Example 2.8-1 Heat Transfer in a Wire Consider the steady-state temperature in a cylindrical wire of radius R that is heated by passage of an electric current and cooled by convective heat transfer to the surrounding air. The local heating rate, H_V, is assumed to be independent of position. This is equivalent to assuming a uniform current density and electrical resistance. For steady conduction in a solid with such a heat source, Eq. (2.4-3) reduces to

$$\nabla^2 T = -\frac{H_V}{k}. \tag{2.8-1}$$

Cylindrical coordinates (r, θ, z) are the natural choice for this problem. The convection boundary condition at the surface of the wire is written as

$$\frac{\partial T}{\partial r} = -\frac{h}{k}(T - T_\infty) \qquad \text{at } r = R, \tag{2.8-2}$$

where T_∞ is the ambient temperature. For simplicity, we assume that h is independent of position. With H_V, k, and h all assumed to be constant, there is nothing to cause the temperature to depend on the angle θ. Thus, we conclude that the temperature field is axisymmetric. It follows that Eq. (2.5-15) is applicable and that the second boundary condition in r is

$$\frac{\partial T}{\partial r} = 0 \qquad \text{at } r = 0. \tag{2.8-3}$$

If the wire is very long, and nothing is done to cause the temperature at the ends to differ, then there is also no reason for the temperature to depend on z. It is apparent now that all of the physical conditions can be satisfied by a temperature field which depends only on r.

Assuming now that $T = T(r)$ only, Table 2-2 is used to rewrite Eq. (2.8-1) as

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = -\frac{H_V}{k}. \tag{2.8-4}$$

This second-order equation requires two boundary conditions in r, which are given already by Eqs. (2.8-2) and (2.8-3). The temperature is determined by first integrating Eq. (2.8-4) to give

$$r\frac{dT}{dr} = -\frac{H_V r^2}{2k} + C_1. \tag{2.8-5}$$

The symmetry condition [Eq. (2.8-3)] indicates that the constant C_1 must be zero. A second integration yields

$$T = -\frac{H_V r^2}{4k} + C_2, \tag{2.8-6}$$

where C_2 is another constant. Substituting this result into the convective boundary condition at the surface [Eq. (2.8-2)] gives

$$-\frac{H_V R}{2k} = -\frac{h}{k}\left(-\frac{H_V R^2}{4k} + C_2 - T_\infty\right). \tag{2.8-7}$$

Solving for C_2, the temperature is found to be

$$T - T_\infty = \frac{H_V R^2}{4k}\left[1 - \left(\frac{r}{R}\right)^2\right] + \frac{H_V R}{2h}. \tag{2.8-8}$$

Thus, the temperature at the surface of the wire exceeds the ambient value by the amount $H_V R/2h$, and the temperature at the center of the wire is elevated further by an amount $H_V R^2/4k$.

The behavior of the temperature is revealed more clearly by using dimensionless quantities defined as

$$\Theta = \frac{T - T_\infty}{T_c - T_\infty}, \qquad \eta = \frac{r}{R}, \qquad \mathrm{Bi} = \frac{hR}{k}, \tag{2.8-9}$$

where $T_c = T(0)$ is the temperature at the center of the wire and Bi is the *Biot number*. By definition, Θ ranges from unity at the center of the wire to zero in the bulk air. Equation (2.8-8) is rewritten now as

$$\Theta = \frac{2 + \mathrm{Bi}(1 - \eta^2)}{2 + \mathrm{Bi}} \tag{2.8-10}$$

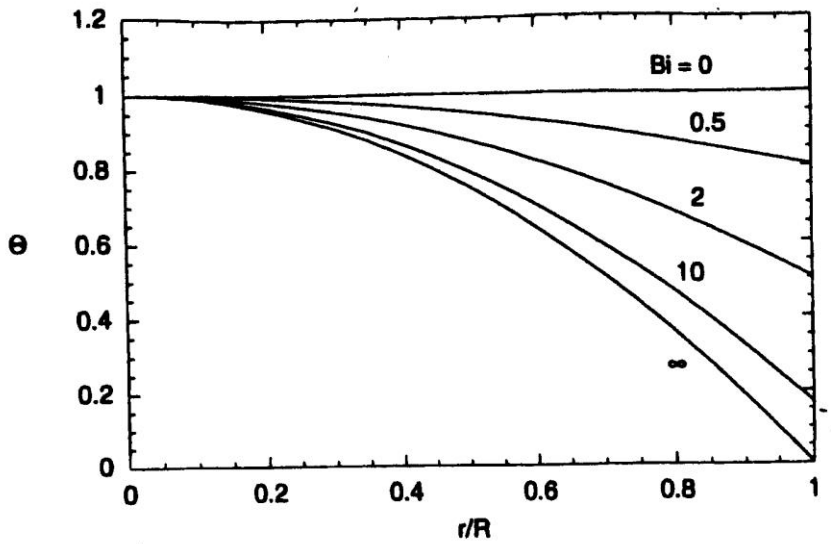


Figure 2-5. Temperature profile in an electrically heated wire, as a function of the Biot number. The plot is based on Eq. (2.8-10).

Figure 2-5 shows the dimensionless temperature profile for several values of Bi. For Bi<<1, radial heat conduction in the wire is so fast that the wire is nearly isothermal, and the main temperature drop is in the air. For Bi>>1, convective heat transfer in the air is so rapid that the external temperature drop is negligible, and the temperature at the wire surface is very close to the ambient value. Thus, the Biot number represents the ratio of the heat transfer resistance within the wire to that within the surrounding air. The significance of Biot numbers for heat or mass transfer is discussed further in Chapter 3.

DEEN, M. Analysis of Transport Phenomena. Oxford University Press, New York, 1998.

Example 2.8-2 Diffusion in a Binary Gas with a Heterogeneous Reaction This example illustrates the use of Fick's law for a binary gas, and it also shows how the reaction rate can influence the boundary condition used at a catalytic surface. The system to be considered is shown in Fig. 2-6. A stagnant gas film of thickness L is in contact with a surface which catalyzes the irreversible reaction, $A \rightarrow mB$. The reaction rate follows nth-order kinetics ($n>0$), as given by

$$R_{SA} = -k_{sn}C_A^n, \tag{2.8-11}$$

where k_{sn} is a constant. It is assumed that C_A depends on y only and that its value at $y=0$ is fixed at C_{A0}. It is assumed also that the gas is isothermal and isobaric, so that the total molar concentra-

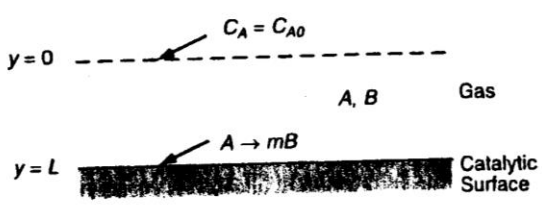


Figure 2-6. Diffusion in a binary gas with a heterogeneous reaction.

tion (C) is constant. Unless the molecular weights of A and B are identical (i.e., unless $m=1$), the total mass density (ρ) will not be constant under these conditions.

Before using species conservation or Fick's law, we first see what can be learned from the continuity equation. For this steady, one-dimensional system with variable ρ, Eq. (2.3-1) becomes

$$\frac{d(\rho v_y)}{dy} = 0. \tag{2.8-12}$$

Thus, ρv_y is independent of y. Because the catalytic surface is assumed to be impermeable (i.e., $v_y=0$ at $y=L$), we conclude that the mass-average velocity is zero throughout the gas film. The main consequence of this is that $J_{iy}=N_{iy}$ for both species.

Most of the results in Section 2.6 cannot be used here because ρ is not constant. However, we can apply Eq. (2.6-1) to both species. It follows from the stated assumptions that

$$\frac{dN_{Ay}}{dy} = 0 = \frac{dN_{By}}{dy}. \tag{2.8-13}$$

There is no reaction term in Eq. (2.8-13) because there is no *homogeneous* reaction. This equation indicates that both fluxes are independent of position, so that evaluating the flux ratio at any location determines the ratio for all y. Using Eq. (2.7-3) together with $J_{iy}=N_{iy}$, we obtain

$$N_{By} = -mN_{Ay}. \tag{2.8-14}$$

No further consideration of species B is necessary, because N_{By} can be obtained from N_{Ay} and because C_B was assumed to have no effect on the reaction kinetics.

In selecting a form of Fick's law it is advantageous to employ an expression which involves C rather than ρ, because it is C which is assumed to be constant. Thus, we adopt Eq. (D) of Table 1-3, which requires that we use the *molar-average* velocity as the reference frame. From Tables 1-2 and 1-3, the total flux of A is given by

$$N_{Ay} = x_A(N_{Ay}+N_{By}) - CD_{AB}\frac{dx_A}{dy}. \tag{2.8-15}$$

Using Eq. (2.8-14) to eliminate N_{By} from this expression, we obtain

$$N_{Ay} = x_A N_{Ay}(1-m) - CD_{AB}\frac{dx_A}{dy}. \tag{2.8-16}$$

The convective flux of A in this reference frame is defined as $C_A v_y^{(M)}$, so that Eq. (2.8-16) implies that $v_y^{(M)} = N_{Ay}(1-m)/C$. Thus, the molar-average velocity does not vanish unless $m=1$, even though the mass-average velocity is zero for all stoichiometries. This indicates that there is a convective flux here when using the molar-average velocity, but not when using the mass-average velocity!

Rearranging Eq. (2.8-16) to solve for N_{Ay} gives

$$N_{Ay} = -\frac{CD_{AB}}{[1-x_A(1-m)]}\frac{dx_A}{dy}. \tag{2.8-17}$$

Taking advantage now of the assumed constancy of C, Eq. (2.8-17) becomes

$$N_{Ay} = -\frac{D_{AB}}{[1-(C_A/C)(1-m)]}\frac{dC_A}{dy}. \tag{2.8-18}$$

The flux of A at the catalytic surface is directly related to the reaction rate at the surface. From Eq. (2.7-2),

$$N_{Ay}(L) = -R_{SA} = k_{sn}[C_A(L)]^n. \tag{2.8-19}$$

Because N_{Ay} has been shown to be independent of position, Eqs. (2.8-18) and (2.8-19) can be equated to give

$$\frac{dC_A}{dy} = -\left(\frac{k_{sn}}{D_{AB}}\right)[C_A(L)]^n\left[1 - \frac{C_A}{C}(1-m)\right], \qquad C_A(0) = C_{A0}. \tag{2.8-20}$$

Before integrating Eq. (2.8-20) to determine the concentration profile, we introduce the dimensionless quantities

$$\theta = \frac{C_A}{C_{A0}}, \qquad \eta = \frac{y}{L}, \qquad \mathrm{Da} = \frac{k_{sn}C_{A0}^{\,n-1}L}{D_{AB}}, \qquad \phi = \frac{C_A}{C_{A0}}\bigg|_{y=L}. \tag{2.8-21}$$

The parameter Da is the *Damköhler number*, which is seen to be the ratio of a reaction velocity ($k_{sn}C_{A0}^{\,n-1}$) to a diffusion velocity (D_{AB}/L). Thus, it is a measure of the intrinsic rate of reaction relative to that of diffusion.[1] The reactant concentration at the catalytic surface, which is an unknown constant, is denoted as ϕ. The governing equation in dimensionless form is then

$$\frac{d\theta}{d\eta} = -\mathrm{Da}\,\phi^n[1 - x_{A0}(1-m)\theta], \qquad \theta(0) = 1, \tag{2.8-22}$$

where x_{A0} is the (known) mole fraction of A at $\eta = 0$.

Equation (2.8-22) is separable and can be integrated from $\eta = 0$ to $\eta = 1$ to obtain implicit expressions for $\phi = \theta(1)$, the surface concentration of the reactant. The results, which depend on the reaction stoichiometry, are

$$\mathrm{Da}\,\phi^n = \begin{cases} \dfrac{1}{x_{A0}(1-m)} \ln\left[\dfrac{1 - x_{A0}(1-m)\phi}{1 - x_{A0}(1-m)}\right], & m \neq 1, \\ 1 - \phi, & m = 1. \end{cases} \tag{2.8-23}$$

Inspection of Eq. (2.8-23) reveals that $\phi \to 1$ as $\mathrm{Da} \to 0$. In this case the reaction is slow relative to diffusion, so that the reaction is the controlling step and the reactant concentration is nearly uniform throughout the film. At the other extreme, as $\mathrm{Da} \to \infty$, the process is controlled entirely by mass transfer and $\phi \to 0$. That is, the concentration at the surface approaches zero.

Concentration profiles for an equimolar, second-order reaction ($m = 1$, $n = 2$) at several values of Da are shown in Fig. 2-7. The transition from kinetic to diffusion control as Da is increased is evident. Also noteworthy is the qualitative similarity between this plot and Fig. 2-5. In both situations the parameter can be interpreted as the ratio of two resistances in series, those for internal and external heat transfer (Bi) or those for diffusion and reaction (Da).

The foregoing results indicate that if we were interested only in diffusion-controlled conditions ($\mathrm{Da} \to \infty$), then we could replace Eq. (2.8-19) by $C_A = 0$ at $y = L$, or (in dimensionless form)

$$\theta(1) = 0. \tag{2.8-24}$$

This type of *fast-reaction boundary condition* holds for any irreversible, diffusion-controlled, heterogeneous reaction. When this simple condition applies, the reaction rate law does not enter into the problem. The accuracy of Eq. (2.8-24) for the present problem is judged most easily for an equimolar reaction ($m = 1$), in which case Eq. (2.8-23) indicates that

[1] There are no fewer than five dimensionless groups named after Damköhler, two of which involve reaction rates. The one which compares rates of reaction and diffusion is sometimes called "Damköhler group II." Because it is the only type of Damköhler number used in this book, no other identifier is added to the symbol Da. An extensive tabulation of named dimensionless groups pertinent to chemical engineering is given in Catchpole and Fulford (1966).

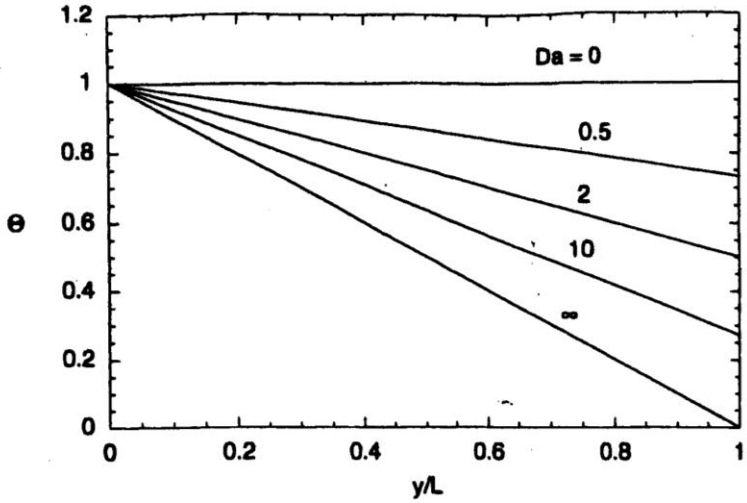


Figure 2-7. Concentration profiles for diffusion through a gas film with a heterogeneous reaction, showing the effect of the Damköhler number. The curves were obtained by solving Eq. (2.8-22) with $m=1$ and $n=2$.

$$\theta(1) \rightarrow \mathrm{Da}^{-1/n} \qquad (\mathrm{Da} \rightarrow \infty,\ m=1). \tag{2.8-25}$$

For first-order kinetics ($n=1$) and $\mathrm{Da}>10^2$, this implies that $\theta(1)<0.01$. Because the reactant flux (and therefore the reaction rate) varies as $1-\theta(1)$, the simplified boundary condition will lead to an error of <1% under these conditions. For second-order kinetics ($n=2$), $\mathrm{Da}>10^4$ is needed to maintain this small level of error.

Example 2.8-3 Diffusion in a Dilute Liquid Solution with a Heterogeneous Reaction We reconsider the situation of Example 2.8-2, but with a dilute liquid solution in place of the gas. One consequence of having a liquid is that we can assume constant ρ. Another key feature of the dilute liquid solution is that it is pseudobinary from a diffusional standpoint. Thus, the product B will have negligible influence on the diffusion of the reactant A for any stoichiometry. Because species B is assumed not to affect the reaction kinetics either, it need not be considered at all.

The previous conclusion that $v_y=0$ remains valid for the liquid. Using this information and Eq. (2.6-4), the flux of A is given by

$$N_{Ay}=J_{Ay}=-D_A\frac{dC_A}{dy}. \tag{2.8-26}$$

This may be contrasted with Eqs. (2.8-16) or (2.8-18) for the gas-phase problem; there is no convection now. From Table 2-3, the conservation equation for species A is

$$\frac{d^2C_A}{dy^2}=0. \tag{2.8-27}$$

Accordingly, C_A is linear in y for all values of m. Converting to the dimensionless variables defined by Eq. (2.8-21) and proceeding much as before, it is found that the concentration of A at the catalytic surface is governed by

$$\mathrm{Da}\phi^n=1-\phi \tag{2.8-28}$$

For the gas, this result held only for $m=1$, corresponding to equimolar counterdiffusion [see Eq. (2.8-23)]. For the liquid, it is valid for all stoichiometries.

Example 6.2-3 Flow of Two Immiscible Fluids in a Parallel-Plate Channel A simple type of two-phase flow occurs when immiscible fluids occupy distinct layers in a parallel-plate channel, as depicted in Fig. 6–3. The density and viscosity of fluid 1 (ρ_1 and μ_1) may differ from those of fluid 2 (ρ_2 and μ_2). It is desired to determine the steady, fully developed velocities of the two fluids, which are denoted as $v_x^1(y)$ and $v_x^2(y)$, respectively.

The Navier–Stokes equation for each phase reduces to

$$\frac{d^2 v_x^{(i)}}{dy^2} = \frac{1}{\mu_i}\frac{d\mathcal{P}^{(i)}}{dx}. \tag{6.2-24}$$

Integrating this twice gives

$$v_x^{(i)}(y) = \frac{1}{\mu_i}\frac{d\mathcal{P}^{(i)}}{dx}\frac{y^2}{2} + a_i y + b_i, \tag{6.2-25}$$

where a_i and b_i are constants. These four constants are determined by the conditions

$$v_x^{(1)}(H_1) = 0, \tag{6.2-26}$$

$$v_x^{(2)}(-H_2) = 0, \tag{6.2-27}$$

$$v_x^{(1)}(0) = v_x^{(2)}(0), \tag{6.2-28}$$

$$\mu_1 \frac{dv_x^{(1)}}{dy}(0) = \mu_2 \frac{dv_x^{(2)}}{dy}(0). \tag{6.2-29}$$

Equations (6.2-26) and (6.2-27) are the usual no-slip conditions at the solid surfaces, whereas Eqs. (6.2-28) and (6.2-29) express the matching of the tangential components of velocity and stress at the fluid–fluid interface. It is convenient to introduce the constants

$$u_i \equiv -\frac{H_i^2}{2\mu_i}\frac{d\mathcal{P}^{(i)}}{dx}, \tag{6.2-30}$$

$$K \equiv \left(\frac{\mu_1}{\mu_2}\right)\left(\frac{H_2}{H_1}\right), \tag{6.2-31}$$

where u_i has units of velocity [compare with Eq. (6.2-9)] and K is dimensionless. The velocities in the two fluids are written as

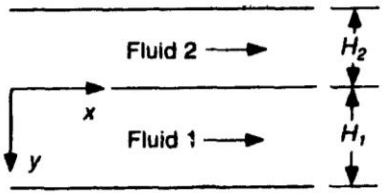


Figure 6-3. Flow of two immiscible fluids in a parallel-plate channel.

$$v_x^{(1)}(y) = u_1\left[1 - \left(\frac{y}{H_1}\right)^2\right] + \left(\frac{u_2 - u_1}{1+K}\right)\left[1 - \left(\frac{y}{H_1}\right)\right], \tag{6.2-32}$$

$$v_x^{(2)}(y) = u_1\left[1 - \left(\frac{u_2}{u_1}\right)\left(\frac{y}{H_2}\right)^2\right] + \left(\frac{u_2 - u_1}{1+K}\right)\left[1 - K\left(\frac{y}{H_2}\right)\right], \tag{6.2-33}$$

where u_i and K are assumed to be known.

What remains is to relate the pressures in the two fluids. In Examples 6.2-1 and 6.2-2 it was seen that for fully developed flow of a single, incompressible fluid in a channel of known dimensions, specifying the mean velocity was the same as setting the axial gradient of the dynamic pressure [see Eqs. (6.2-10), (6.2-19), and (6.2-22)]. Extra care is needed in the present problem, because $d\mathcal{P}^{(i)}/dx$, while constant within each fluid, is generally not the same in the two phases. The constraints on the two-phase flow are revealed by considering the actual pressure, P. For the general case of a channel inclined at an arbitrary angle, $P^{(i)} = P^{(i)}(x, y, z)$ even in the absence of flow, because of static pressure variations. However, from the definition of the dynamic pressure, Eq. (5.8-1), it follows that for this flow

$$\frac{\partial P^{(i)}}{\partial x} = \frac{d\mathcal{P}^{(i)}}{dx} + \rho_i g_x. \tag{6.2-34}$$

Thus, the constancy of $d\mathcal{P}^{(i)}/dx$ implies that $\partial P^{(i)}/\partial x$ too is constant within each phase. The final piece of information needed comes from the normal stress balance at the fluid–fluid interface, based on Eq. (5.7-9). Given that the interface is flat and that there are no normal viscous stresses (i.e., $\tau_{yy} = 0$), the values of P there must match. We conclude that $\partial P/\partial x$ has the same constant value throughout both fluids. To emphasize that there is only one independent pressure gradient, Eq. (6.2-30) is rewritten as

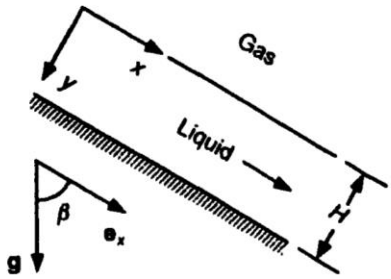
$$u_i = -\frac{H_i^2}{2\mu_i}\left(\frac{\partial P}{\partial x} - \rho_i g_x\right). \tag{6.2-35}$$

The need to consider actual pressure, and not just dynamic pressure, is typical of problems involving fluid–fluid interfaces.

Example 6.2-4 Flow of a Liquid Film Down an Inclined Surface With reference to Fig. 6-4, the objective is to determine the velocity of a liquid film flowing down a surface which is oriented at an angle β relative to vertical. The film thickness (H) is assumed to be constant, making it possible to have steady, fully developed flow. Thus, it is assumed that $v_x = v_x(y)$ only and $v_y = v_z = 0$, from which it follows (as before) that $\mathcal{P} = \mathcal{P}(x)$ only and that $d\mathcal{P}/dx$ is constant.

The falling liquid film can be viewed as a special case of the two-fluid problem in Example 6.2-3, in which fluid 1 is now the liquid and fluid 2 the gas. Assuming that the gas occupies a

Figure 6-4. Flow of a liquid film down an inclined surface.



space at least as thick as the liquid film, and recalling that a typical ratio of liquid to gas viscosities is $\sim 10^2$ (Chapter 1), the parameter K defined by Eq. (6.2-31) will be extremely large. Assuming also that the gas pressure is uniform, the reasoning leading to Eq. (6.2-35) indicates that $\partial P/\partial x = 0$ in the liquid. Noting that $g_x = g \cos \beta$, it follows from Eqs. (6.2-32) and (6.2-35) that the liquid velocity is

$$v_x(y) = \frac{H^2 \rho_L g \cos \beta}{2\mu_L}\left[1 - \left(\frac{y}{H}\right)^2\right] \tag{6.2-36}$$

where the subscript L denotes liquid properties. The velocity profile can be rewritten in terms of the mean velocity (U) as

$$v_x(y) = \frac{3}{2}U\left[1 - \left(\frac{y}{H}\right)^2\right], \tag{6.2-37}$$

$$U = \frac{H^2 \rho_L g \cos \beta}{3\mu_L}. \tag{6.2-38}$$

Notice that Eq. (6.2-37) is exactly the same as the result obtained in Example 6.2-1 for flow in a parallel-plate channel of half-width H.

Assuming that $K \to \infty$, as done in deriving Eq. (6.2-36), is the same as neglecting the shear stress exerted by the gas on the liquid. As discussed in Section 5.7, this is a common approximation at gas–liquid interfaces, and it has the effect of making the liquid velocity independent of the gas properties. It is readily confirmed that Eq. (6.2-36) is obtained also by solving

$$\frac{d^2 v_x}{dy^2} = \frac{1}{\mu_L}\frac{d\mathcal{P}}{dx} = -\frac{\rho_L g \cos \beta}{\mu_L} \tag{6.2-39}$$

with the boundary conditions

$$\frac{dv_x}{dy}(0) = 0, \qquad v_x(H) = 0. \tag{6.2-40}$$

This is clearly the preferred approach if one is interested only in the liquid, in that it avoids having to determine the velocity field in the gas.

DEEN, M. Analysis of Transport Phenomena. Oxford University Press, New York, 1998.

TABLE 1-2
Flux of Species i in Various Reference Frames and Units for a Mixture of n Components

Definitions of fluxes:

Reference velocity	Molar units	Mass units
$\mathbf{0}$	$\mathbf{N}_i$	$\mathbf{n}_i$
$\mathbf{v}$	$\mathbf{J}_i$	$\mathbf{j}_i$
$\mathbf{v}^{(M)}$	$\mathbf{J}_i^{(M)}$	$\mathbf{j}_i^{(M)}$

Flux relationships:

$\mathbf{N}_i = C_i\mathbf{v} + \mathbf{J}_i = C_i\mathbf{v}^{(M)} + \mathbf{J}_i^{(M)}$,	$\sum_{i=1}^{n} \mathbf{N}_i = C\mathbf{v}^{(M)}$,	$\sum_{i=1}^{n} \mathbf{J}_i^{(M)} = \mathbf{0}$
$\mathbf{n}_i = \rho_i\mathbf{v} + \mathbf{j}_i = \rho_i\mathbf{v}^{(M)} + \mathbf{j}_i^{(M)}$,	$\sum_{i=1}^{n} \mathbf{n}_i = \rho\mathbf{v}$,	$\sum_{i=1}^{n} \mathbf{j}_i = \mathbf{0}$

TABLE 1-3
Fick's Law for Binary Mixtures of A and B

Reference velocity	Mass units		Molar units	
$\mathbf{v}$	$\mathbf{j}_A = -\rho D_{AB}\nabla\omega_A$	(A)	$\mathbf{J}_A = -\frac{\rho D_{AB}}{M_A}\nabla\omega_A$	(B)
$\mathbf{v}^{(M)}$	$\mathbf{j}_A^{(M)} = -CM_A D_{AB}\nabla x_A$	(C)	$\mathbf{J}_A^{(M)} = -CD_{AB}\nabla x_A$	(D)

TABLE 2-3
Species Conservation Equations for a Binary or Pseudobinary Mixture in Rectangular, Cylindrical, and Spherical Coordinates[a]

Rectangular: $C_i = C_i(x, y, z, t)$

$$\frac{\partial C_i}{\partial t} + v_x\frac{\partial C_i}{\partial x} + v_y\frac{\partial C_i}{\partial y} + v_z\frac{\partial C_i}{\partial z} = D_i\left[\frac{\partial^2 C_i}{\partial x^2} + \frac{\partial^2 C_i}{\partial y^2} + \frac{\partial^2 C_i}{\partial z^2}\right] + R_{vi}$$

Cylindrical: $C_i = C_i(r, \theta, z, t)$

$$\frac{\partial C_i}{\partial t} + v_r\frac{\partial C_i}{\partial r} + \frac{v_\theta}{r}\frac{\partial C_i}{\partial \theta} + v_z\frac{\partial C_i}{\partial z} = D_i\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C_i}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 C_i}{\partial \theta^2} + \frac{\partial^2 C_i}{\partial z^2}\right] + R_{vi}$$

Spherical: $C_i = C_i(r, \theta, \phi, t)$

$$\frac{\partial C_i}{\partial t} + v_r\frac{\partial C_i}{\partial r} + \frac{v_\theta}{r}\frac{\partial C_i}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial C_i}{\partial \phi} = D_i\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial C_i}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial C_i}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 C_i}{\partial \phi^2}\right] + R_{vi}$$

[a] It is assumed that ρ and D_i are constant, where D_i is the binary or pseudobinary diffusivity.

DEEN, M. Analysis of Transport Phenomena. Oxford University Press, New York, 1998.

PQI-5776 Fenômenos de Transporte I
Lista de Exercícios 3

Em relação ao Exemplo 2.8-1 (Deen, 1998):

(1) Quais são as palavras-chave no enunciado do problema para se poder admitir que o perfil de temperatura é unidimensional $T = T(r)$?

(2) Deduzir a expressão para a constante C_2 da equação (2.8-6) a partir da condição de contorno dada por (2.8-2).

(3) Qual a temperatura na superfície externa do fio para $h \to \infty$? E para $h \to 0$? Justificar as respostas e explicar os seus significados físicos.

(4) Deduzir a equação (2.8-10).

(5) Dar o significado físico do número de Biot. Qual o significado para $Bi \to 0$?

(6) Para um fio de cobre exposto a um ambiente sem vento, qual dos perfis de temperatura da figura 2.5 representa melhor a situação ?

Em relação ao Exemplo 2.8-2 (Deen, 1998):

(7) A relação entre os fluxos molares dos componentes A e B depende: (a) só da cinética da reação ? (b) só da estequiometria da reação ? (c) só dos fenômenos de transporte ? (d) da combinação dos itens acima (dizer quais) ? Justificar a resposta.

(8) O transporte convectivo do A é provocado: (a) só pela velocidade do A ? (b) só pela velocidade do B ? (c) pelo fluxo dos dois ? Justificar a resposta.

(9) Determinar o fluxo mássico da mistura ao longo do filme de gás.

(10) Deduzir a equação (2.8-23).

(11) Deduzir a equação do perfil de concentração do A (C_A) em função da posição y para o caso de $m = 1$ e $n = 2$.

(12) Se a reação fosse reversível, qual parte do equacionamento do problema seria alterada ? E se a reação fosse irreversível instantânea ?

(13) Dar o significado físico do número de Damköhler. Quais os significados físicos para $Da \to 0$ e para $Da \to \infty$? Em qual das duas situações, a concentração do componente A é uniforme ao longo do filme de gás ?

Em relação ao Exemplo 2.8-3 (Deen, 1998):

(14) Quais são as palavras-chave para se poder admitir que a densidade mássica da mistura é constante em relação à posição y ?

(15) Justificar por que se pode aplicar a "lei" de Fick neste exemplo.

(16) Na equação (2.8-26), a difusividade "D_A" é "D_{AB}" ? Justificar a resposta.

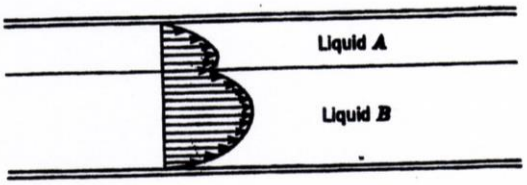
(17) Explicar o motivo de não aparecer o termo convectivo na equação (2.8-26).

Em relação ao Exemplo 6.2-3 (Deen, 1998):

(18) De acordo com a solução apresentada, os dois fluidos precisam ser newtonianos ? Justificar a resposta.

(19) O termo da pressão que aparece no balanço de quantidade de movimento é: pressão estática, pressão dinâmica, pressão de estagnação, pressão termodinâmica, presssão "modificada", pressão de impacto, pressão de Bernoulli, pressão piezométrica ?

(20) Esboçar os perfis de velocidade nos dois fluidos do exemplo 6.2-3 para os casos de: (a) o fluido 2 ser invíscido; (b) o fluido 1 ser invíscido; (c) os dois fluidos serem invíscidos. Justificar sucintamente as respostas.

(21) Responder a questão proposta por Bird; Stewart; Lightfoot, 1960:

Two immiscible liquids *A* and *B* are flowing in laminar flow between two parallel plates. Would there ever be the possibility that the velocity profiles would be of the following form? (Explain briefly the reasons for your answer.)