Teoria di Maxwell

$$\vec{\nabla}\cdot\vec{E} = \rho \qquad \vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{B} = \frac{1}{c}\vec{j} + \frac{1}{c}\frac{\partial\vec{E}}{\partial t} \qquad \vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}$$

$$F_{\mu\nu} = \begin{array}{cc} & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ & \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & B_z & -B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{pmatrix} \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} \end{array}$$

$$j^\mu = (c\rho, \vec{j})$$

$$E_i = F_{0i}$$

$$B_i = \frac{1}{2}\varepsilon_{ijk}B_k$$

$$\boxed{\partial_\nu F^{\mu\nu} = J^\mu} \longrightarrow \partial_\mu J^\mu = 0$$

$$\partial^\lambda F^{\mu\nu} + \partial^\nu F^{\lambda\mu} + \partial^\mu F^{\nu\lambda} = 0$$ identidade de Bianchi

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$$\tilde{F}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$$ dual de Hodge

$$\boxed{\partial_\mu \tilde{F}^{\mu\nu} = 0} \qquad \boxed{F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu}$$

$$\hookrightarrow \quad \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\partial_\mu(\partial_\rho A_\sigma - \partial_\sigma A_\rho) = 0$$

$$\partial_\mu\partial_\rho \qquad \partial_\mu\partial_\sigma$$

$$\partial_\nu \left(\partial^\mu A^\nu - \partial^\nu A^\mu \right) = j^\mu$$

$$\boxed{\Box A^\mu - \partial^\mu(\partial_\nu A^\nu) \pm j^\mu = 0}$$

$$A_\mu \to A_\mu + \underline{\partial_\mu} \alpha \qquad \underline{F_{\mu\nu} \to F_{\mu\nu}}$$

Lagrangiano di Maxwell

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{c} j_\mu A^\mu$$

$$\frac{\delta \mathcal{L}}{\delta \partial_\mu A_\nu} = -\frac{1}{2} F^{\rho\sigma} \frac{\partial F_{\rho\sigma}}{\partial \partial_\mu A_\nu} = -F^{\mu\nu}$$

$$\frac{\delta \mathcal{L}}{\delta A_\nu} = -\frac{1}{c} j^\nu$$

$$\boxed{\partial_\mu F^{\mu\nu} = -\frac{1}{c} j^\nu}$$

$$\mathcal{L} \to \mathcal{L} - \frac{1}{c} j^\mu \partial_\mu \alpha$$

$$= \mathcal{L} - \frac{1}{c}\partial_\mu (j^\mu \alpha) + \frac{1}{c}\alpha\, \underline{\underline{\partial_\mu j^\mu}}$$

$$j^\mu = ec\,\bar{\psi}\gamma^\mu\psi$$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\hbar c\,\bar{\psi}\gamma^\mu \big(\overbrace{\underline{\partial_\mu} - \frac{ie}{\hbar c}A_\mu}^{D_\mu}\big)\psi - mc^2\bar{\psi}\psi$$

$$\psi \to e^{i\alpha}\psi$$

$$D_\mu\psi \to e^{i\alpha}\psi$$

$$\pi^\mu = \frac{\delta \mathcal{L}}{\partial \dot{A}^\mu} = \frac{1}{c}\frac{\delta \mathcal{L}}{\delta\, \partial_0 A_\mu} = -\frac{1}{c} F^{\mu 0} \qquad \Big/\ A^\mu$$

$$\boxed{F^{00} = 0} \qquad \boxed{\pi^0 = 0}$$

A_i $\quad \pi^i = -\frac{1}{c} E_i$

4 A_μ's $\qquad$ 2 graus de liberdade

$$\vec{S} \longleftarrow \qquad \longrightarrow \vec{S}'$$

$$\underset{\vec{k}}{\longrightarrow} \qquad \longrightarrow \vec{k}'$$

Lagrangiana di Fermi (1929)

$$\boxed{\mathcal{L}_F = -\frac{1}{2}(\partial_\nu A_\mu)(\partial^\nu A^\mu) - \frac{1}{c} j^\mu A_\mu}$$

$$\partial_\nu \left(\frac{\delta \mathcal{L}_F}{\delta \partial_\nu A_\mu} \right) = -\partial^2 A_\mu \qquad \frac{\delta \mathcal{L}_F}{\delta A_\mu} = -\frac{1}{c} j^\mu$$

$$\boxed{\Box A_\mu = \frac{1}{c} j_\mu} \qquad \boxed{\partial_\mu A^\mu = 0}$$

gauge di Lorentz

$$A_\mu \to A_\mu + \underline{\partial_\mu \alpha} = A'_\mu$$

$$\underbrace{\partial_\mu A'^\mu}_{=0} = \underbrace{\partial_\mu A^\mu}_{=0} + \underbrace{\partial^2 \alpha}_{=0} = 0$$

$\underline{\vec{\nabla} \cdot \vec{A} = 0}$ Coulomb

$$\boxed{\Box A_\mu = 0} \qquad J^\mu = 0 \qquad k^0 = \frac{\omega_k}{c}$$

$$\frac{\omega_k^2}{c^2} - \vec{k}^2 = 0$$

$$A_\mu = \varepsilon_\mu^r e^{\pm i k \cdot x} \qquad k^2 = 0$$

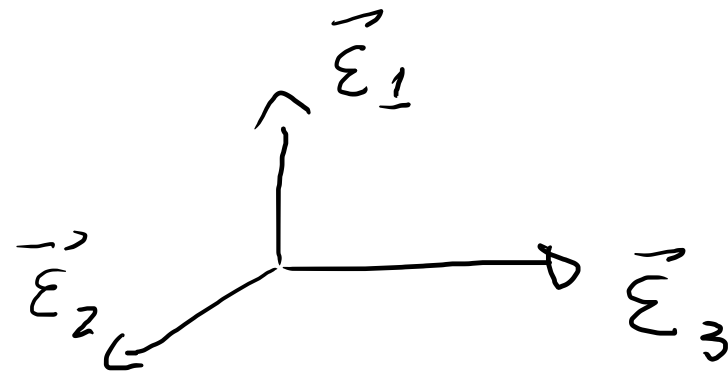
$$\boxed{\begin{aligned} A_\mu(x) &= A_\mu^+(x) + A_\mu^-(x) \\ A^{\mu +}(x) &= \sum_{r, \vec{k}} \sqrt{\frac{\hbar c^2}{V 2\omega_k}}\, \varepsilon_r^\mu(\vec{k})\, a_r(\vec{k})\, e^{-i k \cdot x} \\ A^{\mu -}(x) &= \sum_{r, \vec{k}} \sqrt{\frac{\hbar c^2}{V 2\omega_k}}\, \varepsilon_r^\mu(\vec{k})\, a_r^+(\vec{k})\, e^{i k \cdot x} \end{aligned}}$$

$$\boxed{\varepsilon^\mu_r(\vec{k})\,\varepsilon_{\mu s}(\vec{k}) = -\zeta_r\,\delta_{rs}}$$

$r, s = 0, 1, 2, 3$

$\zeta_0 = -1 \qquad \zeta_1 = \zeta_2 = \zeta_3 = 1$

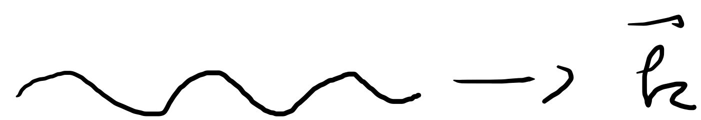
$$\boxed{\sum_r \zeta_r\,\varepsilon^\mu_r(\vec{k})\,\varepsilon^\nu_r(\vec{k}) = -g^{\mu\nu}}$$



$$\varepsilon^\mu_0 = n^\mu = (1, 0, 0, 0)$$

$$\varepsilon^\mu_r = (0, \vec{\varepsilon}_r(\vec{k})) \qquad r = 1, 2, 3$$

$$\vec{\varepsilon}_3(\vec{k}) = \frac{\vec{k}}{|\vec{k}|}$$



$$\boxed{\varepsilon_3^{\mu}(\vec{k}) = \frac{k^{\mu} - (k \cdot n)\, n^{\mu}}{\left[(k \cdot n)^2 - k^2\right]^{1/2}}}$$

$$(1, 0, 0, 0)$$

$$\shortparallel$$

$$k^{\mu} - (k \cdot n)\, n^{\mu} = k^{\mu} - k^{0} n^{\mu}$$

$$= (0, \vec{k})$$

$$(k \cdot n)^2 - k^2 = k_0^2 - (k_0^2 - \vec{k}^2) = \vec{k}^2$$

$$\Pi^{\mu} = \frac{\delta \mathcal{L}_F}{\delta \dot{A}_{\mu}} = -\frac{1}{c^2}\dot{A}^{\mu}$$

Quantização canônica

$$\pi^{\mu} = -\frac{1}{c^2}\dot{A}^{\mu}$$

$$[A^{\mu}(\vec{x},t), A^{\nu}(\vec{y},t)] = 0$$

$$[\dot{A}_{\mu}(\vec{x},t), \dot{A}^{\nu}(\vec{y},t)] = 0$$

$$[A^{\mu}(\vec{x},t), \dot{A}^{\nu}(\vec{y},t)] = -i\hbar c^2 g^{\mu\nu}\delta^{(3)}(\vec{x}-\vec{y})$$

$$\partial_0 A^\mu - i k_0 A^\mu = \sum_{r,\vec{k}} \sqrt{\frac{\hbar c^2}{V 2\omega_k}}\, \varepsilon_r^\mu(\vec{k}) \Big[$$

$$a_r(\vec{k})(-ik_0)e^{-ik\cdot x} + a_r^+(\vec{k})\, \cancel{ik_0}\, e^{ikx}$$

$$\quad\;\; \| -ik_0 \| \qquad\qquad\qquad \| \cancel{-ik_0} \|$$

$$= 2 \sum_{r,\vec{k}} \sqrt{\frac{\hbar c^2}{V 2\omega_k}}\, \varepsilon_r^\mu(\vec{k})(-ik_0)\, a_r\, e^{-ik\cdot x}$$

$$\int d^3x\, e^{ik'x}\left[\partial_0 A^\mu - ik_0 A^\mu\right] = -2ik_0 \sum_{r,\vec{k}} \varepsilon_r^\mu(\vec{k})\, a_r(\vec{k})$$

$$\int d^3x\, e^{-i(k-k')x}$$

$$\underline{k - k'}$$

$$\vec{k} = \vec{k}' \qquad k_0 = k_0'$$

$$\boxed{\frac{1}{V}\int d^3x\, e^{i\vec{\alpha}\cdot\vec{x}} = \delta_{\vec{\alpha},0}}$$

$$= -2 i k_0 \sum_r \sqrt{\frac{\hbar c^2}{2 V \omega_{k'}}} \, \varepsilon^\mu_r(\vec{k}')\, a_r(\vec{k}')\, V\, e^{-i(k_0 - k'_0)x}$$

$$k_0 = \frac{\omega_k}{c}$$

$$\left(\varepsilon^\mu_s(\vec{k})\right) \underbrace{\sum_r \varepsilon^\mu_r(\vec{k})\, a_r(\vec{k})}_{} = \frac{i c \quad \varepsilon^\mu_s(\vec{k})}{\sqrt{2 V \omega_k \hbar c^2}} \int d^3x\, e^{ik\cdot x} \left[\partial_0 A^\mu - i k_0 A_\mu\right]$$

$$- \delta_{rs} \, \rho_r$$

$$\boxed{a_r(\vec{k}) = - \frac{\rho_r \, \varepsilon_{\mu, r}(\vec{k})}{\sqrt{2 V \omega_k \hbar c^2}} \int d^3x\, e^{ik\cdot x} \left[i \dot{A}^\mu + \omega_k A^\mu\right]}$$

$$\frac{1}{\rho_r} = \rho_r = \pm 1$$

$$x = (ct, \vec{x})$$

$$y = (ct, \vec{y})$$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \lambda (\partial_\mu A^\mu)^2$$

$$\mathcal{L}(+) \qquad \Phi = 0$$

$$\mathcal{L}' = \mathcal{L}(+) + \lambda \Phi \qquad A_\mu \to A_\mu + \partial_\mu \alpha$$

E-L + $\quad \Phi = 0 \qquad \partial^2 \alpha = 0$

$$Q_{BRST}^2 = 0 \qquad Q |phys\rangle = 0 \qquad (a_0 - a_3) |phys\rangle = 0$$

$$\left[a_r(\vec{k}), a_s(\vec{k}')\right] = \frac{\rho_r \rho_s \, \varepsilon_{\mu,r}(\vec{k}) \, \varepsilon_{\nu,s}(\vec{k}')}{2V\hbar c^2 \sqrt{\omega_k \omega_{k'}}} \, *$$

$$* \int d^3x \int d^3x' \, e^{ik\cdot x} \, e^{ik'x'} \, *$$

$$* \Big[\underbrace{i\dot{A}^\mu(x) + \omega_k A^\mu(x), \; i \dot{A}^\nu}_{=0}(x') + \overbrace{\omega_{k'} A^\nu(x') \Big]}^{=0}$$

$$= \bigcirc \left(i\omega_{k'} i\hbar c^2 \cancel{g^{\mu\nu}} \, \delta(\vec{x}-\vec{y}) + i\omega_k \cancel{(-i\hbar c^2) g^{\mu\nu}} \, \delta(\vec{x}-\vec{y}) \right)$$

$$\int d^3x \, d^3x' \, \delta(\vec{x}-\vec{y}) \, e^{ik\cdot x} \, e^{ik'x'} = e^{i(\omega_k + \omega_{k'})t} \underbrace{\int d^3x \, e^{-i(\vec{k}+\vec{k}')\cdot\vec{x}}}_{V \, \delta_{\vec{k},-\vec{k}'}}$$

$$[a_r(\vec{k}), a_s(\vec{k}')] = 0$$

$$[a_r^+(\vec{k}), a_s^+(\vec{k}')] = 0$$

$$[a_r(\vec{k}), a_s^+(\vec{k}')] = \frac{g_r g_s \varepsilon_{\mu,r}(\vec{k}) \varepsilon_{\nu,s}(\vec{k}')}{2 V \hbar c^2 \sqrt{\omega_k \omega_{k'}}} \times$$

$$\times \int d^3x \int d^3x' \, e^{i k \cdot x} \, e^{-i k' y} \times$$

$$\times [\overbrace{i \dot{A}^\mu(x,t) + \omega_k A^\mu(x,t)}, \underbrace{-i \dot{A}^\nu(x',t) + \omega_{k'} A^\nu(x',t)}]$$

$$= \bigcirc \left(i \omega_{k'} \, i\hbar c^2 g^{\mu\nu} \delta(\vec{x}-\vec{x}') + i \omega_k \, i\hbar c^2 g^{\mu\nu} \delta(\vec{x}-\vec{x}') \right)$$

$$= - \rho_r \rho_s \, \varepsilon_{\mu,r}(\vec{k}) \, \varepsilon_{s,\nu}(\vec{k}') \, g^{\mu\nu} \, \frac{\cancel{(\omega_k + \omega_{k'})}}{\cancel{2\sqrt{\omega_k \omega_{k'}}}} \, e^{i(k_0 - k'_0)t} \, \delta_{\vec{k},\vec{k}'}$$

$$- \rho_r \, \delta_{r,s}$$

$$\boxed{[a_r(\vec{k}), a_s^+(\vec{k}')] = \rho_r \, \delta_{r,s} \, \delta_{\vec{k},\vec{k}'}}$$

ρ_0

$r = 1, 2$ transv.

$r = 3$ long.

$r = 0$ escalar

A_μ = spin 1 + A_0