqn. (3), and de Broglie's eqn. (1). This result can be compared with Rayleigh's criterion for angular resolution of telescopes and microscopes based on diffraction theory, namely (*9*)

$$\Delta\theta_R = 1.22\lambda/D \qquad (16)$$

To convert $\Delta\theta_R$ to distance resolvable by a microscope, multiply it by the focal length of the objective lens. Sonar imagers and microscopes based on sound waves are limited by the same equation.

Scattering of Radiation by Nuclei, Atoms, or Small Crystals

These cases are exactly analogous to the diffraction of waves by an objective lens, as suggested by Figure 2. If an X-ray photon passes through the electron cloud of an atom or a fast meson encounters the region of a nucleus, the interaction leads to scattering of the incident radiation. Just as in the previous paragraph, there is a characteristic scattering angle at which destructive interference between wavelets scattered from different parts of the scatterer begins to attenuate the scattered intensity. This angle corresponds to the characteristic lateral kick associated with localization prior to scattering. That is, a particle scattered by an atom or a nucleus must have encountered that atom or nucleus and, hence, it is known to have been somewhere within Δy, the breadth of the scatterer. Accordingly, eqn. (15) can be applied. Turning this around, if λ is known and $\Delta\theta_R$ (the breadth of the scattering pattern) is measured, the diameter of the atom or nucleus causing the scattering can be quickly estimated. Identical reasoning relates the breadth of Debye-Scherrer rings of X-rays or electrons diffracted by powdered crystals to the diameter of those crystals.

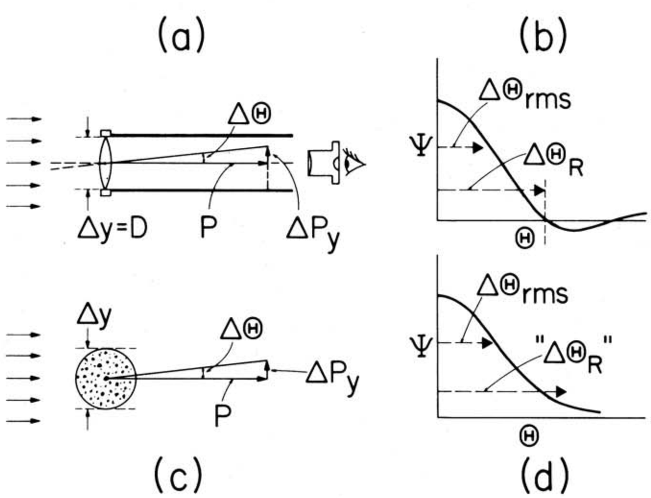

Figure 2. (a) Photon, momentum p, entering objective lens of telescope. Uncertainty relation makes $\Delta p_y \approx h/\Delta y$ nonzero, thereby blurring focal spot by $\Delta\theta \approx \Delta p_y/p$. (b) Photon amplitude for point source, at image plane, broadened by diffraction. $\Delta\theta_R$ is Rayleigh resolution limit, $\Delta\theta_{rms}$ is root-mean-square blur. (c) X-ray (meson) scattered by atom (nucleus). (d) Scattered amplitude as a function of scattering angle.

Zero-Point Motions and Atomic Size

It is widely appreciated that zero-point (irreducible) motions can be considered a manifestation of the uncertainty principle. To show this for a simple harmonic oscillator let us associate the root-mean-square amplitude of motion $\langle x^2\rangle^{1/2}$ with the indeterminacy Δx of the oscillator. If Δx were very small, the corresponding momentum spread $\Delta p_x \approx (h/4\pi)/\Delta x$ would be large, implying a substantial kinetic energy

$$T = mv^2/2 = p^2/2m \qquad (17a)$$

$$\approx (\Delta p_x)^2/2m \approx (h/4\pi\Delta x)^2/2m \qquad (17b)$$

because the characteristic momentum p back and forth must be comparable to the spread Δp_x. On the other hand, if Δx were very large, Δp_x would become small, but the characteristic potential energy

$$V = kx^2/2 \approx k(\Delta x)^2/2 \qquad (18)$$

would become large. The minimum (irreducible) total energy $E = T + V$ can readily be shown (by setting $dE/d(\Delta x) = 0$) to yield

$$\Delta x \approx (h/4\pi)^{1/2}(km)^{-1/4} \qquad (19)$$

and

$$E_0 \approx (h/2)(k/4\pi^2 m)^{1/2} = h\nu_0/2 \qquad (20)$$

which turn out to be the exact quantum results.

An analogous treatment of the electron cloud corresponding to the lowest (1s, zero-point state) of the hydrogen atom affords a quick order of magnitude estimate of the atomic size and energy. In this three-dimensional problem with

$$\Delta x \approx \Delta y \approx \Delta z \approx \Delta r$$

and

$$T = (p_x{}^2 + p_y{}^2 + p_z{}^2)/2m$$

$$\approx 3(\Delta p_x)^2/2m$$

we adopt eqn. (17b) for sake of simplicity and invoke

$$V = -e^2/4\pi\epsilon_0 r$$

$$\approx -e^2/4\pi\epsilon_0(\Delta x)$$

for the characteristic potential energy. The minimum allowable energy then corresponds to

$$\Delta x \approx 3(h/4\pi)^2(4\pi\epsilon_0/me^2)$$

or ¾ of the Bohr radius, while the estimated energy is within 50% of the exact value.

Sharpness of Spectral Lines

States with short lifetimes have ill-defined energies and display broad lines when probed by absorption or emission spectroscopy, as indicated by eqns. (11) and (12). Near the other end of the scale are relatively long-lived nuclear states excited by soft γ rays (e.g., for ^{57}Fe). For favorable cases studied by recoilless resonance absorption, the ratio $\Delta\nu/\nu$ may be so small that the line width and profile can be measured using the Doppler effect. Scanning the spectrum is accomplished by stepping the relative velocity between emitter and absorber by increments of mere fractions of a millimeter per second! The technique, known as Mössbauer spectroscopy (*10*), has found applications ranging from studies of the electronic structure of molecules to tests of general relativity theory.

Perspectives on Quantum Processes

Many quantum phenomena that seem singular and contrary to our physical expectations can at least be rationalized in terms of the uncertainty relations. For example, everyone knows that electron spins point either "up" or "down." However, spin angular momentum can never point exactly straight up or down. Why not? It can for classical gyroscopes. How is it that particles can "tunnel" into classically forbidden regions where their energy is less than the potential energy barrier? In Compton scattering, how can a free electron absorb a photon briefly and then emit it, suffering a recoil when, classically, energy and momentum cannot simultaneously be conserved in such an absorption? How can an electron in the 1*s* state of hydrogen migrate all around the atom to produce a spherically symmetric probability distribution even though its angular momentum is known to be zero? Zero angular momentum implies purely radial, not angular, velocity. Some insight into all of these problems can be provided by applying the uncertainty principle. Let us pursue the last question to review some points worth emphasizing.

Suppose we imagine a 1*s* electron to drop with purely radial motion toward the nucleus along, say, the z direction. Recall that the nucleus is very small and exerts a powerful attractive force to speed up the electron. While kinetic energy acquired in the fall allows the electron to coast freely through the nucleus, our picture suggests that the electron, during its encounter with the nucleus, is close to the center of the nucleus (i.e., Δx and Δy are small). Therefore, we are forced to assume that the sideward momentum spreads Δp_x and Δp_y are enormous. That is, the wave packet corresponding to the electron is strongly diffracted in all directions perpendicular to the z direction (cf. Fig. 2c). This diffraction carries the probability distribution away from the original trajectory, and after many passes through the nuclear region, diffraction blurring completely obliterates any sense of the original direction. While such a picture, taken too literally, cannot accurately describe a pure 1*s* state it does help to rationalize the spherical symmetry. Note that no net angular momentum is involved in the circulation of charge because as much probability density scatters clockwise as counter-clockwise.

However, does the electron *itself* scatter in several directions at once? Do individual electrons diffracted by a pair of man-made slits in interference experiments go through *both* slits simultaneously? At a given time can a single electron exist over a distribution of positions Δx, possess a spread Δp_x of momentum values, and experience an energy blur, ΔE? Our successful order of magnitude treatments do nothing to exclude such possibilities. Certainly the *wave function* associated with electrons manifests such dispersion. Here it is crucial to stress that the spread in space of a wave function propagating away from a region of small Δy (Fig. 2c) means much more than spreading the probability of encountering an electron over a larger region of space. It means that the spreading wave front can, for example, pass through widely separated slits and, on recombination, produce double slit interference fringes signifying that the wave component transmitted through one slit is "coherent" with the component transmitted through the other. In other words, the wave front is not a simple either/or probability prescription. Separated components along the front mutually recognize each other's kinship.

What does the electron, a real particle (*11*), *really* do? Why do we always retreat to the wave function, a human invention, when pointed questions are asked? For one thing, the behavior of an electron out of eyeshot is a metaphysical question inherently unanswerable by experiment if the uncertainty principle is correct. As Schroedinger said (*12*), "We have taken over from previous theory the idea of a particle and all the technical language concerning it. This idea is inadequate. It constantly drives our mind to ask for information which has obviously no significance." Wheeler, a protégé of Bohr, declared (*13*), "It is wrong to attribute a tangibility to the [particle] in all its travel from the point of entry to its [detection]." On the other hand, it is counterproductive to disparage the wave function, which fills in the gaps for us to the extent that they can be filled. Born wrote (*11*), "The question of whether the waves are something 'real' or a fiction to describe and predict phenomena in a convenient way is a matter of taste. I personally like to regard a probability wave . . . as a real thing, certainly as more than a tool for mathematic calculations . . . Quite generally, how could we rely on probability predictions if by this notion we do not refer to something real and objective?" In the third of a century since Born expressed this view, Ψ has emerged as the most fundamental concept of quantum theory (*14*). To grasp the way it interrelates probability distributions of position and momentum is to comprehend the uncertainty principle.

Acknowledgment

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