THE PACKING OF PARTICLES[1]

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ABSTRACT

It is axiomatic that the mode of packing of very large volumes of particles of uniform shape and size is independent of the size of the particles, provided they are large enough for the effect of electrostatic forces, air films, etc., to be negligible. An apparatus is described, in which equal true volumes of approximately spherical particles, ranging in diameter from 0.2 to 0.0035 inch, pack practically to the same apparent volume. This apparatus was used in studying the packing of mixtures of two and three sizes of particles. By plotting the data so obtained in diagrams of a particularly convenient character, it is shown that the apparent volumes of mixtures containing unit real volume of solid fall between limiting values which can be calculated from simple assumptions, and that their deviation from these limits depends in a definite manner upon the diameter ratios of the component particles. The conditions governing the application of the results of the study to ceramic technology are pointed out.

I. Introduction

(1) Importance of Particle Packing A very large proportion of the natural and manufactured materials with which we have to deal are aggregates, or contain aggregates, of particles of several sizes or shapes. An understanding of the packing of particles is of great importance, therefore, in almost every branch of science and industry. The ceramic industry is not an exception to this general statement because practically all ceramic materials, with the probable exception of glasses and glazes, consist of aggregates of natural or artificial minerals. It is well known that a number of important physical properties of refractories, grinding wheels, terra cotta, etc., depend to a considerable extent on the closeness of packing of the aggregate.

(2) Scope of Study A study of the packing of particles was initiated in the fall of 1923 in the Department of Electrochemistry of the University of Toronto. During the first year the study was limited to the packing of approximately spherical particles of sizes greater than 200-mesh (0.0029 in.) and to mixtures containing not more than three sizes of particles. This first year's work forms the basis for the present paper. The information obtained in later work, in which the effect of particle shape, air film, etc., was investigated, will be presented in a future paper.

II. Apparatus

(3) Requirements The closeness of packing of very large volumes of particles of uniform shape and size is independent of the size of the particles, provided electrostatic and air-film effects are

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negligible. In studies of particle packing, therefore, it is a decided advantage if the apparatus is such that the packing of uniform particles is independent of their size. In studies of mixtures of particles of different sizes, it is also important to have an apparatus in which segregation can be avoided. The first part of the study was devoted, therefore, to the development of an apparatus and method of packing which met the foregoing requirements.

(4) Description A diagram of the apparatus which was developed is shown in Fig. 1. It consisted essentially of a cylinder, a piston, and a cam and pin. The cylinder was made from a seamless brass tube, 14 inches high, $2^1/_{16}$ inches internal diameter, and $2^3/_8$ inches external diameter. This was set on a copper base, $3^3/_8$ inches in diameter and $^1/_2$ inch thick. The piston consisted of a brass tube, 8 inches high and $2^1/_{32}$ inches external diameter, closed at the lower end. The piston was graduated in such a way that the volume of a mixture of particles in the cylinder could be determined by placing the piston on top of the mixture and reading at the top of the cylinder. The cam and pin arrangement was such that at each revolution of the cam, the cylinder was slowly elevated and then suffered an unimpeded drop of about 3 inches on to a felt pad shown in the diagram. The cam was driven by an electric motor at such a speed that the cylinder was bumped approximately 25 times a minute.

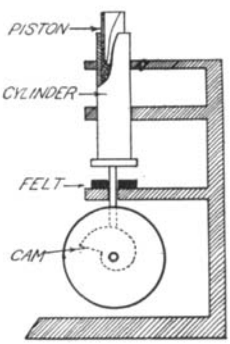

FIG. 1.—Apparatus for packing particles.

(5) Operation In packing a mixture of particles, bumping was continued until a minimum volume was obtained. This required from 30 minutes to three hours, depending on the mixture. When packing mixtures of very fine and very coarse particles, segregation of the fine particles in the bottom of the cylinder sometimes occurred. To overcome this difficulty, the coarse particles were put in the cylinder first and the fine particles placed on top. On bumping such a combination, the volume decreased to the minimum value as the fine and coarse particles mixed and then increased again as segregation took place. No segregation was evident at the minimum volume which was taken to be the packed volume of the mixture.

III. Particles of Uniform Size and Shape

(6) Materials In experiments with aggregates composed of particles of uniform size and shape, lead shot, steel ball bearings, round California beans, poppy seeds, and round washed sand were used. The lead shot and steel ball bearings were spherical; the beans, poppy seeds, and sand were approximately spherical. The fine sand was more angular than the coarse sand, due to the tendency of angular particles to concentrate in fine portions when screening. The materials chosen covered a wide range of specific gravity. The specific gravity of the lead shot was 11.25; of the fine sand, 2.75; of the medium sand, 2.76; and of the coarse sand, 2.69.

(7) Data The data obtained from experiments with particles of uniform size are reported in Table I. The true volume of each cylinder of particles which was packed in the apparatus was calculated from the weight of the particles and their specific gravity. The "per cent void" and *Va*, the apparent volume occupied by unit true volume of the particles, were calculated from the true volume and the minimum apparent volume on packing.

TABLE I

DATA OBTAINED BY PACKING PARTICLES OF UNIFORM SIZE AND SHAPE

Material			Diameter (in.)	% Void*	*Va*†
Lead shot			0.19	36.9	1.58
"			0.15	36.9	1.58
"			0.11	37.0	1.59
"			0.07	36.9	1.58
Steel ball bearings			0.312	39.2	1.64
Round Calif. beans			0.27 (approx.)	37.5	1.59
Poppy seeds			0.04 "	39.8	1.66
Round washed sand:	4–6	mesh	0.176	37.7	1.61
	20–28	"	0.028	38.2	1.62
	48–65	"	0.0097	38.6	1.63
	150–200	"	0.0035	42.5	1.74

* Percentage of apparent volume not occupied by solid particles.
† The apparent volume containing unit true volume of solid particles.

(8) Conclusions The data in Table I show that particles of uniform shape and size could be packed practically to the same extent over a range of diameters extending from 0.312 to 0.0035 inch. In addition, the closeness of packing was not affected by the specific gravity of the material used. The apparatus, therefore, met the requirements given in paragraph 3.

The data also show that spherical particles tend to pack to a per cent void of about 40. It is interesting to compare this value with that calculated for spheres piled in regular arrangements.

When spheres are piled in a regular cubic arrangement, in which each sphere touches six other spheres, the per cent void is 48 and *Va* is 1.910; when spheres are piled in close hexagonal formation, in which there are twelve contacts per sphere, the per cent void is 26 and the *Va* is 1.350. The observed values for per cent void fall between these two values, so the packing was neither cubic nor close hexagonal. When spheres are piled so that the packing in any horizontal layer is hexagonal but in any vertical section is cubic, the calculated per cent void is 39.5 and *Va* is 1.653. These do not differ greatly from the observed values, and it was noticed, particularly with the large spheres, that a vertically cubic and horizontally hexagonal packing was the prevailing tendency. The packing which was secured in the apparatus may be regarded, therefore, as a mixture of cubic and hexagonal packing which approached a regular arrangement, hexagonal in the horizontal direction and cubic vertically.

IV. Mixtures of Particles of Two Sizes

(9) Materials Three grades of round, washed sand, which will be differentiated as "coarse," "medium," and "fine," were obtained by sieving through Tyler sieves. The coarse sand passed through a sieve with a 0.185-inch opening and was caught on one with an opening of 0.168 inch; the medium sand passed through a 0.0328-inch opening and rested on a sieve with a 0.0232-inch opening; the fine sand passed through 0.0041-inch openings and was caught on 0.0029-inch openings. The diameters 0.176, 0.028, and 0.0035 inch were assigned to the coarse, medium, and fine sand, respectively. These diameters were in the proportion 50.5:8:1. The values of Va for the three two-size systems, coarse-fine, coarse-medium, and medium-fine, were determined by the apparatus and procedure described in Chapter II. The mixtures were made up on the basis of real volumes, the densities of the coarse, medium, and fine sand being 2.69, 2.76, and 2.75.

(10) Data The data obtained by experiments with mixtures of two sizes of sand are given in Table II. These data are presented in two forms, the more usual form in which the per cent void and the per cent composition by true volume of each mix is reported; and a more convenient form for calculation and plotting, in which the volume of Va is reported instead of per cent void and the composition of the mixes is given in terms of x, y, and z, x being the true volume of coarse, y the true volume of medium, and z the true volume of fine, in unit true volume of mix. The value of Va which corresponds to a certain per cent void can be calculated readily by means of the equation

$$Va = \frac{100}{(100-\text{per cent void})} \tag{1}$$

TABLE II

(A) COARSE–FINE SYSTEM

Coarse (% by true volume)	Fine (% by true volume)	Void (% by volume)	x	z	Va*
100	0	37.7	1.0	0.0	1.605
90	10	31.5	0.9	0.1	1.460
80	20	25.1	0.8	0.2	1.335
70	30	18.5	0.7	0.3	1.227
60	40	23.6	0.6	0.4	1.309
50	50	27.9	0.5	0.5	1.387
40	60	31.6	0.4	0.6	1.462
20	80	37.8	0.2	0.8	1.608
0	100	42.5	0.0	1.0	1.739

(B) Coarse–Medium System

Coarse (% by true volume)	Medium (% by true volume)	Void (% by volume)	x	y	Va*
100	0	37.7	1.0	0.0	1.605
90	10	32.0	0.9	0.1	1.471
80	20	27.8	0.8	0.2	1.385
70	30	26.2	0.7	0.3	1.355
60	40	27.0	0.6	0.4	1.370
50	50	28.2	0.5	0.5	1.393
40	60	30.2	0.4	0.6	1.433
20	80	34.8	0.2	0.8	1.534
0	100	38.2	0.0	1.0	1.618

(C) Medium–Fine System

Medium (% by true volume)	Fine (% by true volume)	Void (% by volume)	y	z	Va*
100	0	38.2	1.0	0.0	1.618
90	10	32.8	0.9	0.1	1.488
80	20	29.0	0.8	0.2	1.408
70	30	26.7	0.7	0.3	1.364
60	40	27.6	0.6	0.4	1.381
50	50	30.2	0.5	0.5	1.433
40	60	33.0	0.4	0.6	1.493
30	70	35.6	0.3	0.7	1.553
20	80	38.6	0.2	0.8	1.629
10	90	40.4	0.1	0.9	1.678
0	100	42.5	0.0	1.0	1.739

* The apparent volume occupied by unit true volume of mix when packed in the apparatus shown in Fig. 1.

The values of Va given in Table II are plotted in Figs. 2, 3, and 4. In each of these figures, the composition scale at the base shows the composition in terms of true volumes, the total true volume being unity.

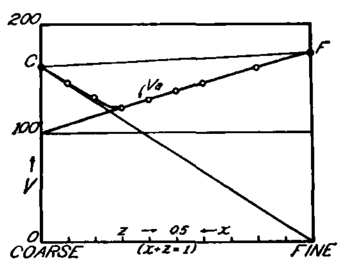

Fig. 2.—Packing in two size systems: Diameter ratio = 50.5.

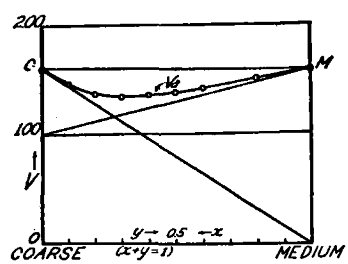

Fig 3.—Packing in two-size systems: Diameter ratio = 6.3.

The perpendicular scale is a scale of volume, V. Experimentally determined values of Va are shown by small circles through which a smooth curve has been drawn. The horizontal line at $V = 1.00$ gives the true volume of the mixture, and the vertical distances from this line to the Va curve give the volumes of air in the different mixes for unit true volume of solid. The straight lines joining the values of Va for the end members of the series, *e.g.*, the line, *CF*, in Fig. 2, gives the apparent volume of the constituents before mixing; the perpendicular distances from this line to the *Va* curve show the shrinkage on mixing.

It will be observed that the *Va* curves in Figs. 2, 3, and 4 are festoons. When the ratio between the diameter of the large and small particles is large, *e.g.*, Fig. 2, the festoon has a very decided droop; when the diameter ratio is small, *e.g.*, Figs. 3 and 4, this droop is much less.

(11) Limiting Cases The relation between the diameter ratios of two sizes of sand and the shape of their *Va* curve can be shown by considering certain limiting cases. Using Fig. 2 for illustration, it is evident that if the diameter ratio is unity, *i.e.*, the coarse and fine particles have the same diameter, there will be no shrinkage on mixing and the *Va* curve will be the straight line, *CF*. On the other hand, if the coarse particles have a diameter which is infinitely greater than that of the fine particles, a different *Va* curve will be obtained. On adding fine particles to a quantity of coarse particles, the fine will pack in the voids in the coarse and the apparent volume of the mix will be equal to that of the coarse particles. The *Va* curve corresponding to this state of affairs is a straight line joining the point *C* in Fig. 2 with the right hand lower corner of the diagram. On adding coarse particles to a large quantity of fines, the solid coarse particles will be immersed in the fines and the *Va* curve will be the straight line joining *F*, Fig. 2, to the point 1.00 at the left side of the diagram. These straight lines cross at *R* and thus *CRF* gives the *Va* curve for the case when the coarse particles are infinitely larger than the fine particles.

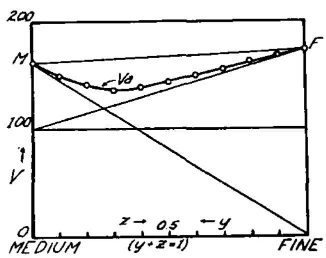

FIG. 4.—Packing in two-size systems: Diameter ratio = 8.0.

The experimental *Va* curves will fall between the line, *CF*, and the trough-shaped curve, *CRF*. As the diameter ratio approaches 1.00, the *Va* curve will approach *CF;* as it increases to large values, the *Va* curve will approach *CRF*. It will be observed that in Fig. 2, where the diameter ratio is 50.5, the *Va* curve approaches closely to *CRF*, and that in Figs. 2 and 3, where the diameter ratios are 6.3 and 8, respectively, the *Va* curves come about half-way between the curves for the limiting cases.

(12) Diameter Ratio To obtain quantitative information as to how the minimum *Va* for different two-size mixes depended on their diameter ratio, the *Va* values of a number of mixes containing 30% by true volume of the smaller particles were determined and are given in Table III.

TABLE III

MIXTURES OF TWO SIZES CONTAINING 30% FINE BY TRUE VOLUME

Material	Diameter ratio	Void (% by volume)	Va
Lead shot	1.000	36.9	1.585
" "	1.266	36.9	1.585
" "	1.460	36.1	1.570
" "	1.712	35.5	1.550
" "	2.110	34.7	1.535
" "	2.71	32.5	1.485
" "	4.13	28.6	1.405
Steel and lead shot	6.80	24.6	1.33
Sand	1.00	37.7	1.605
"	6.29	26.2	1.36
"	50.0	18.5	1.23

The data given in Table III are plotted in Fig. 5 where the abscissae give the ratio of the diameter of the coarse to the diameter of the fine particles and the ordinates of the Va curve give the values of Va for a 30% fine mixture. It will be seen that as the diameter ratio increases, the value of Va rapidly approaches the value 1.19 calculated for an infinite ratio.

(13) Conclusions When experimental Va curves for mixtures of rounded sand are plotted on a diagram of the type shown in Fig. 2, they give the line, CF, when the diameter ratio coarse: fine

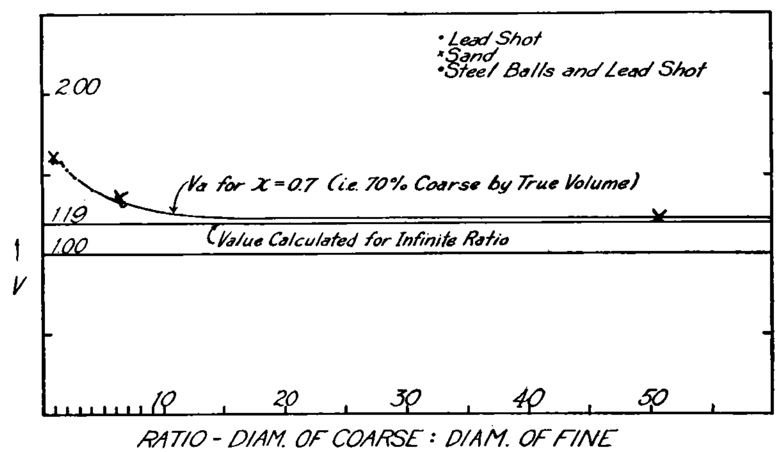

FIG. 5.—Packing in two-size systems: Effect of diameter ratio.

is unity and rapidly approach the shape, CRF, as the diameter ratio increases, the rate of approach being shown in Fig. 5.

V. Mixtures of Particles of Three Sizes

(14) Data The coarse, medium, and fine sands described in paragraph 9 were used in determining the value of Va for a series of mixes containing three sizes of particles. The compositions of these mixes were so chosen that they gave a number of uniformly distributed points on the usual equilateral diagram. The compositions were expressed by means of x, y, and z, where x gives the true volume of coarse, y the true volume of medium, and z, the true volume of fine, in unit true volume of mix The values of Va obtained for the different mixes are given in Table IV. By combining Tables II and IV, data for the complete series of three-size mixes are obtained.

TABLE IV

THREE-SIZE SYSTEMS

x	y	z	Void (% by volume)	Va*
0.8	0.1	0.1	25.8	1.348
0.7	0.1	0.2	15.5	1.183
0.6	0.1	0.3	19.7	1.245
0.5	0.1	0.4	24.2	1.319
0.4	0.1	0.5	28.4	1.397
0.2	0.1	0.7	35.6	1.553
0.7	0.2	0.1	16.8	1.202
0.6	0.2	0.2	17.4	1.211
0.5	0.2	0.3	20.6	1.259
0.4	0.2	0.4	25.4	1.340
0.3	0.2	0.5	28.8	1.404
0.2	0.2	0.6	32.3	1.477
0.6	0.3	0.1	20.0	1.250
0.5	0.3	0.2	18.4	1.225
0.4	0.3	0.3	20.7	1.261
0.3	0.3	0.4	25.8	1.348
0.2	0.3	0.5	29.5	1.418
0.5	0.4	0.1	21.8	1.279
0.4	0.4	0.2	20.1	1.252
0.3	0.4	0.3	22.4	1.289
0.2	0.4	0.4	26.2	1.355
0.1	0.4	0.5	29.8	1.424
0.4	0.5	0.1	24.2	1.319
0.3	0.5	0.2	22.4	1.289
0.2	0.5	0.3	23.6	1.309
0.1	0.5	0.4	26.7	1.364
0.3	0.6	0.1	27.0	1.370
0.2	0.6	0.2	23.8	1.312
0.1	0.6	0.3	24.6	1.326
0.2	0.7	0.1	29.4	1.416
0.1	0.7	0.2	25.8	1.348
0.1	0.8	0.1	31.2	1.453

* The apparent volume occupied by unit true volume of mix when packed in the apparatus shown in Fig. 1.

(15) Limiting Cases In the case of three-size systems it will be more convenient to discuss limiting cases first and experimental surfaces later. As in the two-size system, the limiting cases are (1) where the coarse, medium, and fine particles have the same diameter, and (2) where the coarse particles are very large in comparison with the medium particles and the medium particles are very large as compared with the fine particles. In Fig. 6, the equilateral triangle labelled coarse, medium, and fine provides a field for plotting compositions, and volumes are measured by means of perpendicular ordinates erected on the triangle.

If the points C, M, and F give the values of Va for the coarse, medium, and fine particles, respectively, then the Va surface for mixtures of coarse, medium, and fine for the limiting case where their diameters are equal will be the plane passing through C, M, and F. The Va surface for the other limiting case when the diameter ratios are infinitely large is the shaded surface formed by the three planes in Fig. 6. By joining the point C to the points where $V = O$ at the medium and fine corners of the triangle, F to the points where $V = 1.00$ at the coarse and medium corners, and M to the point $V = O$ at the fine corner and to the point $V = 1.00$ at the coarse corner, these three planes can be obtained graphically and give the binary minimum points O, N, and R. By joining O to the point $V = 1.00$ at the coarse corner, N to the point $V = O$ at the fine corner, and R to the point Q where the line joining C to $V = O$ at the medium corner cuts the line $V =$ 1.00, the figure can be completed.

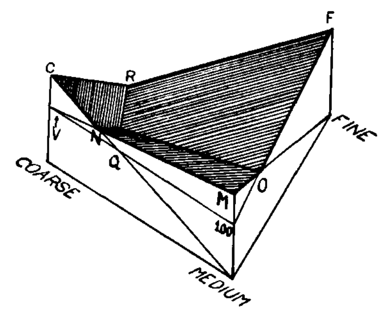

FIG. 6.—Packing in three-size systems: Calculated Va surface for very large diameter ratios.

The experimentally determined Va surfaces should fall between the surface formed by the single plane passing through C, M, and F and the surface formed by the shaded portions of the three planes shown in the figure.

(16) Analytical Expressions Since it is difficult to work with solid models, it is convenient to have analytical expressions for the planes described in the preceding paragraph. Since compositions have been expressed in terms of x, y, and z, where $x + y + z = 1$, these analytical expressions are relatively simple.

If the point C has the height a, M the height b, and F the height c, then the equation for the single plane through C, M, and F is

$$ax + by + cz = v \tag{2}$$

where v is the height of the ordinate to a point in the plane.

If the three planes which form the minimum Va surface are labeled

C, M, and F in correspondence to the points through which they pass, then their equations are

$$v_C = ax \tag{3}$$

$$v_M = x + by \tag{4}$$

$$v_F = x + y + cz \tag{5}$$

where v_C, v_M, and v_F are the ordinates of points on the C, M, and F planes, respectively.

The minimum value of v which is obtained at the junction of the three planes, C, M, and F, is given by

$$v = \frac{abc}{ab + bc + ca - (a + b + c) + 1} \tag{6}$$

If the coarse, medium, and fine particles by themselves have the same value of Va, then $a = b = c$ and the minimum apparent volume is given by

$$v = \frac{a^3}{(1-a)^3 + a^3} \tag{7}$$

For $a = 1.605$, v is 1.057 and gives the value of Va which could be approached by packing coarse, medium, and fine sand with very large diameter ratios in the proportion $x = 0.66$, $y = 0.25$, and $z = 0.09$.

(17) Experimental Surface In order to compare the experimentally determined Va surface with the limiting surfaces derived in the preceding paragraphs, Fig. 7 was prepared. The contours shown in broken line in the figure are those of the experimental Va surface. The method used in plotting these contours may be of interest.

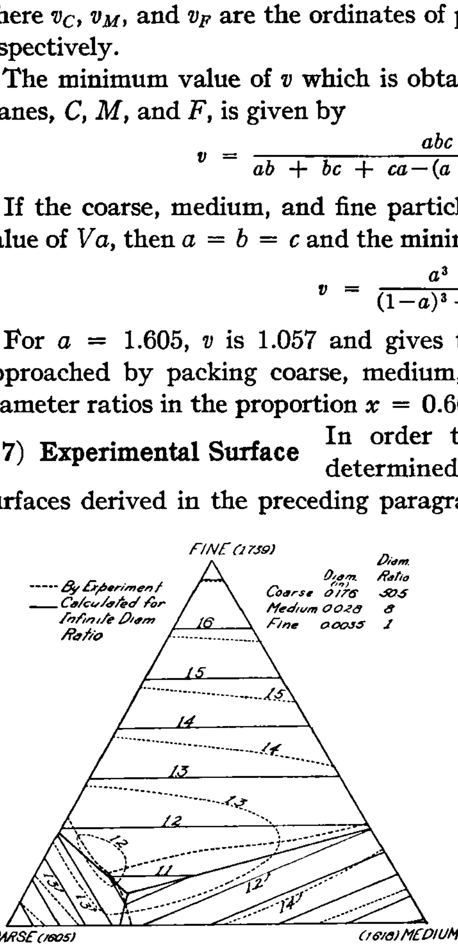

FIG. 7.—Packing in three-size systems: Comparison of experimental Va surface with that calculated for large diameter ratios.

Note: The numerals in this figure should show decimal point after first integer.

The experimental mixes were such that a number of straight lines could be drawn through the points representing their compositions on an equilateral triangle. For each such line a diagram similar to that shown in Fig. 2 was constructed and the compositions for values of Va equal to 1.1, 1.2, 1.3, etc., were found by interpolation. By plotting these compositions in Fig. 7 a large number of points for each contour line was obtained.

The contours shown in full line in Fig. 7 are those of the Va surface calculated for the limiting case in which the coarse, medium, and fines differ infinitely in diameter. It will be noticed that the calculated and experimental contour lines agree very well along the coarse-fine side of the diagram where the diameter ratio is 50.5 and that they differ in other parts of the diagram where the diameter ratios are smaller by an amount which might be predicted from the curve in Fig. 5.

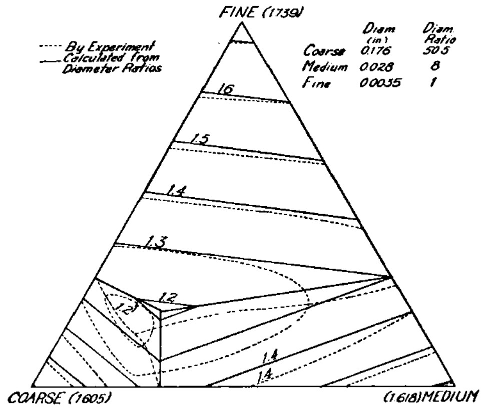

FIG. 8.—Packing in three-size systems: Comparison of experimental Va surface with that calculated from actual diameter ratios.

Note: The numerals in this figure should show decimal point after first integer.

If advantage is taken of the knowledge that the diameters of the coarse, medium, and fine were in the ratio 50.5:8:1, the contours shown in full line in Fig. 8 can be calculated, using Fig. 5 and linear interpolation. These agree very well with the experimentally determined contours shown in the same figure in broken line.

(18) Conclusions By an extension of the methods used in interpreting the data obtained with two-size systems, it is possible to calculate the corresponding limiting cases for three-size systems. Experimental values fall between the values for the limiting cases and differ from them by an amount which can be predicted from the diameter ratios of the particles used.

VI. Mixtures of Particles of Four or More Sizes

(19) Analytic Extension The symmetric nature of the analytic expressions given in paragraph 16 indicates a way in which the methods used in interpreting two- and three-size systems can be extended to include systems of four and more sizes of particles.

Let us suppose that n sizes of particles are designated 1, 2, 3...... n in order of increasing fineness. Let $x_1, x_2, x_3 \ldots\ldots x_n$ be used in expressing the composition by true volume of mixtures of these particles, the total true volume being unity, *i.e.*, $x_1 + x_2 + x_3 + \ldots\ldots x_n = 1$. Then if $a_1, a_2, a_3 \ldots\ldots a_n$ be the values of Va for each size when packed alone, the Va for any mixture in the limiting case in which no shrinkage occurs can be calculated from the equation

$$v = a_1x_1 + a_2x_2 \ldots\ldots a_nx_n \tag{8}$$

For the other limiting case, in which the diameter ratios are infinite, the Va of any mix can be calculated from the equation in the following which gives the largest value of v.

$$\begin{aligned}
v_1 &= a_1x_1 \\
v_2 &= x_1 + a_2x_2 \\
v_3 &= x_1 + x_2 + a_3x_3 \\
v_4 &= x_1 + x_2 + x_3 + a_4x_4 \\
&\text{Etc.}
\end{aligned} \tag{9}$$

By introducing into the above equations linear terms which depend on the diameter ratios involved, it should be possible to predict the value of Va for a mixture of particles of four or more diameters with approximately the accuracy shown in Fig. 8. By fitting curves to the experimental Va curves for two-size systems, closer agreement might be secured, but the gain in accuracy would probably be more than offset by the extra calculation involved.

It may be that further work will make it possible to calculate the packing properties of a mixture of particles from its sieve analysis. This possibility is now under investigation.

VII. Application to Ceramics

(20) Relation to Strength, Texture, Etc. With ceramic bodies which are made from a relatively inert aggregate, such as calcined clay, quartz, etc., and a flux or bond, such as clay, lime, etc., the density with which the aggregate packs may have a pronounced effect on the properties of the product. If a close-packing aggregate is obtained, the amount of bond can be reduced and the strength of the product will more nearly approach that of the aggregate material. The packing of the aggregate, therefore, may influence the strength, texture, refractoriness, color, and other properties of the product.

It should not be assumed, however, that the aggregate which gives the closest packing will necessarily give the best product. Other factors must be taken into consideration. The most important are (1) the solubility of the fine aggregate in the flux, (2) the relative strength of the aggregate and bond, (3) the importance of surface effects, etc. As an example, consider the relation between aggregate packing and the strength of a product. If the aggregate is relatively soluble in the bond, the fine particles in the aggregate will dissolve and change the packing of the aggregate. If the aggregate material is stronger than the bond, close packing will usually give high strength; if the bond is stronger than the aggregate, high strength will be associated with open packing. In some cases, the strength of a product depends to a considerable extent on the area of contact of the aggregate and bond. When this is true, the fine particles in the aggregate have a disproportionate effect on strength.

VIII. Summary

(21) Summary and Conclusions (1) The value of Va, the apparent volume per unit true volume, was determined for systems of one, two, and three sizes of roughly spherical particles when packed in an apparatus in which the packing of particles of uniform size and shape was independent of their diameter.

(2) For one-size systems Va was approximately 1.60.

(3) With two-size and three-size systems, the shape of the Va curve or surface depended on the diameter ratios. It could be calculated directly for the limiting cases in which the ratios were very large (approximately >15) or approached unity; for other ratios, the Va curve or surface could be calculated approximately by using an empirical relation between the minimum Va and the diameter ratio of two-size systems.

(4) The method of calculation was capable of extension to systems of four or more sizes.

(5) Calculated minimum values of Va for systems of one or more sizes of particles having very large diameter ratios, and packing individually to a Va of 1.605 (percentage void = 37.7), follow. The corresponding percentage voids are also given:

Number of sizes	Calculated minimum Va*	Calculated minimum percentage void
1	1.605	37.7
2	1.166	14.2
3	1.057	5.4
4	1.021	2.0
5	1.008	0.8

* The apparent volume occupied by unit true volume.

(6) In applying packing theory to ceramic problems the relative strength, colors, shrinkages, etc., of the aggregate and bond must be considered as well as the solubility of the aggregate in the bond and the importance of the surface area of the aggregate.