Escola Politécnica da Universidade de São Paulo
Departamento de Engenharia Mecatrônica e de Sistemas Mecânicos - PMR

Prova 1

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Aula 7

Equivalentes Discretos de Sistemas Contínuos

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PMR 3409 – Controle II

$F(s) \rightarrow$ CONTROLE
FILTRO
EQ. DIF.

$\xrightarrow{?}$ $F(z)$
MELHOR REPRESENTA

$G(s) \longrightarrow$

$s \rightarrow \frac{z-1}{Tz}$ (backward)

$s \rightarrow \frac{2}{T} \frac{z-1}{z+1}$ (tustin)

$$G(s) = \frac{1}{s^2+2s+1}$$

exemplos python

```python
import numpy as np
import cmath as cm
import control
from scipy import signal
import matplotlib.pyplot as plt
from control.matlab import *


num = [1]
den = [1,2,1]

G = tf(num,den)
Gdb_temp = signal.cont2discrete((num,den),1,method='backward_diff')

Gdt = c2d(G,1,method='tustin')
#Gdm = c2d(G,1,method='matched')
Gdb = tf(Gdb_temp[0][0],Gdb_temp[1],1)


plt.figure(1)
yc, tc = step(G)
ydt, tdt = step(Gdt)
#ydm, tdm = step(Gdm)
ydb, tdb = step(Gdb)

plt.plot(tc,yc)
plt.plot(tdt,ydt,'o')
#plt.plot(tdm,ydm,'x')
plt.plot(tdb,ydb,'+')
```

4) MATCHED = MAPEAMENTO POLOS E ZEROS

$$\frac{s+1}{s^2+2s+1}$$

2 ZEROS $\begin{pmatrix} -1 \\ \infty \end{pmatrix}$

$\hookrightarrow \begin{pmatrix} -1 \\ -1 \end{pmatrix}$ 2 POLOS

$\left(\frac{s}{\infty}+1\right)$

a) TODOS POLOS DE G(s) SÃO MAPEADOS

$\boxed{z = e^{sT}}$

se $s = -a \rightarrow z = e^{-aT}$

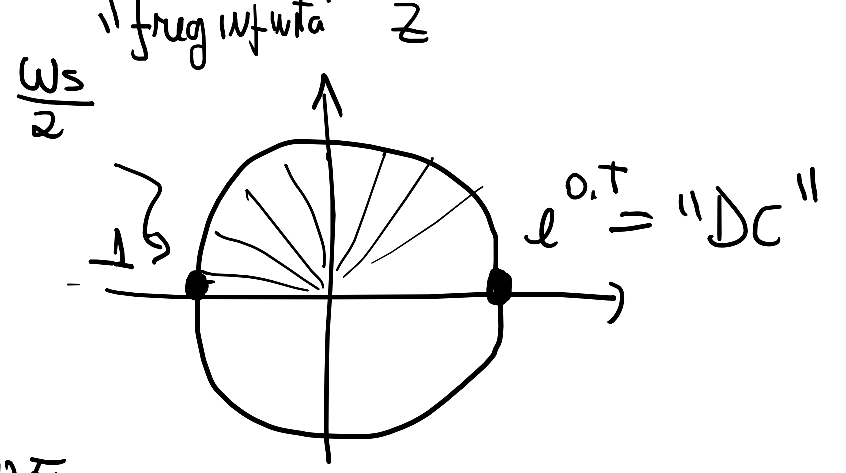
$s = -a + bj \rightarrow z = r e^{j\theta}$

$\hookrightarrow r = e^{-aT}$

$\theta = b.T$

b) TODOS OS ZEROS FINITOS MAPEADOS POR $z = e^{sT}$

c) ZEROS G(s) EM $s = \infty$ SÃO MAPEADOS EM $z = -1$



OBS) UM ZERO DE G(s) INFINITO É MAPEADO EM $z = \infty$, OU SEJA, G(z) TERÁ UM ZERO A MENOS QUE POLOS.

DESTA FORMA, EXPANSÃO $G(z)$ EM POTÊNCIAS DE z^{-1}, NÃO TERÁ TERMO CONSTANTE, OU SEJA, NÃO HÁ TRANSMISSÃO DIRETA ENTRADA-SAÍDA (ATRASO DE CÁLCULO)

d) GANHOS $G(s)\big|_{s=0} = G(z)\big|_{z=1}$

EX) $H(s) = \dfrac{a}{s+a}$

Polo $= -a \rightarrow z = e^{-aT}$

zero $= \infty \rightarrow \begin{cases} z = -1 & \text{OP1} \\ z = \infty & \text{(OBS: 1 ZERO MAPEADO EM } \infty\text{)} \quad \text{OP2} \end{cases}$

OP1)

$$H(z) = \frac{(z+1)}{(z-e^{-aT})} \mathbb{K}$$

$$H(s=0) = H(z=1)$$

$$1 = \frac{2}{1-e^{-aT}} \cdot \mathbb{K} \Rightarrow \mathbb{K} = \frac{1-e^{-aT}}{2}$$

$$H(z) = \mathbb{K} \cdot \frac{z+1}{z-e^{-aT}} \Rightarrow \mathbb{K} \cdot \frac{1+z^{-1}}{1-z^{-1}e^{-aT}} = \frac{y}{u}$$

$$\boxed{y[k] = \mathbb{K}u[k] - \mathbb{K}u[k-1] + e^{-aT}y[k-1]}$$

b) $H(z) = \dfrac{K}{z - e^{-aT}} \Rightarrow K = 1 - e^{-aT}$ ∞∞

$H(s=0) = H(z=+1)$

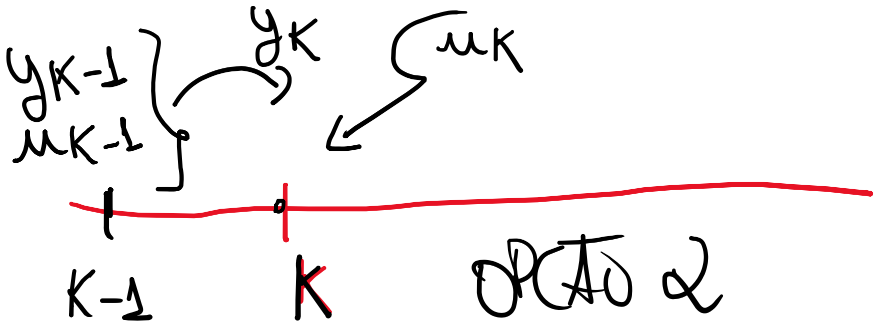
$$H(z) = \frac{1-e^{-aT}}{z-e^{-aT}} = \frac{K}{z-e^{-aT}} = \frac{K z^{-1}}{1 - z^{-1} e^{-aT}}$$

$$\rightarrow \boxed{y[k] = K u[k-1] + y[k-1]\, e^{-aT}}$$

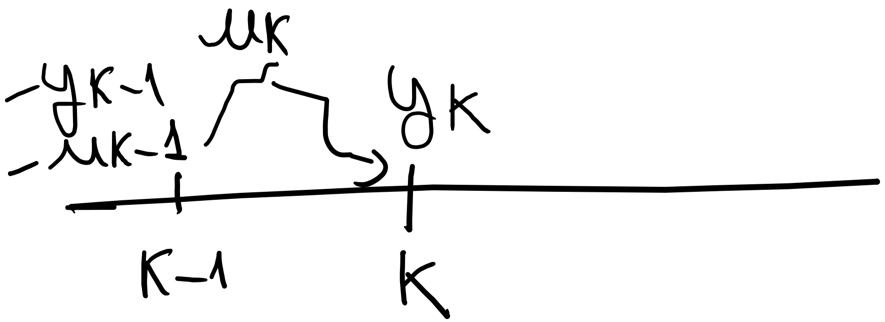
OPÇÃO 2) UM ~~ZERO~~ INFINITO MAPEADO EM ∞

$$\boxed{y[k] = K u[k] - K u[k-1] + e^{-aT} y[k-1]}$$

OPÇÃO 1) TODOS ZEROS INFINITOS MAPEADOS EM −1



OPÇÃO 2



Ex) $G(s) = \dfrac{1}{s^2+2s+1} \Rightarrow$ 2 ZEROS IMPRÓPRIOS
2 POLOS $(-1; -1)$

$G(z) = \dfrac{K.(z+1) \circ\circ\circ}{(z-e^{-T}).(z-e^{-T})}$ 1 ZERO IMPRÓPRIO

$T = 1 seg$

$e^{-T} = 0{,}367 \Rightarrow$ $G(z) = \dfrac{K.(z+1)}{(z-0{,}367)^2} = \dfrac{K(z+1)}{z^2-0{,}736z+0{,}134}$

$G(z=1) = G(s=0) \Rightarrow \dfrac{2K}{1-0{,}736+0{,}134} = 1 \Rightarrow K = 0{,}2 \Rightarrow$ $\boxed{G(z) = \dfrac{0{,}2(z+1)}{z^2-0{,}736z+0{,}134}}$

$$G(s) = \frac{1}{s+1} \rightsquigarrow G(z) = \frac{1-e^{-T}}{z-e^{-T}}$$

OPÇÃO 2

OBS: PARECE QUE O C2D PYTHON NÃO MAPEIA NENHUM ZERO IMPRÓPRIO

$$G(s) = \left(\frac{s}{100}+1\right)^2$$

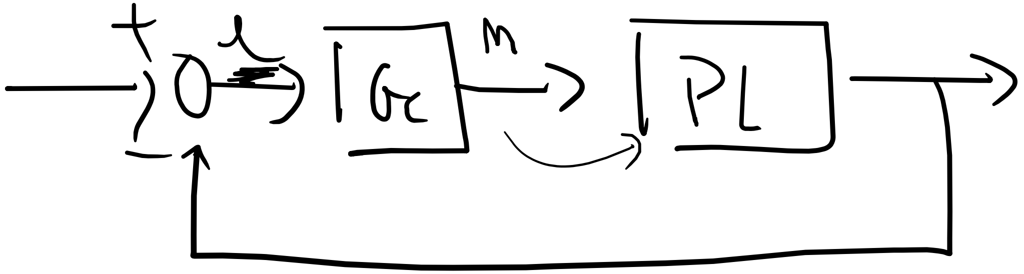
$$\frac{1}{s^2+2s+1} \quad 100$$

In [109]: Gdm
Out[109]:

$$\frac{0.3996}{z^2 - 0.7358z + 0.1353} \quad dt = 1$$

$$\Rightarrow \frac{0{,}39z^{-2}}{1-0{,}73z^{-1}+0{,}13z^{-2}} = \frac{Y}{U}$$

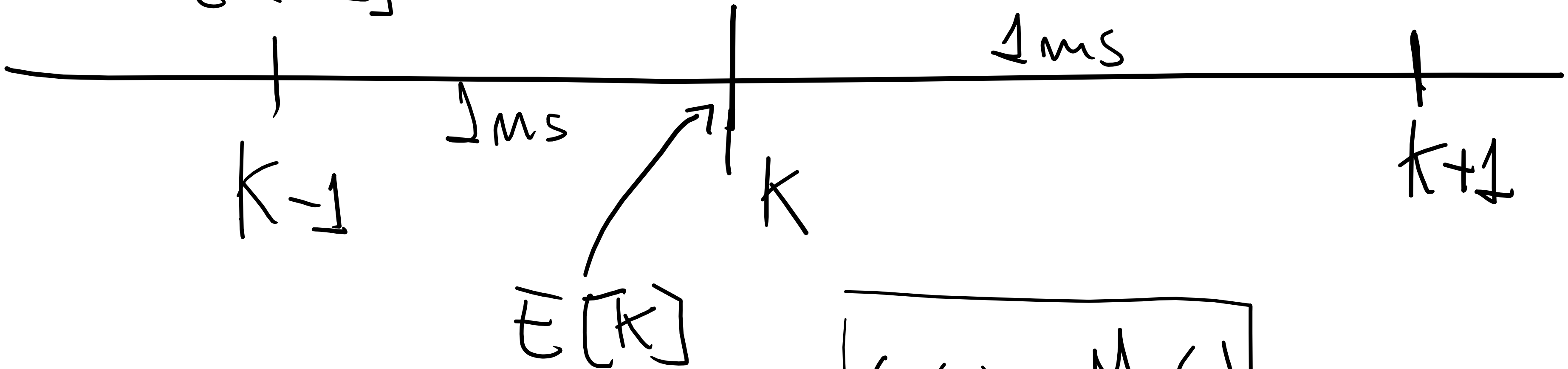
$$y[k] = 0{,}39 . u[k-2] + 0{,}73y[k-1] + 0{,}13y[k-2$$



$M[k-1]$

$E[k-1]$

$$\underline{M[k] = f\left(E[k-1], M[k-1], E[k]\right)}$$



$$\boxed{G_c(z) = \frac{M(z)}{E(z)}}$$

$$\frac{1}{s+1} \longrightarrow \text{backward} \quad s \to \frac{z-1}{tz}$$

$$\frac{1}{\frac{z-1}{z}+1} = \frac{z}{z-1+z} = \frac{z}{2z-1} =$$

$$\frac{1}{2-z^{-1}} = \frac{y}{u} = \frac{0{,}5}{1-0{,}5z^{-1}}$$

$$\boxed{y[k] = 0{,}5u[k] + 0{,}5y[k-1]}$$

TUSTIN

$$\frac{0.3333z + 0.3333}{z - 0.3333} \quad dt = 1$$

$$\frac{0{,}33+0{,}33z^{-1}}{1-0{,}33z^{-1}} = \frac{y}{u} \Rightarrow y[k] = 0{,}33u[k] + 0{,}33u[k-1] + 0{,}33y[k-1]$$