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Parameter determination of the Compressible Packing Model (CPM) for concrete application

Mokrane Bala [a],*, Rachid Zentar [a], Pascal Boustingorry [b]

[a] Institut Mines Telecom Lille-Douai, LGCgE-GCE, 764 bd Lahure, BP 10838, 59508 Douai, France
[b] CHRYSO France, 7 rue de l'Europe - Z.I, 45300 Sermaises du Loiret, France

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ABSTRACT

The packing density of granular materials is in the centre of interest of many industries. For concrete industry, the packing density of aggregates is the most representative physical parameter of the granular mixtures: it combines the grain size and the morphology of grains. Compressible Packing Model (CPM) accurately predicts packing density by involving three parameters: wall effect coefficient, loosening effect coefficient and compaction index. The identification of these parameters has been made on elementary granular classes (narrow particle size distributions) which can be time consuming and difficult to achieve. In this research work, we have elaborate a strategy to determine CPM parameters for wide particle size distribution of granular mixtures. This allows reducing the number of tests to optimize packing density of granular mixtures. The results of the modelling show a good agreement with the experiment. The work will be undertaken on two typologies of grains: crushed and rolled aggregates.

1. Introduction

In the field of civil engineering, concrete is an indispensable raw material which often offering the best technical and economic choice for the construction of buildings and infrastructures. Aggregates in concrete reach up to 80% of the total volume and come in different sizes and shapes. The remaining volume is occupied by a cement paste. Aggregates can arise from natural, artificial or recycled (industrial by-product and demolition source materials). The most important property in the granular mixture of concrete is its packing density. Packing density optimization for the granular mixtures allows not only minimizing the quantity of cement incorporated in the concrete but also enhances its performance and durability [1,2].

In conventional concrete mix design methods, granular mixtures are determined empirically, often from particle size distribution curves (ideal distribution curves), as for the works of Fuller & Thompson (1907), Andreasen & Andersen (1929), Faury (1958) and Dreux (1970) [3,4]. These methods make it possible to determine the ideal proportions of each grain size to approach the maximum packing density of the mix, but do not allow to predict accurately the packing density [3] and may require several series of experiments for the optimization of the granular mix [2,5].

With the emergence of modern concretes and special concretes in the 20th century (such as High Performance Concretes (HPC), Self-Compacting Concrete (SCC)…), concrete mix design by "ideal" particle size distribution curves proved to be difficult or unsuccessful [2,5]. For these concretes, several objectives are aimed at the same time: obtaining high workability, without risk of segregation and good mechanical properties. Therefore, a wide variety of distributions are possible and there is no ideal curve that suits all requirements [3].

In order to appear to these difficulties, several models have been developed to predict the packing density of a granular mixture. Modelling of packing density was developed first by highlighting the interaction effects between grains of different sizes, in particular by the work of Caquot (1937), which shows the major influence of the wall effect on granular mixtures [3]. It has prompted researchers to study granular classes in pairs. Inspired by Mooney viscosity model (1950), Stovall studied binary mixtures, with and without interaction. He has developed the linear packing model [6] for granular mixtures with multiple classes, taking into account both the wall effects and the loosening effect between the granular classes of different sizes. The loosening effect appears when a small grain is inserted in a dominant large grains population and the wall effect appears when some quantities of large grains isolated are immersed in fine grains agglomerate [7].

The linear packing model was refined to build the virtual packing model that predicted virtual packing density (i.e. an orderly packing of grains with the least voids), including mixtures of grains of the same size but of different shapes. The virtual packing density for a mixture of “n” granular fractions where the class (i) is dominant, is given by Eq. 1 [5]. It involves the volume proportions of each of the granular classes (y_i), their packing density when they are arranged separately (β_i),

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* Corresponding author.
E-mail address: mokrane.bala@imt-lille-douai.fr (M. Bala).

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Table 1
Origin and nature of the studied aggregates.

Type of aggregates	Origin	Granular class d/D	Density [t/m^3]
Crushed	Boulonnais	Sand 0/4	2.69
		Gravel 4/10	2.67
		Gravel 12/20	2.67
Rolled	Chevrières	Sand 0/4	2.55
		Gravel 4/10	2.43
	Decize	Gravel 11/22	2.54

the loosening effect coefficient (a_{ij}) given by Eq. 2 and the wall effect coefficient (b_{ji}) given by Eq. 3 [5]. "e" is the void ratio of the granular mixture defined as the ratio of void and solid fraction (given by Eq. 4, $\varnothing$ is the solid fraction).

$$\gamma = \gamma_i = \frac{\beta_i}{1-\sum_{j=1}^{i-1} y_j\left(1-\beta_i+\beta_i b_{ji}\left(1-\frac{1}{\beta_j}\right)\right)-\sum_{j=i+1}^{n} y_j\left(1-a_{ij}\frac{\beta_i}{\beta_j}\right)} \tag{1}$$

$$\mathrm{a_{ij}} = \beta_\mathrm{j}\left(\left|\frac{\partial \mathrm{e}}{\partial \mathrm{y_j}}\right|_{\mathrm{y_j}=0}+\frac{1}{\beta_\mathrm{i}}\right) \tag{2}$$

$$\mathrm{b_{ji}} = \frac{\frac{1}{\beta_\mathrm{j}}-1-\left|\frac{\partial \mathrm{e}}{\partial \mathrm{y_j}}\right|_{\mathrm{y_j}=1}}{\frac{1}{\beta_\mathrm{i}}-1} \tag{3}$$

$$e = \frac{1-\varnothing}{\varnothing} \tag{4}$$

A first attempt to predict the packing density by the Solid Suspension Model [8,9] was limited by two defects related to the notion of reference viscosity (considered as a description of the degree of compaction of the system) and the interaction functions that were not satisfactory in their mathematical form [10]. This compaction energy was introduced later in CPM model by the concept of the compaction index "K". This model makes it possible to predict the real packing density of a mixture of several granular classes from the knowledge of the compactness of each one-dimensional class and the energy of the setting up. The real packing density depends on the compaction energy. The real packing density "C" of a mixture of aggregates is connected to "K" by the expression given in Eq. 5. Moreover, simplified formulas (Eq. 6 and Eq. 7) of the granular interaction coefficients (a and b) was proposed by de Larrard [5] after calibration of the CPM on different series of experimental data.

$$K = \sum_{i=1}^{n} K_i = \sum_{i=1}^{n} \frac{y_i/\beta_i}{\frac{1}{C}-\frac{1}{\gamma_i}} \tag{5}$$

$$a_{i,j} = \sqrt{1-\left(1-\frac{d_j}{d_i}\right)^{1,02}} \tag{6}$$

Table 2
Experimental program of binary mixtures.

Type of aggregates	Granular class d/D	Binary mixtures	D_1/D_2 ratio
Crushed	Sand 0/4	SC 0/4 + GC 12/20	5
	Gravel 4/10	SC 0/4 + GC 4/10	2.5
	Gravel 12/20	GC 4/10 + GC 12/20	2
Rolled	Sand 0/4	SR 0/4 + GR 11/22	5
	Gravel 4/10	SR 0/4 + GR 4/10	2.5
	Gravel 11/22	GR 4/10 + GR 11/22	2

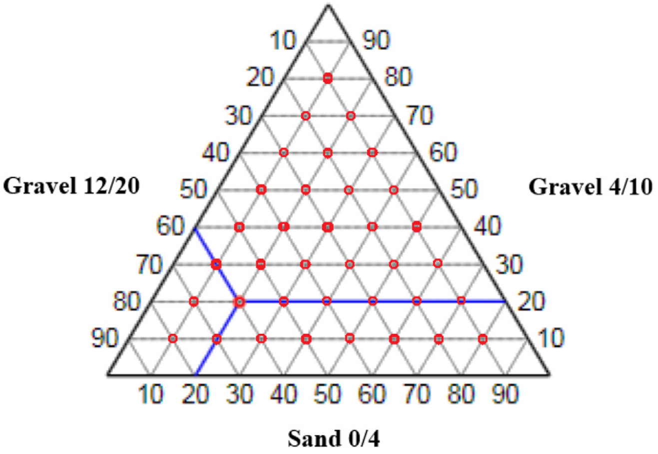


Fig. 1. Experimental program of ternary mixtures (Crushed aggregates).

$$b_{j,i} = 1-\left(1-\frac{d_i}{d_j}\right)^{1,5} \tag{7}$$

In the framework of the CPM, the prediction of the packing density is therefore possible after determination of the coefficients a, b (by the calculation of the void index at the limits of coarse grains dominant and fine grains dominant) and K. These parameters are calibrated on binary mixtures of aggregates of different natures (crushed and rolled) [5,11]. The error between measured and calculated packing densities after calibration does not exceed 0.77% for rolled aggregates and 1.71% for crushed aggregates [5].

In order to improve the prediction of the packing density, several models and calculation approaches have been proposed [12–18]. Some authors introduced new interaction parameters [15,17]. Roquier [17] introduced the interference effect that occurs when coarse grains, in a growing number, become too close to each other by trapping some fine grains in small spaces between them [3,17]. However, despite these various attempts to improve the packing density prediction, the CPM remains among the most accurate models and the most simplest to apply [19,20]. Moutassem [19] had compared 9 packing density models and he found that the CPM predict correctly the packing density of granular mixtures used in concrete.

Other research works has focused on the CPM parameters in order to propose optimized values for a_{ij}, b_{ji} and K [11,20]. The work of Lecomte [11] has slightly improved the calculation of the interaction coefficients through new simplified formulas of calculation. Indeed, the simplified formulas of the interaction coefficients are probably valid only for one type of aggregates which justifies the differences between the formulas proposed by de Larrard [2] and by Lecomte [11]. In fact, for the CPM, the functions of a_{ij} and b_{ji} have been calibrated on elementary granular classes (d and D are the minimum and maximum grain sizes respectively) while respecting on the one hand a ratio $d_i/D_i > 0.1$ (so that the elementary granular classes are the more unimodal possible) [2,5] and on the other hand a ratio $d_i/d_j < 4$ [2,5]. In the case of granular mixtures optimization for concrete, this work is long and expensive to achieve because it requires a long process of material preparation and sieving elementary granular classes.

In the present paper we will calibrate the CPM parameters in the case of large granular classes with $d_i/D_i < 0.1$ (especially for sand) and for some ratios $d_j/d_i > 4$. This study does not require any sieving work to obtain elementary granular classes that must have narrower particle size distributions. Our goal is to save time and materials in packing density studies of granular mixtures for concrete without affecting the accuracy of the model. First, we will apply the CPM as developed by de Larrard in order to qualify the prediction of the model and determine the most influential parameters. After that, we will optimize the

Table 3
Characteristics of the aggregates of the study.

	Crushed aggregates			Rolled aggregates		
d/D	SC 0/4 (Sand)	GC 4/10 (Gravel)	GC 12/20 (Gravel)	SR 0/4 (Sand)	GR 4/10 (Gravel)	GR 11/22 (Gravel)
Fine content <63 μm [%]	6.2	1.0	0.8	1.3	1.7	0.1
d_{50} [mm]	0.7	7.2	15.2	0.4	7.0	16.0
Fineness modulus (for sands)	3.2	–	–	3.4	–	–
Absolute density [g/cm^3]	2.726	2.735	2.711	2.626	2.648	2.626
Water absorption [%]	0.43	0.59	0.49	1.18	3.06	1.79
Shape and roughness	Angular aggregates, flat and rough surface			Rounded shape, flat or elongated aggregates, smooth surface		

parameters of the CPM for wide particle size distributions of granular mixtures of crushed and rolled aggregates. The coefficients a, b and K will be calibrated on binary mixtures. Finally, we will evaluate the accuracy of the model applied to large granular classes of binary and ternary mixtures compared to previous studies.

2. Materials and methods

In Table 1 are listed the aggregates of the present study and few characteristics. The size, shape and roughness of the grains are the three main parameters that affect the packing density [3,5,21] that has influenced our choice. Rolled aggregates come from "Chevrières" quarry ("Hauts-de-France" region) and "Decize" quarry ("Bourgogne-Franche-Comté" region). The crushed aggregates come from the quarries of "Boulonnais" ("Nord Pas-de-Calais" region). The granular fractions adopted for this study range from sand (0/4 mm) to gravel (4/10 and 10/20 mm). The absolute density of the aggregates was measured according to the European standard NF EN 1097–6 [22] and the particle size analysis was carried out according to the European standard NF EN 933–1 [23].

In order to measure the packing density of the mono-granular classes, the LPC procedure No. 61 was followed [24]. After weighing 7 kg of one mono-granular material, the sample is placed in a cylindrical mould in three equivalent layers. Each layer is subjected to 20 shocks in the shaking table before putting the next layer. The sample is then subjected to 40 shocks under a pressure of 10 kPa (equivalent weight of 20 kg can be placed above the sample). The energy transmitted by the shocks allows the granular material to be into a more dense

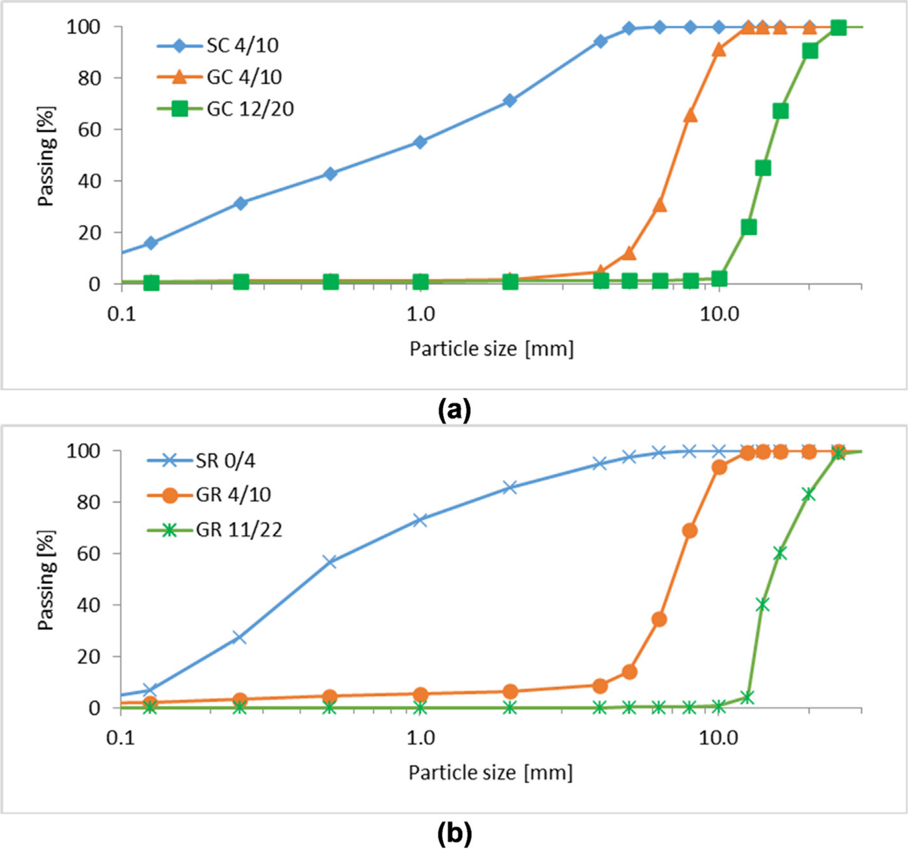
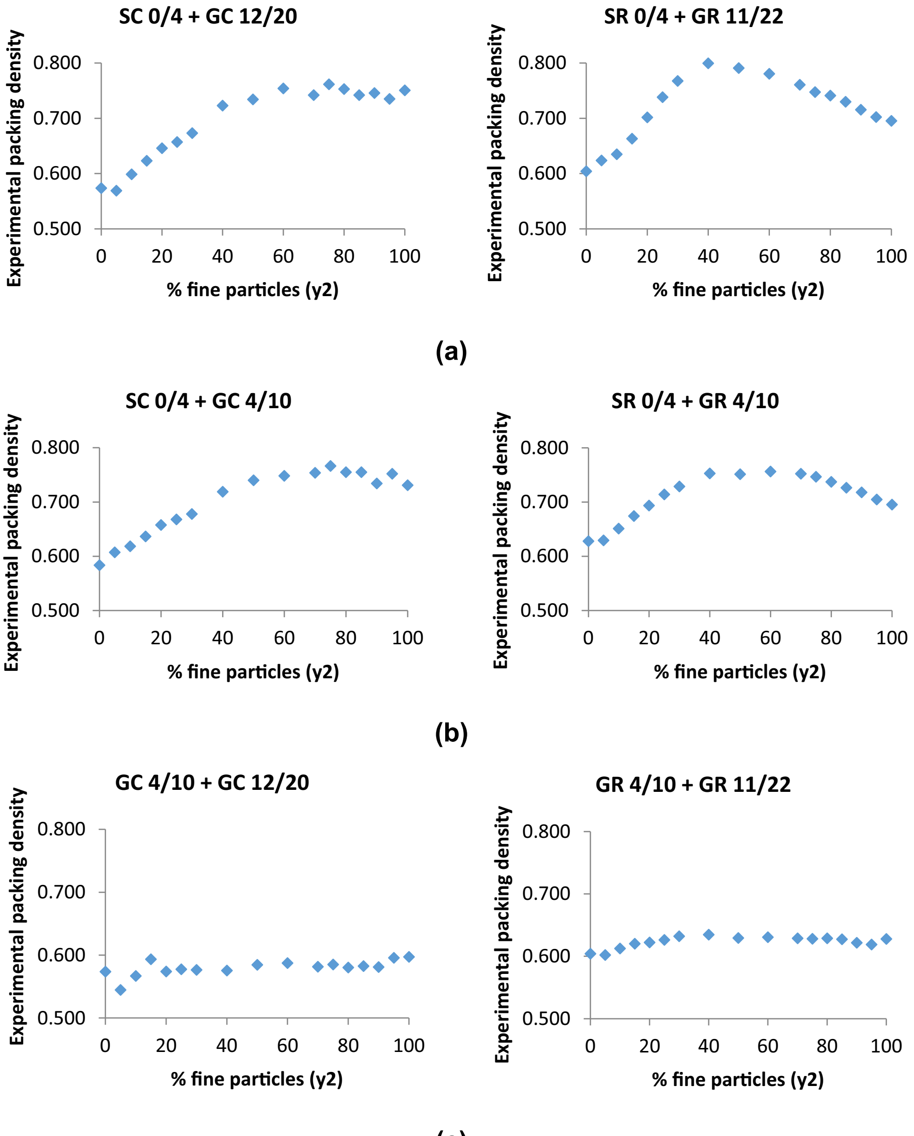


Fig. 2. Particle size distribution of crushed aggregates (a) and rolled aggregates (b).



(a)

(b)

(c)

Fig. 3. Packing density measured by the compaction table of binary mixtures of crushed aggregates (left) and rolled aggregates (right) for: (a) $D_1/D_2 = 5$, (b) $D_1/D_2 = 2.5$ and (c) $D_1/D_2 = 2$.

configuration, which improves its packing density. The compaction index (K) of this process was calibrated after several tests. Using Eq. 5, K = 9 was found [24].

The packing densities were measured on binary granular mixtures of crushed and rolled aggregates to determine the interaction coefficients for each type of aggregates [2,11]. Different combinations between sand and gravels were analysed. As shown in Table 2, considering the maximum nominal diameter of each class, the effect of the size ratio D_1/D_2 and the grain shape on the packing density was experienced. Otherwise, the variation of the proportions of the smaller granular class in the binary mixture equal to 5% in the extreme parts of the curve of packing density and to 10% in the centre of the curve. This is in order to measure the impact of the granular interaction effects that have defined previously (wall effect and loosening effect coefficients). Finally, packing density of ternary granular mixtures was measured for crushed and rolled aggregates. This is in order to evaluate the efficiency of the interaction parameters determined on the binary mixtures. The measurements were realized by increment of 10%. The different combinations of the experimental program on ternary mixtures are shown in Fig. 1.

3. Results and discussions

The main properties of the aggregates used in this study are listed in Table 3. Otherwise, the particle size distribution of the different materials are shown in Fig. 2. Particle size analysis showed a high fines content in crushed sand (5% of grains are smaller than 63 μm and 16% smaller than 125 μm).

Packing densities measurement tests were realized using the compaction table according to LPC procedure No. 61 [24] on binary and ternary mixtures of crushed and rolled aggregates. The different materials are used in their raw state without any prior sieving to remove the fine fraction. For each test, two different measurements of compactness were made. In each case, the average packing density and the variances are calculated. It has been observed through the experimental program that the repeatability of the tests is ensured [25] (maximum standard deviation recorded is 0.017 for crushed aggregates and 0.008 for rolled aggregates). The use of the aggregates in their raw state without any prior sieving (fine particles content up to 7%) did not influence the packing density measurements by compaction table [25].

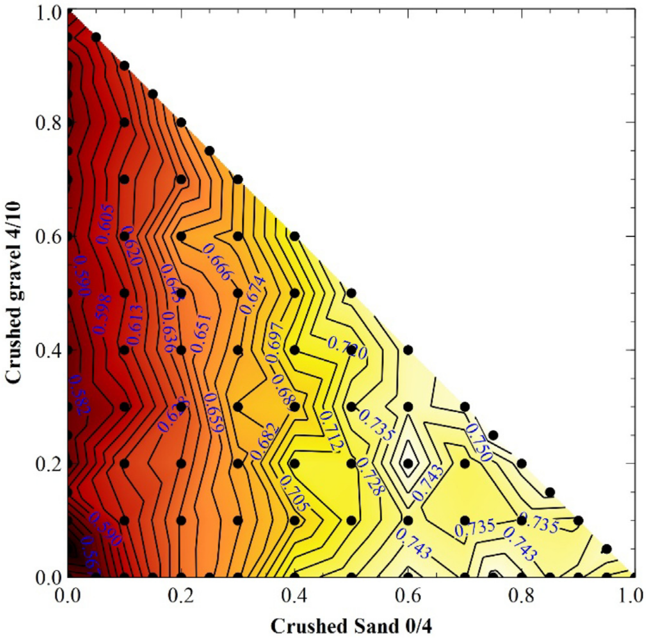


Fig. 4. Packing density measured by the compaction table of ternary mixtures of crushed aggregates.

The evolution curves of the packing density measured by the compaction table as a function of the proportion of the addition of fine material are shown in Fig. 3. From the curves in Fig. 3, it is observed a rapid increase of the packing density when the sizes ratio of the mixed materials is large ($D_1/D_2 = 5$). The packing density curves exhibits an optimum before decreasing when wall effects occur in the area where the small grains material becomes dominant. Even for smaller ratio ($D_1/D_2 = 2.5$), the evolution of the packing density still important (Fig. 3-b). For $D_1/D_2 = 2$, the packing density curve evolution is almost nil (Fig. 3-c). These results show that the packing density optimum of the binary mixtures increases when D_1/D_2 ratio increases from 2 up to 5. This is in agreement with the results found by de Larrard [2,5] and McGeary [26]. Finally, the size ratio $D_1/D_2 = 2$ seems to define the limit for materials which are in total interaction [2,5].

Despite their low fines content, the rolled aggregates (Fig. 3. right) reaches the highest packing density in comparison to crushed aggregates. The packing of rolled aggregates is composed of grains of various sizes (wide particle size distribution) that their shape is close to the sphere. This packing approaches the Apollonian packing (packing of circles) which gives the highest packing density [27,28]. The high fines content (particle content <63 μm) in the crushed aggregates (Fig. 3. left) was not able to counterbalance the shape effects in terms of packing density (Fig. 3a and Fig. 3b).

From these results, we deduce that the maximum packing density that can be achieved on binary mixtures is influenced by the

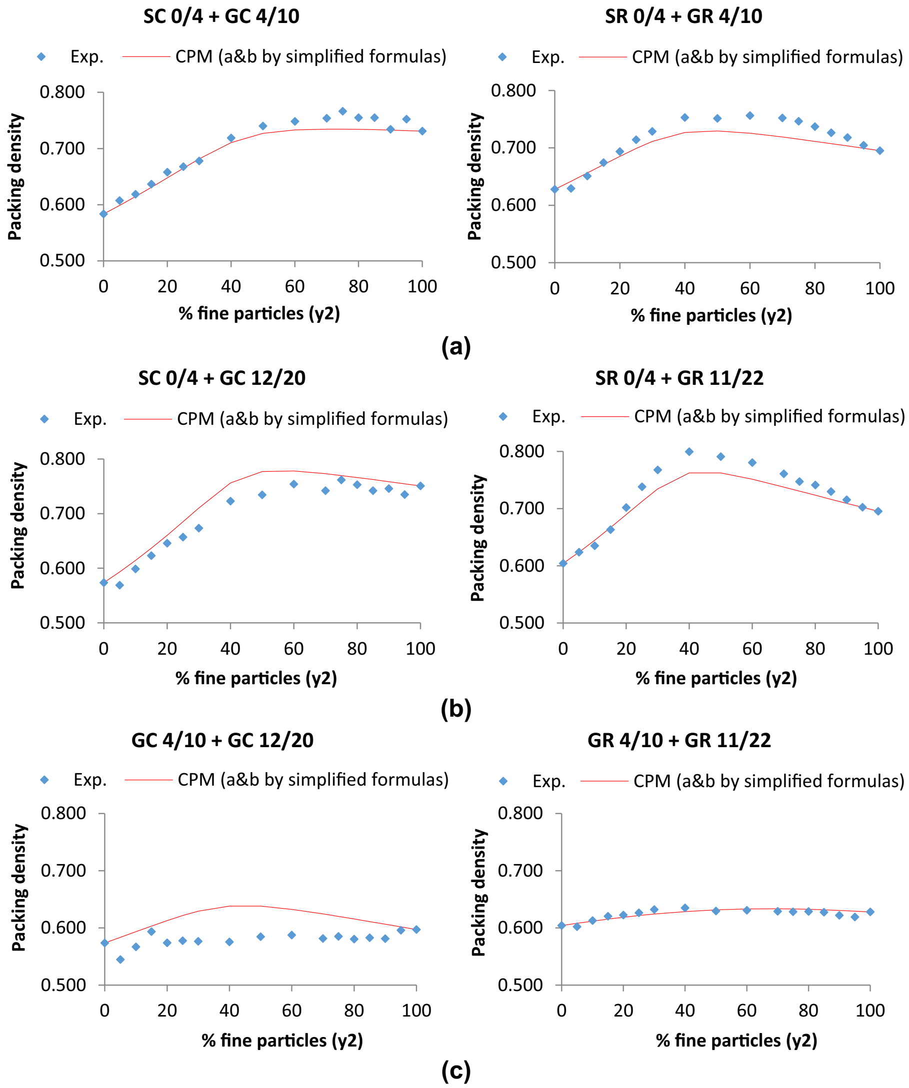


Fig. 5. Measured and modelled packing density of crushed (left) and rolled (right) binary Mixtures via simplified formulas of interaction coefficients for: (a) $D_1/D_2 = 5$, (b) $D_1/D_2 = 2.5$ and (c) $D_1/D_2 = 2$.

Table 4
Parameters used for modelling by simplified formulas of a and b (K = 9).

Type of aggregates	D_1	D_2	Mixtures	D_2/D_1	a & b by simplified formulas			
					a	b	K	Average error
Crushed	10	4	SC 0/4 + GC 4/10	0.40	0.64	0.54	9	1.5%
	20	4	SC 0/4 + GC 10/20	0.20	0.45	0.28		2.8%
	20	10	GC 4/10 + GC 10/20	0.50	0.71	0.65		5.2%
				Average error for crushed aggregates				3.2%
Rolled	10	4	SR + GR 4/10	0.40	0.64	0.54	9	2.2%
	22	4	SR + GR 10/20	0.18	0.43	0.26		2.1%
	22	10	GR 4/10 + GR 10/20	0.45	0.68	0.60		0.7%
				Average error for rolled aggregates				1.7%

combination of the following parameters (regardless of the compaction index): the packing density of the elementary granular classes, the shape and roughness of the grains as well as the fines content and the size ratio. The packing density of binary mixtures can be increased by improving the packing density of the elementary classes as found by de Larrad [2,5]. Similarly, some authors have shown that the packing density increases when the size ratio of elementary classes increases [2,17,29]. The effect of the presence of ultra-fine particles (powders) can also be related to the large size ratio between the powders and coarse grains.

For the ternary mixtures (of crushed aggregates) studied in this research work, the results are shown in Fig. 4. The packing density measured decreases in the area of low sand content (dark area) and increases with the introduction of sand (white area). The presence of intermediate gravel (G 4/10) with the coarser gravel (G 10/20) disturbs the granular mixture because of the observed interaction between these materials as seen in Fig. 3c.

The evolution curves of the experimental packing density for binary and ternary mixtures allow to proceed to the modelling of these packing densities by the CPM in which we will analyse the different parameters of the model which are: the coefficients of granular interactions (a_{ij} and b_{ji}) and the compaction index (K). This work will be undertaken for large granular classes.

In order to determine the packing densities by the CPM, the virtual packing densities are calculated using Eq. 1 where the coefficient of loosening effect (a_{ij}) and wall effect (b_{ji}) have to be determined. In the

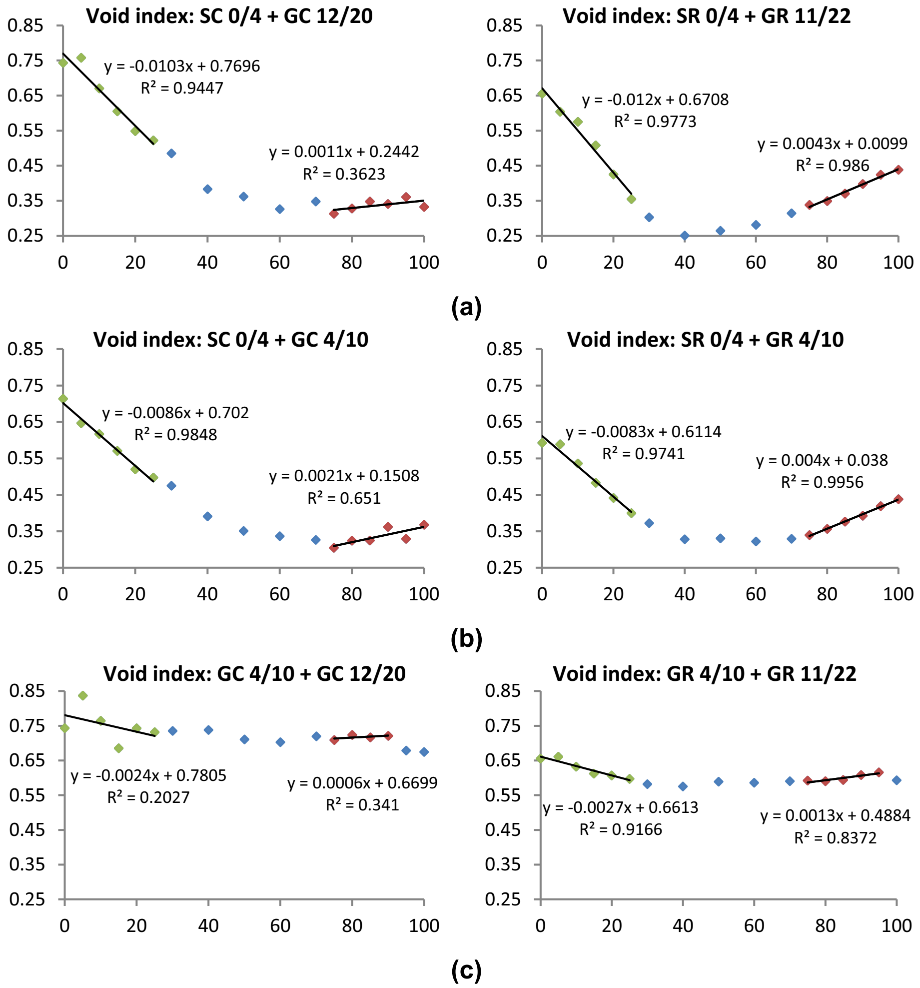


Fig. 6. Determination of the granular interaction coefficients a and b for crushed (left) and rolled (right) aggregates for: (a) $D_1/D_2 = 5$, (b) $D_1/D_2 = 2.5$ and (c) $D_1/D_2 = 2$.

Table 5
Parameters of the modelling by calibrated formulas of a and b (K = 9).

Type of aggregates	D_1	D_2	Mixtures	D_2/D_1	a & b after calibration			
					a	b	K	Average error
Crushed	10	4	SC 0/4 + GC 4/10	0.40	0.62	0.22	9	1.1%
	20	4	SC 0/4 + GC 10/20	0.20	0.54	0.30		2.1%
	20	10	GC 4/10 + GC 10/20	0.50	0.90	0.83		2.9%
				Average error for crushed aggregates				2.0%
Rolled	10	4	SR + GR 4/10	0.40	0.53	0.06	9	1.2%
	22	4	SR + GR 10/20	0.18	0.32	0.01		0.6%
	22	10	GR 4/10 + GR 10/20	0.45	0.87	0.71		1.4%
				Average error for rolled aggregates				1.1%

first method, these parameters are calculated using the simplified formulas (Eq. 6 and Eq. (7)) proposed by de Larard [5]. The real packing density of the mixtures is subsequently calculated through the compaction index which is taken equal to 9 [24]. In the second method and for comparison, the parameters a_{ij} and b_{ji} are determined on the experimental curves and the compaction index value is maintained equal to 9 as in the first method.

The results in terms of experimental data in comparison to the model prediction following the first method are shown in Fig. 5.

In this case the CPM allows to predict experimental packing densities with an average error of 3.2% for crushed aggregates and 1.7% for rolled aggregates. We note through the curves shown in Fig. 5 that the largest differences are found in the area of optimal packing density. In this zone, the packing density is overestimated by the CPM for crushed aggregate mixtures and underestimated in the case of rolled aggregate mixtures. This leads to say that the efficiency of the compaction mode by the compaction table is influenced not only by the particle size as found by Sadok et al. [16,30], but also by the shape of the grains. The parameters a, b and K used in this first modelling are given in Table 4.

In the previous studies performed by de Larrard, an average error of 1.71% for crushed aggregates and 0.77% for rolled aggregates is found [2,5].

In the second method, the parameters of the CPM are determined from experimental data. According to the basic formulas of interaction coefficients (Eq. (2) and Eq. (3)), it could be seen that the slope of the experimental data in terms of void ratio (given by Eq. (4)) versus the percentage of fine material at the origin of the curve and at the extremity of the curve are respectively proportional to the parameters a_{ij} and b_{ji}. These latter are calculated using Eq. (6) and Eq. (7) after calculating β_1, β_2 and β_3 by Eq. (5) (K is taken equal to 9) and evaluating graphically the slope of the curve evolution of the void ratio as a function of the percentage of fine grains. The parameter determination for binary mixtures is highlighted in Fig. 6.

As it could be seen in Fig. 6, due to the discrepancy of the data, for some curves it is difficult to define the slope at the origin or at the extremity of the curve. In this order, the slope is evaluated on 25% of the curve (at the origin and at the extremity) in area considered as linear part. The values of the coefficients a_{ij} and b_{ji} determined (given in Table 5) were compared to those obtained by de Larrard [2,5] and Lecomte [11] as shown on Fig. 7. The results of the modelling are presented in Fig. 8.

As shown in Fig. 7, the coefficients a_{ij} and b_{ji} obtained on large granular classes follow the same evolution as those obtained by de Larrard [2,5] and Lecomte [11]. They increase when the ratio D_2/D_1 increases. This shows that the granular interactions are stronger when the granular classes are of the same size (until total interaction as shown in Fig. 3-c). However, Fig. 7 shows some differences between the interaction coefficients determined experimentally and the functions of the simplified formulas proposed by de Larrard or Lecompte. This allows to say that there is no unique function which makes it possible to calculate the interaction coefficients accurately for different type of aggregates and without causing errors in the prediction of the packing density. Moreover, when we analyse in more details the obtained results from our experimental data we can point out that the values of a_{ij} measured on rolled aggregates are systematically lower than those measured on crushed aggregates. This result could be observed for most data published by de Larard and is in line with the

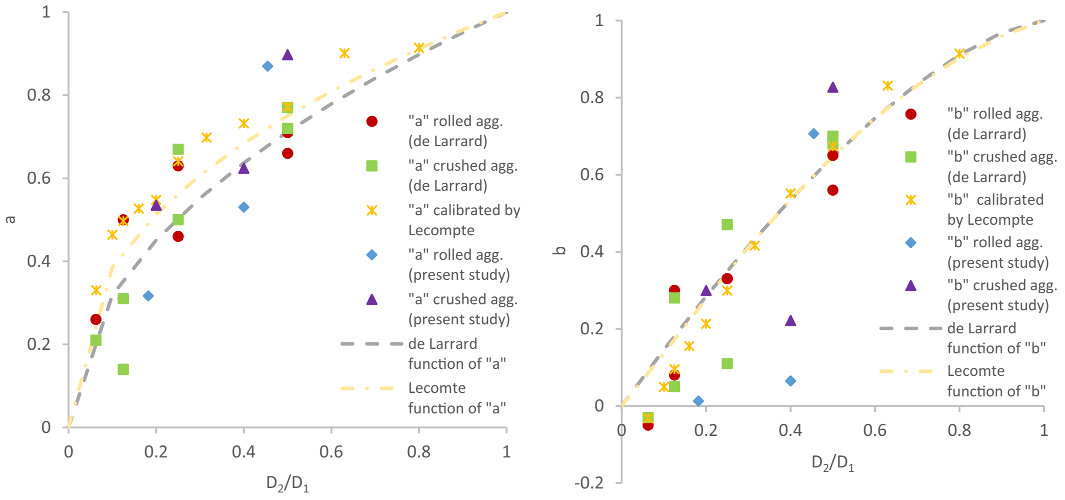


Fig. 7. Interaction coefficients vs. size ratio for crushed and rolled aggregates (comparison between experimental data of the present study and data from the literature): loosening effect coefficient "a" (left) and wall effect coefficient "b" (right).

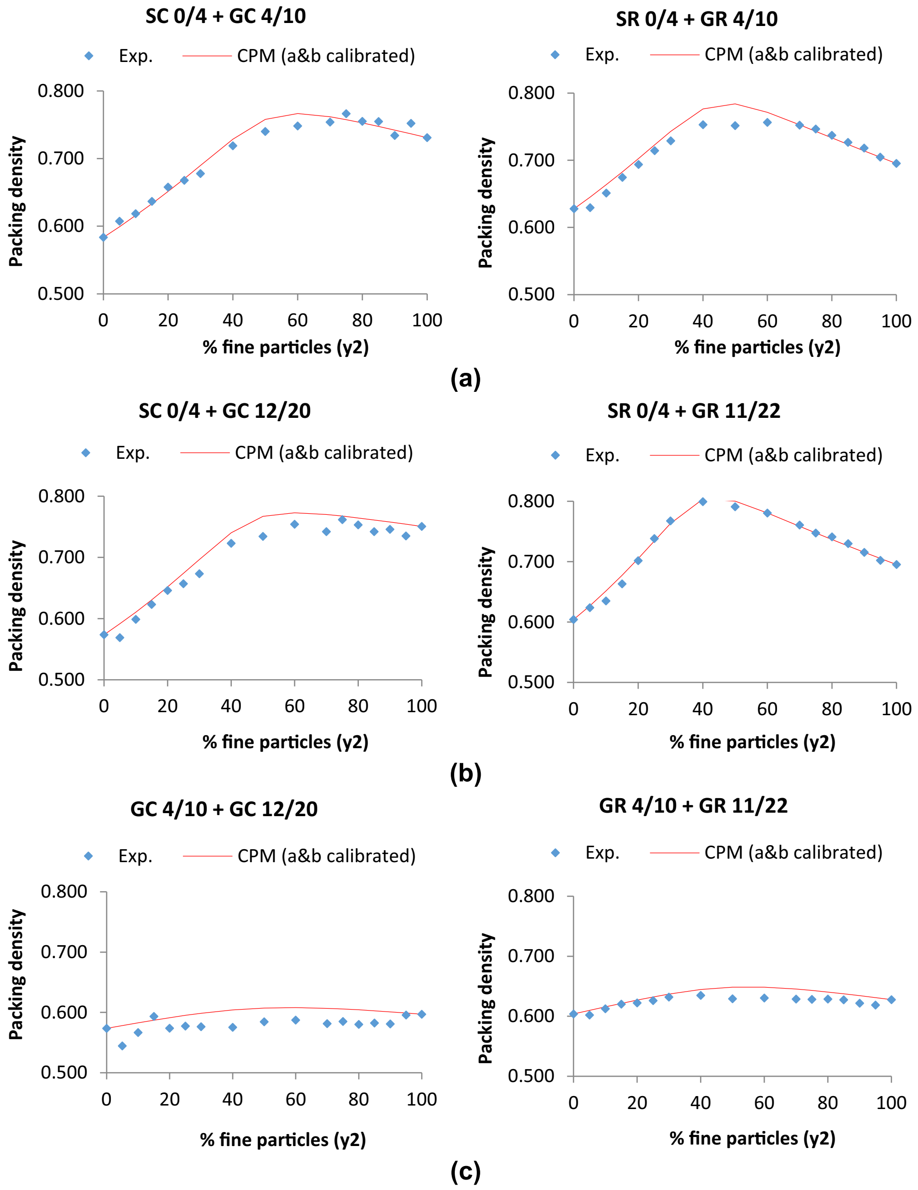


Fig. 8. Measured and modelled packing density of crushed (left) and rolled (right) binary mixtures after calibration of interaction coefficients for: (a) $D_1/D_2 = 5$, (b) $D_1/D_2 = 2.5$ and (c) $D_1/D_2 = 2$.

published paper of Roquier [18]. In this later, the estimates of the loosening effect parameter calculated from two different models (4- parameter CPM as 3PPM) show clearly that the values calculated on rolled aggregates are lower than those calculated for crushed aggregates. It is also interesting to note that for small diameter ratios (D2/D1), the loosening effect parameter evolves very quickly with increasing the diameter ratios.

For the wall effects parameter b_{ji}, in terms of aggregates shape effects, the same conclusions than those addressed for the loosening effect parameter could be drawn. The measured parameters values on rolled

Table 6
Parameters of the modelling after calibration of a, b and K.

Type of aggregates	D_1	D_2	Mixtures	D_2/D_1	a, b and K after calibration			
					a	b	K	Average error
Crushed	10	4	SC 0/4 + GC 4/10	0.40	0.62	0.22	5.5	0.8%
	20	4	SC 0/4 + GC 10/20	0.20	0.54	0.30	3.2	1.1%
	20	10	GC 4/10 + GC 10/20	0.50	0.90	0.83	0.3	1.1%
				Average error for crushed aggregates				1.0%
Rolled	10	4	SR + GR 4/10	0.40	0.53	0.06	4.8	0.8%
	22	4	SR + GR 10/20	0.18	0.32	0.01	8.2	0.6%
	22	10	GR 4/10 + GR 10/20	0.45	0.87	0.71	2.0	0.7%
				Average error for rolled aggregates				0.7%

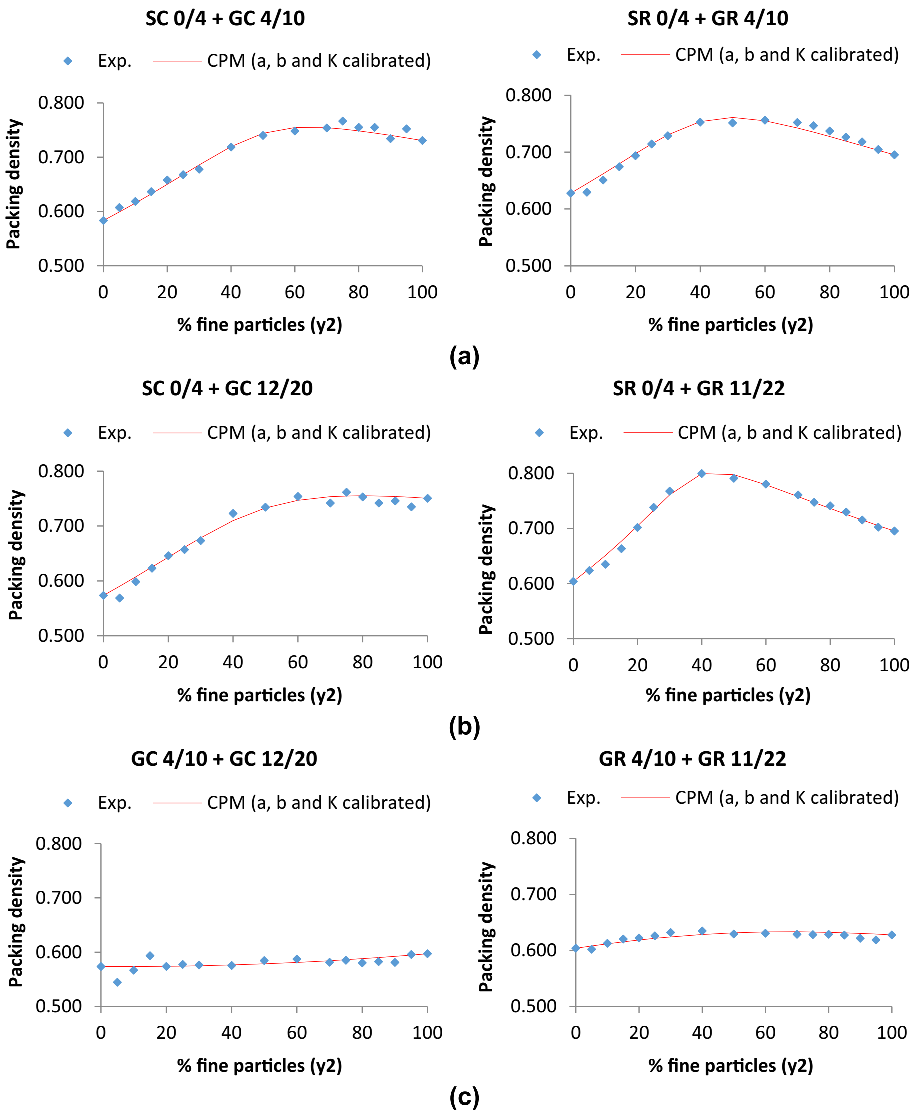


Fig. 9. Measured and modelled packing density of crushed (left) and rolled (right) binary mixtures after calibration of a, b and K for: (a) $D_1/D_2 = 5$, (b) $D_1/D_2 = 2.5$ and (c) $D_1/D_2 = 2$.

aggregates are systematically lower than those measured on crushed aggregates. This is also in line with most published data from de Larrad and other researchers.

The comparisons of absolute values of the wall effect parameter seems to be systematically lower that the values of the loosening effect parameter in the whole range of the diameter ratios.

Finally the comparison of the measured values for both parameters with the calculated values using the proposed relationships by de Larrard [2,5] and Lecomte [11] shows that a big differences could be induced.

Comparisons between the predictions using CPM with identified a_{ij} and b_{ji} on experimental data and experimental results (Fig. 8) show better agreement than in the first method (modelling through simplified formulas of a_{ij} and b_{ji}). The difference between the model prediction and the experimental data decreases to 2.0% for crushed aggregates and 1.1% for rolled aggregates. These errors are similar to those found in the work by de Larrard [2,5]. From these results, we can deduce that identification on elementary granular classes obtained by sieving is unnecessary to achieve the best performances of the CPM. However, it seems necessary to define interaction coefficients on experimental data.

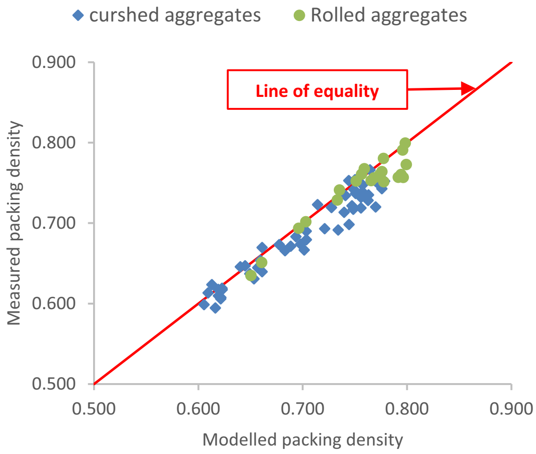


Fig. 10. Modelled vs. Measured packing density of ternary mixtures after calibration of a and b (K = 9).

In order to assess the effect of the compaction index (K) on the prediction of the CPM, in the last section of this study a back calculation of K on experimental data is undertaken. In this procedure, the difference between the model prediction and the experimental data is decreased by changing the value of K. The parameters a_{ij} and b_{ji} are kept as defined in the second method. The obtained results in terms of compaction index and errors between experimental data and prediction of the model is summarised in Table 6 and Fig. 9.

The compaction index obtained after calibration is lower than 9 (especially for crushed aggregates). Different values of the compaction index are obtained (given in Table 6) depending on the granular mixture. The compaction index depends to the experimental process, other values of K was recorded with others experimental processes [31,32]. But for compaction mode by shaking table, K must be constant because the energy used was constant [7]. We conclude that for crushed aggregates the compaction by the shaking table is not effective [25,33]. This is due to their high fines content [5,30]. With regard to the rolled aggregates, the compaction efficiency was better since we could reach a compaction index close to 9. However, the difference is not very significant in terms of packing density prediction, from which we retain the compaction index K = 9 for modelling of ternary mixtures.

In the case of ternary mixtures, the prediction of the experimental data is undertaken using CPM with a_{ij} and b_{ji} as defined in method 2 on binary mixtures. The compaction index, following the observed results on the binary mixtures and the results of several authors [2,3,30] the result is value is fixed to 9. To ease the comparisons, the predicted results versus experiments are shown on Fig. 10.

The errors observed between the model and the experiments for ternary mixtures is about 2.4% for crushed aggregates and 1.6% for rolled aggregates. From the results shown on Fig. 10, we note that in the majority of ternary mixtures of crushed and rolled aggregates, CPM overestimates the packing density.

4. Conclusion and perspectives

In this paper, the prediction of packing density of binary and ternary mixtures of crushed and rolled aggregates was investigated. From this study, we can draw the following conclusions:

For packing density measurements at the compaction table made with materials including the fine fraction (<63 μm), the repeatability of the tests is ensured (maximum standard deviation of 0.015 between two tests). The segregation effects of fine particles can be neglected for crushed and rolled aggregates with fine particles (<63 μm) content up to 7%.

The modelling of the packing density by the CPM have shown that the use of the granular interaction coefficients determined by the simplified formulas is insufficient to have a good prediction.

The back calculation of the compaction index (K) may vary from one mixture to another (especially for crushed aggregates).

It has been demonstrated in this study that the CPM can be applied to large granular classes without creating elementary subclasses. This save time and materials in packing density studies, especially for concrete mix design.

The identification of the model coefficients following the procedure suggested improved the prediction of the CPM.

The experimental program and the modelization realized allow to improve the precision of the CPM in the determination of the packing density of granular mixtures for concrete. In the perspective of this study, the packing density will be exploited to go back to the yield stress and the compressive strength of concrete following the approaches of Chateau et al. [34] and de Larrard [2,35] respectevly by combining the results of this study with the study of the properties of the cement paste.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References

[1] P.J. Andersen, Particle packing and concrete properties, Mater. Sci. Concr. II (1995) 111–146.
[2] F. de Larrard, Concrete Mixture Proportioning: A Scientific Approach, CRC Press, 1999.
[3] G. Roquier, Etude de la compacité optimale des mélanges granulaires binaires: classe granulaire dominante, effet de paroi, effet de desserrement, PhD Thesis, Paris Est 2016.
[4] S.A. Fennis, J.C. Walraven, Using particle packing technology for sustainable concrete mixture design, Heron 57 (2012) (2012) 2.
[5] F. de Larrard, Structures granulaires et formulation des bétons, Etudes Rech. Lab. Ponts Chaussées, OA 34, 414, 2000.
[6] T. Stovall, F. de Larrard, and M. Buil, Linear packing density model of grain mixtures, Powder Technol., vol. 48, no. 1, pp. 1–12, Sep. 1986.
[7] M. Miraoui, R. Zentar, N.-E. Abriak, Road material basis in dredged sediment and basic oxygen furnace steel slag, Constr. Build. Mater. 30 (2012) 309–319.
[8] T. Sedran, F. de Larrard, D. Angot, Prévision de la compacité des mélanges granulaires par le modèle de suspension solide -I- Fondements théoriques et étalonnage du modèle, 194, Bull. Liaison Lab. Ponts Chaussées, 1994.
[9] T. Sedran, F. de Larrard, D. Angot, Prévision de la compacité des mélanges granulaires par le modèle de suspension solide -II- Validation, cas des mélanges confinés, 194, Bull. Liaison Lab. Ponts Chaussées, 1994.
[10] F. de Larrard, T. Sedran, Une nouvelle approche de la formulation des bétons, Annales du BTP 6 (1999) 39–54.
[11] A. Lecomte, A. Zennir, F. de Larrard, Modèle de suspension solide et formulation de bétons calcaires en Lorraine, Bull.Lab. Ponts Chaussées, 1997 41–52.
[12] W. Toufar, M. Born, E. Klose, Contribution of optimisation of components of different density in polydispersed particles systems, Freib. Bookl. A 558 (1976) 29–44.
[13] J.D. Dewar, Ready-mixed concrete mix design, Munic. Eng. 3 (1986) 35–43.
[14] M. Reisi, D.M. Nejad, A numerical method to predict packing density of aggregates in concrete, Adv. Mater. Res. 337 (2011) 313–316.
[15] A.K.H. Kwan, K.W. Chan, V. Wong, A 3-parameter particle packing model incorporating the wedging effect, Powder Technol. 237 (Mar. 2013) 172–179.
[16] A. Sadok, R. Zentar, N.-E. Abriak, Genetic programming for granular compactness modelling, Eur. J. Environ. Civ. Eng. (2016) 1–13.
[17] G. Roquier, The 4-parameter compressible packing model (CPM) including a new theory about wall effect and loosening effect for spheres, Powder Technol. 302 (2016) 247–253.
[18] G. Roquier, A theoretical packing density model (TPDM) for ordered and disordered packings, Powder Technol. 344 (2019) 343–362.
[19] F. Moutassem, Assessment of packing density models and optimizing concrete mixtures, Int. J. Civ. Mech. Energy Sci. (IJCMES) 2 (4) (2016) 29–36.
[20] M.R. Jones, L. Zheng, M.D. Newlands, Comparison of particle packing models for proportioning concrete constitutents for minimum voids ratio, Mater. Struct. 35 (5) (2002) 301–309.
[21] A. Lecomte, J.-M. Mechling, Compacité des mélanges et propriétés des grains, Bull.-Lab. Ponts Chaussées, 1999 21–34.
[22] Association Française de Normalisation (AFNOR), Tests for mechanical and physical properties of aggregates - part 6: determination of particle density and water absorption, Standard No. 1097–6, 2014.
[23] Association Française de Normalisation (AFNOR), Tests for geometrical properties of aggrega-tes - part 1: determination of particle size distribution - sieving method, Standard No. 933–1 2012.
[24] V. Ledee, F. de Larrard, T. Sedran, F. Brochu, Essai de compacité des fractions granulaires à la table à secousses: Mode opératoire, Tech. Méthodes Lab. Ponts Chaussées, 2004.
[25] M. Bala, R. Zentar, and P. Boustingorry, Parameter analysis of the compressible packing model for Concrete application, 12th fib International PhD Symposium in Civil Engineering, Prague, Czech Republic, Unpublished results, pp. 1–8.
[26] R.K. McGeary, Mechanical packing of spherical particles, J. Am. Ceram. Soc. 44 (10) (1961) 513–522.
[27] G.W. Delaney, S. Hutzler, T. Aste, Relation between grain shape and fractal properties in random apollonian packing with grain rotation, Phys. Rev. Lett. 101 (12) (2008), 120602.
[28] D. Weaire, T. Aste, The Pursuit of Perfect Packing, CRC Press, 2008.
[29] A.B. Yu, N. Standish, Porosity calculations of multi-component mixtures of spherical particles, Powder Technol. 52 (3) (1987) 233–241.
[30] S. Abdelfeteh, Formulation de matériaux de construction à base de sous-produits industriels avec des méthodes issues de l'intelligence artificielle, PhD Thesis Univ. Lille, 2016 1.

[31] A. Sadok, R. Zentar, N.E. Abriak, Modélisation de la compacité granulaire par une approche d'Apprentissage Automatique, Rencontres Universitaires de Génie Civil (2015).
[32] T.N. Thanh, Valorisation de sédiments marins et fluviaux en technique routière, PhD Thesis Univ. d'Artois - Ecole des Mines de Douai, 2009.
[33] M. Bala, R. Zentar, P. Boustingorry, Etude d'impact de la forme des granulats sur les paramètres du modèle d'empilement compressible, 36èmes Rencontres Universitaires de Génie Civil de l'AUGC, Saint-Etienne, France 2018, pp. 1–4.
[34] X. Chateau, G. Ovarlez, K.L. Trung, Homogenization approach to the behavior of suspensions of noncolloidal particles in yield stress fluids, J. Rheol. 52 (2) (2008) 489–506.
[35] F. de Larrard, A. Belloc, L'influence du granulat sur la résistance à la compression des bétons, 219, Bull. Lab. Ponts Chaussées, 1999.