Resumo:

$$\boxed{\partial_\mu A^\mu = 0}$$

Lagrangiana de Fermi $\mathcal{L} = -\frac{1}{2}\partial_\mu A_\nu \partial^\mu A_\nu \longrightarrow \pi^\mu = -\frac{1}{c^2}\dot{A}^\mu$

Quantização canônica:

$$[A^\mu(\vec{x},t), A^\nu(\vec{y},t)] = [\dot{A}^\mu(\vec{x},t), \dot{A}^\nu(\vec{y},t)] = 0 \qquad [A^\mu(\vec{x},t), \dot{A}^\nu(\vec{y},t)] = -i\hbar c^2 g^{\mu\nu}\delta(\vec{x}-\vec{y})$$

Expansão dos campos $A^\mu = A^{\mu +} + A^{\mu -}$

$$A^{\mu(\pm)} = \sum_{r,\vec{k}} \sqrt{\frac{\hbar c^2}{2V\omega_k}}\, \varepsilon^\mu_r(\vec{k}) \begin{pmatrix} a_r(\vec{k}) \\ a^+_r(\vec{k}) \end{pmatrix} e^{(\mp)i k\cdot x}$$

$$\varepsilon^\mu_r(\vec{k})\,\varepsilon_{s\mu}(\vec{k}) = -\delta_{rs}\,\zeta_r$$

$$\sum_r \zeta_r\, \varepsilon^\mu_r(\vec{k})\,\varepsilon^\nu_r(\vec{k}) = -g^{\mu\nu}$$

$\zeta_0 = -1 \qquad \zeta_i = 1$

$$[a_r(\vec{k}), a_s(\vec{k}')] = [a^+_r(\vec{k}), a^+_s(\vec{k}')] = 0$$

$$[a_r(\vec{k}), a^+_s(\vec{k}')] = \zeta_r\,\delta_{rs}\,\delta_{\vec{k},\vec{k}'}$$

Espaço de Fock: $a_r(\vec{k})|0\rangle = 0 \quad \forall\, r, \vec{k}$

$$H = \int d^3x :\pi^\mu \dot{A}_\mu - \mathcal{L}: = \sum_{\vec{k}} \hbar\omega_k \left[-a_0^\dagger(\vec{k})a_0(\vec{k}) + \sum_{i=1}^{3} a_i^\dagger(\vec{k})a_i(\vec{k}) \right]$$

$\langle\psi|H|\psi\rangle \geq 0$ p/ todo $|\psi\rangle = (a_{r_1}(k_1))^{n_1} \cdots (a_{r_n}(\vec{k}_n))^{n_n}|0\rangle$

Problemas: $|1_{\vec{k},r}\rangle = a_r^\dagger(\vec{k})|0\rangle \longrightarrow \langle 1_{k,r}|1_{k,r}\rangle = \begin{cases} 1 & r = 1,2,3 \\ -1 & r = 0 \end{cases}$

$|\psi\rangle = |1_{k,i}\rangle + |1_{\vec{k}',0}\rangle \longrightarrow \langle\psi|\psi\rangle = 0$

Qualquer estado com número ímpar de fótons $r=0$ tem norma negativa.

Incompatível com MQ !!

$\underline{\partial_\mu A^\mu = 0}$ Fock $\to$ Hilbert

$\langle \psi | \psi \rangle = 0$

$$[\partial_\mu A^\mu(x), A^\nu(y)] = i\hbar c\, \partial_\mu D^{\mu\nu}(x-y) \neq 0$$

$$[0, A^\nu(y)] \neq 0$$

Gupta Bleuler (1950)

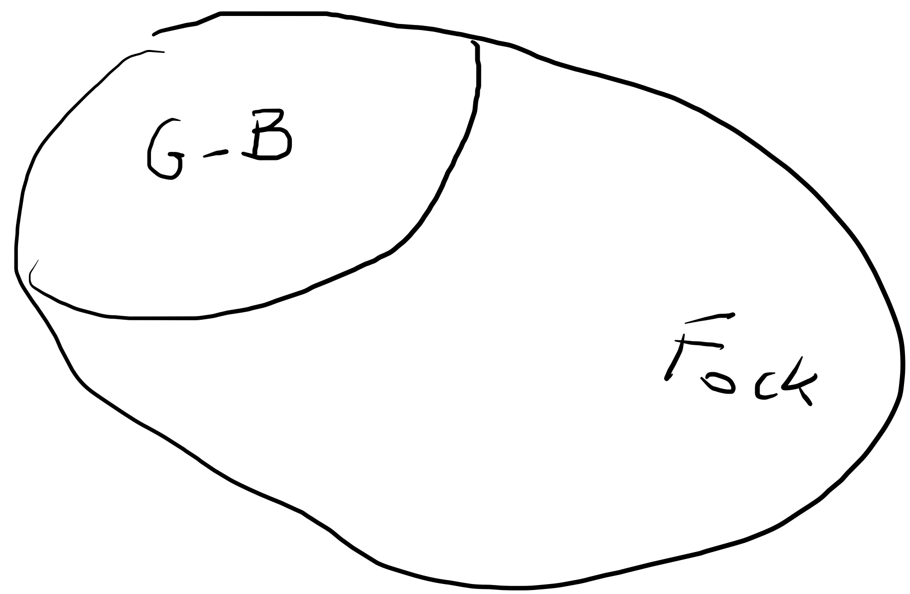
$|\psi\rangle$ é físico $\boxed{\partial_\mu A^{\mu +}(x) |\psi\rangle = 0}$

$$\langle \psi | \partial_\mu A^{\mu -} = 0$$

$$\langle \psi' | \underbrace{\partial_\mu A^\mu}_{0} | \psi \rangle = 0$$

$$= \langle \psi' | \partial_\mu A^{\mu -} + \partial_\mu A^{\mu +} | \psi \rangle = 0$$

$$\partial^\mu A^\mu \neq 0$$



$$\underline{\partial_\mu A^{\mu +}|\psi\rangle = 0} \qquad \varepsilon_0^\mu = (1,0,0,0)$$

$$\underbrace{\partial_\mu A^{\mu +}|\psi\rangle}_{-\vec{k}\cdot\vec{\varepsilon}_r} = \sum_{r,\vec{k}} \sqrt{\frac{\hbar c^2}{2V\omega_k}}\, \varepsilon_r^\mu(\vec{k})\, a_r|\vec{k}\rangle (-ik_\mu)\, e^{-ik\cdot x} |\psi\rangle$$

$$k_\mu \varepsilon_r^\mu = \begin{cases} k_0 & r=0 \\ |\vec{k}| & r=3 \\ 0 & r=1,2 \end{cases} \qquad \varepsilon_i^\mu = (0, \vec{\varepsilon}_i)$$

$$k_0 = |\vec{k}| = \frac{\omega_k}{c}$$

$$= -i\sum_{\vec{k}} \sqrt{\frac{\hbar\omega_k}{2V}} (a_0 - a_3)\, e^{-ikx} |\psi\rangle = 0$$

$$e^{-ik'\cdot x}$$

$$0 = \int d^3x \; e^{ik'x} \partial_\mu A^{\mu +} |\psi\rangle$$

$$= -i \sum_{\vec{k}} \sqrt{\frac{\hbar \omega_k}{2V}} \left[a_0(\vec{k}) - a_3(\vec{k}) \right] e^{i(k'_0 - k_0)x_0}$$

$$\times \int d^3x \; e^{i(\vec{k} - \vec{k}')\vec{x}} |\psi\rangle$$

$$\boxed{\partial_\mu A^{\mu +}_{(x)} |\psi\rangle = 0}$$

$$\boxed{\left[a_0(\vec{k}) - a_3(\vec{k}) \right] |\psi\rangle = 0}$$

$$a_0(\vec{k})|\psi\rangle = a_3(\vec{k})|\psi\rangle$$

$$\langle\psi| a_0^\dagger(\vec{k}) = \langle\psi|a_3^\dagger(\vec{k})$$

$$\langle\psi|\overbrace{-a_0^\dagger(\vec{k})}^{-\langle\psi|a_3^\dagger(\vec{k})} a_0(\vec{k}) + a_3^\dagger(\vec{k})\,\overbrace{a_3(\vec{k})|\psi\rangle}^{\cancel{a_0(\vec{k})|\psi\rangle}}$$

$$= \langle\psi|a_3^\dagger(\vec{k})\left(+a_3(\vec{k}) - a_0(\vec{k})\right)|\psi\rangle$$

$$= 0$$

$$\langle\psi|H|\psi\rangle = \sum_{\vec{k}} \hbar\omega_k \sum_{i=1}^{2} \langle\psi|\underline{a_i^\dagger a_i}|\psi\rangle$$

$|\psi\rangle = ?$ $\qquad [a, a^\dagger] = \varepsilon \qquad \varepsilon = \pm 1$

$$[a, (a^\dagger)^n] = (a^\dagger)^{n-1}\, n\, \varepsilon$$

$$a^n a^{\dagger n} |0\rangle = a^{n-1}\left(a^{\dagger n} a + [a, a^{\dagger n}]\right)|0\rangle$$

$$= n\, \varepsilon\, a^{n-1} a^{\dagger\, n-1} |0\rangle$$

$$= n(n-1)\, \varepsilon^2\, a^{n-2} a^{\dagger\, n-2} |0\rangle$$

$$\vdots$$

$$= n!\, \varepsilon^n |0\rangle$$

$$|n\rangle = a^{\dagger n}|0\rangle \rightarrow \boxed{\langle n | n\rangle = n!\, \varepsilon^n}$$

$$|\phi\rangle = (a_0^+)^{n_0}(a_1^+)^{n_1}(a_2^+)^{n_2}(a_3^+)^{n_3}|0\rangle$$

$$\langle\phi|\phi\rangle = (-1)^{n_0}\, n_0!\, n_1!\, n_2!\, n_3!$$

$$|n_0, n_1, n_2, n_3\rangle = \frac{(a_0^+)^{n_0}\cdots(a_3^+)^{n_3}}{\sqrt{n_0!\cdots n_3!}}|0\rangle$$

$$\big\|\,|n_0 \ldots, n_3\rangle\big\|^2 = \begin{cases} 1 & n_0 \text{ par} \\ -1 & n_0 \text{ impar} \end{cases}$$

$$(a_0 - a_3)|0, n_1, n_2, 0\rangle = 0$$

$$(a_0 - a_3)\big(|1, n_1, n_2, 0\rangle - |0, n_1, n_2, 1\rangle\big) = 0$$

$$(a_0 - a_3)\Big[\tfrac{1}{\sqrt{2}}|2, n_1, n_2, 0\rangle - \sqrt{2}\,|1, n_1, n_1, 1\rangle + \tfrac{1}{\sqrt{2}}|0, n_1, n_2, 2\rangle\Big]$$

$2 \quad a_0^{\dagger 2}|0\rangle$

a_0 $\quad$ a_3 $\quad$ $a_3^{\dagger 2}$

$$-\sqrt{2}\,|1, n_1, n_2, 0\rangle \ ② + \sqrt{2}\,|0, n_1, n_2, 1\rangle \ ① + \sqrt{2}\,|1, n_1, n_2, 0\rangle \ ②$$

$$-\sqrt{2}\,|0, n_1, n_2, 1\rangle \ ①$$

$$= 0$$

$$\boxed{|n_1, n_2, N\rangle = \sum_{l=0}^{N} (-1)^{l} \sqrt{\frac{N!}{(N-l)!\, l!}}\, |N-l, n_1, n_2, l\rangle}$$

$$\boxed{(a_0 - a_3)\,|n_1, n_2, N\rangle = 0}$$

$$\langle N, n_1, n_2 | n_1, n_2, N\rangle = \sum_{l, l'=0}^{N} (-1)^{\overset{2l}{\overline{l+l'}}} \frac{N!}{\sqrt{(N-l)!\, l!\, (N-l')!\, l'!}} \langle N-l', n_1, n_2, l' | N-l, n_1, n_2, l\rangle$$

$$(-1)^{N-l}\, \delta_{l,l'}$$

$$= (-1)^N \underbrace{\sum_{l=0}^{N} (-1)^l \frac{N!}{(N-l)!\, l!}} = (1-1)^N = 0$$

$$(a+b)^N = \sum_{l=0}^{N} \frac{N!}{(N-l)!\, l!} a^{N-l} b^l$$

$$\langle 0, n, n, 0 | 0, n, n, 0 \rangle = \underline{1}$$

$$\langle N, n_1, n_2 | N, n_1, n_2 \rangle = 0 \quad N \neq 0$$

$$(a_0 - a_3)$$

$$|n_1, n_2, N\rangle = \frac{(a_0^+ - a_3^+)^N}{\sqrt{N!}} |0, n_1, n_2, 0\rangle$$

$$= \sum_{l=0}^{N} (-1)^l \frac{\sqrt{N!}}{\sqrt{(N-l)!\,l!}} |N-l, n_1, n_2, l\rangle$$

$$[a_0 - a_3, a_0^+ - a_3^+] = -1 + 1 = 0$$

$$|\psi_N\rangle = \sum_{n_1,n_2} b(n_1,n_2,N)\,|n_1,n_2,N\rangle$$

$$|\psi\rangle = \sum |\psi_N\rangle$$

$$\langle\psi|\psi\rangle = \langle\psi_0|\psi_0\rangle$$

$$\langle\psi|H|\bar{\psi}\rangle = \langle\psi_0|H|\psi_0\rangle$$

$$|\psi\rangle = |0,n_1,n_2,0\rangle$$

Propagador do fóton

$$D_F^{\mu\nu}(x) = -\frac{g^{\mu\nu}}{(2\pi)^4}\int d^4k \, \frac{e^{-ikx}}{k^2 + i\varepsilon}$$

$$= \frac{1}{(2\pi)^4}\int d^4k \, D_F^{\mu\nu}(\vec{k}) \, e^{-ikx}$$

$$\boxed{D_F^{\mu\nu}(\vec{k}) = -\frac{g^{\mu\nu}}{k^2 + i\varepsilon} = \frac{1}{k^2 + i\varepsilon}\sum_{r=0}^{3} \zeta_r \, \varepsilon_r^{\mu}(\vec{k})\, \varepsilon_r^{\nu}(\vec{k})}$$

$$D_F^{\mu\nu}(\bar{k}) = \frac{1}{k^2 + i\varepsilon}\left\{ \sum_{r=1}^{2} \varepsilon_r^{\mu}\, \varepsilon_r^{\nu}(\bar{k}) + \frac{\left[k^{\mu} - (k\cdot n)\, n^{\mu}\right]\left[k^{\nu} - (k\cdot n)\, n^{\nu}\right]}{(k\cdot n)^2 - k^2} - n^{\mu} n^{\nu} \right\}$$

$$\frac{k^{\mu} k^{\nu} - (k\cdot n)\left(k^{\nu} n^{\mu} + k^{\mu} n^{\nu}\right)}{(k\cdot n)^2 - k^2}$$

$$\cancel{k_0^2 - \vec{k}^2}$$

$$\frac{\cancel{(k\cdot n)^2}\, n^{\mu} n^{\nu} - n^{\mu} n^{\nu}\left(\cancel{(k\cdot n)^2} - k^2\right)}{(k\cdot n)^2 - k^2}$$

$${}_{T}D_F^{\mu\nu} = \frac{1}{k^2 + i\epsilon} \sum_{r=1}^{2} \epsilon_r^{\mu}\, \epsilon_r^{\nu}(\vec{k})$$

$${}_{C}D_F^{\mu\nu} = \frac{n^{\mu} n^{\nu}}{(k \cdot n)^2 - k^2}$$

$$\boxed{k^2 \neq 0}$$

$${}_{R}D_F^{\mu\nu} = \frac{1}{k^2 + i\epsilon} \left(\frac{k^{\mu} k^{\nu} - (k \cdot n)(k^{\mu} n^{\nu} + k^{\nu} n^{\mu})}{(k \cdot n)^2 - k^2} \right)$$

$$n^\mu = (1,0,0,0) = g^{\mu 0}$$

$$(k\cdot n)^2 - k^2 = k_0^2 - (k_0^2 - \vec{k}^2) = \underline{\underline{\vec{k}^2}}$$

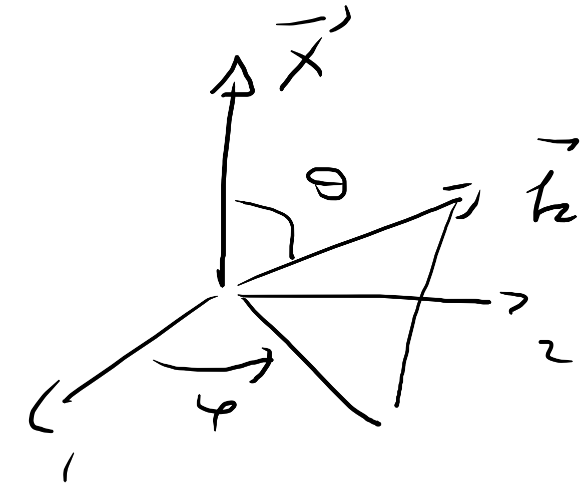
$$_c D_F^{\mu\nu}(x) = \frac{1}{(2\pi)^4}\int d^4k \; {}_c D_F^{\mu\nu}(k)\, e^{-ik\cdot x}$$

$$= \frac{g^{\mu 0} g^{\nu 0}}{(2\pi)^4}\int \frac{d^3k}{\vec{k}^2}\, e^{+i\vec{k}\cdot\vec{x}} \underline{\int dk_0\, e^{-ik_0 x_0}}$$

$$\frac{1}{2\pi}\int dk_0\, e^{-ik_0 x_0} = \delta(x_0)$$

$$d^3\vec{k} = \vec{k}^2 \sin\theta \, d|k| \, d\theta \, d\varphi$$

$$= \vec{k} \, d(\cos\theta) \, d|k| \, d\varphi$$



$$\int \frac{d^3k}{\vec{k}^2} e^{i\vec{k}\cdot\vec{x}} = \int d|k| \, d\varphi \, d(\cos\theta) \, e^{i|\vec{k}||\vec{x}|\cos\theta}$$

$$= 2\pi \int d|k| \, \frac{e^{i|\vec{k}||\vec{x}|} - e^{-i|\vec{k}||x|}}{i|k||x|}$$

$$= 4\pi \int_0^\infty d|k| \, \frac{\sin|k||x|}{|k||x|} = \frac{4\pi}{|x|}\frac{\pi}{2}$$

$$\int_0^\infty \frac{\sin ax}{x} dx = (\operatorname{sign} a)\frac{\pi}{2}$$

$$ {}_{C}D^{\mu\nu}_{F}(x) = \frac{g^{\mu 0} \; g^{\nu 0}}{4\pi |x|} \delta(x^0) $$

$$ \int d^4x \, d^4y \; j_1^{\mu}(x) \; {}_{C}D^{\mu\nu}_{F}(x-y) \; j_2^{\nu}(y) $$

$$ = \frac{1}{4\pi} \int d^4x \, d^4y \; j_1^0(x) \, j_2^0(y) \; \frac{\delta(x^0 - y^0)}{|\vec{x} - \vec{y}|} $$

$$ = \frac{1}{4\pi} \int dx_0 \int \underline{d^3x} \, d^3y \; \frac{\rho_1(x) \; \rho_2(y) \, c^2}{|\vec{x} - \vec{y}|} $$

$$|1_{\vec{k},r}\rangle = a_r^{+}(\vec{k})\,|0\rangle$$

1 fóton

$$\boxed{E^2 = p^2c^2 + m^2c^4}$$

$$\hbar^2\omega_k^2 = \hbar^2\vec{k}^2c^2 \qquad m = 0$$

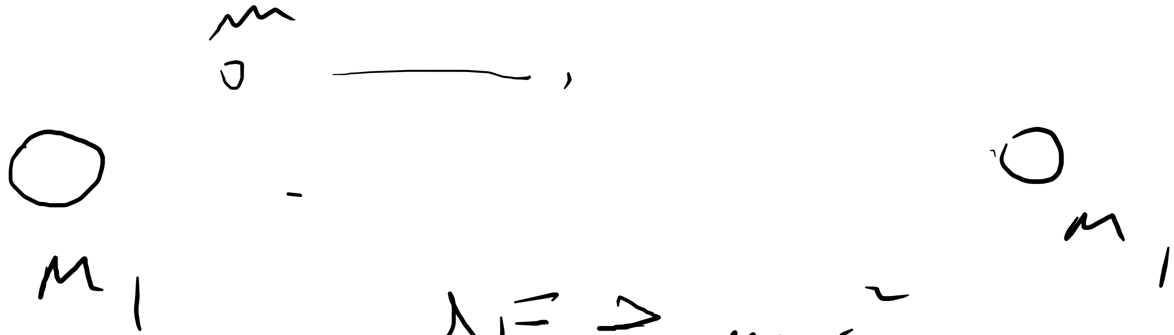
$$\underline{\epsilon^\mu_r \; e^{ik.x}}$$

$$\boxed{\frac{\omega_k}{c} = |\vec{k}|}$$

$$k^2 = 0$$

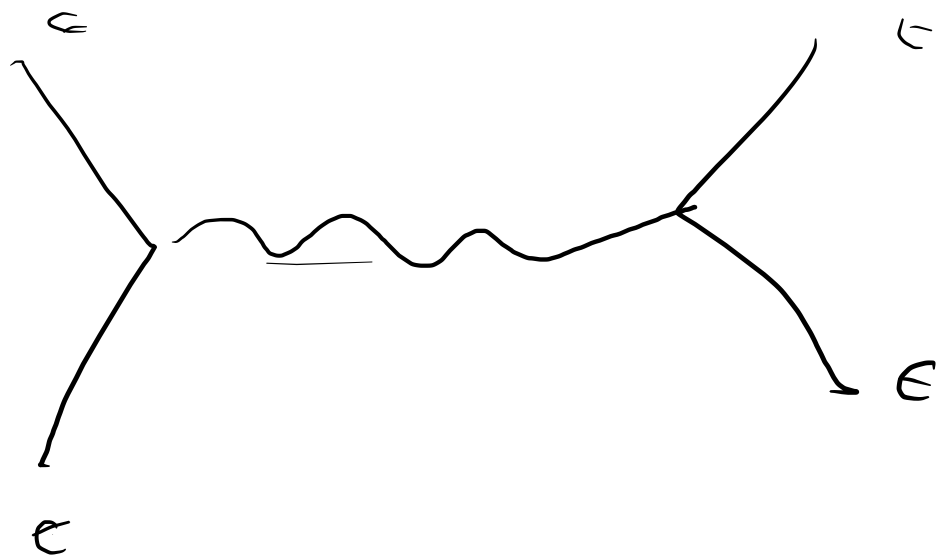
$$\boxed{\Box A_\mu = 0}$$

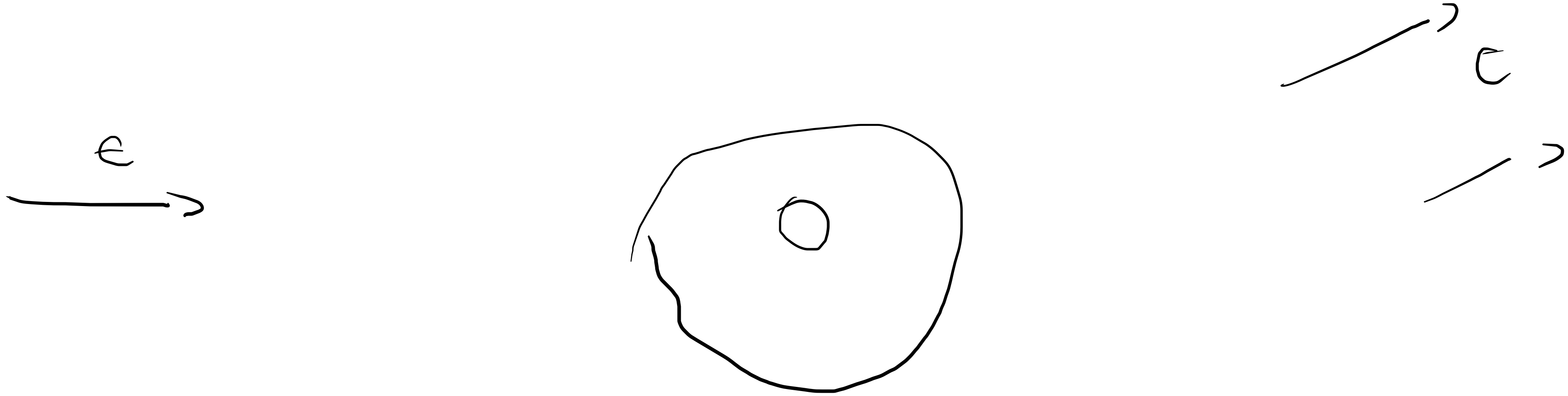
$$k^2 = 0$$



$$\Delta E \geq mc^2$$

$$P^2 = m^2$$





$$\rightarrow \; |\psi\rangle_{asympt}$$

$$\|$$

$$|\psi\rangle = (a^+)^n |0\rangle$$

$$E \sim \hbar\omega_n$$