Aula 14: The Johansen Approach

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Outline

✓ Introduction

Structure of a CGE Model

The Johansen Approach

Stylized Johansen Model

What is a CGE?

Computable, based on data

It has many sectors

And perhaps many regions, primary factors and households

A big database of matrices

Many, simultaneous, equations (hard to solve)

Prices guide demands by agents

Prices determined by supply and demand

Trade focus: elastic foreign demand and supply

General features of CGE models

CGE models include equations specifying:
- market-clearing conditions for commodities and primary factors;
- producers' demands for produced inputs and primary factors;
- final demands (investment, household, export and government);
- the relationship of prices to supply costs and taxes;
- various macroeconomic variables and price indices.

Neo-classical flavor
- demand equations consistent with optimizing behavior (cost minimization, utility maximization);
- competitive markets: producers price at marginal cost.

CGE simplifications

Not much dynamics (leads and lags)

An imposed structure of behavior, based on theory

Neoclassical assumptions (optimizing, competition)

Nesting (separability assumptions)

Why: time series data for huge matrices cannot be found

Theory and assumptions (partially) replace econometrics

What is a CGE model good for?

Analyzing policies that affect different sectors in different ways

The effect of a policy on different:
- Sectors
- Regions
- Factors (Labor, Land, Capital)
- Household types

Policies (tariff or subsidies) that help one sector a lot, and harm all the rest a little

What-if questions

What if productivity in agriculture increased 1%?

What if foreign demand for exports increased 5%?

What if consumer tastes shifted towards imported food?

What if CO_2 emissions were taxed?

What if water became scarce?

A great number of exogenous variables (tax rates, endowments, technical coefficients)

Comparative static models: results show effect of policy shocks only, in terms of changes from initial equilibrium

Comparative-static interpretation of results

Results refer to changes at *some* future point in time
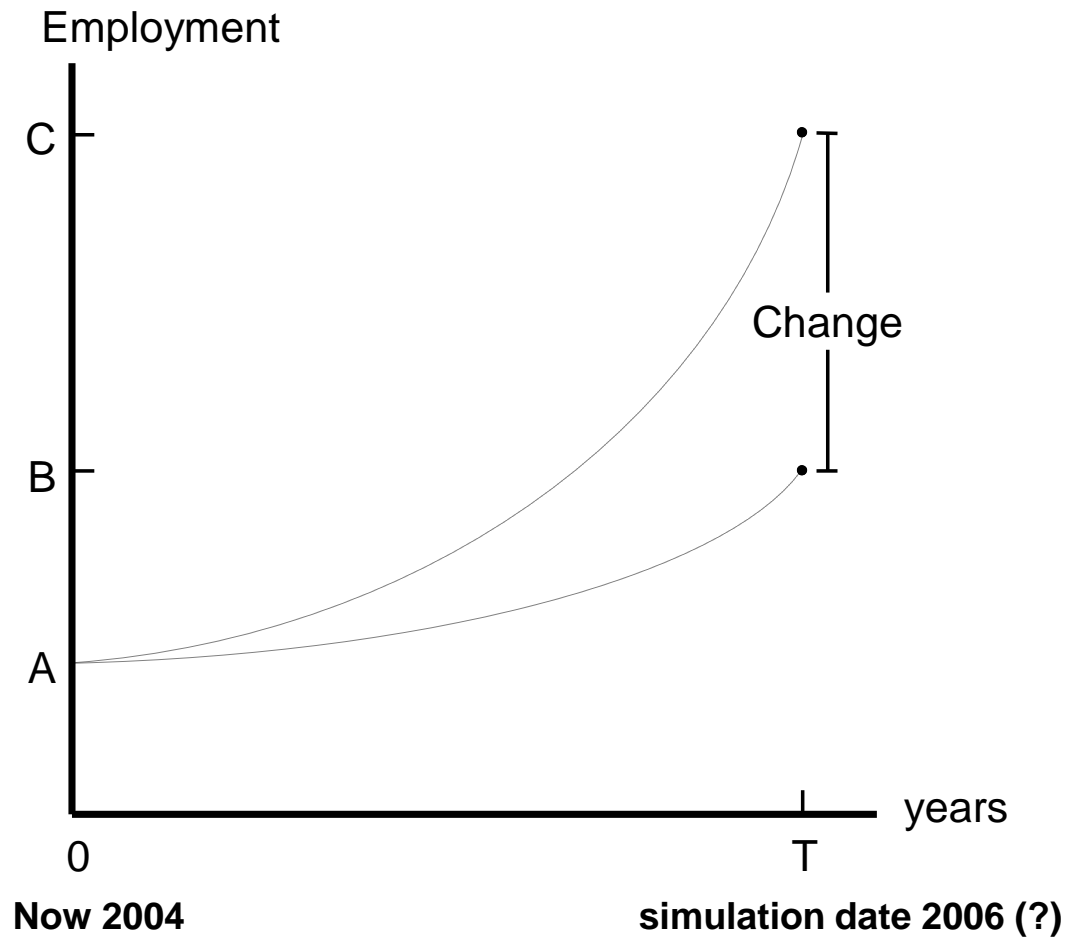
Regional modeling

Intense interest in regional results

Policies which are good for nation but bad for one region may not be politically feasible

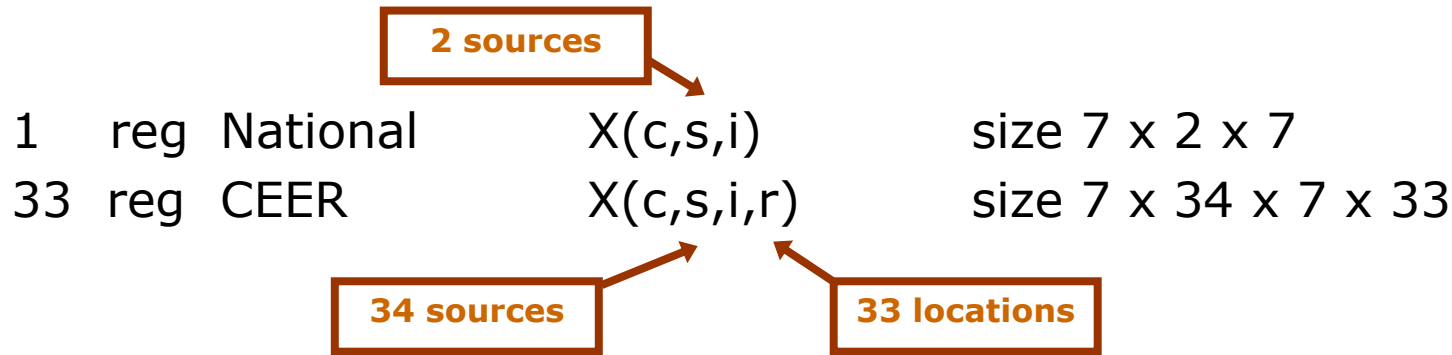Assistance to one region may harm nation

Regional modeling is the largest part of Fipe's Brazilian contract work

- More customers:
  1 national government but 27 state governments

Two approaches: **Bottom-Up** or Top-Down

The bottom-up approach

Simply add a regional subscript (or two) to each variable and data

2 sources

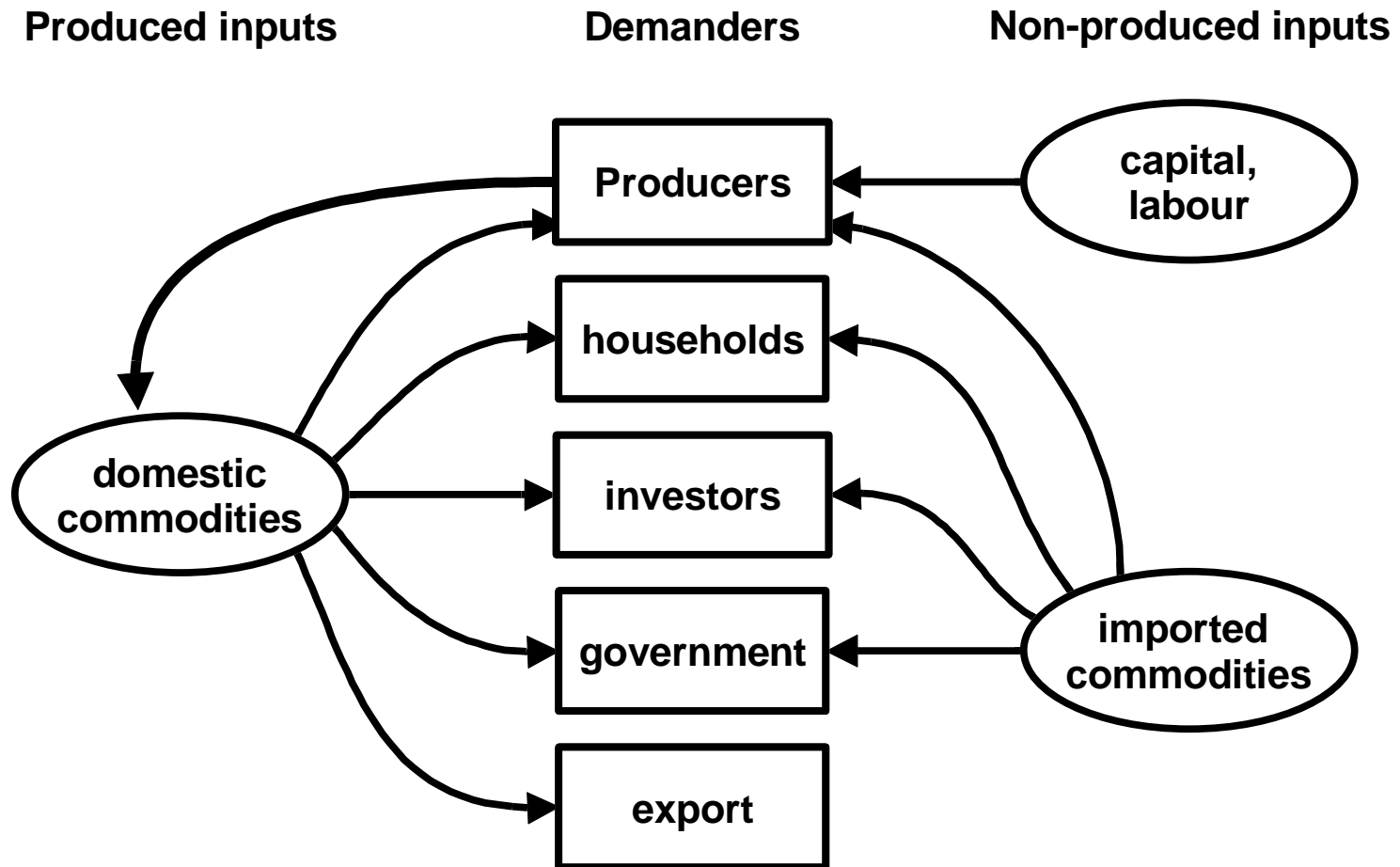|				
1	reg	National	$X(c,s,i)$	size 7 x 2 x 7
33	reg	CEER	$X(c,s,i,r)$	size 7 x 34 x 7 x 33

34 sources   **33 locations**

Database has grown by factor of [34/2]*33 = 561

Number of variables also 561 times bigger

Solve time and memory needs move with SQUARE of model size
(so model needs 300,000 times as much memory and takes 300,000
times longer to solve)

Stylized GE model: material flows

You will learn...

How microeconomic theory – cost-minimizing, utility-maximizing – underlies the equations;

The use of nested production and utility functions:

How input-output data is used in equations;

How model equations are represented in percent change form;

How choice of exogenous variables makes model more flexible;

CGE models mostly similar, so skills will transfer.

Outline

Introduction

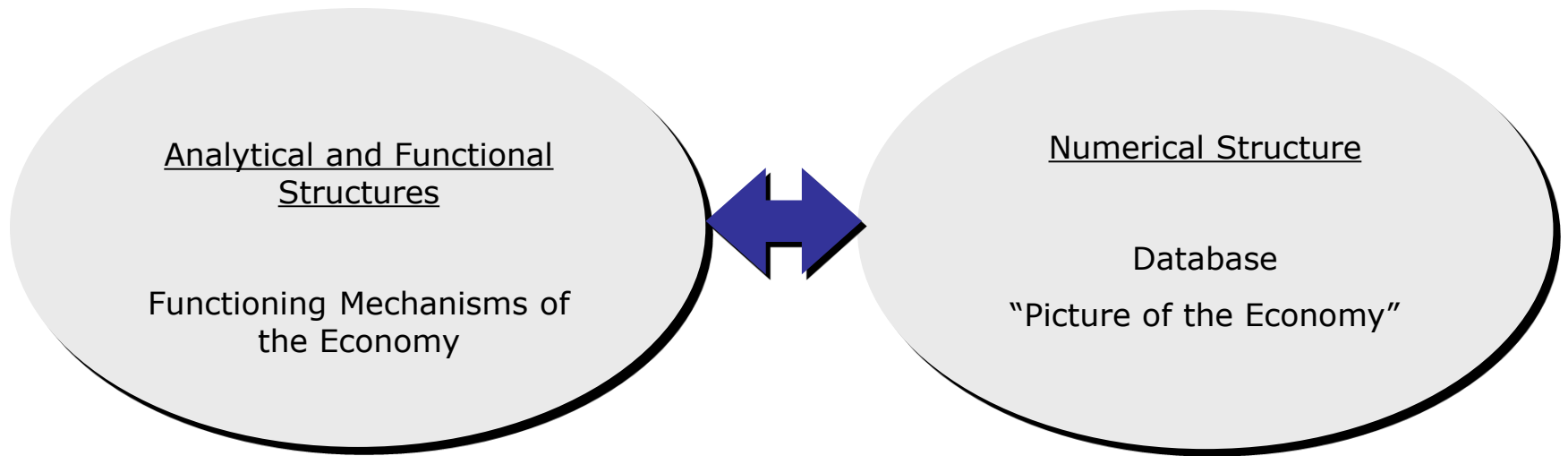✓ **Structure of a CGE Model**

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CGE models – Definition

Analytical and Functional Structures

Functioning Mechanisms of the Economy

Numerical Structure

Database

"Picture of the Economy"

Nested structures in CGE models

In each industry: Output = function of inputs:

output = F (inputs) = F (Labor, Capital, Land, dom. goods, imp. goods)

Separability assumptions simplify the production structure:

output = F (primary factor composite, composite goods)

where: primary factor composite = CES (Labor, Capital, Land)

labor = CES (Various skill grades)

composite good (i) = CES (domestic good (i), imported good (i))

All industries share common production structure.
BUT: Input proportions and behavioral parameters vary.

Nesting is like staged decisions:
<u>First</u> decide how much leather to use – based on output.
<u>Then</u> decide import/domestic proportions, depending on the relative prices of local and foreign leather.
Each nest requires 2 or 3 equations.

Activity Level

Leontief — **top nest**

Good 1 — Good G — Primary Factors — 'Other Costs'

CES — CES — CES — **primary factor nest**

Domestic Good 1 — Imported Good 1 — Domestic Good G — Imported Good G

Armington nest

Land — Labour — Capital

Work upwards

CES — **skill nest**

KEY

Functional Form

Inputs or Outputs

Labour type 1 — Labour type 2 — up to — Labour type O
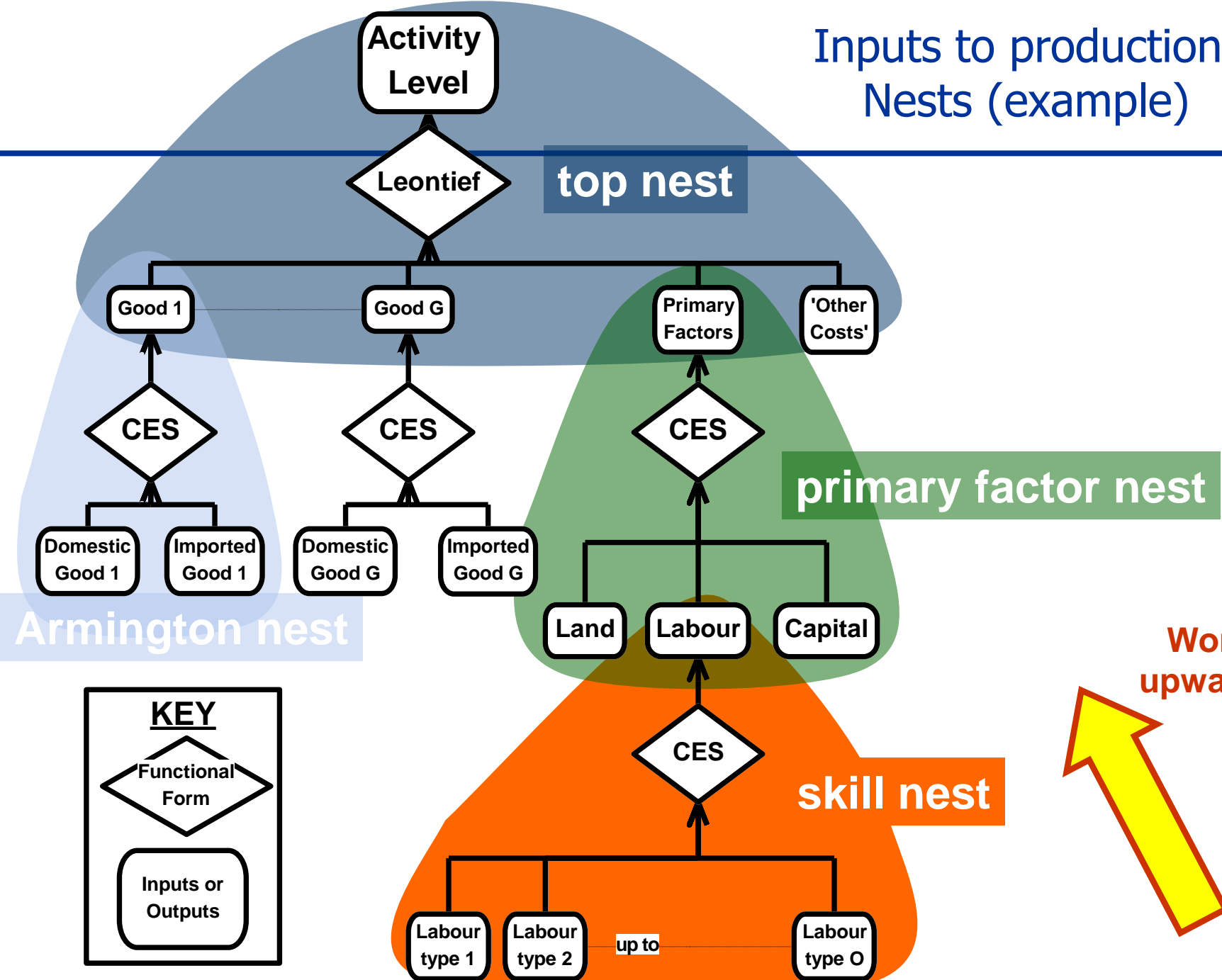
Example 1: CES skill substitution

Skilled X_s

$$X^\alpha = X_s{}^\alpha + X_u{}^\alpha$$

$$0 < \alpha < 1$$

R

C

A

X=15

X=10

B

Cost=$6

Cost=$9

UnSkilled X_u

Effect of price change

Unskilled wages fall relative to skilled wages

Skilled X_s

PR1

PR2

X=10

A

B

A → B

Unskilled X_u
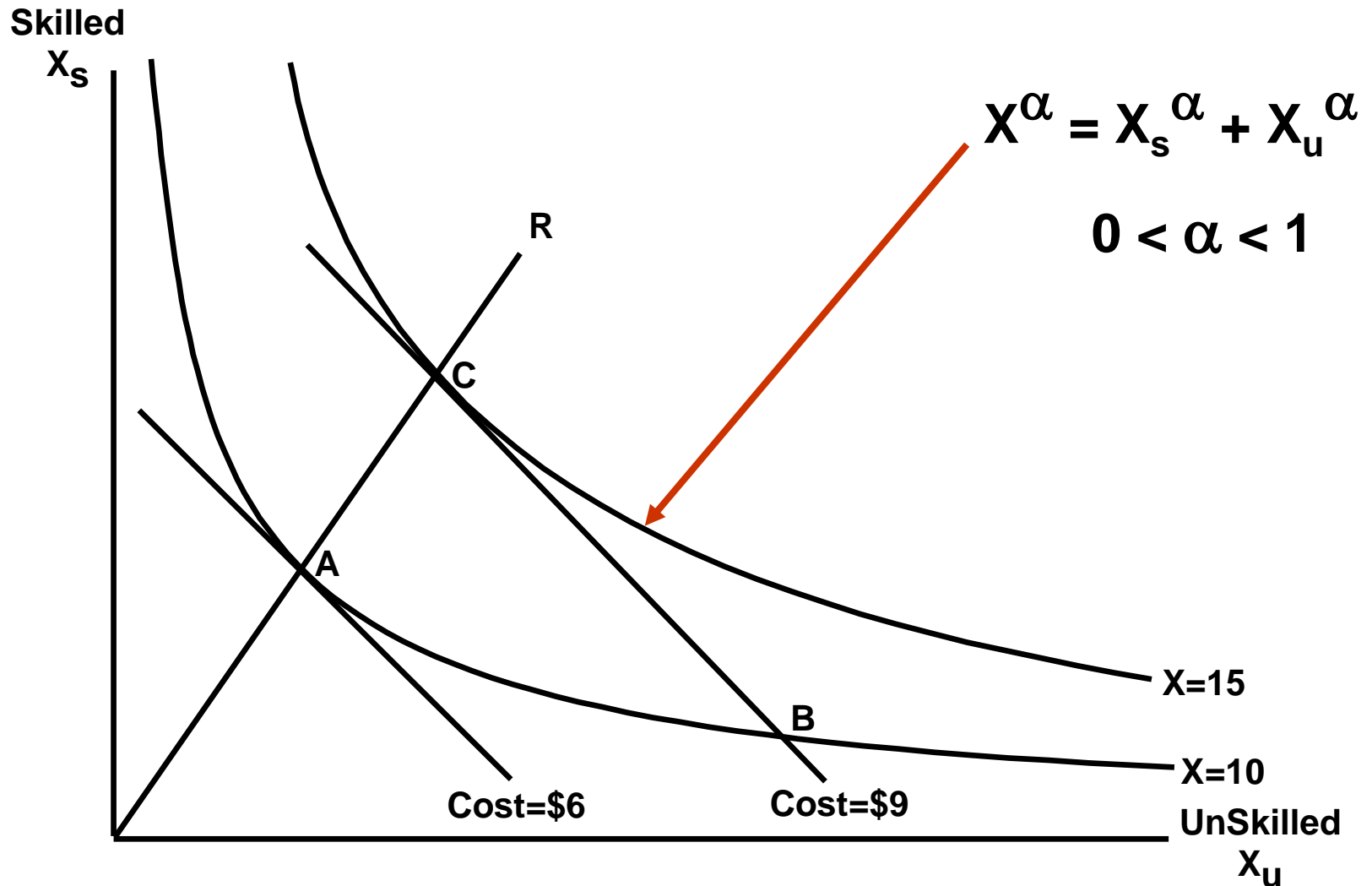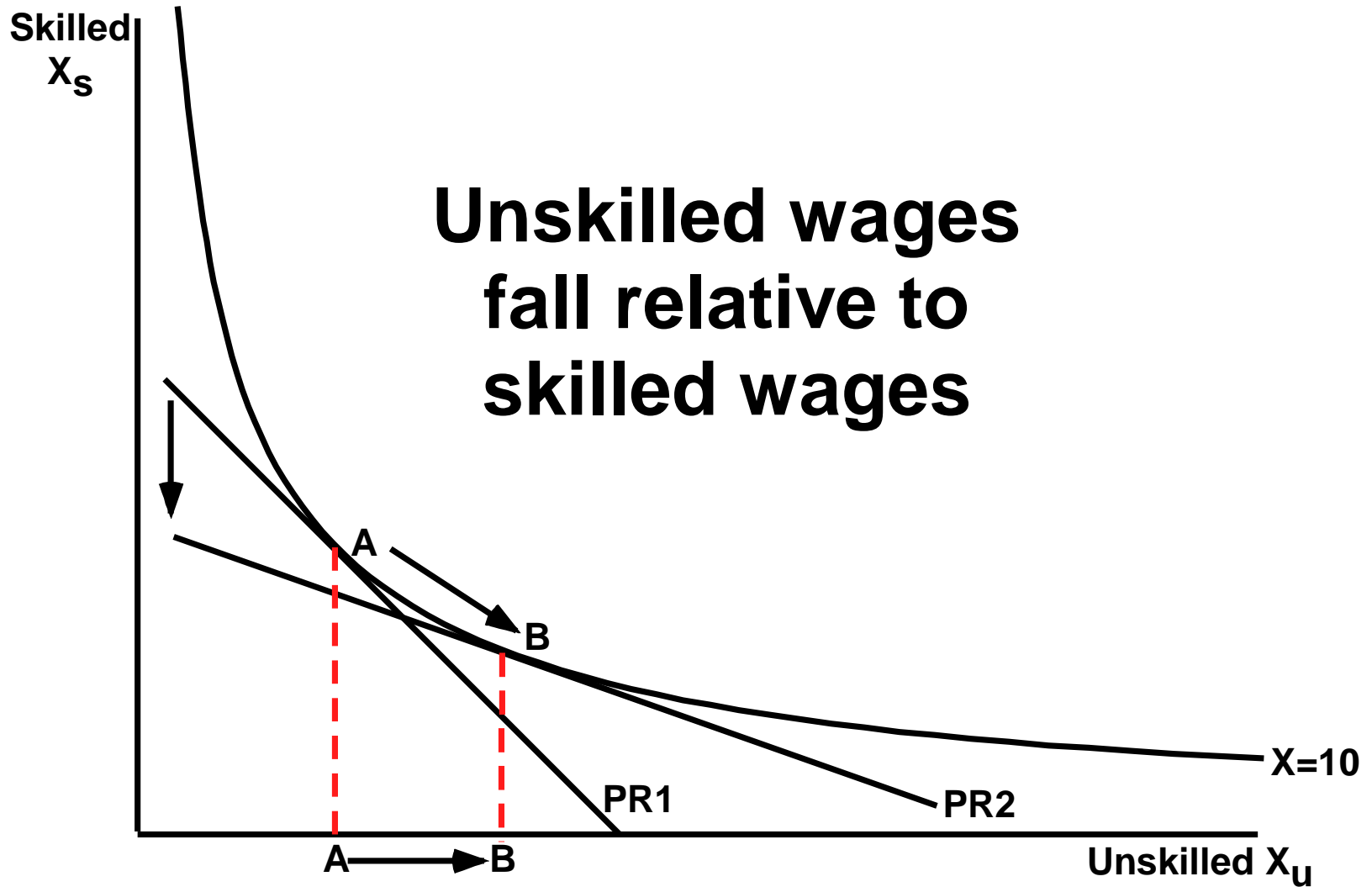
Example 2: Substitution between domestic and imported sources

KEY

Functional Form | Inputs or Outputs

Good 1 — up to — Good C

CES

CES
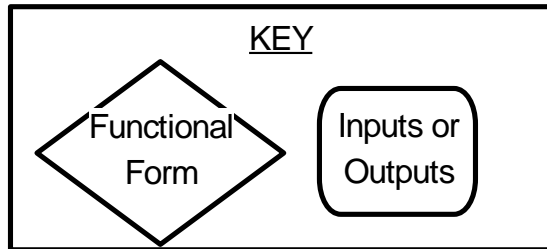
Armington hypothesis:

CES aggregation for different sources

Good-specific parameters

Same structure for all users

Domestic Good 1 | Imported Good 1

Domestic Good C | Imported Good C
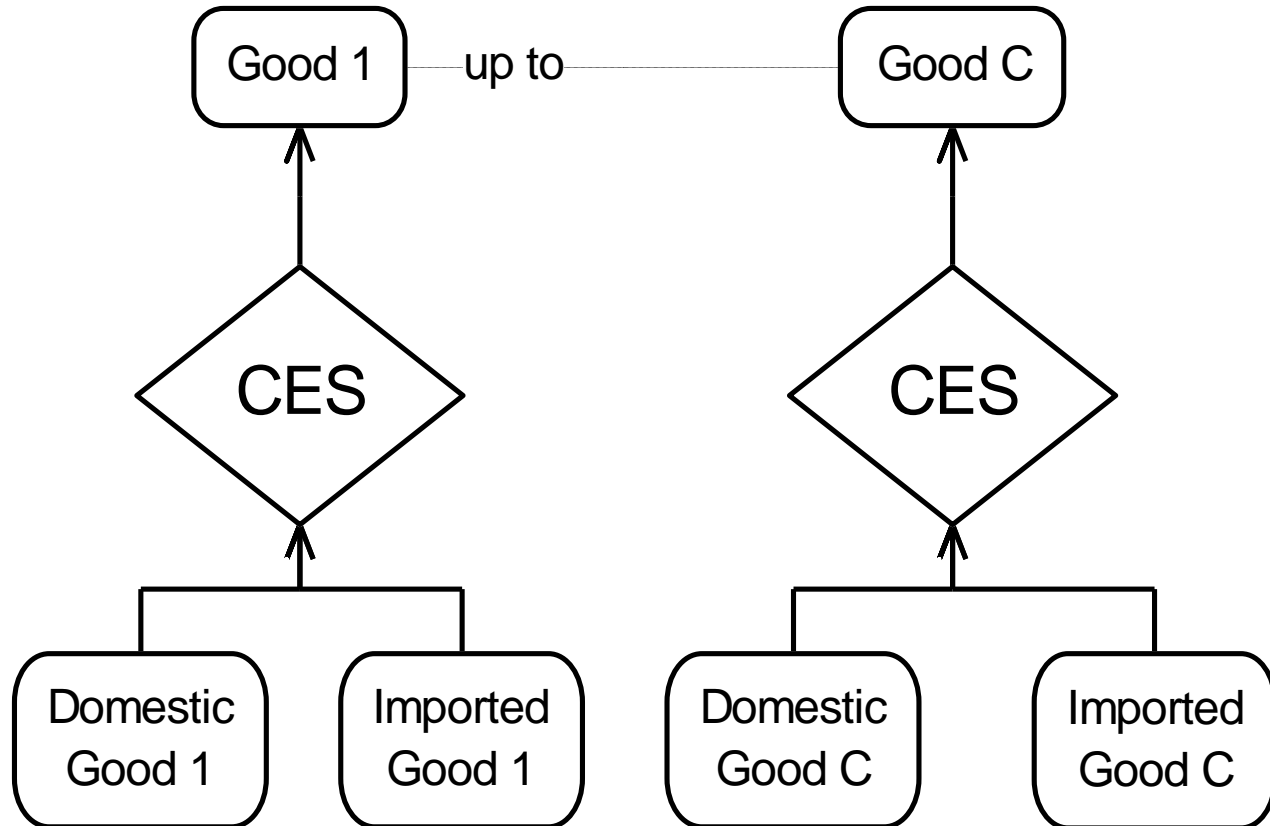
Numerical example

$p = S_d p_d + S_m p_m$ average price of manufactures dom. e imp.

$x_d = x - \sigma(p_d - p)$ demand for domestic manufactures

$x_m = x - \sigma(p_m - p)$ demand for imported manufactures

Let $p_m = -10\%$; $x = p_d = 0$; $S_m = 0.3$ and $\sigma = 2$

Then, <u>cheaper imports</u> imply:

$p = -0.3(-10\%) = -3\%$ *decrease* in the average price of imp. mnf.

$x_d = -2[0 - (-3\%)] = -6\%$ *decrease* in domestic demand

$x_m = -2[-10\% - (-3\%)] = 14\%$ *increase* in import volume

Effects on domestic sales are proportional to S_m e σ

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Johansen models

Class of general equilibrium models in which a equilibrium is a vector *V*, of length *n* satisfying a system of equations

$$F(V) = 0 \qquad\qquad\qquad (1)$$

where *F* is a vector function of length *m*.

We assume that *F* is differentiable and that the number of variables, *n*, exceeds the number of equations *m*.

Johansen's approach is to derive from (1) a system of linear equations in which the variables are changes, percentage changes or changes in the logarithms of the components of *V*.

Illustrative computations

We will assume that the system (1) consists of 2 equations and 3 variables and has the form

$$\begin{cases} V_1^2 V_3 - 1 = 0 \\ V_1 + V_2 - 2 = 0 \end{cases}$$

V_1 and V_2 (endogenous); V_3 (exogenous)

$$\begin{cases} V_1 = V_3^{-1/2} \\ V_2 = 2 - V_3^{-1/2} \end{cases}$$

Illustrative computations (cont.)

Initial solution: $V^I = (V_1^I, V_2^I, V_3^I) = (1, 1, 1)$

What is the effects on V_1 and V_2 of a shift in V_3 from 1 to 1.1?

$$V_1 = 0.9535$$
$$V_2 = 1.0465$$

Johansen approach:
- Complexity and size of the system (1) normally rule out the possibility of deriving from it explicitly solution equations
- Solve a linearized version of (1)

Step by step

Derive from (1) a differential form

$$A(V)v = 0 \qquad\qquad (2)$$

v is usually interpreted as showing percentage changes or changes in the logarithms of the variables V.

Johansen-style computations make use of a initial solution, V^I, with results being reported usually as percentage deviations from this initial solution.

$$A(V) \rightarrow A(V^I) \quad \text{fixed matrix (\"model\")}$$

Derivation of (2) is by total differentiation of (1)

$$\begin{bmatrix} 2V_1V_3 & 0 & V_1^2 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} dV_1 \\ dV_2 \\ dV_3 \end{bmatrix} = 0$$

Se $V = V^I$,

$$A(V^I)v = 0 \tag{3}$$

$$\begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} dV_1 \\ dV_2 \\ dV_3 \end{bmatrix} = 0$$

On choosing variable 3 to be exogenous:

$$\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} dV_1 \\ dV_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} dV_3 = 0$$

$$A_\alpha(V^I)v_\alpha + A_\beta(V^I)v_\beta = 0 \qquad (4)$$

$$\begin{bmatrix} dV_1 \\ dV_2 \end{bmatrix} = -\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 1 \\ 0 \end{bmatrix} dV_3$$

$$v_\alpha = -A_\alpha^{-1}(V^I)A_\beta(V^I)v_\beta \qquad (5)$$

$$\begin{bmatrix} dV_1 \\ dV_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0.5 \end{bmatrix} dV_3$$

$$v_\alpha = B(V^I)v_\beta \qquad (6)$$

Any difference?

<u>Before</u>: V_3↑ 10% → V_1↓ 4.65%, V_2↑ 4.65%

<u>Now</u>: V_3↑ 10% → V_1↓ 5%, V_2↑ 5%

Differences are due to linearization errors…

The operations give us the values of the derivatives or elasticities only for the initial values, V^I, of the variables.

When we move away from V^I, the derivatives or elasticities will change.

In percentage change form:

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = - \begin{bmatrix} 2 & 0 \\ 0.5 & 0.5 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} v_3$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0.5 \end{bmatrix} v_3$$

Linearization error
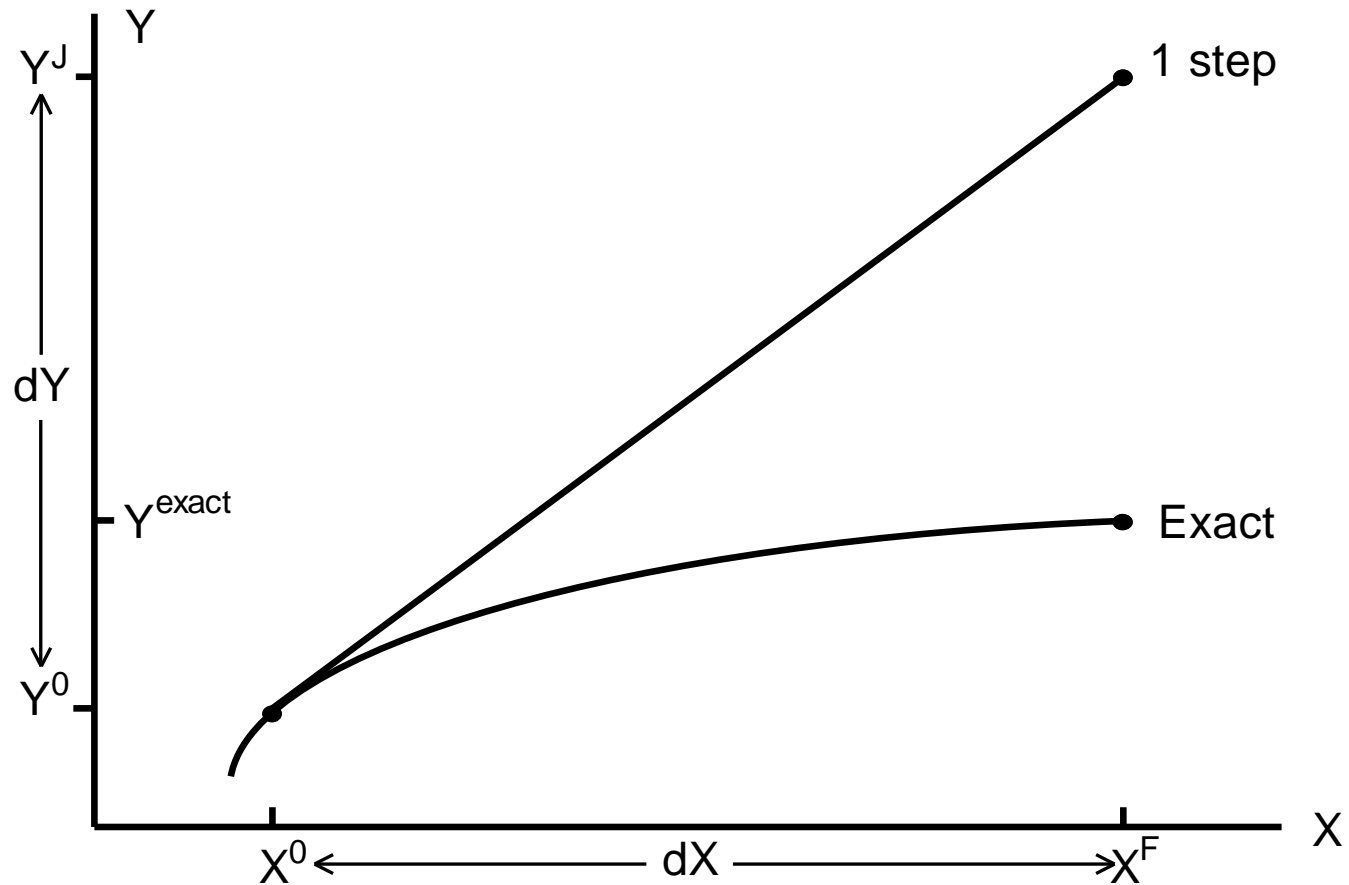
Y^J is *Johansen* estimate
Error is proportionately less for smaller changes

Breaking large changes in X into a number of steps
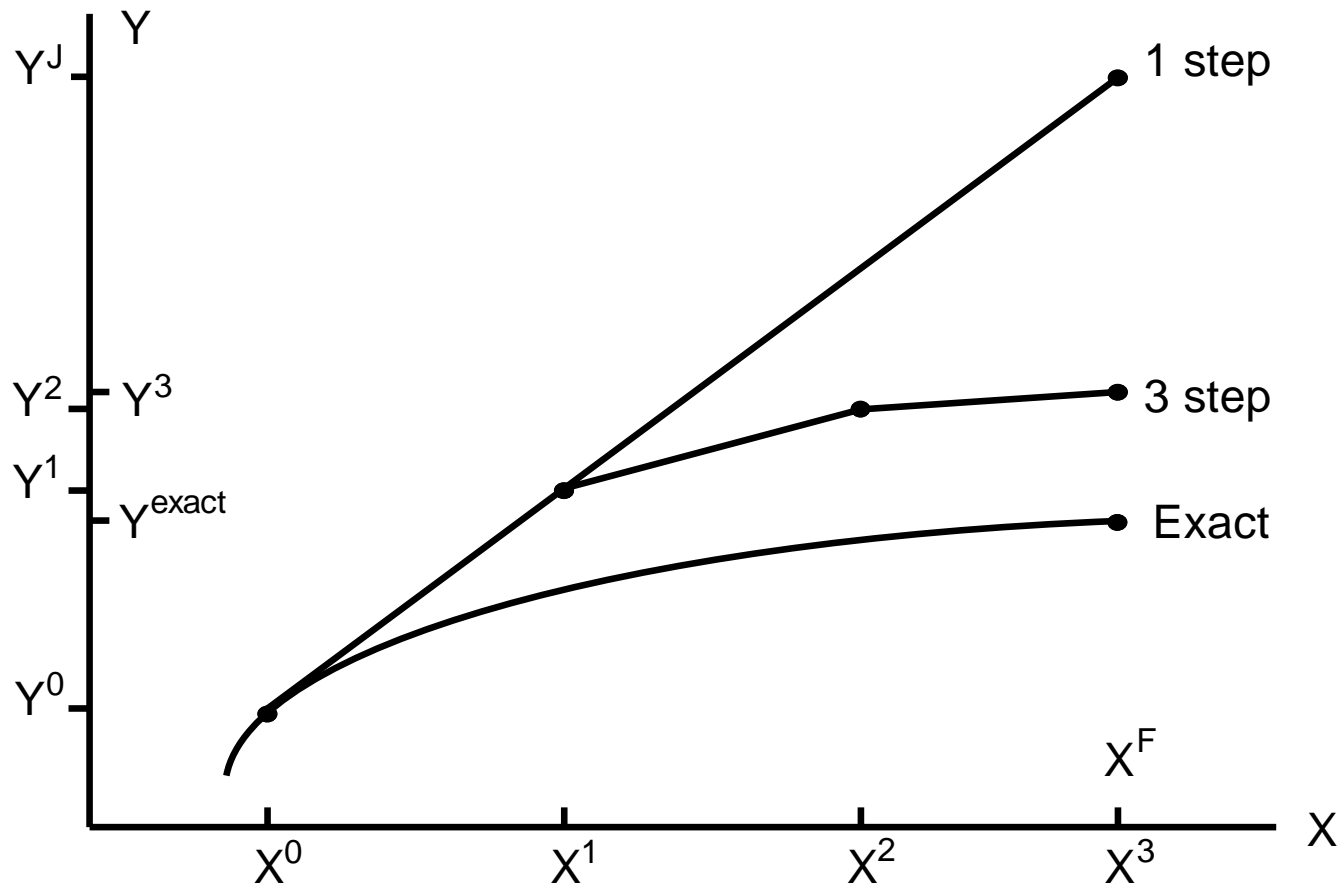
Multi-step process to reduce linearization error

Extrapolating from Johansen and Euler approximations

Method	y	Error
Johansen (1-step)	150%	50%
Euler 2-step	125%	25%
Euler 4-step	112.3%	12.3%
Euler ∞-step (exact)	100%	0

The error follows a rule (*arbitrary* polynomial approximation of continuous function)

Use results from 3 approximate solutions to estimate exact solution + error bound

Summary

1. We start with the model's equations represented in their levels form

2. The equations are linearized: take total differential of each equation

3. Total differential expressions converted to (mostly) % change form

4. Linear equations evaluated at initial solution to the levels model

5. Exogenous variables chosen; model then solved for movements in endogenous variables, given user-specified values for exogenous variables

But, a problem: *linearization error*

Multi-step, extrapolation

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Implementation

1. Development of a theoretical structure

2. Linearization of the model equations

3. Use of input-output data to provide estimates for the relevant cost and sales shares

4. Development of a flexible computer program to manipulate the linear system

Structure of the model

Two good/sectors (1 and 2)

Two primary factors (3 and 4)

One final user – household (0)

Assumptions

Household sector (0)

$$\max_{X_{i0}} \; U = X_{10}^{\alpha_{10}} X_{20}^{\alpha_{20}} \quad \text{subject to} \quad P_1 X_{10} + P_2 X_{20} = Y$$

Producers (j = 1,2)

$$\min_{X_{ij}} \; C_j = \sum_{i=1}^{4} P_i X_{ij} \quad \text{subject to} \quad X_j = A_j X_{1j}^{\alpha_{1j}} X_{2j}^{\alpha_{2j}} X_{3j}^{\alpha_{3j}} X_{4j}^{\alpha_{4j}}$$

$$A_j > 0, \; \sum_{i=1}^{4} \alpha_{ij} = 1$$

Assumptions (cont.)

Value of output equals the value of inputs

$$C_j = P_j X_j = \sum_{i=1}^{4} X_{ij} P_i, \quad j = 1,2$$

Equilibrium conditions

$$\sum_{j=0}^{2} X_{ij} = X_i, \quad i = 1,2$$

$$\sum_{j=1}^{2} X_{ij} = X_i, \quad i = 3,4$$

Household budget equals factor income

Structural form

Share of input i
in total costs of
industry j

$$X_{i0} = \frac{\alpha_{i0}Y}{P_i}, \quad i = 1,2 \tag{1}$$

$$X_{ij} = \frac{\left(\alpha_{ij} Q_j X_j \prod_{t=1}^{4} P_t^{\alpha_{tj}} \right)}{P_i}, \quad i = 1, \cdots, 4 \quad j = 1,2 \tag{2}$$

$$P_j = Q_j \prod P_t^{\alpha_{tj}}, \quad j = 1,2 \tag{3}$$

Structural form (cont.)

$$\sum_{j=0}^{2} X_{ij} = X_i, \quad i = 1,2 \tag{4}$$

$$\sum_{j=1}^{2} X_{ij} = X_i, \quad i = 3,4 \tag{5}$$

$$P_1 = 1 \tag{6}$$

Variables:

$$X_{i0} \quad i = 1,2$$

$$X_{ij} \quad i = 1,\cdots,4 \ \ j = 1,2$$

$$P_i \quad i = 1,\cdots,4$$

$$X_i = 1,\cdots,4$$

$$Y$$

Percent-change equations – examples

Levels form: $A = B + C$

Ordinary
change form: $\Delta A = \Delta B + \Delta C$

Convert to % $A(100.\Delta A/A) = B(100.\Delta B/B) + C(100.\Delta C/C)$
change form: $A\ \underline{a} \qquad\qquad = B\ \underline{b} \qquad\qquad + C\ \underline{c}$

Typically two ways of expressing % change form

Intermediate form: $A\,a = B\,b + C\,c$

Percentage change (share) form: $a = S_b\,b + S_c\,c$
                                               where $S_b = B/A$; $S_c = C/A$

Percent-change equations – examples

Levels form: $A = B\,C$

Ordinary
change form: $\Delta A = \Delta B\,C + \Delta C\,B$

Convert to % $A(100.\Delta A/A) = BC(100.\Delta B/B) + BC(100.\Delta C/C)$
change form: $A\quad a\qquad\quad = BC\quad b\qquad + BC\quad c$

$\qquad\qquad\qquad a\qquad\quad = \qquad b\qquad + \qquad c$

PRACTICE: $X = F\,P^{\varepsilon}$

- Ordinary change and percent change are both linearized
- Linearized equations easier for computers to solve
- % change equations easier for economists to understand: elasticities

Percent-change numerical example

Levels form $Z = X*Y$

Ordinary Change form $\Delta Z = Y*\Delta X + X*\Delta Y$ $[+ \Delta X \Delta Y]$

2nd-order

Multiply by 100: $100*\Delta Z = 100*Y*\Delta X + 100*X*\Delta Y$

Define x = % change in X, so $X*x = 100\Delta X$

$$Z*z = X*Y*x + X*Y*y$$

Divide by $Z=X*Y$ to get:

Percent Change form $z = x + y$

Initially   X=4, Y=5, so Z = X*Y = 20
Suppose x=25%, y=20% [ie, X:4$\Rightarrow$5, Y:5$\Rightarrow$6]
Linear approximation z = x + y gives z = 45%
True answer: 30 = 5*6… = 50% more than original 20
Error 5% is 2nd order term: z = x+y $+ x*y/100$

Note: reduce shocks by a factor of 10, error by factor of 100

Linearized version

$$x_{i0} = y - p_i \quad i = 1,2$$

$$x_{ij} = x_j - \left(p_i - \sum_{t=1}^{4} \alpha_{tj} p_t \right) \quad i = 1, \cdots, 4 \quad j = 1,2$$

$$p_j = \sum_{t=1}^{4} \alpha_{tj} p_t \quad j = 1,2$$

Variables:

$$\sum_{j=0}^{2} x_{ij} \beta_{ij} = x_i \quad i = 1,2$$

$$\sum_{j=1}^{2} x_{ij} \beta_{ij} = x_i \quad i = 3,4$$

$$p_1 = 0$$

$$
\begin{array}{|l|}
\hline
x_{i0} \quad i = 1,2 \\
x_{ij} \quad i = 1, \cdots, 4 \quad j = 1,2 \\
p_i \quad i = 1, \cdots, 4 \\
x_i = 1, \cdots, 4 \\
y \\
\hline
\end{array}
$$

$$\boxed{\beta_{ij} = X_{ij} / X_i, \quad i = 1, \ldots, 4 \quad j = 0,1,2}$$

Initial solution

Input-output data

Convention: one unit of good or factor *i* is the amount which costs $1 in the base period

Parameters of the structural form: $\alpha_{i0}, \alpha_{ij}, Q_j$

Calibration guarantees that structural form equations are satisfied by the suggested values for the parameters

Matrix $A(V^I)$

Matrix $A(V^I)$ can be evaluated using the input-output data

Equivalent to evaluate the coefficients of the model!

Exercise: representation of the matrix $A(V^I)$ of the linearized system

Linearized version

$$x_{i0} = y - p_i \quad i = 1,2$$

$$x_{ij} = x_j - \left(p_i - \sum_{t=1}^{4} \alpha_{tj} p_t \right) \quad i = 1, \cdots, 4 \quad j = 1,2$$

$$p_j = \sum_{t=1}^{4} \alpha_{tj} p_t \quad j = 1,2$$

Variables:

$$\sum_{j=0}^{2} x_{ij} \beta_{ij} = x_i \quad i = 1,2$$

$$\sum_{j=1}^{2} x_{ij} \beta_{ij} = x_i \quad i = 3,4$$

$$p_1 = 0$$

$$\begin{array}{ll} x_{i0} & i = 1,2 \\ x_{ij} & i = 1,\cdots,4 \quad j = 1,2 \\ p_i & i = 1,\cdots,4 \\ x_i & = 1,\cdots,4 \\ y & \end{array}$$

$$\beta_{ij} = X_{ij} / X_i, \quad i = 1,...,4 \quad j = 0,1,2$$

Input-output data

Table E3.3.1
Input-Output Data (Flows in dollars)

		Industry		Households	Total Sales
		1	2		
Commodity	1	4	2	2	8
	2	2	6	4	12
Primary Factors	Labor 3	1	3		4
	Capital 4	1	1		2
Production		8	12	6	

17 columns

Table E3.4.2

Answer to Exercise 3.4: The Transpose of the Matrix $A(V^I)$ for the Stylized Johansen Model*

19 rows

	(E3.2.1)		(E3.2.2)										(E3.2.3)		(E3.2.4)	(E3.2.5)	(E3.2.6)
y	-1	-1															
x_{10}	1												-.25				
x_{20}		1												-.3			
x_{11}			1										-.5				
x_{21}				1										-.16			
x_{31}					1										-.25		
x_{41}						1										-.5	
x_{12}							1						-.25				
x_{22}								1						-.5			
x_{32}									1						-.75		
x_{42}										1						-.5	
x_1			-1	-1	-1	-1							1				
x_2							-1	-1	-1	-1				1			
x_3															1		
x_4																1	
p_1	1		.5	-.5	-.5	-.5	.83	-.16	-.16	-.16	.5	-.16					1
p_2		1	-.25	.75	-.25	-.25	-.5	.5	-.5	-.5	-.25	.5					
p_3			-.125	-.125	.875	-.125	-.25	-.25	.75	-.25	-.125	-.25					
p_4			-.125	-.125	-.125	.875	-.083	-.083	-.083	.916	-.125	-.083					

*   For typographical convenience we have listed the columns of $A(V^I)$ as rows. Numbers of the form .83, .16, etc. are to be read as .8333..., .1666..., etc.

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The model

A(VI)

	y	x10	x20	x11	x21	x31	x41	x12	x22	x32	x42	x1	x2	x3	x4	p1	p2	p3	p4
	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
	-1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
	0	0	0	1	0	0	0	0	0	0	0	-1	0	0	0	0.5	-0.25	-0.125	-0.125
	0	0	0	0	1	0	0	0	0	0	0	-1	0	0	0	-0.5	0.75	-0.125	-0.125
	0	0	0	0	0	1	0	0	0	0	0	-1	0	0	0	-0.5	-0.25	0.875	-0.125
	0	0	0	0	0	0	1	0	0	0	0	-1	0	0	0	-0.5	-0.25	-0.125	0.875
	0	0	0	0	0	0	0	1	0	0	0	0	-1	0	0	0.833	-0.5	-0.25	-0.0833
	0	0	0	0	0	0	0	0	1	0	0	0	-1	0	0	-0.166	0.5	-0.25	-0.0833
	0	0	0	0	0	0	0	0	0	1	0	0	-1	0	0	-0.166	-0.5	0.75	-0.0833
	0	0	0	0	0	0	0	0	0	0	1	0	-1	0	0	-0.166	-0.5	-0.25	0.9166
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.5	-0.25	-0.125	-0.125
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-0.166	0.5	-0.25	-0.0833
	0	-0.25	0	-0.5	0	0	0	-0.25	0	0	0	1	0	0	0	0	0	0	0
	0	0	-0.333	0	-0.166	0	0	0	-0.5	0	0	0	1	0	0	0	0	0	0
	0	0	0	0	0	-0.25	0	0	0	-0.75	0	0	0	1	0	0	0	0	0
	0	0	0	0	0	0	-0.5	0	0	0	-0.5	0	0	0	1	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0

Aalfa

	y	x10	x20	x11	x21	x31	x41	x12	x22	x32	x42	x1	x2	p1	p2	p3	p4	Abeta	x3	x4
	-1	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0		0	0
	-1	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0		0	0
	0	0	0	1	0	0	0	0	0	0	0	-1	0	0.5	-0.25	-0.125	-0.125		0	0
	0	0	0	0	1	0	0	0	0	0	0	-1	0	-0.5	0.75	-0.125	-0.125		0	0
	0	0	0	0	0	1	0	0	0	0	0	-1	0	-0.5	-0.25	0.875	-0.125		0	0
	0	0	0	0	0	0	1	0	0	0	0	-1	0	-0.5	-0.25	-0.125	0.875		0	0
	0	0	0	0	0	0	0	1	0	0	0	0	-1	0.833	-0.5	-0.25	-0.0833		0	0
	0	0	0	0	0	0	0	0	1	0	0	0	-1	-0.166	0.5	-0.25	-0.0833		0	0
	0	0	0	0	0	0	0	0	0	1	0	0	-1	-0.166	-0.5	0.75	-0.0833		0	0
	0	0	0	0	0	0	0	0	0	0	1	0	-1	-0.166	-0.5	-0.25	0.9166		0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0.5	-0.25	-0.125	-0.125		0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	-0.166	0.5	-0.25	-0.0833		0	0
	0	-0.25	0	-0.5	0	0	0	-0.25	0	0	0	1	0	0	0	0	0		0	0
	0	0	-0.333	0	-0.166	0	0	0	-0.5	0	0	0	1	0	0	0	0		0	0
	0	0	0	0	0	-0.25	0	0	0	-0.75	0	0	0	0	0	0	0		1	0
	0	0	0	0	0	0	-0.5	0	0	0	-0.5	0	0	0	0	0	0		0	1
	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0		0	0

Solution matrix

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0.340	0.660	0.680	0.329	0.150	0.200	0.340	0.991	0.451	0.200	1.042	-0.778	1.359	1.982	0.601	0.401	-1.000
-0.660	0.660	0.680	0.329	0.150	0.200	0.340	0.991	0.451	0.200	1.042	-0.778	1.359	1.982	0.601	0.401	0.000
0.330	-0.330	0.660	0.334	0.175	0.150	0.330	1.006	0.526	0.150	-0.518	0.392	1.319	2.012	0.701	0.301	-0.001
-0.260	0.260	-1.520	0.130	0.150	0.200	-0.260	0.390	0.450	0.200	3.441	0.421	-1.040	0.781	0.601	0.400	-0.001
-0.270	0.270	-0.540	-0.865	0.175	0.150	-0.270	0.405	0.525	0.150	1.881	1.591	-1.080	0.811	0.701	0.300	-0.002
-0.300	0.300	-0.600	0.150	-0.750	0.000	-0.300	0.451	0.750	0.000	1.201	-0.899	-1.200	0.901	1.001	0.000	0.000
-0.200	0.200	-0.400	0.100	0.000	-0.500	-0.200	0.300	0.000	0.500	0.801	-0.599	-0.800	0.601	0.000	1.000	0.000
0.140	-0.140	0.280	-0.070	0.150	0.200	-0.860	-0.210	0.450	0.200	1.840	1.620	0.560	-0.421	0.600	0.400	-0.002
0.130	-0.130	0.260	-0.065	0.175	0.150	0.130	-1.195	0.525	0.150	0.280	2.790	0.520	-0.390	0.700	0.300	-0.002
0.100	-0.100	0.200	-0.050	0.250	0.000	0.100	-0.150	-0.250	0.000	-0.400	0.300	0.400	-0.300	1.000	0.000	0.000
0.200	-0.200	0.400	-0.100	0.000	0.500	0.200	-0.300	0.000	-0.500	-0.801	0.599	0.800	-0.601	0.000	1.000	0.000
-0.260	0.260	-0.520	0.130	0.150	0.200	-0.260	0.390	0.450	0.200	2.441	0.421	-1.040	0.781	0.601	0.400	-0.001
0.130	-0.130	0.260	-0.065	0.175	0.150	0.130	-0.195	0.525	0.150	0.280	1.790	0.520	-0.390	0.700	0.300	-0.002
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.000
0.010	-0.010	0.020	-0.005	-0.025	0.050	0.010	-0.015	-0.075	0.050	1.560	-1.170	0.040	-0.030	-0.100	0.100	-0.999
0.040	-0.040	0.080	-0.020	-0.100	0.200	0.040	-0.060	-0.300	0.200	2.240	1.320	0.160	-0.120	-0.400	0.400	-1.001
-0.060	0.060	-0.120	0.030	0.150	-0.300	-0.060	0.090	0.450	-0.300	2.640	1.020	-0.240	0.180	0.600	-0.600	-1.001

B(VI)

0.60	0.40	y
0.60	0.40	x10
0.70	0.30	x20
0.60	0.40	x11
0.70	0.30	x21
1.00	0.00	x31
0.00	1.00	x41
0.60	0.40	x12
0.70	0.30	x22
1.00	0.00	x32
0.00	1.00	x42
0.60	0.40	x1
0.70	0.30	x2
0.00	0.00	p1
-0.10	0.10	p2
-0.40	0.40	p3
0.60	-0.60	p4

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Notes and Problems in Microeconomic Theory
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- 2.3. The linear expenditure system
- 4.2. Properties of the Cobb-Douglas production function
- 4.20. The CES production function

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- Cap. 3. The Johansen Approach