Quantum Theory of Many-Body systems in Condensed Matter (4302112) 2020

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Today's class: *Kubo formula for conductivity.*

- Current operator.
- Conductivity tensor.
- Linear response: Current-current correlation function.
- Conductance

Current operator

Hamiltonian in equilibirum: $$H_0 = \frac{1}{2m}\sum_\sigma \int d\vec{r}\,\hat{\psi}^+_\sigma(\hat{r})\left(\frac{\hbar}{i}\nabla_r + e\hat{A}(\vec{r})\right)^2 \hat{\psi}_\sigma(\vec{r})$$

In terms of the *current density*: $$\hat{H}_0 = \hat{K} + e\vec{A}\cdot\left(\mathbf{J}^\nabla + \frac{\mathbf{J}^A}{2}\right)$$

[redacted]

$$\mathbf{J}^\nabla = \frac{i\hbar}{2m}\sum_\sigma \left[\left(\nabla\psi^\dagger_\sigma(\vec{r})\right)\psi_\sigma(\vec{r}) - \psi^\dagger_\sigma(\vec{r})\left(\nabla\psi_\sigma(\vec{r})\right)\right]$$ “paramagnetic”

$$\mathbf{J}^A = \frac{e}{m}\vec{A}(\vec{r})\sum_\sigma \psi^\dagger_\sigma(\vec{r})\psi_\sigma(\vec{r}) = \frac{e}{m}\vec{A}(\vec{r})\;\rho(\vec{r})$$ “diamagnetic”

Perturbation: $$\vec{A} \to \vec{A} + \delta\vec{A} \Rightarrow \hat{H} = \hat{H}_0 + \delta\hat{H}$$

$$\delta\hat{H} = e\int d\vec{r}\,\mathbf{J}_0(r)\cdot\delta\vec{A}(\vec{r}) \qquad \mathbf{J}_0 = \mathbf{J}^\nabla + \mathbf{J}^A$$

Conductivity and linear response

Perturbation "turned on" at t_0: $\delta \mathbf{E}(\vec{r},t) = -\frac{\partial}{\partial t}\delta \vec{A}(\vec{r},t)$

Current density: $\mathbf{J}(\vec{r},t) = \mathbf{J}_0(\vec{r},t) + \frac{e}{m}\rho(\vec{r})\delta \vec{A}$

Conductivity tensor: $-e\langle \mathbf{J}(\vec{r},t)\rangle = \int d\vec{r}\,'dt'\ \boldsymbol{\sigma}(\vec{r},\vec{r}\,',t')\cdot\delta\mathbf{E}(\vec{r}\,',t')$

Let's focus on a single component: (say, x)

$$-e\langle J(\vec{r},t)\rangle = \int d\vec{r}\,'dt'\ \sigma_{xx}(\vec{r},\vec{r}\,',t')\delta E(\vec{r}\,',t')$$

$$J(\vec{r},t) = J_0(\vec{r},t) + \frac{e}{m}\rho(\vec{r})\delta A$$

$$\delta E(\vec{r},t) = -\frac{\partial}{\partial t}\delta A(\vec{r},t) \Rightarrow \delta E(\vec{r},\omega) = i\omega\delta A(\vec{r},\omega)$$

Kubo formula for the conductivity

Linear response: $$\langle J(\vec{r},t)\rangle = \langle J(\vec{r},t)\rangle_0 - i\int_{t_0}^{\infty} dt'\theta(t-t_0)\left\langle \left[J(\vec{r},t),\delta\hat{H}(t')\right]_- \right\rangle_0$$

Perturbation: $$\delta\hat{H} = e\int d\vec{r}\; J_0(r,t)\delta A(\vec{r},t)$$ with $$\langle J_0(\vec{r},t)\rangle_0 = 0$$

Equilibrium

Kubo formula for the conductivity:

$$\sigma_{xx}(\vec{r},\vec{r}\,',\omega) = \frac{ie^2}{m\omega}\langle\rho(\vec{r})\rangle_0\,\delta(\vec{r}-\vec{r}\,') + \frac{ie^2}{\omega}\chi^R_{J_0J_0}(\vec{r},\vec{r}\,',\omega)$$

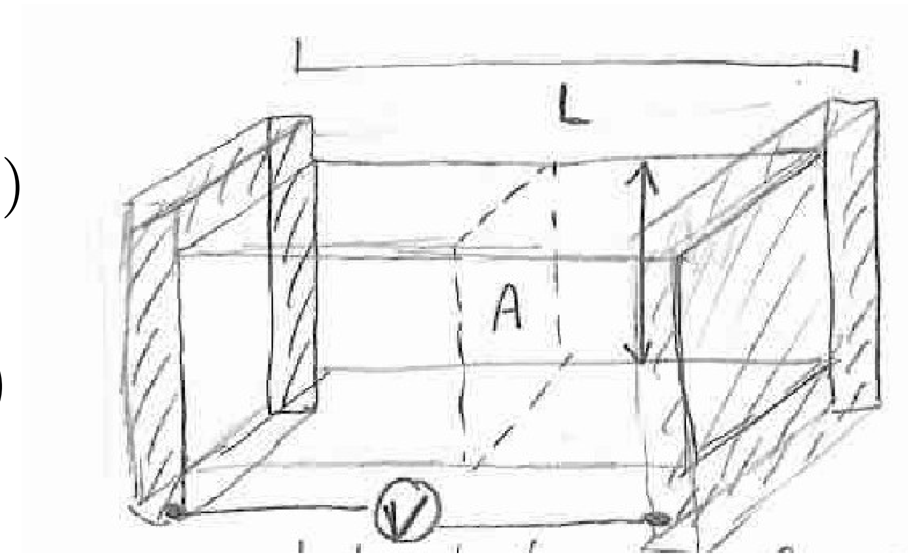
with

$$\chi^R_{J_0J_0}(\vec{r},\vec{r}\,',\omega) = \langle\langle J_0(\vec{r}) : J_0(\vec{r}\,')\rangle\rangle_\omega$$ Current-current correlation function

Conductance

Electric current:

$$\begin{cases} \vec{J}_e(\vec{r},t) = -e\vec{J}(\vec{r},t) \\ I = \int_A dA\, \vec{n}_\perp \cdot \vec{J}_e(\vec{r} \in A, t) \end{cases}$$



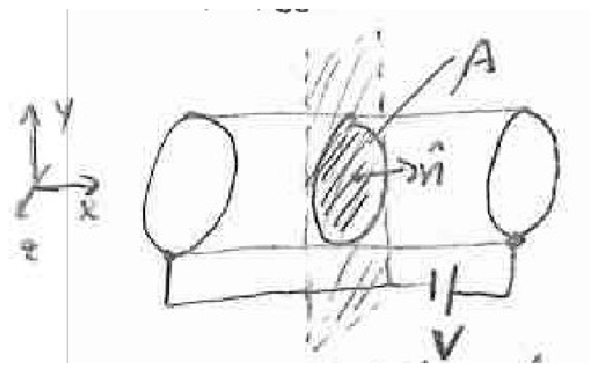
Uniform sample

$$G = \frac{I}{V} = \frac{A}{L}\sigma_{xx}(\omega = 0)$$

dc conductivity

Non-uniform samples

$$I(x) = \int_{A_x} dA\, \vec{n}_\perp \cdot \vec{J}_e(\vec{r} \in A_x)$$



Conductance

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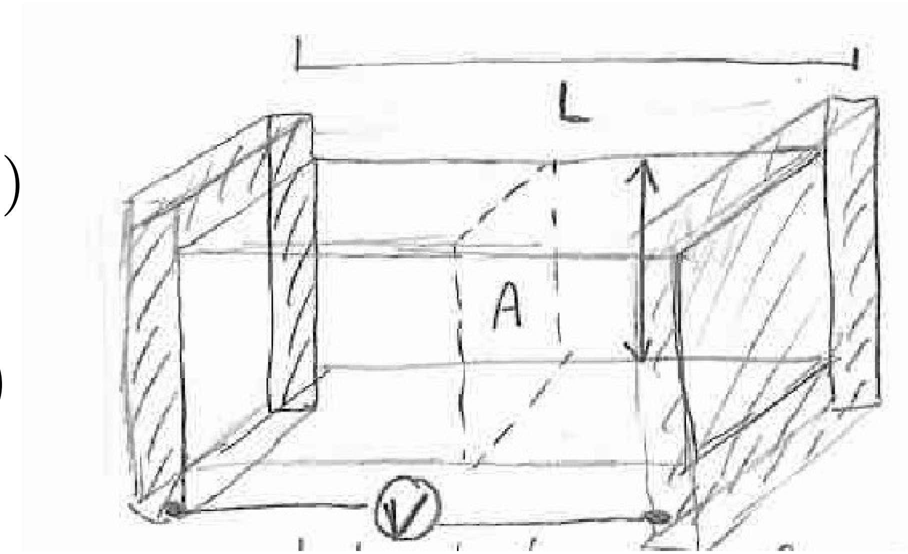

Uniform sample

$$G = \frac{I}{V} = \frac{A}{L}\sigma_{xx}(\omega = 0)$$

dc conductivity

Non-uniform samples

$$I = \int_A dA\, \vec{n}_\perp \cdot \vec{J}$$

$$I(x) = \int_0^L G(x,x')E_x(x')dx'$$

$$G(x,x') = \lim_{\omega\to 0} \mathrm{Re}\, \frac{e}{i\omega}\chi^R_{II}(x,x'\omega)E_x(x')dx'$$