Quantum Theory of Many-Body systems in Condensed Matter (4302112) 2020

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Today’s class: *Electronic transport in the Anderson model*

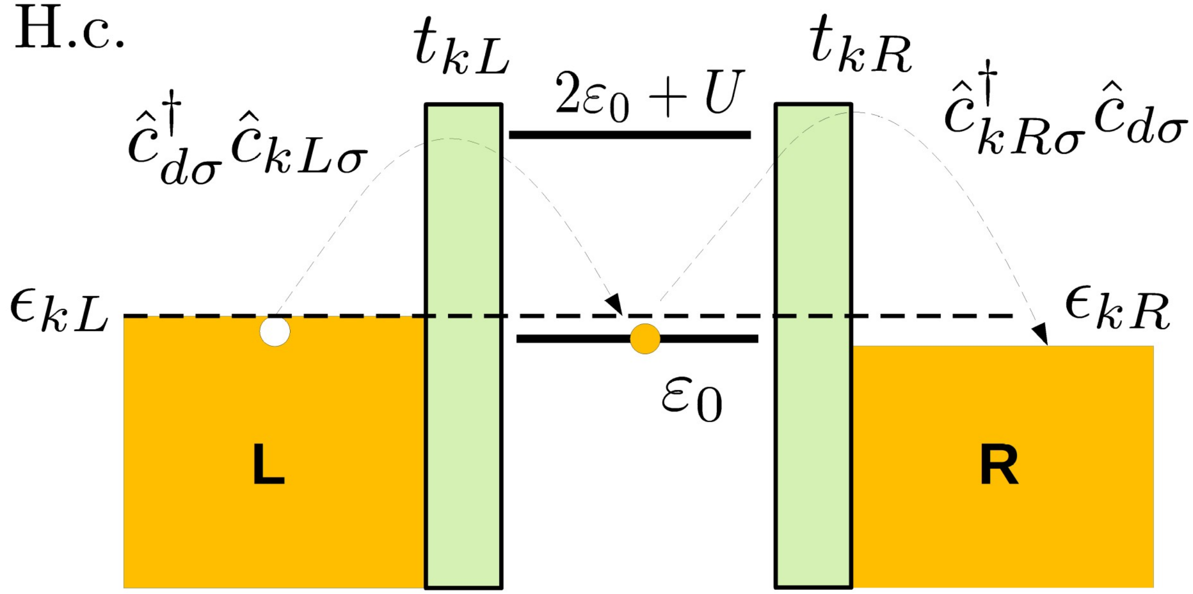
- Anderson model coupled to two metallic leads.
- Current operator
- Conductance for the Anderson model.
- Application: quantum dots in the Coulomb blockade regime.

Anderson model: two metallic baths

$$\hat{H} = \hat{H}_{\text{imp}} + \hat{H}_{\text{coup}} + \hat{H}_{\text{band}} \qquad \hat{n}_{d\sigma} \equiv \hat{c}^{\dagger}_{d\sigma}\hat{c}_{d\sigma}$$

$$\begin{cases} \hat{H}_{\text{imp}} = \sum_{\sigma=\uparrow,\downarrow} \varepsilon_0 \hat{n}_{d\sigma} + U \hat{n}_{d\uparrow}\hat{n}_{d\downarrow} \\ \hat{H}_{\text{coup}} = \sum_{k,\ell=L,R,\sigma} t_{k\ell}\hat{c}^{\dagger}_{d\sigma}\hat{c}_{k\ell\sigma} + \text{H.c.} \\ \hat{H}_{\text{band}} = \sum_{k,\ell=L,R\sigma} \epsilon_{\ell k}\hat{n}_{k\ell\sigma} \end{cases}$$

Fermionic operators (“spinful”):



$$\delta\mu = \epsilon_{kL} - \epsilon_{kR}$$

Current operator

$$\begin{cases} \hat{I}_L = (-e)\dfrac{d}{dt}\hat{n}_L(t) = -e\dfrac{d}{dt}\sum_{k,\sigma}\hat{n}_{kL\sigma}(t) \\ \hat{I}_R = -(-e)\dfrac{d}{dt}\hat{n}_R(t) = +e\dfrac{d}{dt}\sum_{k,\sigma}\hat{n}_{kR\sigma}(t) \end{cases}$$

Current that "leaves" *L*

Current that "enters" *R*

$$I = \hat{I}_L = \hat{I}_R \Rightarrow I = \alpha\hat{I}_L + (1-\alpha)\hat{I}_R$$

Conservation of current

(Assignment)

$$\begin{cases} \hat{I}_L = i\sum_{k,\sigma} t_L^* \hat{c}^\dagger_{kL\sigma}\hat{c}_{d\sigma} - t_L\hat{c}^\dagger_{d\sigma}\hat{c}_{kL\sigma} \\ \hat{I}_R = (-i)\sum_{k,\sigma} t_R^* \hat{c}^\dagger_{kR\sigma}\hat{c}_{d\sigma} - t_R\hat{c}^\dagger_{d\sigma}\hat{c}_{kR\sigma} \end{cases}$$

$$\hat{I}_R = (-i)\sum_{k,\sigma} t_R^* \hat{c}^\dagger_{kR\sigma}\hat{c}_{d\sigma} - t_R\hat{c}^\dagger_{d\sigma}\hat{c}_{kR\sigma}$$

Conductance in linear response

$$G = \lim_{\omega \to 0} \text{Re}\ \frac{ie}{\omega} \chi^R_{II}(\omega) = \lim_{\omega \to 0} \frac{-e}{\omega} \text{Im}\ \chi^R_{II}(\omega)$$

$$\chi^R_{II}(\omega) = \int_{-\infty}^{\infty} dt\ e^{i\omega^+ t} \left(-i\theta(t) \left\langle \left[\hat{I}(t), \hat{I}(0) \right]_- \right\rangle_0 \right)$$

It turns out:

[redacted] $$\text{Im}\ \chi^R_{II}(\omega) = (2\pi)|\bar{t}|^2 \sum_\sigma \int_{-\infty}^{\infty} d\omega' A_{d\sigma}(\omega') \left[A_{kA\sigma}(\omega' + \omega) \left(n_F(\omega' + \omega) - n_F(\omega') \right) - A_{kA\sigma}(\omega' - \omega) \left(n_F(\omega' - \omega) - n_F(\omega') \right) \right]$$

$$A_\nu(\omega) = \frac{-\ \text{Im}\ G^R_\nu(\omega)}{\pi}$$

$$-iG^<_\nu(\omega) = 2\pi A_\nu(\omega) n_F(\omega)$$

Fluctuation-dissipation theorem

$$\bar{t} = \frac{t_L t_R}{\sqrt{|t_L|^2 + |t_R|^2}}$$

Landauer-like equation (Meir-Wingreen)

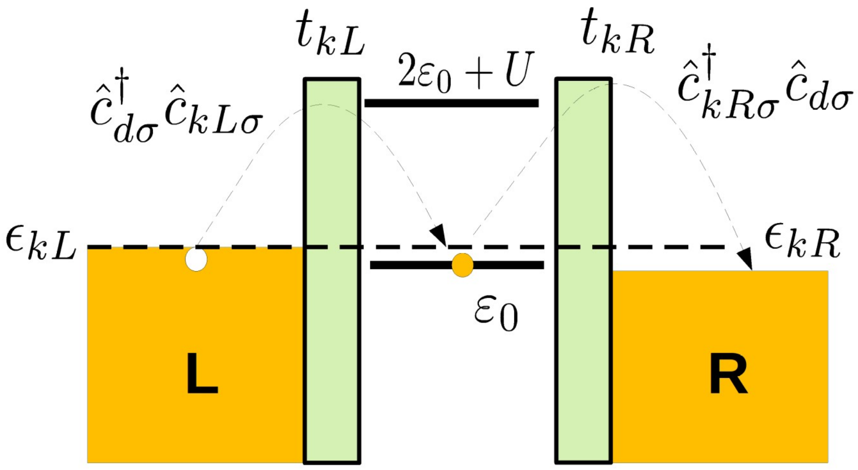
$$G = 2\pi e^2 |\bar{t}|^2 \sum_{k\sigma} A_{d\sigma}\left(\varepsilon_k\right) \left(- \frac{\partial n_F\left(\varepsilon_k\right)}{\partial \varepsilon_k} \right)$$

$$\sum_{k\sigma} \to \int d\varepsilon \rho(\varepsilon) \qquad \Gamma_\ell(\varepsilon) = 2\pi |t_\ell|^2 \rho(\varepsilon) \qquad \bar{\Gamma} = \frac{\Gamma_L \Gamma_R}{\sqrt{|\Gamma_L|^2 + |\Gamma_R|^2}}$$

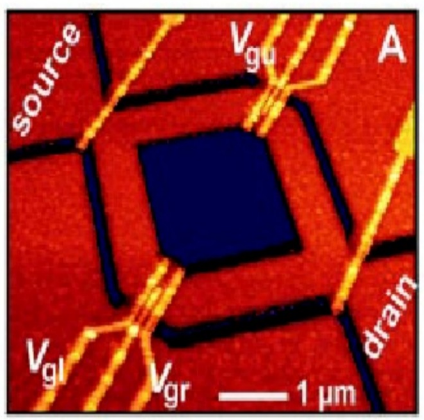
$$G = e^2 \sum_{\sigma} \int_{-\infty}^{\infty} d\varepsilon\, \bar{\Gamma}(\varepsilon) A_{d\sigma}\left(\varepsilon\right) \left(- \frac{\partial n_F\left(\varepsilon\right)}{\partial \varepsilon} \right)$$

Meir-Wingreen formula.

Application: quantum dots in the CB regime.



$$\delta\mu = \epsilon_{kL} - \epsilon_{kR}$$



van der Wiel et al., *Science* **289** 2105 (2000).