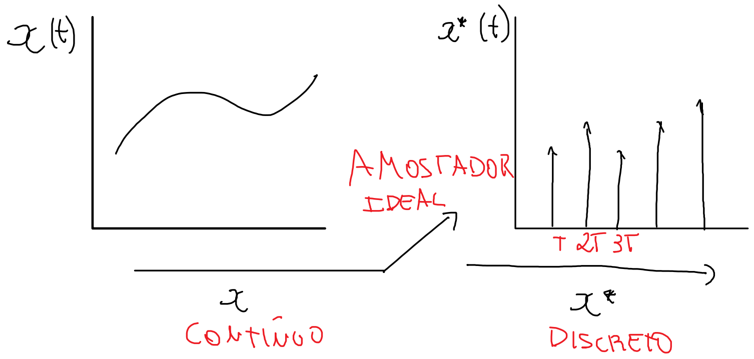
Mapeamento Contínuo-Discreto

quinta-feira, 17 de setembro de 2020 18:00

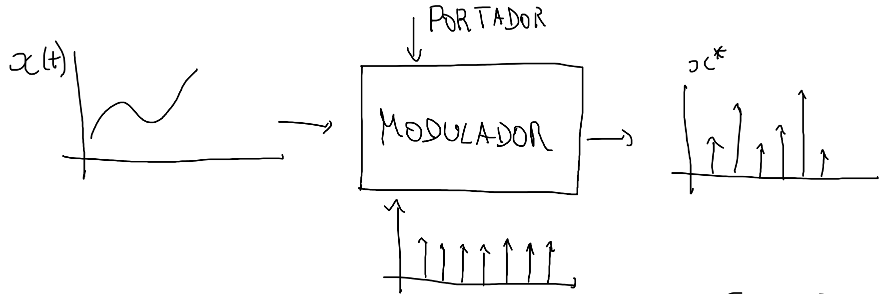
PROVAR QUE

$$\boxed{z = e^{Ts}}$$



$$x^*(t) = \sum_{0}^{\infty} x(KT) \cdot \delta(t - KT)$$

$$x^*(t) = x(0)\,\delta(0) + x(T) \cdot \delta(t-T) + \ldots + x(KT)\,\delta(t-KT)$$



$$X^*(s) = \mathcal{L}\left[x^*(t)\right] = x(0) \cdot \underbrace{\mathcal{L}\left[\delta(t)\right]} + x(T)\,\mathcal{L}\left[\delta(t-T)\right] + \ldots$$

$$= x(0) + x(T) \cdot e^{-Ts} + x(2T) \cdot e^{-2Ts} \ldots$$

$$\boxed{X^*(s) = \sum_{K=0}^{\infty} x(KT) \cdot e^{-KTs}} \rightarrow \text{TRANSF. LAPLACE } x^*$$

$$\boxed{X(z) = \sum_{K=0}^{\infty} x(KT) \cdot z^{-K}} \rightarrow \text{TRANSF Z } x^*$$

SE $\boxed{e^{Ts} = z}$ OU $\boxed{s = \frac{1}{T} \ln z}$

$$\boxed{X^*(s)\Big|_{s=\frac{1}{T}\ln z} = X(z)}$$

Transformada de Laplace do sinal amostrado é igual à Transformada Z do sinal original, com a substituição s=1/ln(z) ou z=exp(Ts)

PENSANDO "FISICAMENTE"

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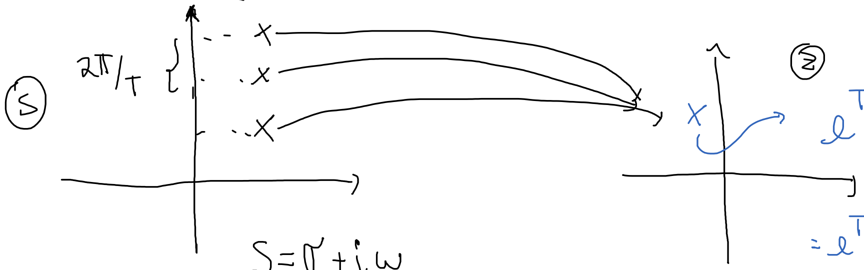
z^{-1} = ATRASO DE T

e^{-Ts} = ATRASO DE T

$\Rightarrow z^{-1} = e^{-Ts}$

MAPEAMENTO PLANO s ↔ PLANO z

$$z = e^{Ts}$$

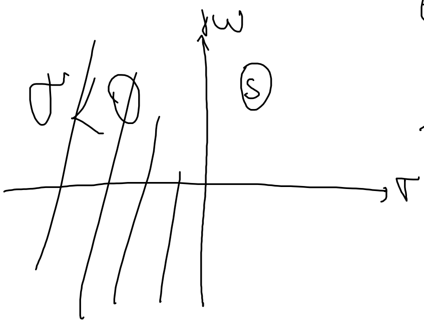


$$e^{T(\sigma + j\omega)} = e^{T\sigma} \cdot e^{T\omega j} = e^{T\sigma} \cdot e^{(T\omega + 2\pi k) j}$$

$$s = \sigma + j\omega$$

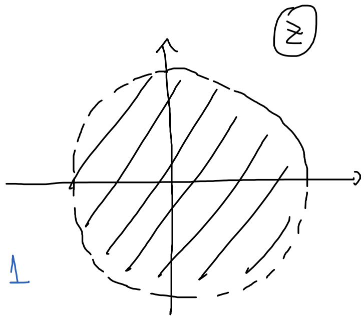
VÁRIOS POLOS COM PARTE IMAGINÁRIA DISTANCIADAS 2π/T EM s → MAPEADOS EM ÚNICO POLO z

ESTABILIDADE

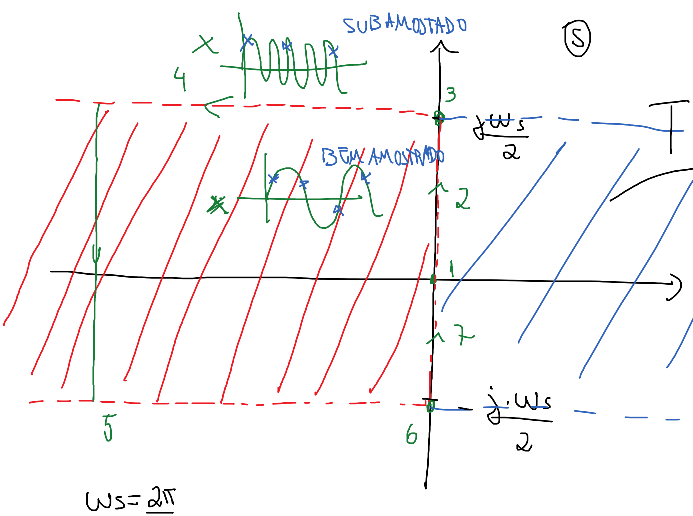


$$z = e^{T(\sigma + j\omega)} = e^{T\sigma} \cdot \overbrace{e^{Tj\omega}}^{1}$$

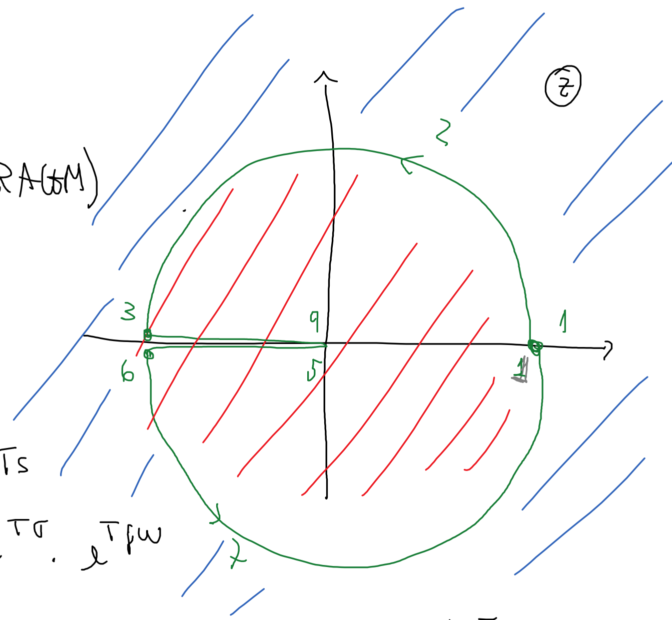
$|z| = e^{T\sigma} \Rightarrow$ se $\sigma < 0 \Rightarrow |z| < 1$ (ESTAB.)



FAIXA PRINCIPAL



T (AMOSTRAGEM)



$z = e^{Ts}$

$z = e^{T\sigma} \cdot e^{Tj\omega}$

$\omega_s = \frac{2\pi}{T}$

$\frac{\omega_s}{2}$ = METADE FREQ AMOSTRAGEM = ω_N

TODA FAIXA PRINCIPAL DEFINIDA DE $-\frac{j\omega_s}{2}$ A $+\frac{j\omega_s}{2}$ É MAPEADA NO INTERIOR CÍRCULO UNITÁRIO

$-\frac{j\omega_s}{2}$ A $+\frac{j\omega_s}{2}$ É MAPEADA NO CÍRCULO UNITÁRIO SE $\sigma < 0$, E NO EXTERIOR DO CÍRCULO UNITÁRIO SE $\sigma > 0$

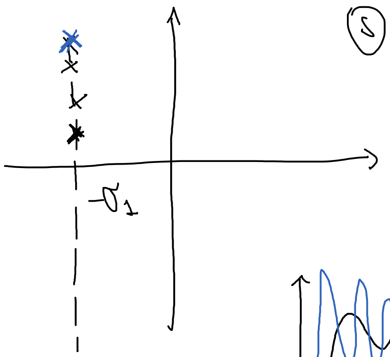
OBS) SE O SISTEMA ESTIVER DEVIDAMENTE PROJETADO

$\Rightarrow \quad \omega_{MAX} < \frac{1}{2}\omega_s$ (TEOREMA DE NYQUIST)

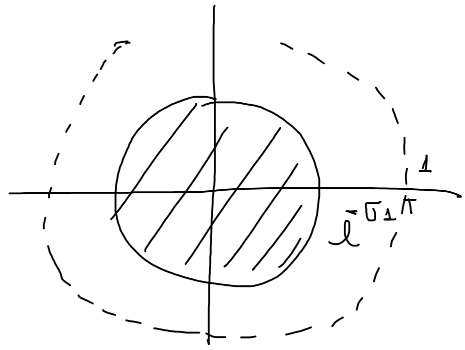
$\Rightarrow$ ESTAREMOS SEMPRE NA FAIXA PRINCIPAL

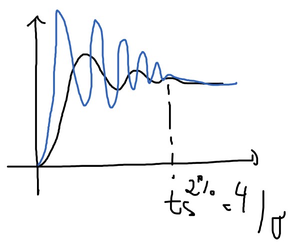
LUGARES GEOMÉTRICOS

1) ATENUAÇÃO CONSTANTE

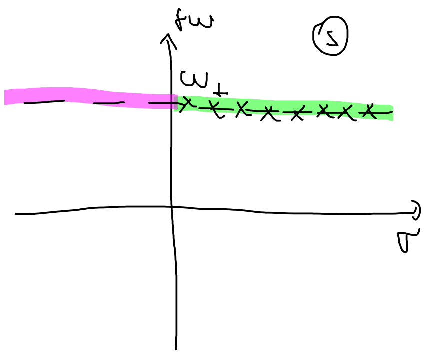


$z = e^{-\sigma_1 T} \cdot e^{j\omega T}$ $0 \sim \infty$

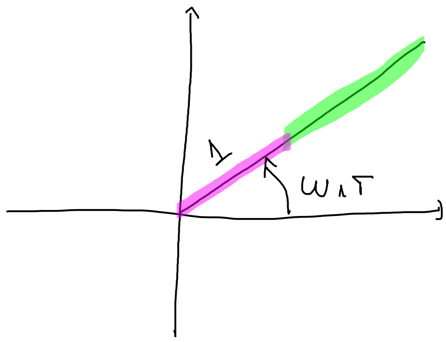




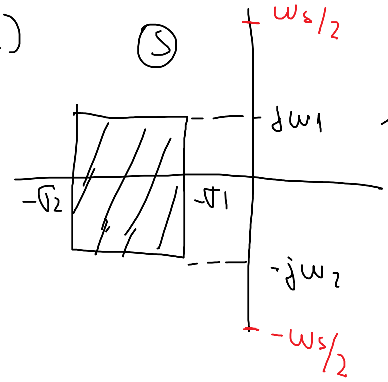
2) FREQ CONSTANTE

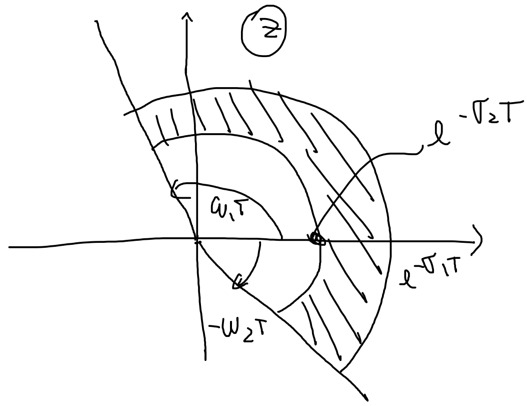


$e^{T\sigma} \cdot e^{j\omega_1 T}$

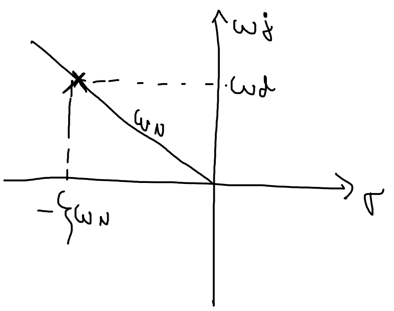


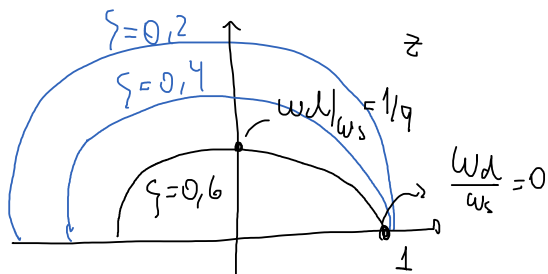
Ex)

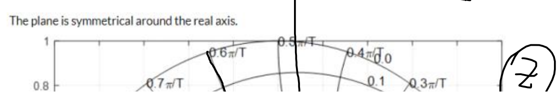




3) AMORT CONSTANTE

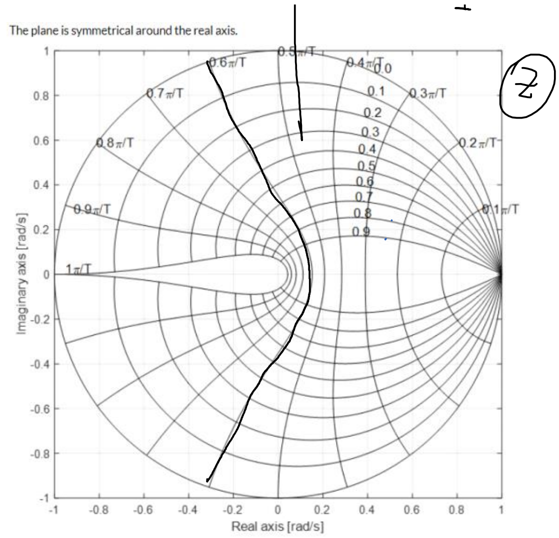




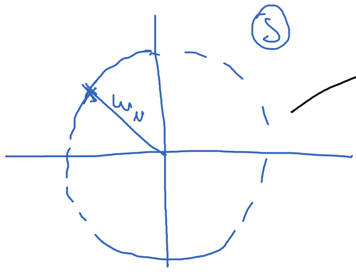


$$\begin{cases} s = -\zeta\omega_N + j\omega_N\sqrt{1-\zeta^2} \\ z = e^{Ts} \end{cases}$$

$\zeta = cte$ e $\omega_N \in (0, \infty)$



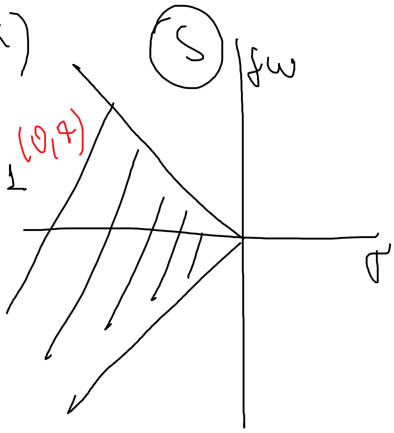
4) FREQ NATURAL CONSTANTE

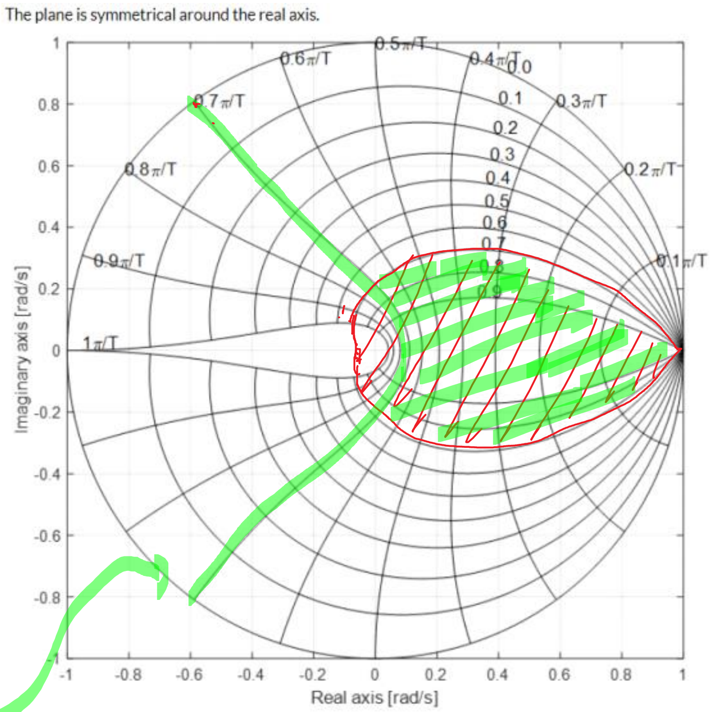


CHEGA NAS LINHAS AO LADO

Ex)

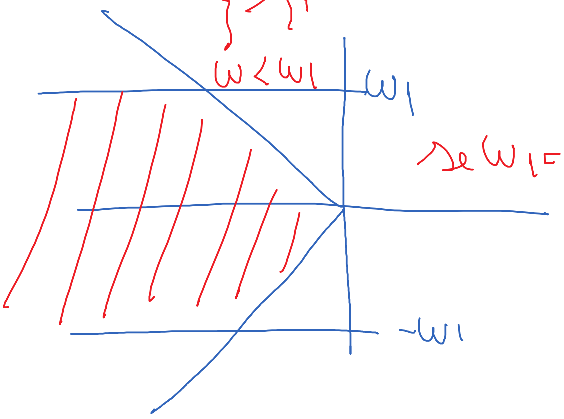
$\zeta > \zeta_1$ (0,7)





$\zeta > \zeta_1$

$\omega < \omega_1$



se $\omega_1 = \dfrac{0{,}7\pi}{T}$