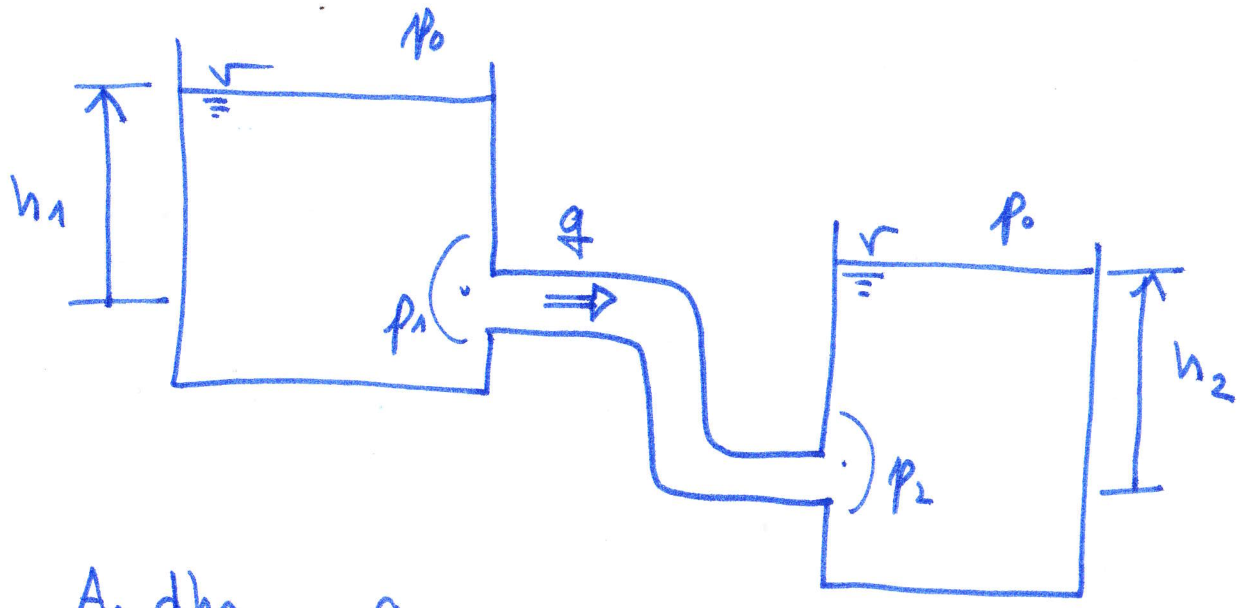


$$A_1 \frac{dh_1}{dt} = -q$$

$$A_2 \frac{dh_2}{dt} = +q$$

$$p_1 - p_2 = R q^2$$

$$p_1 = p_0 + \rho g h_1$$

$$p_2 = p_0 + \rho g h_2$$

$$R = \frac{K}{2 g A^2}$$

$$K = \left(\sum K_s\right) + K_L$$

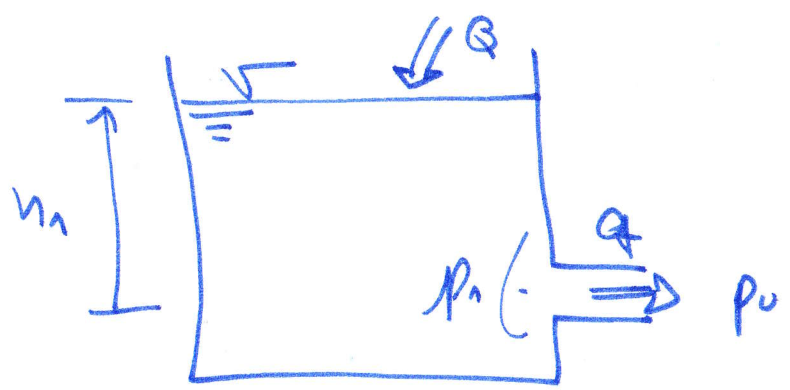
$$K_L = f \frac{L}{D}$$

$$f = f(Re, \epsilon)$$

$$\frac{1}{\sqrt{f}} = -2 \log\left(\frac{e}{3{,}7 \cdot D} + \frac{2{,}51}{Re\sqrt{f}}\right)$$

$$s = \frac{1}{\sqrt{f}}$$

Sistema Hidráulico Linearizado.



$$A_1 \frac{dh_1}{dt} = -q + Q$$

$$p_1 = p_0 + \rho g h_1$$

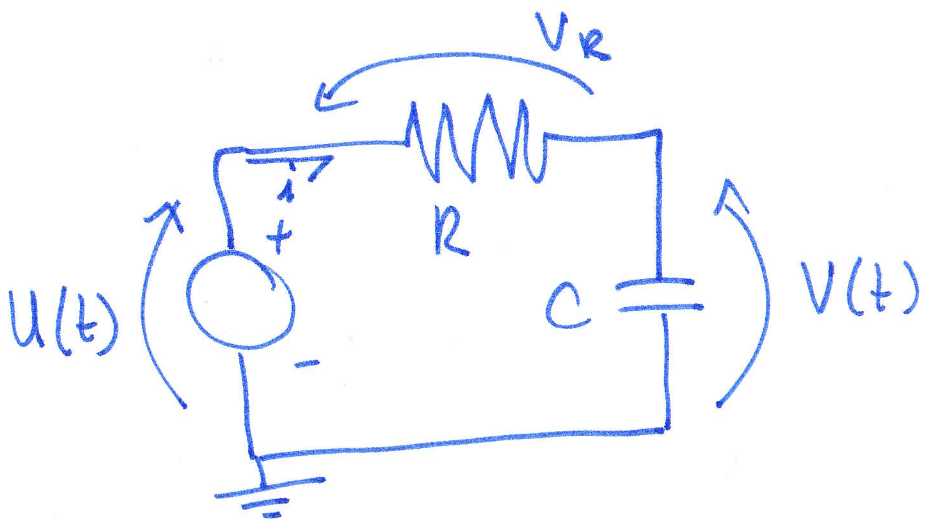
$$p_1 - p_0 = R q^2 \xrightarrow[\text{linearização}]{} p_1 - p_0 = R\, q_0\, q \Rightarrow \frac{p_1 - p_0}{R q_0} = q$$

$q_0 =$ médio (q) no ponto de operação.

$$A_1 \frac{dh_1}{dt} = -\frac{\rho g}{R q_0} h_1 + Q(t) \Rightarrow \boxed{A_1 \frac{dh_1}{dt} + \frac{\rho g}{R q_0} h_1 = Q(t)}$$

Reg. Perm: $\frac{dh_1}{dt} = 0 \Rightarrow q_0 = Q$

Sistema Elettrico:



$V_R = Ri$ resistor

$i = C \frac{dV}{dt}$ capacitor

$U = V_R + V$ Kirchhoff

$Ri + V = U(t)$

$$\boxed{RC \frac{dV}{dt} + V = U(t)}$$

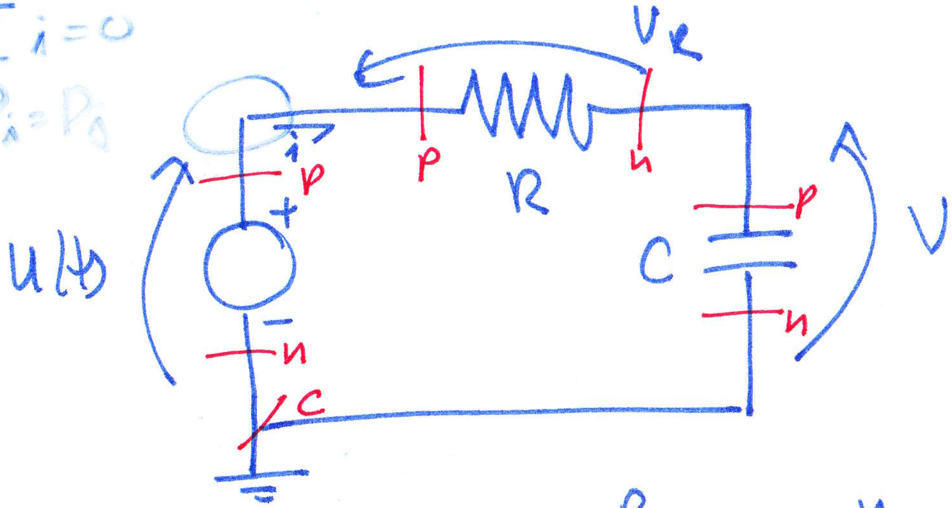
$RC \frac{dV}{dt} + V = 0 \Rightarrow V(t) = V_0 e^{-t/\tau}$ | $\tau = RC$

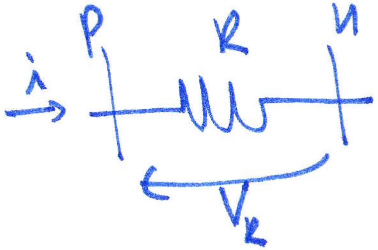
costante di tempo

$\Sigma i = 0$

$P_i = P_j$

Modelagem p/ Componentes.



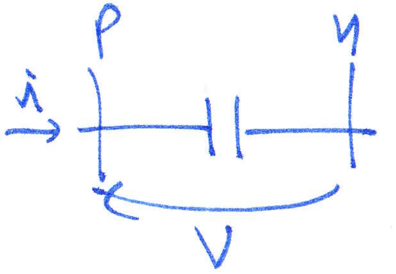


$V_R = R\,i$

$i = p.i$

$V_R = p.v - n.v$

$p.i + n.i = 0$

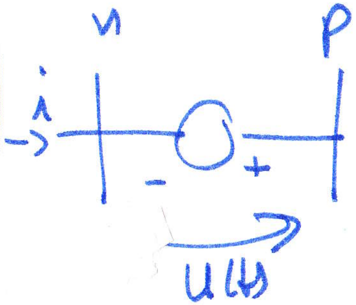


$i = C\dfrac{dV}{dt}$

$i = p.i$

$V = p.v - n.v$

$p.i + n.i = 0$

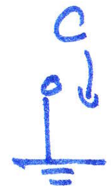


$u(t) = \begin{cases} 0 & t < t_0 \\ U_0 & t \geq t_0 \end{cases}$

$i = n.i$

$U = p.v - n.v$

$p.i + n.i = 0$



$c.v = 0$

$\forall i \in \mathbb{R}$

$$\begin{cases} m_1 c \dfrac{dT_1}{dt} + \dfrac{kA}{L}(T_1 - T_2) = P_0 \\[2ex] m_2 c \dfrac{dT_2}{dt} - \dfrac{kA}{L}(T_1 - T_2) = 0 \end{cases}$$

$$P_0 = \begin{cases} 0 & | \; t \leq t_0 \\ P_0 & | \; t > t_0 \end{cases} .$$

p/ $t_0 = 0$ //

Solução particular:

i) $T_1 = C_1$

$T_2 = C_2$ $\Rightarrow$ $\dfrac{kA}{L}(C_1 - C_2) = P_0$

$-\dfrac{kA}{L}(C_1 - C_2) = 0 \Rightarrow C_1 = C_2 \Rightarrow P_0 = 0$

$\therefore$ não é solução !!

ii) $T_1 = C_1 e^{st} \Rightarrow \dfrac{dT_1}{dt} = s\, C_1 e^{st}$

$T_2 = C_2 e^{st} \Rightarrow \dfrac{dT_2}{dt} = s\, C_2 e^{st}$

número   exp.

$$\left[\left(\frac{hA}{L} + m_1 c s\right) C_1 - \frac{hA}{L} C_2\right] e^{st} = P_0 \leftarrow \text{constante.}$$

$$\left[-\frac{hA}{L} C_1 + \left(\frac{hA}{L} + m_2 c s\right) C_2\right] e^{st} = 0$$

$e^{st} \neq 0 \quad \therefore \quad \nexists$ solução exponencial.

iii) $T_1(t) = A_1 + B_1 t \Rightarrow \dfrac{dT_1}{dt} = B_1$

$T_2(t) = A_2 + B_2 t \Rightarrow \dfrac{dT_2}{dt} = B_2$

$$m_1 c\, B_1 + \frac{kA}{L}\left(A_1 + B_1 t - A_2 - B_2 t\right) = P_0$$

$$m_2 c\, B_2 - \frac{kA}{L}\left(A_1 + B_1 t - A_2 - B_2 t\right) = 0$$

$B_1 \neq B_2 \Rightarrow$ termo esquerdo cresce linearmente $\therefore$ não é solução;

$\therefore \quad B_1 = B_2 = B$

$$\begin{cases} m_1 c B + \dfrac{kA}{L}(A_1 - A_2) = P_0 \\ m_2 c B - \dfrac{kA}{L}(A_1 - A_2) = 0 \end{cases} \qquad A_1 - A_2 = C$$

$$m_1 c B + \frac{kA}{L} C = P_0$$

$$m_2 c B - \frac{kA}{L} C = 0$$

$$(+) \quad (m_1 + m_2) c B = P_0 \quad \Rightarrow \quad B = \frac{P_0}{(m_1+m_2) c} \;//$$

$$C = \frac{L}{kA} \cdot m_2 c B \quad \Rightarrow \quad C = \frac{m_2}{m_1+m_2} \cdot \frac{L}{kA} P_0 \;//$$

$$T_1(t) = A_1 + Bt$$

$$T_2(t) = A_2 + Bt$$

t.q. $A_1 - A_2 = C$ //

$$B = \frac{P_0}{(m_1+m_2) c} \;//$$

Verificação:

$$m_1 c \frac{dT_1}{dt} + \frac{kA}{L}(T_1 - T_2) = P_0$$

$$m_1 c B + \frac{kA}{L}\overbrace{(A_1 - A_2)}^{C} = P_0$$

$$\frac{m_1 \cancel{c}\, P_0}{(m_1+m_2)\cancel{c}} + \frac{\cancel{kA}}{\cancel{L}}\,\frac{m_2}{m_1+m_2}\,\frac{\cancel{L}}{\cancel{kA}}P_0 = \frac{m_1+m_2}{m_1+m_2}P_0 = P_0 \quad //$$

$$m_2 c \frac{dT_2}{dt} - \frac{kA}{L}(T_1 - T_2) = 0$$

$$m_2 c B - \frac{kA}{L}\overbrace{(A_1 - A_2)}^{C} = 0$$

$$\frac{m_2 \cancel{c}\, P_0}{(m_1+m_2)\cancel{c}} - \frac{\cancel{kA}}{\cancel{L}}\,\frac{m_2}{m_1+m_2}\,\frac{\cancel{L}}{\cancel{kA}}P_0 = \frac{m_2-m_2}{m_1+m_2}P_0 = 0 \quad //$$

$$m_1 c \frac{dT_1}{dt} + \frac{hA}{L}(T_1 - T_2) = 0$$

$$m_2 c \frac{dT_2}{dt} - \frac{hA}{L}(T_1 - T_2) = 0$$

$$T_1(t) = C_1 e^{-st} \Rightarrow \frac{dT_1}{dt} = -s C_1 e^{-st}$$

$$T_2(t) = C_2 e^{-st} \Rightarrow \frac{dT_2}{dt} = -s C_2 e^{-st}$$

$$\left[\left(\frac{hA}{L} - m_1 c s\right) C_1 - \frac{hA}{L} C_2\right] e^{-st} = 0$$

$$\left[-\frac{hA}{L} C_1 + \left(\frac{hA}{L} - m_2 c s\right) C_2\right] e^{-st} = 0$$

$$e^{-st} \neq 0$$

$$\begin{bmatrix} \left(\frac{kA}{L} - m_1 c s\right) & -\frac{kA}{L} \\ -\frac{kA}{L} & \left(\frac{kA}{L} - m_2 c s\right) \end{bmatrix} \begin{Bmatrix} C_1 \\ C_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

solução trivial $C_1 = 0 = C_2$

$$\det\left(\begin{bmatrix} \left(\frac{kA}{L} - m_1 c s\right) & -\frac{kA}{L} \\ -\frac{kA}{L} & \left(\frac{kA}{L} - m_2 c s\right) \end{bmatrix} \right) = 0$$

$$\left(\frac{kA}{L} - m_1 c s\right)\left(\frac{kA}{L} - m_2 c s\right) - \left(\frac{kA}{L}\right)^2 = 0$$

caso $m_1 = m_2$ vem:

$$\left(\frac{kA}{L} - mcs\right)^2 = \left(\frac{kA}{L}\right)^2$$

$$\frac{kA}{L} - mcs = \pm \frac{kA}{L}$$

$$\frac{kA}{L} - mcs = \frac{kA}{L} \Rightarrow \quad s = 0 \text{ //}$$

$$\frac{kA}{L} - mcs = -\frac{kA}{L} \Rightarrow \quad mcs = \frac{2kA}{L} \Rightarrow s = \frac{2kA}{mcL} \text{ //}$$

$$\therefore \quad s_1 = 0 \text{ //}$$

$$s_2 = \frac{2kA}{mcL} \text{ //}$$

$s = s_1$

$$\begin{bmatrix} \frac{kA}{L} - mc\,0 & -\frac{kA}{L} \\ -\frac{kA}{L} & \frac{kA}{L} - mc\,0 \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$a_1 = a_2 \;//$$

$s = s_2$

$$\begin{bmatrix} \frac{kA}{L} - \cancel{mc}\,\frac{2kA}{\cancel{mc}\,L} & -\frac{kA}{L} \\ -\frac{kA}{L} & \frac{kA}{L} - \cancel{mc}\,\frac{2kA}{\cancel{mc}\,L} \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$a_1 = -a_2 \;//$$

$\therefore \quad s_1 = 0 \quad \Rightarrow \quad C_1 = C_2 = K_1$

$s_2 = \dfrac{2hA}{mcL} \quad \Rightarrow \quad C_1 = -C_2 = K_2$

$T_1(t) = K_1 e^{0t} + K_2 e^{-s_2 t}$ //

$T_2(t) = K_1 e^{0t} - K_2 e^{-s_2 t}$ //

$$\boxed{T_1(t) = K_1 + K_2 e^{-s_2 t}}$$

$$\boxed{T_2(t) = K_1 - K_2 e^{-s_2 t}}$$

Solução homogênea.

Solução Homogénea:

$$T_1(t) = K_1 + K_2 e^{-s_2 t}$$

$$T_2(t) = K_1 - K_2 e^{-s_2 t}$$

$$s_2 = \frac{2kA}{mcL}$$ //

Solução Particular:

$$T_1(t) = A_1 + Bt$$

$$T_2(t) = A_2 + Bt$$

$$A_1 - A_2 = \frac{m_2}{m_1+m_2} \frac{L}{kA} P_0$$

$$B = \frac{1}{(m_1+m_2)c} \cdot P_0$$

Solução Geral:

$$T_1(t) = (K_1 + A_1) + Bt + K_2 e^{-s_2 t}$$

$$T_2(t) = (K_1 + A_2) + Bt - K_2 e^{-s_2 t}$$

C.I :

$$T_1(0) = T_{01}$$

$$T_2(0) = T_{02}$$

$$T_{01} = (K_1 + A_1) + B \cdot 0 + K_2 e^0$$

$$T_{02} = (K_1 + A_2) + B \cdot 0 - K_2 e^0$$

$$T_{01} = (K_1 + A_1) + K_2$$

$$T_{02} = (K_1 + A_2) - K_2$$

$$T_{01} - T_{02} = (A_1 - A_2) + 2K_2 \Rightarrow \boxed{C = T_{01} - T_{02} - 2K_2}$$

$$2K_2 = (T_{01} - T_{02}) - \frac{m_2}{m_1 + m_2} \frac{L}{kA} P_0$$

$$K_2 = \frac{T_{01} - T_{02}}{2} - \frac{m_2}{2(m_1 + m_2)} \frac{L}{kA} P_0$$

$$T_{01} + T_{02} = 2K_1 + A_1 + A_2$$

$$A_1 - A_2 = \frac{m_2}{m_1+m_2} \frac{L}{hA} p_0$$

$K_1 = 0$
(livre)

$$2A_1 = T_{01} + T_{02} + \frac{m_2}{m_1+m_2} \frac{L}{hA} p_0$$

$$A_1 = \frac{T_{01}+T_{02}}{2} + \frac{m_2}{2(m_1+m_2)} \frac{L}{hA} p_0 \quad //$$

$$A_2 = \frac{T_{01}+T_{02}}{2} - \frac{m_2}{2(m_1+m_2)} \frac{L}{hA} p_0 \quad //$$

Pauth!

$$T_1(t) = \frac{T_{01}+T_{02}}{2} + \frac{m_2}{2(m_1+m_2)} \frac{L}{kA} P_0$$

$$+ \frac{1}{(m_1+m_2)c} t P_0 + \left[\frac{T_{01}-T_{02}}{2} - \frac{m_2}{2(m_1+m_2} \frac{L}{kA} P_0\right] e^{-s_2 t}$$

$$T_2(t) = \frac{T_{01}+T_{02}}{2} - \frac{m_2}{2(m_1+m_2)} \frac{L}{kA} P_0$$

$$+ \frac{1}{(m_1+m_2)c} t P_0 - \left[\frac{T_{01}-T_{02}}{2} - \frac{m_2}{2(m_1+m_2)} \frac{L}{kA} P_0\right] e^{-s_2 t}$$