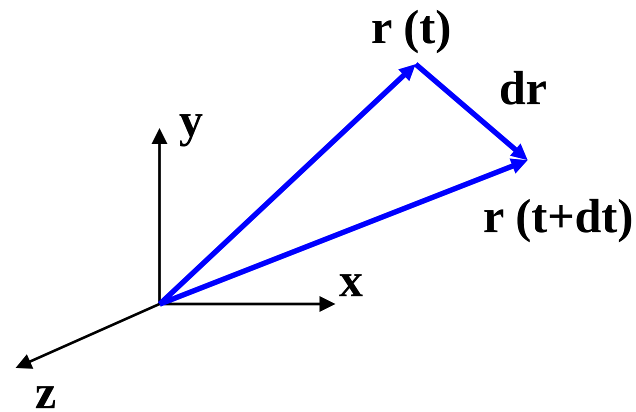
Derivadas no tempo- Observações

- **MEIO CONTÍNUO – CAMPO**

PROPRIEDADE : $\varphi\left(\vec{r}(t), t\right)$



$$d\varphi = \left(\frac{d\varphi}{dt}\right)_{\vec{r}} dt + \left(\frac{d\varphi}{d\vec{r}}\right)_{t} d\vec{r}$$

$$d\varphi = \left(\frac{d\varphi}{dt}\right)_{\vec{r}} dt + \operatorname{gr\vec{a}d}\varphi \bullet d\vec{r}$$

$$\frac{d\varphi}{dt} = \left(\frac{\partial\varphi}{\partial t}\right) + \operatorname{gr\vec{a}d}\varphi \bullet \vec{w}$$

$$\vec{w} = \frac{d\vec{r}}{dt}$$

Velocidade de observação

Derivadas no tempo- Observações

- EULER
  - Equacionamento usual em meios contínuos
  - Definição do regime: permanente /transiente

- LAGRANGE:
  - **Modelo Físico** – Equações Constitutivas
  - Equacionamento – Alguns sistemas multifásicos

Derivadas no tempo- Observações

Observação	Observação	$\left(\frac{d\varphi}{dt}\right)$
$\vec{w} = 0$	**EULER** VARIAÇÃO LOCAL	$\left(\frac{\partial \varphi}{\partial t}\right)_{\vec{r}}$
$\vec{w} = \vec{v}$	**LAGRANGE** MATERIAL OU SUBSTANTIVA	$\left(\frac{D\varphi}{Dt}\right) = \frac{\partial \varphi}{\partial t} + \vec{v}.\vec{grad}\varphi$
$\vec{w}$	TOTAL	$\left(\frac{d\varphi}{dt}\right) = \frac{\partial \varphi}{\partial t} + \vec{w}.\vec{grad}\varphi$
$\vec{w} = \vec{v}_k$	SUBSTANTIVA Em relação ao componente k	$\left(\frac{d^k\varphi}{dt}\right) = \frac{\partial \varphi}{\partial t} + \vec{v}_k.\vec{grad}\varphi$

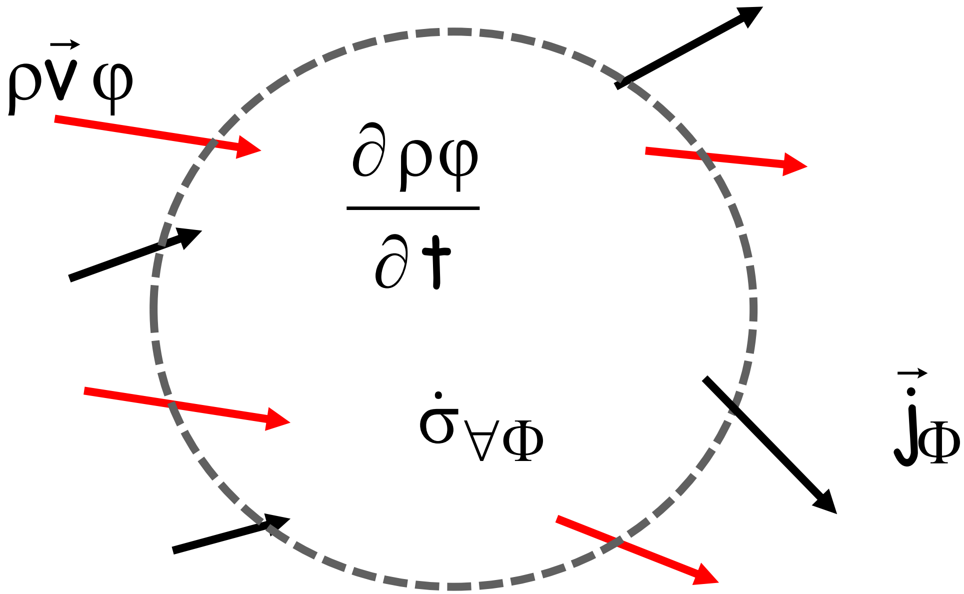
- * Lagrange (Substantiva - molar): $\left(\frac{D^*\varphi}{Dt}\right) = \frac{\partial \varphi}{\partial t} + \vec{v}^*.\vec{grad}\varphi$

OPERADOR

$$\boxed{\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{v}.\vec{grad}}$$

Equação de Conservação

$$\rho\frac{D\varphi}{Dt} = \frac{\partial \rho\varphi}{\partial t} + \mathrm{div}\,\rho\vec{v}\,\varphi = -\mathrm{div}\,\vec{j}_{\Phi} + \dot{\sigma}_{\forall\Phi}$$



$$\varphi = \frac{\partial\Phi}{\partial m} \Rightarrow \Phi = \int \varphi\, dm = \Phi = \int \varphi\, \rho dV$$

Equação de Conservação

$$\boxed{\varphi = \frac{\partial \Phi}{\partial \mathrm{m}} \Rightarrow \Phi = \int \varphi \, dm = \Phi = \int \varphi \, \rho dV}$$

$$\rho \frac{D\varphi}{Dt} = \frac{\partial \rho\varphi}{\partial t} + \mathrm{div}\, \rho\vec{v}\, \varphi = -\mathrm{div}\vec{j}_{\Phi} + \dot{\sigma}_{\forall\Phi}$$

propriedade	Φ	unidades	φ	unidades
carga elétrica	Σ	C	ε	C/kg
entropia	S	J/K	s	J/(K.kg)
massa de um constituinte i	m_i	kg de i	ω_i	kgi/kg
massa total	$m = \sum m_i$	kg	1	-
quantidade de movimento	$m\vec{v}$	kg.m/s	$\vec{v}$	m/s
momentum angular	$m\,\vec{r}.\vec{v}$	kg.m^2/s	$\vec{r}.\vec{v}$	m^2/s
energia cinética	E_c	J	e_c	J/kg
energia potencial	E_P	J	e_P	J/kg
energia interna	U	J	u	J/kg
Entalpia ~ C_pT	$\sim \; mC_pT$	J	C_pT	J/kg

Equação da Continuidade

Princípio da conservação da massa

$$\Phi = \mathrm{m} \Rightarrow \varphi = \frac{\partial \Phi}{\partial \mathrm{m}} = \frac{\partial \Phi}{\partial \rho V} = 1$$

$$\rho \frac{D1}{Dt} = \frac{\partial \rho}{\partial t} + div\, \rho \vec{v} = -div\, \vec{j}_m + \dot{\sigma}_{V,m} \quad = 0$$

$$\boxed{\frac{\partial \rho}{\partial t} + \mathrm{div}\, \rho \vec{\mathrm{v}} = 0}$$

Equação da Continuidade

Princípio da conservação da massa

EULER:

$$\boxed{\frac{\partial \rho}{\partial t} + \operatorname{div} \rho\vec{v} = 0}$$

$$\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + \vec{v}.\operatorname{gr\vec{a}d}\,\rho = -\operatorname{div}\rho\vec{v} + \vec{v}.\operatorname{gr\vec{a}d}\,\rho$$

$$\frac{D\rho}{Dt} = -\rho \operatorname{div}\vec{v} - \vec{v}.\operatorname{gr\vec{a}d}\,\rho + \vec{v}.\operatorname{gr\vec{a}d}\,\rho = -\rho \operatorname{div}\vec{v}$$

LAGRANGE:

Derivada material

$$\boxed{\frac{D\rho}{Dt} = -\rho \operatorname{div}\vec{v}}$$

Equação da Continuidade

Compressível e Incompressível

$$\boxed{\frac{D\rho}{Dt} = -\rho \,\mathrm{div}\,\vec{v}}$$

Observando-se uma massa ***m constante* – *Lagrange*** **-** em um volume ***V*:**

$$\frac{D\rho}{Dt} = \frac{D(m/V)}{Dt} = -\frac{m}{V^2}\frac{DV}{Dt} = -\rho\frac{1}{V}\frac{DV}{Dt} \Rightarrow \boxed{-\frac{1}{V}\frac{DV}{Dt} = \mathrm{div}\,\vec{v}}$$

Definição de Escoamento incompressível:

$$\boxed{\frac{D\rho}{Dt} = 0 \quad \Rightarrow \quad \mathrm{div}\;\vec{v} = 0}$$

ρ constante ?

$$\boxed{\frac{\partial\rho}{\partial t} + \mathrm{div}\,\rho\vec{v} = \frac{\partial\rho}{\partial t} + \rho\,\mathrm{div}\,\vec{v} + \vec{v}.\mathrm{gr\vec{a}d}\,\rho = 0}$$

$$\boxed{\Rightarrow \frac{\partial\rho}{\partial t} + \vec{v}.\mathrm{gr\vec{a}d}\rho = 0}$$

BALANÇO LOCAL DA PROPRIEDADE φ

Derivada material de φ

$$\left[\frac{D\varphi}{Dt} = \frac{\partial \varphi}{\partial t} + \vec{v}.\vec{\text{grad}}\,\varphi\right] \times \rho \qquad (1)$$

Continuidade

$$0 = \left[\frac{\partial \rho}{\partial t} + \text{div}\,\rho\vec{v}\right] \times \varphi \qquad (2)$$

(2) +(1):

$$\rho\frac{D\varphi}{Dt} = \underbrace{\rho\frac{\partial \varphi}{\partial t} + \varphi\frac{\partial \rho}{\partial t}}_{\frac{\partial \rho\varphi}{\partial t}} + \underbrace{\rho\,\vec{v}.\vec{\text{grad}}\,\varphi + \varphi\text{div}\,\rho\vec{v}}_{\text{div}\,\rho\varphi\vec{v}} = \frac{\partial \rho\varphi}{\partial t} + \text{div}\,\rho\varphi\vec{v}$$

Balanço microscópico de φ:

$$\boxed{\rho\frac{D\varphi}{Dt} = \frac{\partial \rho\varphi}{\partial t} + \text{div}\left(\rho\varphi\,\vec{v}\right) = -\text{div}\,\vec{j}_{\phi} + \dot{\sigma}_{\forall\Phi}}$$

Balanço microscópico de φ:

LAGRANGE EULER CONVECÇÃO DIFUSÃO PRODUÇÃO

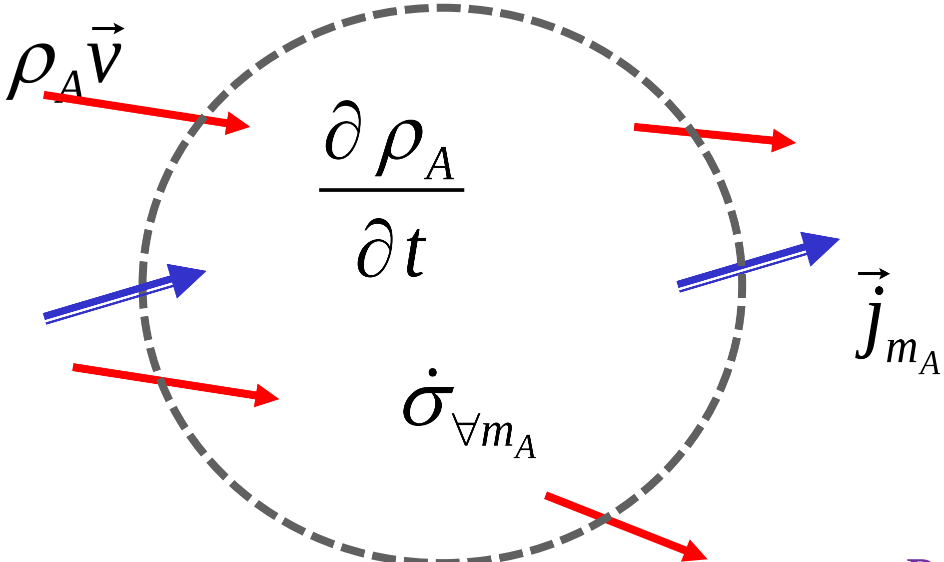
$$\rho\frac{D\varphi}{Dt}=\frac{\partial\rho\varphi}{\partial t}+\mathrm{div}\,\rho\vec{v}\varphi = -\mathrm{div}\,\vec{j}_{\Phi}+\dot{\sigma}_{\forall_{\Phi}}$$

VARIAÇÃO TEMPORAL ESCOAMENTO "LEIS" E MODELOS

$$\rho\frac{D\varphi}{Dt}=\rho\frac{\partial\varphi}{\partial t}+\underbrace{\varphi\frac{\partial\rho}{\partial t}+\varphi\,\mathrm{div}\,\rho\vec{v}}_{=0,\,\mathrm{continuidade}}+\rho\vec{v}.\mathrm{gr\vec{a}d}\,\varphi = -\mathrm{div}\,\vec{j}_{\Phi}+\dot{\sigma}_{\forall_{\Phi}}$$

$$\boxed{\rho\frac{D\varphi}{Dt}=\rho\frac{\partial\varphi}{\partial t}+\rho\vec{v}.\mathrm{gr\vec{a}d}\,\varphi = -\mathrm{div}\,\vec{j}_{\Phi}+\dot{\sigma}_{\forall_{\Phi}}}$$

Equação da Continuidade para a espécie A



Reação química kg A/(m^3 s)

$$\frac{\partial \rho_A}{\partial t} = -\operatorname{div} \vec{n}_A + \dot{r}_{\forall A}$$

Fluxo mássico total de A (kg A/(m^2 s)), convecção + difusão:

$$\boxed{\begin{aligned} \vec{n}_A &= \rho_A \vec{v} + \vec{j}_A \\ \vec{n}_A &= \omega_A \vec{n} + \vec{j}_A \end{aligned}}$$

Fluxo mássico total

Equação da Continuidade para a espécie A

$$\Phi = m_A \Rightarrow \varphi = \frac{\partial \Phi}{\partial m} = \omega_A$$

Fração mássica de A

$$\boxed{\rho_A = \rho \omega_A}$$

$$\rho \frac{D\omega_A}{Dt} = \frac{\partial \rho_A}{\partial t} + \operatorname{div} \rho_A \vec{v} = -\operatorname{div} \vec{j}_A + \dot{\sigma}_{\forall A}$$

difusão reação química

$$\frac{\partial \rho_A}{\partial t} = -\operatorname{div} \underbrace{\left(\rho_A \vec{v} + \vec{j}_A \right)}_{\vec{n}_A} + \dot{r}_{\forall A}$$

Fluxo mássico total de A (kg A/(m^2 s)), convecção + difusão:

$$\vec{n}_A = \rho_A \vec{v} + \vec{j}_A$$

$$\vec{n} = \rho \vec{v}$$

$$\vec{n}_A = \frac{\rho_A}{\rho} \vec{n} + \vec{j}_A = \omega_A \vec{n} + \vec{j}_A$$

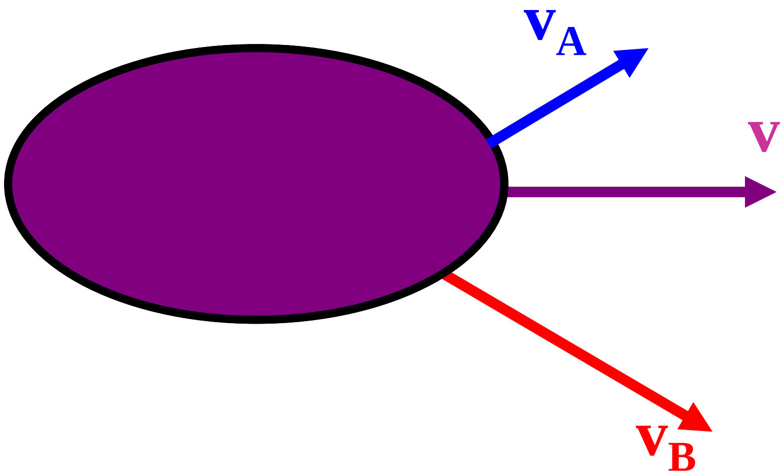
FLUXOS MÁSSICOS: DIFUSIVOS E CONVECTIVOS

Sistema Binário

ρ_A - kg A/m^3

$$\vec{v}_A = \frac{\vec{n}_A}{\rho_A}$$

$$\vec{v}_B = \frac{\vec{n}_B}{\rho_B}$$



DIFUSÃO (Definição)

Fluxo (difusivo) em relação ao centro mássico

$$\vec{n}_A = \rho_A \vec{v}_A = \rho_A \vec{v} + \vec{j}_A \Rightarrow \boxed{\vec{j}_A = \rho_A \left(\vec{v}_A - \vec{v} \right)}$$ kg A/(m^2 s)

VELOCIDADE DO CENTRO DE MASSA

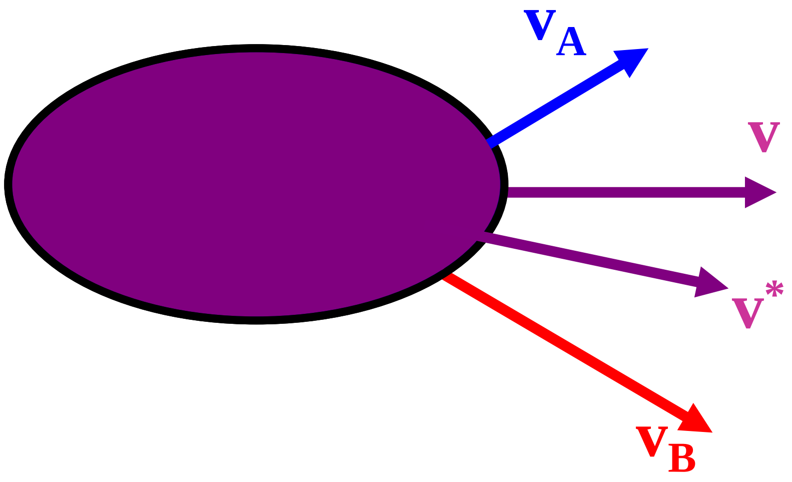
$$\vec{n} = \rho \vec{v} = \rho_A \vec{v}_A + \rho_B \vec{v}_B \Rightarrow \boxed{\vec{v} = \omega_A \vec{v}_A + \omega_B \vec{v}_B}$$

FLUXOS MOLARES: DIFUSIVOS E CONVECTIVOS
Sistema Binário

C_A - kgmol A/m^3

$$\vec{v}_A = \frac{\vec{N}_A}{C_A}$$

$$\vec{v}_B = \frac{\vec{N}_B}{C_B}$$



DIFUSÃO (Definição)

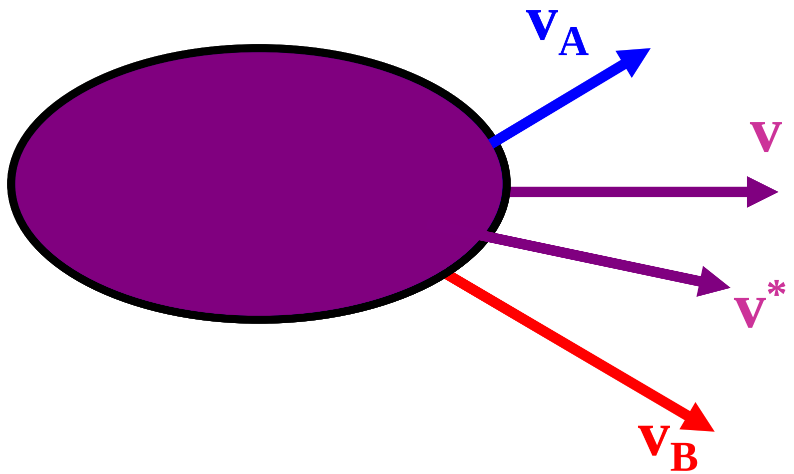
Fluxo (difusivo) em relação ao centro molar

$$\vec{N}_A = C_A\vec{v}_A = C_A\vec{v}^* + \vec{J}_A^* \Rightarrow \boxed{\vec{J}_A^* = C_A\left(\vec{v}_A - \vec{v}^*\right)}$$ gmol A/(m^2 s)

VELOCIDADE DO CENTRO MOLAR

$$\vec{N} = C\vec{v}^* = C_A\vec{v}_A + C_B\vec{v}_B \Rightarrow \boxed{\vec{v}^* = x_A\vec{v}_A + x_B\vec{v}_B}$$

FLUXOS MOLARES: DIFUSIVOS E CONVECTIVOS



$$\vec{v} - \vec{v}^* = \omega_A\left(\vec{v}_A - \vec{v}^*\right) + \omega_B\left(\vec{v}_B - \vec{v}^*\right)$$

Generalizando-se para sistema multicomponente:

$$\vec{v} - \vec{v}^* = \sum_{\alpha=1}^{N} \omega_\alpha\left(\vec{v}_\alpha - \vec{v}^*\right)$$

$$\vec{v}^* - \vec{v} = \sum_{\alpha=1}^{N} x_\alpha\left(\vec{v}_\alpha - \vec{v}\right)$$

FLUXOS MÁSSICOS E MOLARES: DIFUSIVOS E CONVECTIVOS - BIRD

Table 17.8-1 Notation for Mass and Molar Fluxes*

Quantity	With respect to stationary axes		With respect to mass average velocity $\mathbf{v}$		With respect to molar average velocity $\mathbf{v}^*$	
Velocity of species α (cm/s)	$\mathbf{v}_\alpha$	(A)	$\mathbf{v}_\alpha - \mathbf{v}$	(B)	$\mathbf{v}_\alpha - \mathbf{v}^*$	(C)
Mass flux of species α (g/cm^2 s)	$\mathbf{n}_\alpha = \rho_\alpha \mathbf{v}_\alpha$	(D)	$\mathbf{j}_\alpha = \rho_\alpha(\mathbf{v}_\alpha - \mathbf{v})$	(E)	$\mathbf{j}_\alpha^* = \rho_\alpha(\mathbf{v}_\alpha - \mathbf{v}^*)$	(F)
Molar flux of species α (g-moles/cm^2 s)	$\mathbf{N}_\alpha = c_\alpha \mathbf{v}_\alpha$	(G)	$\mathbf{J}_\alpha = c_\alpha(\mathbf{v}_\alpha - \mathbf{v})$	(H)	$\mathbf{J}_\alpha^* = c_\alpha(\mathbf{v}_\alpha - \mathbf{v}^*)$	(I)
Sums of mass fluxes	$\sum_{\alpha=1}^{N} \mathbf{n}_\alpha = \rho \mathbf{v}$	(J)	$\sum_{\alpha=1}^{N} \mathbf{j}_\alpha = 0$	(K)	$\sum_{\alpha=1}^{N} \mathbf{j}_\alpha^* = \rho(\mathbf{v} - \mathbf{v}^*)$	(L)
Sums of molar fluxes	$\sum_{\alpha=1}^{N} \mathbf{N}_\alpha = c\mathbf{v}^*$	(M)	$\sum_{\alpha=1}^{N} \mathbf{J}_\alpha = c(\mathbf{v}^* - \mathbf{v})$	(N)	$\sum_{\alpha=1}^{N} \mathbf{J}_\alpha^* = 0$	(O)
Relations between mass and molar fluxes	$\mathbf{n}_\alpha = M_\alpha \mathbf{N}_\alpha$	(P)	$\mathbf{j}_\alpha = M_\alpha \mathbf{J}_\alpha$	(Q)	$\mathbf{j}_\alpha^* = M_\alpha \mathbf{J}_\alpha^*$	(R)
Interrelations among mass fluxes	$\mathbf{n}_\alpha = \mathbf{j}_\alpha + \rho_\alpha \mathbf{v}$	(S)	$\mathbf{j}_\alpha = \mathbf{n}_\alpha - \omega_\alpha \sum_{\beta=1}^{N} \mathbf{n}_\beta$	(T)	$\mathbf{j}_\alpha^* = \mathbf{n}_\alpha - x_\alpha \sum_{\beta=1}^{N} \frac{M_\alpha}{M_\beta} \mathbf{n}_\beta$	(U)
Interrelations among molar fluxes	$\mathbf{N}_\alpha = \mathbf{J}_\alpha^* + c_\alpha \mathbf{v}^*$	(V)	$\mathbf{J}_\alpha = \mathbf{N}_\alpha - \omega_\alpha \sum_{\beta=1}^{N} \frac{M_\beta}{M_\alpha} \mathbf{N}_\beta$	(W)	$\mathbf{J}_\alpha^* = \mathbf{N}_\alpha - x_\alpha \sum_{\beta=1}^{N} \mathbf{N}_\beta$	(X)

* Entries in the shaded boxes, involving the "hybrid fluxes" $\mathbf{j}_\alpha^*$ and $\mathbf{J}_\alpha$, are seldom needed; they are included only for the sake of completeness.

Equações Constitutivas

"LEIS" E MODELOS FÍSICOS

Modelo Físico → Observação Lagrangeana (material)

$$\rho \frac{D\varphi}{Dt} = -\mathrm{div}\,\vec{j}_{\Phi} + \dot{\sigma}_{\forall_{\Phi}}$$

VARIAÇÃO DIFUSÃO PRODUÇÃO

Equações Constitutivas

"LEI DE FICK" – DIFUSÃO DE MASSA

$$\varphi = \omega_A$$

$$\rho \frac{D\omega_A}{Dt} = -\operatorname{div} \vec{j}_A + \dot{\sigma}_{\forall A}$$

Difusão : "Lei" de Fick - Sistema binário (A/B)

$$\vec{j}_A = -\rho\, D_{AB}\, \vec{\text{grad}}\, \omega_A$$

ω_A : fração mássica

$$\vec{J}^*_A = -C\, D_{AB}\, \vec{\text{grad}}\, x_A$$

x_A : fração molar

$\mathbf{D_{AB}}$: Difusividade de A em B

Produção: "Leis" Cinéticas – Velocidade de reação - r_A

$$\dot{\sigma}_{\forall A} = \dot{r}_i \cong \nu_{ij} K_j \prod_i \omega_i^{\nu_{ij}}$$

Equação da Continuidade para espécie A:

"LEI DE FICK" – DIFUSÃO DE MASSA

$$\rho\frac{D\omega_A}{Dt} = \frac{\partial\rho\omega_A}{\partial t} + \operatorname{div}\rho\vec{v}\omega_A = -\operatorname{div}\left(-\rho D_{AB}\,\vec{\operatorname{grad}}\,\omega_A\right) + \dot{\sigma}_{\forall A}$$

$$\rho D_{AB} \sim cte$$

$$\boxed{\rho\frac{D\omega_A}{Dt} = \frac{\partial\rho_A}{\partial t} + \operatorname{div}\rho_A\vec{v} = \rho D_{AB}\operatorname{lap}\omega_A + \dot{\sigma}_{\forall A}}$$

A equação acima, para os diferentes sistemas de coordenadas, está apresentada no apêndice B.11 (Bird 2002).

$$\boxed{\rho\frac{D\omega_A}{Dt} = \rho\frac{\partial\omega_A}{\partial t} + \rho\vec{v}.\vec{\operatorname{grad}}\omega_A = \rho D_{AB}\operatorname{lap}\omega_A + \dot{\sigma}_{\forall A}}$$

Equações Constitutivas

″LEI DE FOURIER″ – DIFUSÃO DE CALOR

$$\Phi = mc_p T \Rightarrow \varphi = \frac{\partial \Phi}{\partial m} = c_p T$$

$$\rho \frac{Dc_p T}{Dt} = -\mathrm{div}\, \vec{j}_{c_p T} + \dot{\sigma}_{\forall_{c_p T}}$$

Difusão de Calor : "Lei″de FOURIER

$$\vec{j}_{c_p T} = \vec{q}\,'' = -k\ \vec{\mathrm{grad}}\, T$$

k : condutividade térmica

Produção: q’’’ - p.exemplo - = $\mu \Phi_v$

Equação de Energia

″LEI DE FOURIER″ – DIFUSÃO DE CALOR

$$\rho \frac{DC_P T}{Dt} = \rho \frac{\partial (C_P T)}{\partial t} + \rho \vec{v}.\mathrm{gr\vec{a}d}(C_P T) = -\mathrm{div}(-k\, \mathrm{gr\vec{a}d}\, T) + q'''$$

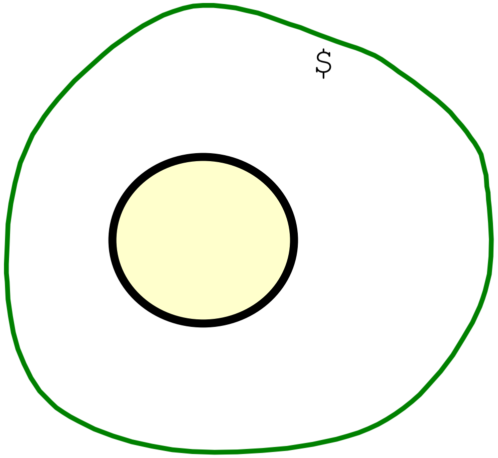
k e C_P: constantes

$$\boxed{\frac{DT}{Dt} = \frac{\partial T}{\partial t} + \vec{v}.\mathrm{gr\vec{a}d}\, T = \alpha\, \mathrm{lap} T + \frac{q'''}{\rho C_P}}$$

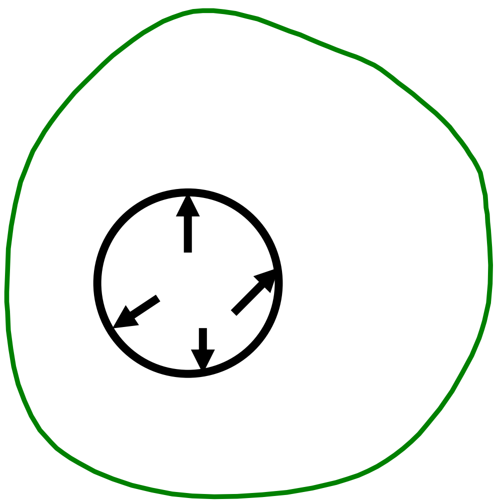
$\boldsymbol{\alpha} = k/\rho C_P$: difusividade térmica

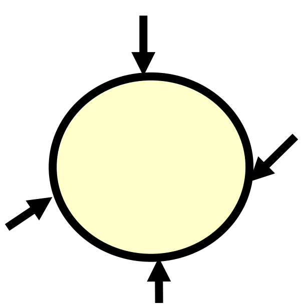
Equações Constitutivas

CAUCHY

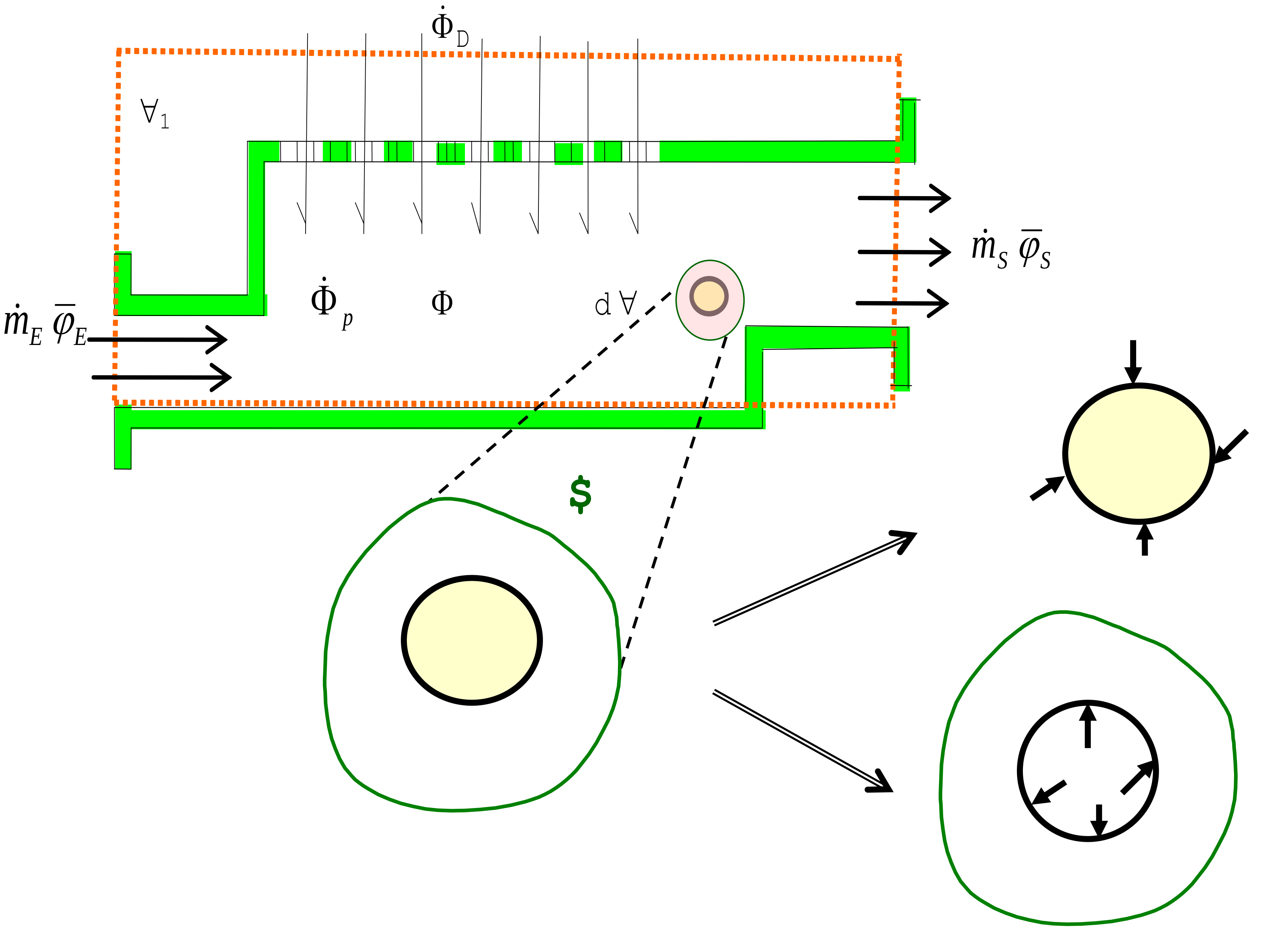


VOLUME DE FLUIDO



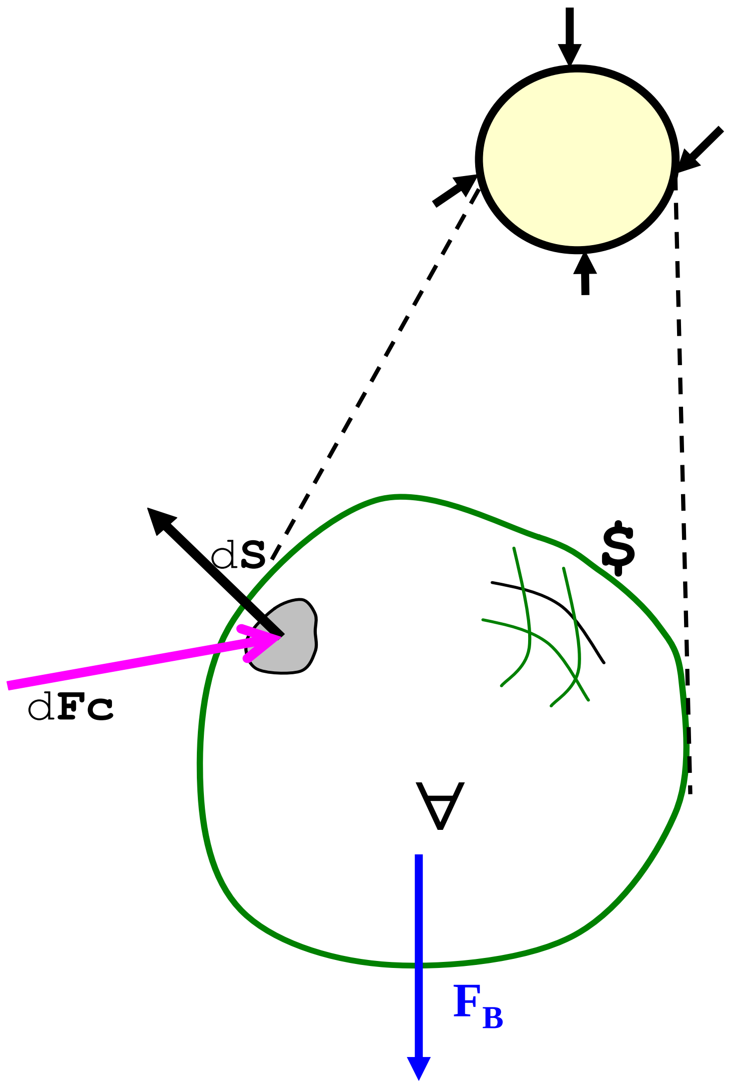


Equações Constitutivas – Escoamento - CAUCHY



Equações Constitutivas

FORÇA DE CONTATO E DE CAMPO



F_B - força de campo – produção

F_C - força de contato - difusão

Equações Constitutivas

FORÇA DE CONTATO E DE CAMPO

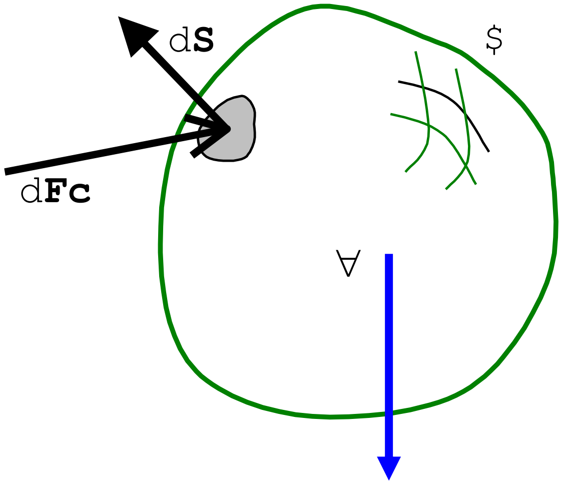
Força de campo/volume: GRAVIDADE $\vec{F}_B = \rho \vec{g}$

O tensor das **Tensões**: $\vec{\vec{\pi}} = -\dfrac{d\vec{F}_C}{d\vec{S}}$

$$\vec{F}_C = \int_{\$} d\vec{F}_C = -\int_{\$} \vec{\vec{\pi}} . d\vec{S} = -\int_{\forall} d\vec{i}v\, \vec{\vec{\pi}}\, dV$$

F_B - força de campo – produção

F_C - força de contato - difusão



Equações Constitutivas

LEI DE NEWTON – QUANTIDADE MOVIMENTO

$$\Phi = m\vec{v} \Rightarrow \varphi = \frac{\partial \Phi}{\partial m} = \vec{v}$$

$$\rho \frac{D\vec{v}}{Dt} = \vec{F}_B + \vec{F}_{C\forall}$$

$$\vec{F}_B = \rho \vec{g}$$

$$\vec{F}_C = \int_{\$} d\vec{F}_C = -\int_{\$} \vec{\vec{\pi}} . d\vec{S} = -\int_{\forall} d\vec{i}v\, \vec{\vec{\pi}}\, dV$$

$$F_{C\forall} = \frac{d\vec{F}_C}{d\forall} = -d\vec{i}v\, \vec{\vec{\pi}}$$

$$\boxed{\rho \frac{D\vec{v}}{Dt} = \rho \vec{g} - d\vec{i}v\, \vec{\vec{\pi}}}$$

Equações Constitutivas

Decomposição do Tensor das **TENSÕES** π

$$\vec{\vec{\pi}} = p\vec{\vec{\delta}} + \vec{\vec{\tau}} \quad \begin{cases} \text{tensões normais:} & \pi_{ii} = p + \tau_{ii} \\ \text{tensões de cisalhamento:} & \pi_{ij} = \tau_{ij} \end{cases}$$

π_{ij} : **componente da força (de contato) na direção j por unidade de área perpendicular à direção i.**

$$\vec{\vec{\tau}} = p_v\vec{\vec{\delta}} + \vec{\vec{\tau}}^{\overset{\circ}{S}} + \vec{\vec{\tau}}^{a}$$

pressão viscosa **simétrico normalizado** **anti-simétrico**

Equações Constitutivas

Decomposição do Tensor das **TENSÕES**

$$\vec{\vec{\pi}} = p\vec{\vec{\delta}} + \vec{\vec{\tau}}$$

$$\vec{\vec{\tau}} = p_v\vec{\vec{\delta}} + \vec{\vec{\tau}}^{\overset{o}{S}} + \vec{\vec{\tau}}^{a}$$

pressão viscosa — **simétrico normalizado** — **anti-simétrico**

$$\begin{bmatrix} \tau_{11} & \tau_{21} & \tau_{31} \\ \tau_{21} & \tau_{22} & \tau_{23} \\ \tau_{31} & \tau_{32} & \tau_{33} \end{bmatrix} = \frac{\tau_{11}+\tau_{22}+\tau_{33}}{3}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} \frac{2\tau_{11}-\tau_{22}-\tau_{33}}{3} & \frac{\tau_{21}+\tau_{12}}{2} & \frac{\tau_{31}+\tau_{13}}{2} \\ \frac{\tau_{12}+\tau_{21}}{2} & \frac{2\tau_{22}-\tau_{11}-\tau_{33}}{3} & \frac{\tau_{32}+\tau_{23}}{2} \\ \frac{\tau_{13}+\tau_{31}}{2} & \frac{\tau_{23}+\tau_{32}}{2} & \frac{2\tau_{33}-\tau_{11}-\tau_{22}}{3} \end{bmatrix} + \begin{bmatrix} 0 & \frac{\tau_{21}-\tau_{12}}{2} & \frac{\tau_{31}-\tau_{13}}{2} \\ \frac{\tau_{12}-\tau_{21}}{2} & 0 & \frac{\tau_{32}-\tau_{23}}{2} \\ \frac{\tau_{13}-\tau_{31}}{2} & \frac{\tau_{23}-\tau_{32}}{2} & 0 \end{bmatrix}$$

Equações Constitutivas

Decomposição do Tensor **grad v**

$$\mathrm{gr\vec{a}d}\,\vec{v} = \frac{1}{3}\left(\mathrm{div}\,\vec{v}\right)\vec{\vec{\delta}} + (\mathrm{gr\vec{a}\overset{o}{d}}\,\vec{v})^{S} + (\mathrm{gr\vec{a}d}\,\vec{v})^{A}$$

traço — **simétrico normalizado** — **anti-simétrico**

dilatação — **cisalhamento** — **rotação**

$$(\mathrm{gr\vec{a}\overset{o}{d}}\,\vec{v})^{S} = \frac{1}{2}\left(\mathrm{gr\vec{a}d}\,\vec{v} + (\mathrm{gr\vec{a}d}\,\vec{v})^{T}\right) - \frac{1}{3}\left(\mathrm{div}\,\vec{v}\right)\vec{\vec{\delta}}$$

$$(\mathrm{gr\vec{a}d}\,\vec{v})^{A} = \frac{1}{2}\left(\mathrm{gr\vec{a}d}\,\vec{v} - (\mathrm{gr\vec{a}d}\,\vec{v})^{T}\right)$$

Equações Constitutivas

Lei de NEWTON generalizada

$$\vec{\vec{\tau}} = -\vec{\vec{\vec{\vec{\mu}}}} : \operatorname{gr\vec{a}d} \vec{v} \qquad ; \qquad \tau_{ij} = -\sum_k \sum_l \mu_{ijkl} \frac{\partial v_k}{\partial x_k}$$

μ_{ijkl} - Tensor dos coeficientes de viscosidade com 81 termos!

$$\vec{\vec{\tau}} = p_v \vec{\vec{\delta}} + \vec{\vec{\tau}}^{\overset{o}{S}} + \vec{\vec{\tau}}^a$$

Equações Constitutivas

Lei de NEWTON generalizada

$$\boxed{\vec{\vec{\tau}} = p_v \vec{\vec{\delta}} + \overset{o}{\vec{\vec{\tau}}}{}^{S} + \vec{\vec{\tau}}^{a}}$$

$$p_v = -\kappa \operatorname{div} \vec{v}$$

$$\vec{\vec{\tau}}^{a} = -\eta_R \left(\operatorname{rot} \vec{v} - 2\vec{\Omega} \right)$$

$$\overset{o}{\vec{\vec{\tau}}}{}^{S} = -2\mu \, (\overset{o}{\operatorname{gr\vec{a}d}} \, \vec{v})^{S}$$

Viscosidades

κ - "bulk" - dilatacional - compressibilidade

η_R-"rotational"- diferença entre a rotação do fluido e a rotação intrínsica da partícula

μ - "shear" - cisalhamento.

Equações Constitutivas

Lei de NEWTON generalizada

$$\vec{\vec{\tau}} = -\kappa\left(\mathrm{div}\,\vec{v}\right)\vec{\vec{\delta}} - 2\mu\,(\overset{o}{\mathrm{gr\vec{a}d}}\,\vec{v})^{S} - \eta_R \underbrace{\left(\mathrm{rot}\,\vec{v} - 2\vec{\Omega}\right)}_{\approx 0}$$

Substituindo-se:

$$(\overset{o}{\mathrm{gr\vec{a}d}}\,\vec{v})^{S} = (\mathrm{gr\vec{a}d}\,\vec{v})^{S} - \frac{1}{3}\left(\mathrm{div}\,\vec{v}\right)\vec{\vec{\delta}}$$

$$\boxed{\vec{\vec{\tau}} = \left(\frac{2}{3}\mu - \kappa\right)\left(\mathrm{div}\,\vec{v}\right)\vec{\vec{\delta}} - 2\mu\,(\mathrm{gr\vec{a}d}\,\vec{v})^{S}}$$

$$\vec{\vec{\tau}} = \left(\frac{2}{3}\mu - \kappa\right)\left(\mathrm{div}\,\vec{v}\right)\vec{\vec{\delta}} - \mu\left(\mathrm{gr\vec{a}d}\,\vec{v} + (\mathrm{gr\vec{a}d}\,\vec{v})^{T}\right)$$

Equações Constitutivas

NAVIER- STOKES

$$\vec{\vec{\tau}} = -\mu\left(\mathrm{gr\vec{a}d}\,\vec{v} + (\mathrm{gr\vec{a}d}\,\vec{v})^{T}\right) + \left(\frac{2}{3}\mu - \cancel{\kappa}\right)\cancel{(\mathrm{div}\,\vec{v})}\,\vec{\vec{\delta}}$$

Escoamento incompressível: $\boxed{\mathrm{div}\,\vec{v} = 0}$

$$\vec{\vec{\tau}} = -\mu\left(\mathrm{gr\vec{a}d}\,\vec{v} + (\mathrm{gr\vec{a}d}\,\vec{v})^{T}\right)$$

$$\mathrm{div}\,\vec{\vec{\tau}} = -\mu\,\mathrm{div}\left(\mathrm{gr\vec{a}d}\,\vec{v} + (\mathrm{gr\vec{a}d}\,\vec{v})^{T}\right) = -\mu\left(\mathrm{div}\,\mathrm{gr\vec{a}d}\,\vec{v} + \underbrace{\mathrm{div}\,(\mathrm{gr\vec{a}d}\,\vec{v})^{T}}_{=0}\right)$$

Escoamento incompressível e newtoniano

$$\boxed{\mathrm{div}\,\vec{\vec{\tau}} = -\mu\,\mathrm{lap}\,\vec{v}}$$

Equações Constitutivas

NAVIER- STOKES

$$\boxed{\rho\frac{D\vec{v}}{Dt}=\rho\vec{g}-d\vec{i}v\,\vec{\vec{\pi}}}$$

$$\vec{\vec{\pi}}=p\vec{\vec{\delta}}+\vec{\vec{\tau}}$$

$$\operatorname{div}\vec{\vec{\pi}}=\operatorname{div}\left(p\vec{\vec{\delta}}\right)+\operatorname{div}\vec{\vec{\tau}}=\operatorname{gr\vec{a}d}p+\operatorname{div}\vec{\vec{\tau}}$$

$$\boxed{\rho\frac{D\vec{v}}{Dt}=\rho\vec{g}-\operatorname{gr\vec{a}d}p-\operatorname{div}\vec{\vec{\tau}}}$$

Equações Constitutivas

NAVIER- STOKES

$$\boxed{\rho\frac{D\vec{v}}{Dt} = \rho\vec{g} - \vec{\text{grad}}\,p - \text{div}\,\vec{\vec{\tau}}}$$

$$\boxed{\text{div}\,\vec{\vec{\tau}} = -\mu\,\text{lap}\,\vec{v}}$$

NAVIER - STOKES

$$\boxed{\rho\frac{D\vec{v}}{Dt} = \rho\frac{\partial\vec{v}}{\partial t} + \rho\,\vec{v}.\vec{\text{grad}}\vec{v} = \rho\vec{g} - \vec{\text{grad}}\,p + \mu\,\text{lap}\,\vec{v}}$$

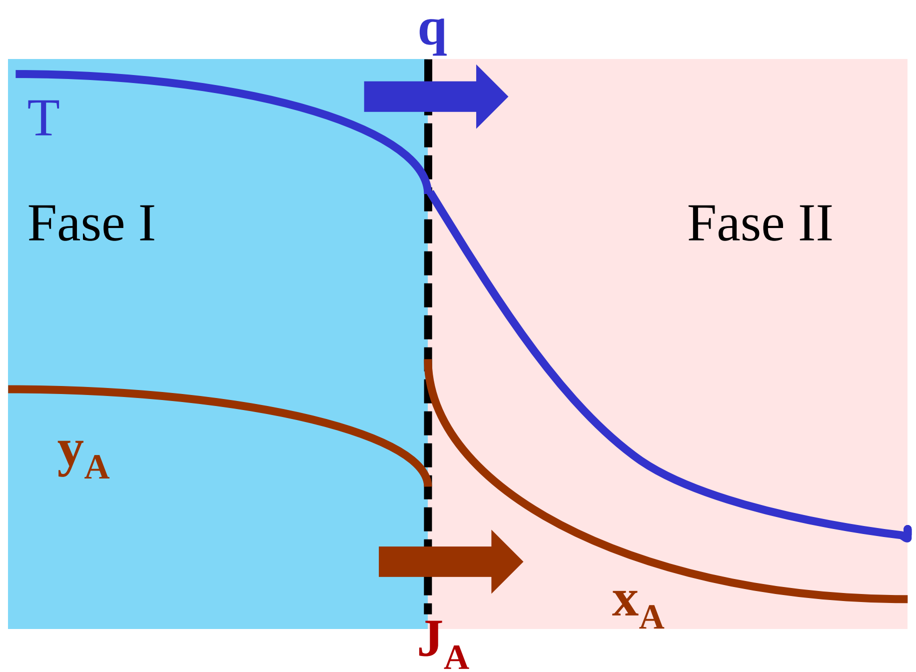
Escoamento incompressível e newtoniano

Equações constitutivas - Mecanismos complexos

Difusão de Calor - Transporte simultâneo de calor e massa

$$\boxed{\vec{q} = -\vec{\vec{k}}.\,\mathrm{gr\vec{a}d}\,T + \sum \vec{j}_i \hat{H}_i + \vec{q}^{\,x}}$$

DIFUSÃO DE CALOR — **CONDUÇÃO FOURIER** — **DIFUSÃO** — **TERMODIFUSÃO DUFOUR**



$$\boxed{-\left(k.\,\mathrm{gr\vec{a}d}\,T\right)_I + \vec{j}_A \hat{H}_{A,I} = -\left(k.\,\mathrm{gr\vec{a}d}\,T\right)_{II} + \vec{j}_A \hat{H}_{A,II}}$$

Equações constitutivas - Mecanismos complexos

Difusão de Massa - Difusão de Calor

$$\vec{J}_i = -D_i\left[\frac{C_i}{RT}\left(\mathrm{gr\vec{a}d}\,\mu_i\right)_{T,P} + \frac{1}{RT}\left(\varphi_{vi} - w_i\right)\mathrm{gr\vec{a}d}\,P - \frac{\rho_i}{RT}\left(\vec{g}_i - \sum_{k=1}^{n} w_k \vec{g}_k\right)\right] + \frac{D_i^{(T)}}{M_i}\,\frac{\mathrm{gr\vec{a}d}\,T}{T}$$

DIFUSÃO	**ORDINÁRIA FICK**	**PRESSÃO**	**FORÇADA**	**TÉRMICA SORET**

φ_{vi} = fração volumétrica de i

w_i = fração mássica de i

ρ_i = concentração mássica de i

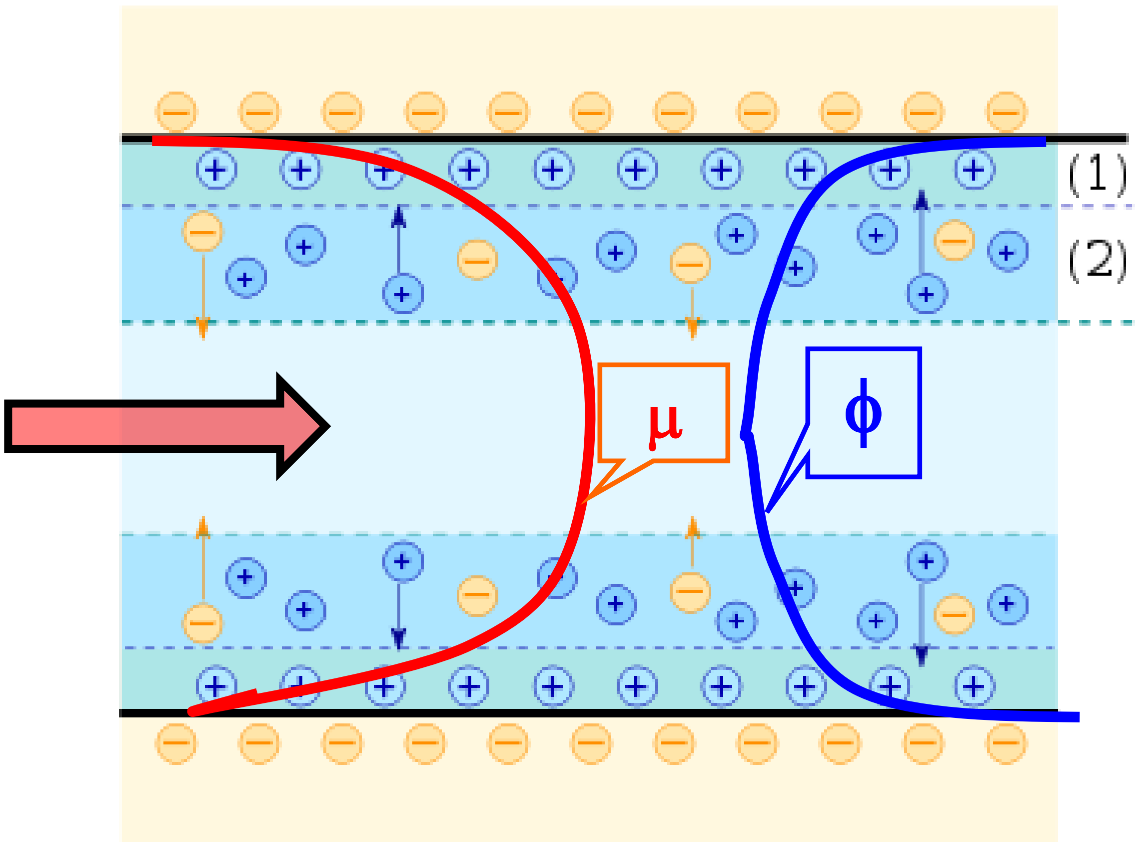
C_i = concentração molar de i

$\mathbf{g}_i$ = força de campo exercida em i

$D_i^{(T)}$ = coeficiente de termodifusão

$(\mu_i)_{T,P}$ = potencial químico , T e P ctes

Difusão de Massa – Eletroquímica 1



Difusão de Massa – Eletroquímica - 2

$$\vec{J}_i = -D_i\left[\boxed{\frac{C_i}{RT}\left(\text{gr}\vec{\text{a}}\text{d}\,\mu_i\right)_{T,P}} + \frac{1}{RT}\left(\varphi_{vi} - \cancel{w_i}\right)\text{gr}\vec{\text{a}}\text{d}\,P - \boxed{\frac{\rho_i}{RT}\left(\vec{g}_i - \sum_{k=1}^{n} w_k \vec{g}_k\right)}\right] + \cancel{\frac{D_i^{(T)}}{M_i}\frac{\text{gr}\vec{\text{a}}\text{d}\,T}{T}}$$

DIFUSÃO ORDINÁRIA CAMPO ELÉTRICO

$$\boxed{\vec{F}_e = q\vec{E} \Rightarrow \vec{F}_e^M = -z_i\Im\,\text{gr}\vec{\text{a}}\text{d}\,\phi \Rightarrow \vec{g}_i^e = -\frac{z_i\Im}{M_i}\text{gr}\vec{\text{a}}\text{d}\,\phi}$$

$$\boxed{\begin{aligned}&\Im = 96520\,C/mol\\ &z_i = valência\\ &\vec{E} = -gr\vec{a}d\,\Phi \quad (V/m)\end{aligned}}$$

$$\boxed{\frac{\rho_i}{RT}\left(\vec{g}_i - \sum_{k=1}^{n} w_k \vec{g}_k\right)} = \frac{\rho_i}{RT}\left(-\frac{z_i\Im}{M_i}\text{gr}\vec{\text{a}}\text{d}\,\phi + \sum_{k=1}^{n} w_k \frac{z_k\Im}{M_k}\text{gr}\vec{\text{a}}\text{d}\,\phi\right) =$$

$$= -\frac{\rho_i}{RT}\frac{z_i\Im}{M_i}\text{gr}\vec{\text{a}}\text{d}\,\phi\left(1 - \frac{M_i}{z_i}\sum_{k=1}^{n}\frac{w_k z_k}{M_k}\right) = \boxed{-\frac{\rho_i}{RT}\frac{z_i\Im}{M_i}\text{gr}\vec{\text{a}}\text{d}\,\phi}\left(1 - \underbrace{\frac{M_i}{Mz_i}\sum_{k=1}^{n} x_k z_k}_{\cong 0\text{, diluída}}\right)$$

$$w_k = \frac{x_k M_k}{M}$$

Difusão de Massa - Eletroquímica - 3

$$\boxed{\frac{\rho_i}{RT}\left(\vec{g}_i - \sum_{k=1}^{n} w_k \vec{g}_k\right) = -\frac{\rho_i}{RT}\frac{z_i \Im}{M_i}\,\mathrm{gr\vec{a}d}\,\phi = -\frac{C_i}{RT} z_i \Im\,\mathrm{gr\vec{a}d}\,\phi}$$

$$\boxed{\frac{\rho_i}{RT}\left(\vec{g}_i - \sum_{k=1}^{n} w_k \vec{g}_k\right) = -\frac{C_i}{RT} z_i \Im\,\mathrm{gr\vec{a}d}\,\phi}$$

$$\boxed{\vec{J}_i = -D_i\left[\boxed{\frac{C_i}{RT}\left(\mathrm{gr\vec{a}d}\,\mu_i\right)_{T,P}} + \frac{1}{RT}\cancel{\left(\varphi_{vi} - w_i\right)\mathrm{gr\vec{a}d}\,P} - \boxed{\frac{\rho_i}{RT}\left(\vec{g}_i - \sum_{k=1}^{n} w_k \vec{g}_k\right)}\right] + \cancel{\frac{D_i^{(T)}}{M_i}\frac{\mathrm{gr\vec{a}d}\,T}{T}}}$$

$$\boxed{\mu_i = \mu_{i0} + RT\ln x_i}$$

$$\boxed{\frac{C_i}{RT}\left(\mathrm{gr\vec{a}d}\,\mu_i\right)_{T,P} = \frac{C_i}{RT}\left[\mathrm{gr\vec{a}d}\left(RT\ln x_i\right)\right]_{T,P} = C_i\frac{\mathrm{gr\vec{a}d}\,x_i}{x_i} = Cx_i\frac{\mathrm{gr\vec{a}d}\,x_i}{x_i} = \mathrm{gr\vec{a}d}\,C_i}$$

$$\boxed{\frac{C_i}{RT}\left(\mathrm{gr\vec{a}d}\,\mu_i\right)_{T,P} = \mathrm{gr\vec{a}d}\,C_i}$$

Difusão de Massa - Eletroquímica - 4

$$\vec{J}_i = -D_i\left[\frac{C_i}{RT}\left(\vec{\text{grad}}\,\mu_i\right)_{T,P} - \frac{\rho_i}{RT}\left(\vec{g}_i - \sum_{k=1}^{n} w_k \vec{g}_k\right)\right]$$

$$\frac{C_i}{RT}\left(\vec{\text{grad}}\,\mu_i\right)_{T,P} = \vec{\text{grad}}\,C_i$$

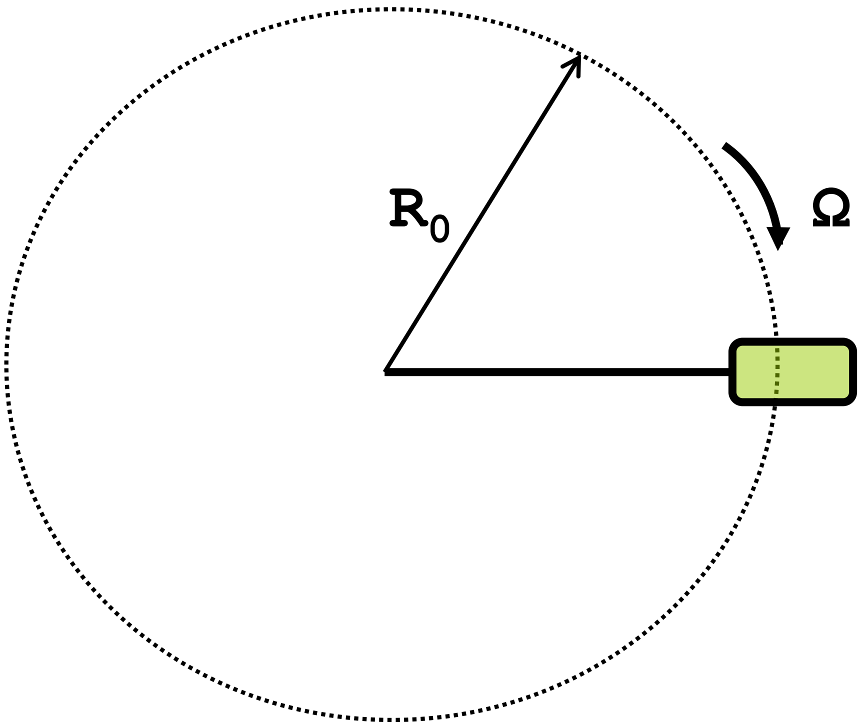
$$\frac{\rho_i}{RT}\left(\vec{g}_i - \sum_{k=1}^{n} w_k \vec{g}_k\right) = -\frac{C_i}{RT} z_i \mathfrak{I}\,\vec{\text{grad}}\,\phi$$

Equação de Nernst - Planck

$$\vec{J}_i = -D_i\left[\vec{\text{grad}}\,C_i + C_i z_i \frac{\mathfrak{I}}{RT}\vec{\text{grad}}\,\phi\right]$$

$$\frac{RT}{\mathfrak{I}} = 25{,}7\,mV$$

Centrifugação - 1



$$v_r = 0 \;\; ; \;\; v_z = 0 \;\; ; \; v_\theta = v_\theta = \vec{\Omega} \times \vec{r} = \Omega r$$

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r v_r)}{\partial \theta}\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial^2 v_\theta}{\partial \theta} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_r$$

$$\boxed{\frac{\partial p}{\partial r} = \rho \frac{v_\theta^2}{r} = \rho \frac{(\Omega r)^2}{r} = \rho \Omega^2 r}$$

Centrifugação - sistema binário - 2

$$\vec{J}_i = -D_i\left[\boxed{\frac{C_i}{RT}\left(\mathrm{gr\vec{a}d}\,\mu_i\right)_{T,P}} + \boxed{\frac{1}{RT}\left(\varphi_{vi} - w_i\right)\mathrm{gr\vec{a}d}\,P} - \frac{\rho_i}{RT}\left(\vec{g}_i - \cancel{\sum_{k=1}^{n} w_k \vec{g}_k}\right)\right] + \cancel{\frac{D_i^{(T)}}{M_i}\frac{\mathrm{gr\vec{a}d}\,T}{T}}$$

$$\frac{C_i}{RT}\left(\mathrm{gr\vec{a}d}\,\mu_A\right)_{T,P} = \mathrm{gr\vec{a}d}\,C_A = C\,\mathrm{gr\vec{a}d}\,x_A = C\frac{dx_A}{dr}$$

$$\varphi_{vA} = \frac{V_A}{V} = \frac{n_A}{V}\frac{V_A}{n_A} = \frac{m_A}{VM_A}\overline{V}_A = \rho\frac{m_A}{m}\frac{\overline{V}_A}{M_A} = \rho\,w_A\frac{\overline{V}_A}{M_A}$$

$$\frac{\partial p}{\partial r} = \rho\Omega^2 r$$

$$\frac{1}{RT}\left(\varphi_{vi} - w_i\right)\mathrm{gr\vec{a}d}\,P = \frac{1}{RT}\left(\rho\,w_A\frac{\overline{V}_A}{M_A} - w_A\right)\rho\Omega^2 r =$$

$$= \frac{\rho w_A}{RTM_A}\left(\rho\,\overline{V}_A - M_A\right)\Omega^2 r = \frac{Cx_A}{RT}\left(\rho\,\overline{V}_A - M_A\right)\Omega^2 r$$

Centrifugação - sistema binário - 3

$$\vec{J}_i = -D_i \left[\boxed{\frac{C_i}{RT}\left(\vec{\text{grad}}\,\mu_i\right)_{T,P}} + \boxed{\frac{1}{RT}\left(\varphi_{vi} - w_i\right)\vec{\text{grad}}\,P} \right]$$

$$\frac{C_A}{RT}\left(\vec{\text{grad}}\,\mu_A\right)_{T,P} = C\frac{dx_A}{dr}$$

$$\frac{1}{RT}\left(\varphi_{vA} - w_A\right)\vec{\text{grad}}\,P = \frac{Cx_A}{RT}\left(\rho\,\overline{V}_A - M_A\right)\Omega^2 r$$

$$\vec{J}_A = -D_A\left[C\frac{dx_A}{dr} + \frac{Cx_A}{RT}\left(\rho\,\overline{V}_A - M_A\right)\Omega^2 r\right]$$

Centrifugação - 4

$$\vec{J}_A = -D_A\left[C\frac{d\,x_A}{d\,r} + \frac{x_A}{RT}\left(\rho\,\overline{V}_A - M_A\right)\Omega^2 r\right]$$

No equilíbrio e sob ação centrífuga: $\vec{J}_A = 0 \;\; ; \;\; \vec{J}_B = 0$

$$\frac{d\,x_A}{d\,r} = -\frac{x_A}{RT}\left(\rho\,\overline{V}_A - M_A\right)\Omega^2 r \qquad (1)$$

$$\frac{d\,x_B}{d\,r} = -\frac{x_B}{RT}\left(\rho\,\overline{V}_B - M_B\right)\Omega^2 r \qquad (2)$$

(2) x $\overline{V}_A$ - (1) x $\overline{V}_B$

$$\boxed{\overline{V}_A\frac{d\,x_B}{x_B} - \overline{V}_B\frac{d\,x_A}{x_A} = \frac{1}{RT}\left(M_A\overline{V}_B - M_B\overline{V}_A\right)\Omega^2 r dr}$$

Centrifugação - 5

$$\boxed{\overline{V}_A \frac{d\,x_B}{x_B} - \overline{V}_B \frac{d\,x_A}{x_A} = \frac{1}{RT}\left(M_A \overline{V}_B - M_B \overline{V}_A\right)\Omega^2 r dr}$$

$$\overline{V}_A \int_{x_{B1}}^{x_{B2}} \frac{d\,x_B}{x_B} - \overline{V}_B \int_{x_{A1}}^{x_{A2}} \frac{d\,x_A}{x_A} = \frac{1}{RT}\left(M_A \overline{V}_B - M_B \overline{V}_A\right)\Omega^2 \int_{r_1}^{r_2} r\,dr$$

$$\boxed{\overline{V}_A \ln \frac{x_{B1}}{x_{B2}} - \overline{V}_B \ln \frac{x_{A1}}{x_{A2}} = \frac{1}{RT}\left(M_A \overline{V}_B - M_B \overline{V}_A\right)\Omega^2 \frac{\left(r_1^2 - r_2^2\right)}{2}}$$

Equações Constitutivas
Fluidos Não Newtonianos

Newtoniano e incompressível:

$$\vec{\vec{\tau}} = -\mu\left(gr\vec{a}d\,\vec{v} + (gr\vec{a}d\,\vec{v})^T\right) = -\mu\,\dot{\vec{\vec{\gamma}}}$$

Não Newtoniano e incompressível:

$$\begin{cases} \vec{\vec{\tau}} = -\eta\left(gr\vec{a}d\,\vec{v} + (gr\vec{a}d\,\vec{v})^T\right) = -\eta\,\dot{\vec{\vec{\gamma}}} \\ \eta = \eta(\dot{\gamma}) \qquad\qquad \dot{\gamma} = \sqrt{\frac{1}{2}\dot{\vec{\vec{\gamma}}} : \dot{\vec{\vec{\gamma}}}} \end{cases}$$

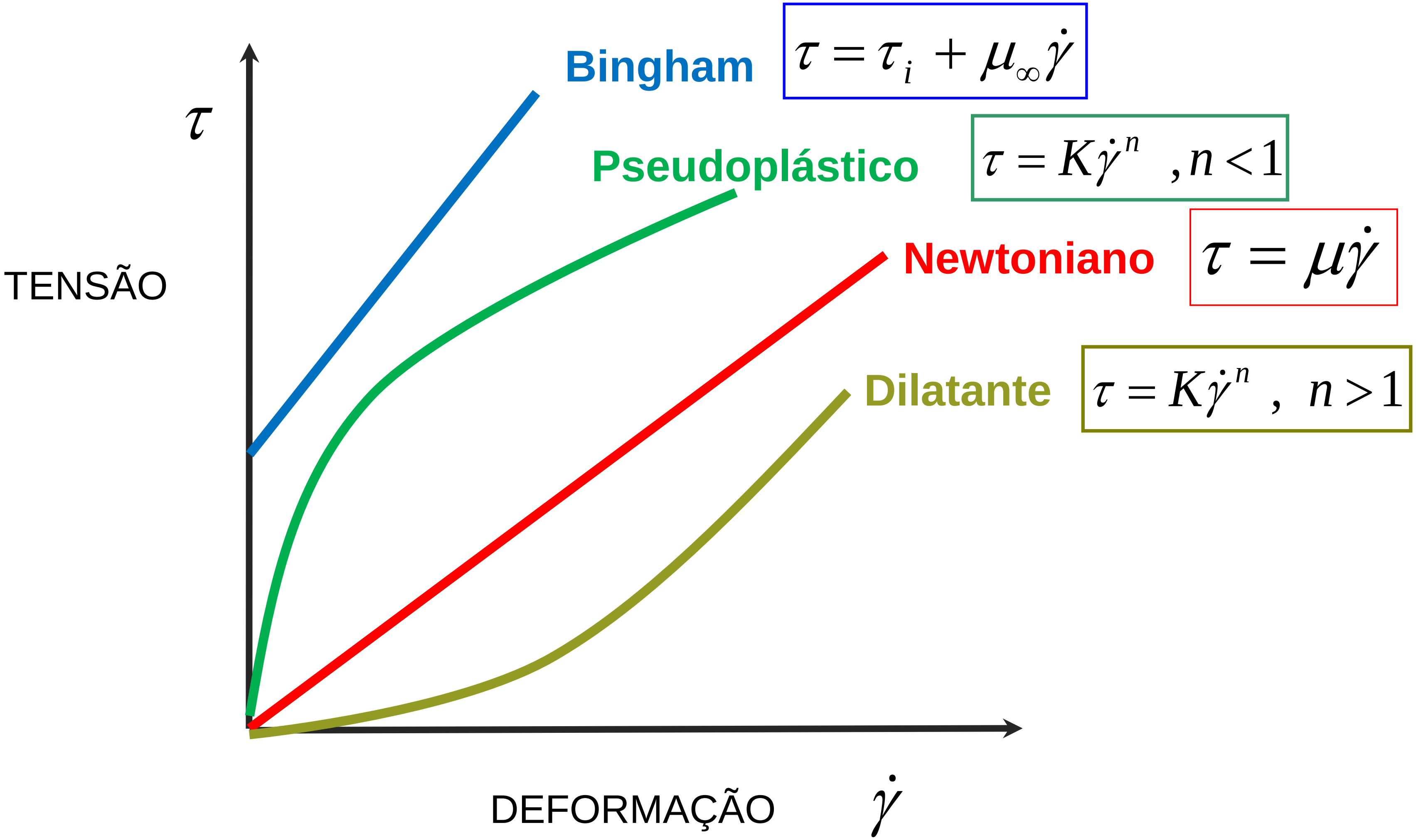
Modelo: Lei de potência

$$\eta = m\dot{\gamma}^{n-1}$$

Quantidade de movimento:

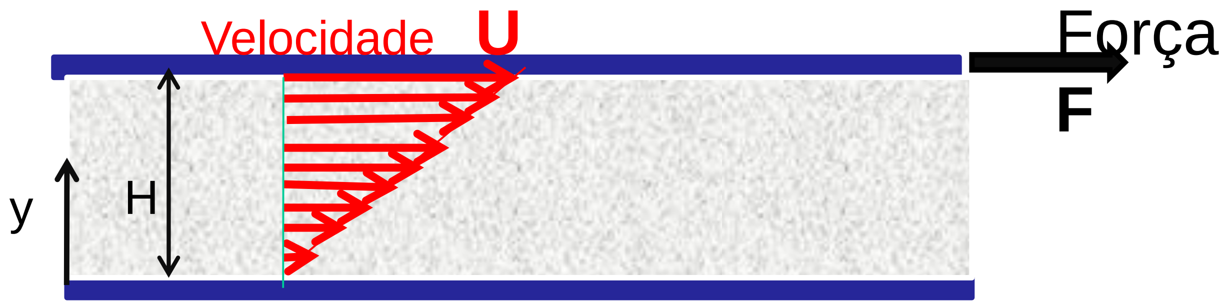
$$\rho\frac{D\vec{v}}{Dt} = \rho\frac{\partial\vec{v}}{\partial t} + \rho\,\vec{v}.gr\vec{a}d\vec{v} = \rho\vec{g} - gr\vec{a}d\;p - div\vec{\vec{\tau}}$$

Reologia

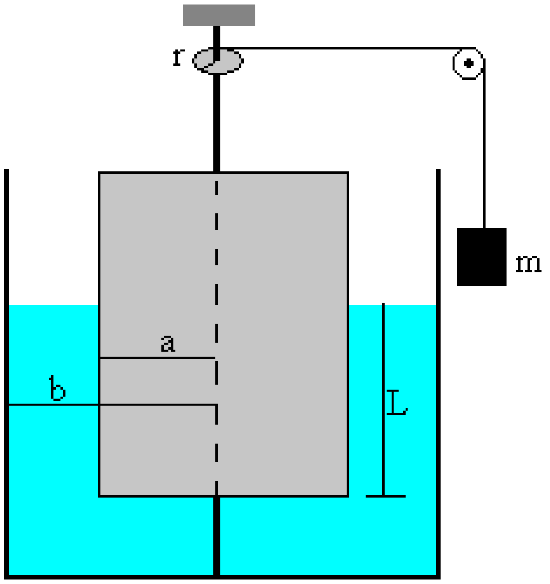


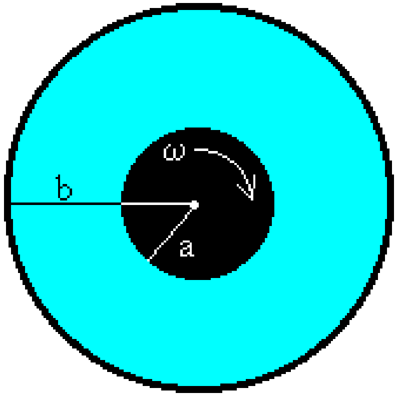
Bingham
$\tau = \tau_i + \mu_\infty \dot{\gamma}$
τ
Pseudoplástico
$\tau = K\dot{\gamma}^n \quad , n < 1$
Newtoniano
$\tau = \mu\dot{\gamma}$
TENSÃO
Dilatante
$\tau = K\dot{\gamma}^n \; , \; n > 1$
DEFORMAÇÃO
$\dot{\gamma}$

Reômetro



Reômetro de
Cilindros Coaxiais





FENÔMENOS DE TRANSPORTE

$$\boxed{\frac{\partial \rho}{\partial t} + \operatorname{div} \rho \vec{v} = 0}$$

$$\frac{D\rho}{Dt} = -\rho \operatorname{div} \vec{v}$$

$$\rho \frac{D\omega_A}{Dt} = \frac{\partial \rho_A}{\partial t} + \operatorname{div} \rho_A \vec{v} = \rho D_{AB} \operatorname{lap}\omega_A + \dot{\sigma}_{\forall A}$$

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + \vec{v}.\vec{\operatorname{grad}}\, T = \alpha \operatorname{lap}T + \frac{q'''}{\rho C_P}$$

$$\rho \frac{D\vec{v}}{Dt} = \rho \frac{\partial \vec{v}}{\partial t} + \rho\, \vec{v}.\vec{\operatorname{grad}}\vec{v} = \rho \vec{g} - \vec{\operatorname{grad}}\, p + \mu \operatorname{lap} \vec{v}$$

FENÔMENOS DE TRANSPORTE

MODELAGEM E SOLUÇÃO DAS EQUAÇÕES