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Synchronization in Vehicle Routing—A Survey of VRPs with Multiple Synchronization Constraints

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This paper presents a survey of vehicle routing problems with multiple synchronization constraints. These problems exhibit, in addition to the usual task covering constraints, further synchronization requirements between the vehicles, concerning spatial, temporal, and load aspects. They constitute an emerging field in vehicle routing research and are becoming a "hot" topic. The contribution of the paper is threefold: (i) It presents a classification of different types of synchronization. (ii) It discusses the central issues related to the exact and heuristic solution of such problems. (iii) It comprehensively reviews pertinent literature with respect to applications as well as successful solution approaches, and it identifies promising algorithmic avenues.

Key words: survey; vehicle routing; synchronization; coordination; transshipment; trailer
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1. Introduction

Vehicle routing problems (VRPs) constitute one of the great success stories of operational research (OR). They have been the subject of intensive study for more than half a century now. This has led to the publication of thousands of scientific papers and to the foundation of more than one hundred software companies worldwide selling commercial vehicle routing software. This development is certainly because of the intellectual challenge VRPs pose as well as to their practical relevance in logistics and transport. Research on VRPs is incessantly ongoing, stimulated by unsolved theoretical problems and continuous input from logistics practice. One generic class of VRPs that is receiving more and more interest is denoted here as *vehicle routing problems with multiple synchronization constraints* (*VRPMSs*): In classical vehicle routing problems, synchronization is necessary between the vehicles with respect to which vehicle visits which customer. VRPMSs are VRPs that exhibit additional synchronization requirements with regard to spatial, temporal, and load aspects. For the purposes of this survey, the following definition applies:

> *A VRPMS is a vehicle routing problem in which more than one vehicle may or must be used to fulfill a task.*

It will become clear what is meant by this in a moment: In the next section, an example of a particular VRPMS will make the definition concrete.

VRPMSs constitute an emerging field in VRP research and are becoming a "hot" topic; most of the literature surveyed in this paper was published not more than three years ago, and this is a justification for having written the present survey.

The contribution of the paper is threefold: (i) It presents a classification of VRPMSs. (ii) It discusses the central issues related to the exact and heuristic solution of VRPMSs. (iii) It analyzes scientific publications on VRPMSs with respect to applications as well as successful solution approaches and identifies promising algorithmic avenues.

The presentation of the material assumes familiarity with vehicle routing problems (capacitated VRP, VRP with time windows [VRPTW], pickup-and-delivery problem with time windows, capacitated arc routing problem, dial-a-ride problem, etc.) and with the standard modeling and exact and heuristic solution methodologies (mixed-integer programming, branch/cut/price, local/neighborhood search, metaheuristics). If this is not the case, the reader is referred to Toth and Vigo (2002); Golden, Raghavan, and Wasil (2008); Desaulniers, Desrosiers, and Solomon (2005); Funke, Grünert, and Irnich (2005); Røpke (2005); and Gendreau and Potvin (2010).

The present paper is a strongly abbreviated version of a much more extensive technical report on the subject (Drexl 2011). The reader is referred to this report for further details on any topic discussed in the present paper.

The rest of the paper is structured as follows. The next section gives a concrete example of an archetypal VRPMS and exemplifies the different types of synchronization identified in this survey. In §3, a classification of synchronization (henceforth abbreviated by "s.") is given. Section 4 points out the difficulties concerning the formal modeling and the solution of VRPMSs. Section 5 gives an annotated bibliography of relevant publications by type of s. with respect to applications, models, and algorithms, and it briefly identifies related fields that may offer fruitful input for further study of VRPMSs. Finally, §6 summarizes the central findings of the literature review and proposes promising directions for future research.

2. A Concrete Example: The Vehicle Routing Problem with Trailers and Transshipments

The *vehicle routing problem with trailers and transshipments* (VRPTT) was chosen as a concrete example of a VRPMS because it contains all types of synchronization relevant in this paper. The VRPTT as presented here is a simplified version of the underlying real-world problem. A description of the complete problem can be found in Drexl (2007). The research on the VRPTT was motivated by the problem of raw milk collection in Southern Bavaria, Germany: The milk is collected from farmers and is transported to a dairy plant (the *depot*) every day by a heterogeneous fleet of vehicles stationed at the depot; see Figure 1.

The vehicles differ with respect to two orthogonal criteria: First, lorries and tractors are *autonomous vehicles*, able to move in time and space on their own, whereas drawbar trailers and semi-trailers are *nonautonomous vehicles*, which can move in time on their own but must be pulled by a compatible autonomous vehicle to move in space. Second, lorries and drawbar trailers are *task vehicles*, technically equipped to visit customers and collect supply, whereas tractors and semi-trailers are not; they can only be used as *support vehicles*, that is, as mobile depots to which the task vehicles can transfer load. The load transfers can be carried out at *transshipment locations* (TLs) such as parking places.

Most farmers can only be visited by a lorry without a trailer (a *single* lorry) and are hence called *lorry customers*. The other farmers can be visited by a lorry with or without a trailer and are called *trailer customers*. All customers must be visited exactly once by exactly one task lorry and by at most one task trailer.

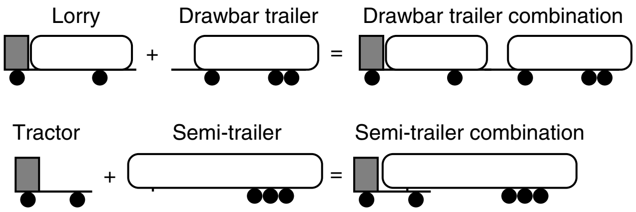

Figure 1 **VRPTT Fleet**

There may be time windows at the customers as well as at the TLs.

All vehicles start and end their routes at the depot. There is no fixed assignment of a trailer to a lorry or of a semi-trailer to a tractor. Any nonautonomous vehicle may be pulled, on the whole or on a part of its itinerary, by any compatible autonomous vehicle. What is more, any vehicle may transfer its load partially or completely to any other vehicle at any TL arbitrarily often. For technical reasons, at any TL, only one transshipment can be performed at a time, and during any transshipment, only one *active* vehicle can transfer load to one *passive* vehicle. Moreover, the time a transshipment takes depends on the amount of load transferred. An example route plan, which for simplicity does not contain support vehicles, is depicted in Figure 2.

In the example, lorry 1, together with the trailer, starts at the depot, goes to a TL, decouples the trailer there, visits two lorry customers, returns to the trailer, transfers some load, leaves the trailer there, and returns to the depot via two lorry and two trailer customers. Lorry 2 starts at the depot, visits two lorry customers, couples the trailer (after lorry 1 has performed its load transfer), visits a trailer customer, decouples the trailer at another TL, possibly performs a load transfer, visits some lorry customers, returns to the trailer, re-couples it, and pulls it back to the depot via a trailer customer. Meanwhile, lorry 3 also starts at the depot, visits some lorry customers, transfers some load to the trailer while lorry 2 is visiting the three lorry customers at bottom right, and returns to the depot via another lorry customer. The two TLs in the center of the figure are not used.

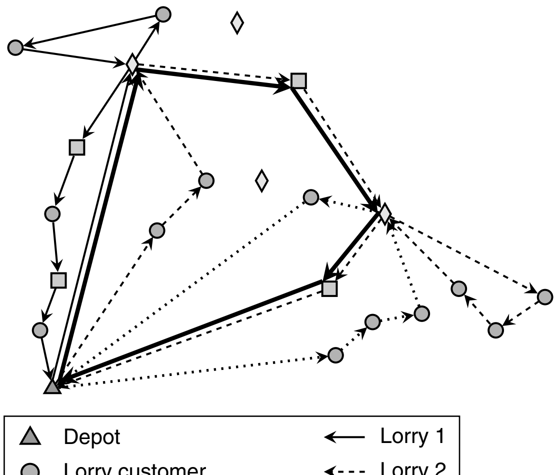

Figure 2 **VRPTT Example Route Plan**

Tractors pull semi-trailers from the depot to TLs, where they either decouple the semi-trailers and return later (in the meantime pulling other semi-trailers) or wait until a semi-trailer has received enough load from other vehicles to be pulled back to the depot. Note that tractors have a capacity of zero and cannot visit any customers; nevertheless, they are useful.

All vehicles may return to the depot for unloading and start new routes arbitrarily often. Vehicles need not carry any load when returning to the depot. Lorries (tractors) need not bring back a drawbar trailer (semi-trailer), neither one they may have pulled when leaving the depot nor any other.

The problem is to devise routings of minimal total costs for all vehicles (some of which may not be needed), such that the complete supply of all customers is collected and delivered to the depot.

The VRPTT has the following properties in connection with synchronization:

(i) Customers may be visited by two vehicles, a lorry and a trailer, at the same time.

(ii) Trailers are nonautonomous vehicles that must be pulled by autonomous vehicles to move in space.

(iii) Trailers may be pulled by different autonomous vehicles on their itinerary.

(iv) Support vehicles cannot visit any customers.

(v) Transshipments are possible between arbitrary vehicles.

(vi) At TLs, only one transshipment can be performed at a time.

A transshipment is defined by

(i) The *location* where the transshipment takes place
(ii) The *point in time* when the transshipment begins
(iii) The *active vehicle*, which transfers all or part of its load
(iv) The *passive vehicle*, which receives load
(v) The *amount of load* transferred

Hence, the central question in the VRPTT is

> *Which vehicle transfers how much load when, where, and into which other vehicle?*

The main difficulty of the problem is because several vehicles may or must participate in fulfilling a task, that is, in collecting a customer's supply and transporting it to the depot. This leads to a close interdependency between the vehicles. This is not usually the case in vehicle routing problems.

3. A Classification of Synchronization

In standard VRPs, vehicles are independent of one another: A change in one route does not affect any other route. In VRPMSs, by contrast, a change in one route may have effects on other routes; in the worst case, a change in one route may render all other routes infeasible. This is called the *interdependence problem*. In Figure 2, if lorry 1 does not visit the leftmost TL but goes directly to the uppermost lorry customer, the trailer cannot move to the TL, and the other two lorries have no opportunity to transfer load, which may violate their capacity constraints.

Addressing the interdependence problem may require different types of synchronization. The following types are identified in this paper:

(i) *Task synchronization*

The fundamental object types in VRPs are *tasks* and *vehicles*. A task is a mandatory duty, something that must be done and requires zero or more units of some capacity. Tasks may consist of collecting supply at or delivering demand to one location, in picking up load at one location and delivering this load to another location, in visiting a location to render a service, etc. A vehicle is an autonomous or nonautonomous mobile object (lorry, trailer, driver, etc.) that provides zero or more capacity units and may or must be used to fulfill tasks. Task s. means it must be decided which vehicle(s) fulfill each task. Task s. is what differentiates VRPs from single-vehicle routing problems such as traveling salesman or postman problems. As mentioned, in the VRPTT, all customers must be visited exactly once by exactly one task lorry and by at most one task trailer. The fundamental problem of task s. can thus be stated as follows:

> *Each task must be performed exactly once by one or more suitable vehicle(s).*

(ii) *Operation synchronization*

An *operation* is something that may or must be performed by a vehicle at a location or vertex to allow or facilitate the execution of one or more tasks. Operation s. is the s. of operations of *different vehicles* at the *same or different locations* (or vertices) with regard to the *time* at which the vehicles perform their respective operation at the respective location(s). Consequently, operation s. decides on spatial and temporal aspects of tasks. Operation s. may induce *dynamic time windows*. A dynamic time window for execution of an operation depends on the execution of another operation. The computation of a schedule for one vehicle without considering schedules for other vehicles is *not* operation s. In the VRPTT, transshipments are possible only if both the active and the passive vehicle are present at the respective location during the time needed for the load transfer. The dynamic time window for when the passive vehicle involved in a transshipment can start executing its operation, that is, start to receive load, depends on the arrival time of the active vehicle and vice versa, and this arrival time is not given in advance but is determined in the course of the algorithm. The fundamental problem of operation s. can hence be formulated as follows:

> *The offset, that is, the time that may elapse between the start of execution of a specified operation by a suitable vehicle at*

> *a certain vertex and the start of execution of another specified operation by another suitable vehicle at another certain vertex, must lie within a specified finite interval of zero or positive length, both vehicles must be compatible, and the vertices may be the same one or different ones.*

With respect to the consideration of the temporal aspect, three types of operation s. can be distinguished, where the offset is denoted by Δ and the interval within which it must lie by $[a, b]$, $a \leqq b$:

(a) Pure spatial operation s. ($a < b$, $b - a = T =$ length of overall planning horizon $<\infty$ without loss of generality)

This is the case when in a model or an algorithm, the temporal aspect of operation s. is irrelevant or is ignored and only the spatial aspect is taken into account. For example, a model of an application with transshipment possibilities might only ensure that the active and the passive vehicle visit the same transshipment location on their respective routes. The temporal aspect of operation s., that is, the determination of concrete vehicle schedules, can then be considered after such a model has been solved, either by dispatchers who manually assign rendezvous times or by bilateral agreement among the drivers performing the computed routes.

(b) Operation s. with precedences ($a < b$, $b - a < T$)

This is the case when two vehicles must start executing their respective operation at their respective vertex with a variable offset ($a \leqq \Delta \leqq b$).

(c) Exact operation s. ($a = b$)

This means that two vehicles must either start executing their respective operation at their respective vertex at the same time (simultaneous operation s., $0 = a = \Delta = b$) or with a fixed offset of positive cardinality (deferred operation s., $0 \neq a = \Delta = b$).

(iii) *Movement synchronization*

This refers to the fact that *nonautonomous vehicles* require *autonomous vehicles* to move in space or that nonautonomous vehicles have to join to become able to move in space. In both cases, the movements of at least two *elementary vehicles* must be synchronized with respect to space and time, yielding an autonomous *composite vehicle*. In the VRPTT, drawbar trailers must be pulled by compatible lorries and semi-trailers must be pulled by compatible tractors. The fundamental problem of movement s. can therefore be formulated as follows:

> *For a vehicle to be able to move along a certain arc, a different but compatible vehicle must move along the same arc at the same time; that is, both vehicles must leave the tail of the arc at the same time, traverse the arc together, and reach the head of the arc at the same time.*

This is a restrictive definition and excludes deferred execution of tasks along the same arc.

There are two subtypes of movement s.:

(a) Movement s. at the depot

This is the case when two vehicles may join and separate only at the depot, before the start and after the end of a route.

(b) Movement s. en route

This is obviously the case when two vehicles may join and separate at different locations that they may visit during their route.

(iv) *Load synchronization*

The amount of capacity used on vehicles when fulfilling a task or performing an operation must be correctly taken into account. In other words, it must always be ensured that the right amount of load is collected, delivered, or transshipped. In the VRPTT, for each transshipment, it must be decided how much load is to be transferred. The load the active vehicle unloads is exactly equal to the load the passive vehicle receives; no load gets lost. The fundamental problem of load s. can then be formulated as follows:

> *For each vertex with specified negative, zero, or positive demand, the difference between the total amount of load unloaded at the vertex by all active vehicles visiting it and the total amount of load received at the vertex by all passive vehicles visiting it must be equal to the specified demand.*

There are three subtypes of load s.:

(a) Fixed load s.

This is the case when the amount of load that can be delivered, collected, or transferred is fixed in advance, for example, when the application context requires that during a transshipment the active vehicle always unloads completely.

(b) Discretized load s.

This is the case when there is a finite number of possible amounts of load that can be delivered, collected, or transferred.

(c) Continuous load s.

In this case, the amount of load that can be delivered, collected, or transferred may be any real number between zero and the respective obvious upper bound.

(v) *Resource synchronization*

This is necessary when different vehicles compete for *common*, scarce resources. In the VRPTT, transshipment possibilities are limited: Only one active vehicle can perform a load transfer to a certain passive vehicle at a certain location at a certain point in time. The fundamental problem of resource s. can accordingly be formulated as follows:

At any point in time, the total utilization or consumption of a specified resource by all vehicles must be less than or equal to a specified limit.

Remarks on the above classification scheme follow.

The proposed scheme is of course not the only way to categorize synchronization. However, the scheme captures significant aspects of synchronization and allows a structured view of synchronization as well as a structured review of the pertinent literature.

One issue must be pointed out here: The types of synchronization requirements that are present in a real-world application, an academic problem, or a mathematical model are *always a matter of perspective and/or a modeling decision*. This means that the client company in an industry project defines what it considers the "problem" in its application context. If, for example, a company uses a fleet of lorries and trailers that technically allows a free lorry-trailer assignment and arbitrary load transfers, then route planning for this company's fleet could be regarded as a VRPTT. If, however, for corporate policy reasons the company decides to require a fixed lorry-trailer assignment and to exclude the option of load transfers, then the "problem" to be modeled and solved by the OR professional is not a VRPMS, although the "underlying real-world problem" is. Similarly, if an OR professional trying to model and solve a VRPTT in the context of raw milk collection decides to allow only a discrete finite number of load transfer amounts (to be able to apply a certain solution approach), then the resulting model requires discretized load synchronization, whereas the "underlying real-world problem" requires continuous load synchronization. Therefore, concrete examples of synchronization requirements may be subsumed under more than one of the above types. In particular, all types can be *modeled* as resource s. because of the immense generality and flexibility of the resource concept. The subsequent literature review lists a cited paper under those synchronization categories that are captured in the model that is solved.

Also the definition of the tasks is a matter of perspective, a modeling decision. An underlying application may only suggest one definition or an other. For example, Bredström and Rönnqvist (2008) describe an application in homecare staff scheduling, where two nurses must visit a disabled person at the same time for lifting purposes or with a fixed offset to apply medicine after a meal. One option is to say that a task consists of the visits of two nurses. The other option is to say that each visit of a nurse is one task. In the following, it will become clear from the context what a task is in each application.

Operation and movement s. are also denoted *space-time synchronization*. Space-time and load s. are interdependent if the time needed for a load transfer depends on the amount of load transferred (*load-dependent load transfer times*), as in the VRPTT. If splitting of loads is allowed, task s. requires load s.

Synchronization is not restricted to be performed between only two vehicles. The principle extends in a straightforward manner to three or more vehicles/objects and to transshipments in more than one direction at the same location at the same time. See, for example, Recker (1995); Bürckert, Fischer, and Vierke (2000); Rivers (2002); Li, Lim, and Rodrigues (2005); and Cheung et al. (2008).

4. Modeling and Algorithmic Issues

VRPMSs are generalizations of VRPs. As such, the modeling and algorithmic tools developed for VRPs can basically be used for VRPMSs, too, but as this survey will show, additional modeling and solution efforts are in most cases necessary to solve VRPMSs.

Seen from a *modeling perspective*, it is sometimes convenient to introduce complex networks for VRPMSs. For example, vertices do not need to directly correspond to a real-world location only but may instead represent points in space and time or even multidimensional objects such as space-time-operation-vehicle combinations and the like. Several solution approaches for VRPMSs discussed below are based on canny network representations.

When developing mixed integer programming (MIP) models for VRPMSs, all types of s. can be represented as constraints. The unified model by Desaulniers et al. (1998) provides a suitable framework for representing all VRPMSs discussed in this survey.

Seen from an *algorithmic perspective*, the standard exact approach for solving vehicle routing and scheduling problems formulated as MIPs is the branch-and-cut-and-price (BCP) principle, that is, the combination of cut and column generation embedded in branch-and-bound (Desaulniers, Desrosiers, and Solomon 2005). In column generation terminology (see, for example, Lübbecke and Desrosiers 2005), the synchronization constraints are basically *coupling* or *linking* or *joint* constraints, which go into the master problem and which provide dual prices guiding the generation of new variables/columns by the solution of the subproblem. The solution of the master problem is mostly not too difficult for VRPs. The difficulty lies in solving the sub- or pricing problem, which has the structure of an elementary shortest path problem with resource constraints (ESPPRC; see Irnich and Desaulniers 2005). This problem is usually solved by a so-called labeling algorithm based on dynamic programming. Desaulniers et al. (1998) give properties the subproblem has to fulfill so that such a labeling algorithm can be applied. As pointed out in Drexl (2007), unfortunately these properties are not fulfilled for the VRPTT subproblem (and also not for other VRPMSs). Other approaches for the exact solution of the ESPPRC subproblem have not been successful up to now (Jepsen, Petersen, and Spoorendonk 2008). Therefore, the exact solution of many types of VRPMSs, most notably the VRPTT, is an open research topic.

The standard heuristic approach for solving large-scale real-world rich VRPs is based on local search (Funke, Grünert, and Irnich 2005) and/or large neighborhood search (Røpke 2005) embedded in a metaheuristic (Gendreau and Potvin 2010). Basically, local search procedures for VRPs exploit routes being independent of one another so that changes to one route (or two routes in the case of a swap move etc.) do not affect other routes. The interdependence problem encountered in VRPMSs precisely means that routes in VRPMSs *are* affected by changes to other routes. This is relevant for the feasibility of other routes as well as the objective function value of a solution. A change in one route may make all other routes infeasible, so the evaluation of a move may require checking the feasibility of all other routes. If, as usual in VRPs, the objective is to minimize the overall distance traveled or the number of vehicles used, the evaluation of the overall objective function remains easy also for VRPMSs because the contribution of each route to the objective function remains independent of other routes. If, however, the objective is, for example, to minimize the maximal route duration or the duration of the execution of the complete route plan, then to recompute the objective function value after a move in a standard VRP still requires only the recomputation of the schedule of the modified route(s), whereas in VRPMSs, the interdependence problem may require the rescheduling of all routes.

A fundamental observation in standard VRPs is that a given solution in form of a set of vehicle routes, or, in other words, a vehicle flow, completely determines the path each request takes, be the latter a simple demand or supply request in a classical VRP or a pickup-and-delivery request. This is because a request is transported by exactly one vehicle and only this vehicle visits the corresponding request location(s). When transshipments are possible, this is no longer the case because the vehicle picking up a request need not necessarily transport it to the depot/delivery location. In single-commodity problems, that is, when a homogeneous, substitutable good is to be transported, as in the VRPTT, this is not an issue. In multicommodity problems—that is, pickup-and-delivery problems where each request consists in the transport of a unique, nonsubstitutable commodity such as a parcel or a letter between a dedicated pickup and a dedicated delivery location—though, the problem of determining *request leg sequences* arises. For example, if request r is to transport some good from location r^+ to location r^-, and if it is possible to transship the request at a TL l, then it must be decided whether to transport the request over one leg $r^+ \to r^-$ with one vehicle or over two legs $r^+ \to l$ and $l \to r^-$ with two vehicles. (Note that if a leg for some request r is from location l_1 to location l_2, this does not mean that the vehicle k transporting r from l_1 to l_2 drives directly from l_1 to l_2: After picking up r at l_1, k may visit an arbitrary number of locations before reaching l_2. However, r will stay on k from l_1 to l_2.) Using the second possibility obviously induces a dynamic time window for the second leg because the second leg cannot be performed before the first one is finished. Consequently, if there is a change in the route performing the first leg before visiting l or in the route performing the second leg after visiting l, the respective other route is affected. This effect may further propagate and may, in the worst case, affect all routes. In particular, such a change may be the insertion or the removal of a leg sequence or a leg in insertion heuristics.

The determination of leg sequences need not be the first step in a heuristic, nor need they be determined explicitly at all. In MIP approaches, the decision variables must be such that request leg sequences are either explicitly modeled by decision variables or can be reconstructed from a solution.

Because the VRPTT contains all aspects of s. identified in §3, a successful exact or heuristic solution procedure for the VRPTT would provide a general procedure for VRPMSs. However, no powerful solution algorithm for the VRPTT yet exists. Moreover, it is probable that more specialized algorithms for VRPMSs with fewer synchronization requirements are easier to develop and lead to better solution quality and shorter computation times. Nevertheless, two research goals for the near future concerning VRPMSs are (i) the development of an exact branch-and-cut-and-price algorithm for the VRPTT capable of solving problems larger than the tiny instances described in Drexl (2007) and (ii) the development of a metaheuristic combining large neighborhood search and local search capable of solving real-world VRPTTs. Therefore, with respect to modeling and algorithmic approaches surveyed in this paper, the focus will be on such methods. The literature survey below will pay special attention to how the abovementioned two central issues, the solution of the pricing problem in BCP algorithms and the solution of the interdependence problem in (local search) heuristics, are addressed. Other potentially promising ways for solving VRPMSs and their potential advantages and drawbacks will nevertheless be outlined.

5. Literature Survey

As its title implies, this paper is a survey on synchronization in *vehicle routing* problems. Therefore, problems which are not VRPs are not considered and neither are VRPs without multiple synchronization constraints.

With respect to applications of VRPMSs, or, put differently, with regard to the *causes* for the existence of multiple synchronization constraints in a problem, four main types were identified as a result of the literature review:

(i) the possibility of splitting the pickup or the delivery of load at a customer between several visits by several vehicles
(ii) the possibility or requirement of transshipment of load or transfer of persons
(iii) the requirement of simultaneous presence of vehicles at a location to render a service
(iv) the existence of nonautonomous vehicles

With respect to the types of s. that appear in a problem in addition to task s., a considerable number of applications and publications was found for the following:

(i) Load s.
(ii) Resource s.
(iii) Operation s.
(iv) Movement s.
(v) Operation and load s.
(vi) Movement, operation, and load s.

The subsequent literature review is structured by subtype of s. Although the literature on VRPMSs is not yet as extensive as that on the VRP or the VRP with time windows (VRPTW), it is beyond the scope of this survey to give a detailed review of the cited references. Therefore, in what follows, problems with load and resource s. are only briefly discussed before giving an annotated bibliography and describing in greater detail recurring and successful modeling and solution approaches for the different subtypes of operation and movement s. At the end of the section, related fields that might be of interest are briefly sketched. Moreover, to provide a unified treatment, the appendix contains a glossary of terms and a summary of abbreviations.

5.1. Load Synchronization

VRPMSs with exclusively task and load s. are known in the literature as *split delivery VRPs*. In these problems, it is allowed that several vehicles visit a customer, each delivering (collecting) a part of the customer's demand (supply). Load s. is necessary at the customer vertices, even though no transshipments are allowed. For more information on the split delivery VRP or pickup-and-delivery problem (PDP), the reader is referred to Hooker and Natraj (1995); Chen, Golden, and Wasil (2007); Archetti and Speranza (2008); Nowak, Ergun, and White (2008); Schönberger et al. (2009); Desaulniers (2010); Derigs, Li, and Vogel (2010); and Hennig (2010).

5.2. Resource Synchronization

Resource s. was introduced under the name *intertour resource constraints* in Hempsch and Irnich (2008). There, a generic model for representing rich VRPs with resource s. is developed. This model is based on the unified framework by Irnich (2008) and uses the giant-route representation (Christofides and Eilon 1969; Funke, Grünert, and Irnich 2005) and the concept of resource-constrained shortest paths (Irnich and Desaulniers 2005). The innovative idea is that the giant route is considered as one single, resource-constrained shortest path. By doing so, efficient solution procedures for local search developed for VRPs without resource s. can be used also for VRPs with resource s.

Examples of resource s. abound. Hempsch and Irnich (2008) mention a limited number of docking stations at depots, a limited number of routes with certain properties such as distance or duration, time-varying sorting capacities at mail-sorting centers, and the allocation of a limited fleet to several depots. Two further exemplary contributions are Ebben, van der Heijden, and van Harten (2005) and El Hachemi, Gendreau, and Rousseau (2011b). Ebben, van der Heijden, and van Harten (2005) study the scheduling of automated guided vehicles (AGVs) at an airport. Transport requests have to be fulfilled by AGVs capable of performing one request at a time. There are several scarce resources to be considered: the number of docks for (un)loading cargo, parking places for currently unused AGVs, and cargo storage space. El Hachemi, Gendreau, and Rousseau (2011b) study an application in the context of forest management. Vehicles have to transport wood from forest areas to mills. For loading the wood onto the vehicles, special loading machines, which are capable of loading one vehicle at a time, are necessary, and there is only one machine available per area.

It must be noted that it is highly difficult to find publications considering VRPs with resource s. because it usually cannot be deduced from the title or the keywords whether or not resource s. is relevant in a paper.

5.3. Pure Spatial Operation Synchronization

Table 1 gives an overview of research on pure spatial operation s. The applications described in the referenced papers all consider transshipment possibilities. This obviously introduces interdependencies between routes and makes operation s. nontrivial even when the time aspect is neglected.

(Note: In many papers listed in the tables in §5, MIP models are used only to precisely define the respective problem, but no MIP-based algorithm is developed. Thus, often the column "MIP variable type" contains an entry for a paper even though the corresponding column "Solution approach(es)" lists only heuristic approaches.)

An important class of VRPMSs is that of *multi-echelon* (or *N-echelon*) *vehicle routing problems*, which

Table 1 Research on Pure Spatial Operation Synchronization

Paper	Problem type(s) and application(s)	Objects to synchronize	Types of s.	Solution approach(es)	MIP variable type
Russell and Morrel (1986)	VRP: School bus routing	Buses	Pure spatial operation, fixed load	Heuristic hierarchical decomposition	—
Baker, Franz, and Sweigart (1993)	PDP: Passenger transport	Small buses	Pure spatial operation, fixed load	Standard MIP solver	Arc
Rivers (2002)	VRP: Bitumen delivery, mid-air refueling of aircraft, school bus routing; PDP with transshipments	Abstract, autonomous vehicles	Pure spatial operation, continuously split load	Cluster-first-route-second, local search	—
Amaya, Langevin, and Trépanier (2007)	CARP: Road marking	1 task and 1 support vehicle	Pure spatial operation, fixed load	Branch-and-cut	Arc
Amaya, Langevin, and Trépanier (2010)	CARP: Road marking	1 task and 1 support vehicle	Pure spatial operation, fixed load	Branch-and-cut, heuristic route-first-cluster-second	Arc
Gonzalez-Feliu et al. (2008); Perboli, Tadei, and Vigo (2008)	N-echelon VRP	Task and support lorries	Pure spatial operation, fixed load	Branch-and-cut	Arc
Perboli, Tadei, and Masoero (2009); Perboli, Tadei, and Tadei (2010)	2-echelon VRP	Task and support lorries	Pure spatial operation, fixed load	Branch-and-cut	Arc
Gonzalez-Feliu (2009)	N-echelon LRP	Task and support lorries	Pure spatial operation, fixed load	—	Path
Ambrosino and Scutellà (2005)	N-echelon LRP	Task and support lorries	Pure spatial operation, fixed load	Standard MIP solver	Arc
Crainic et al. (2010)	2-echelon VRP	Task and support lorries	Pure spatial operation, fixed load	Hierarchical decomposition, cluster-first-route-second, multi-start local search	—
Nguyen, Prins, and Prodhon (2010)	2-echelon LRP	Task and support lorries	Pure spatial operation, fixed load	Hierarchical decomposition, hybrid GRASP and evolutionary/iterated local search	—
Boccia et al. (2010)	2-echelon LRP	Task and support lorries	Pure spatial operation, fixed load	Hierarchical decomposition, tabu search	—

is formally introduced in Gonzalez-Feliu et al. (2008) and Perboli, Tadei, and Vigo (2008), where these terms are used for the first time. The basic idea behind this problem class is that customers are not delivered directly from a central depot but via N legs in an N-stage distribution network. An N-stage distribution network contains $N+1$ levels of location. Echelon or stage $n \in \{1, \ldots, N\}$ considers transports from location level $n-1$ to n. For each stage n, there are dedicated vehicles that can only visit the locations defining stage n. This means that only the vehicles of stage N are task vehicles, that is, are allowed to visit customers; all other vehicles are support vehicles. All vehicles are autonomous. Load transfers are only possible between vehicles of different stages. The difference with distribution network design problems is that for each vehicle in the problem, a complete route is computed. The general N-echelon *location-routing problem* (LRP), which is studied by several authors, differs from the N-echelon VRP in that the latter, contrary to the former, considers fixed costs for opening a TL.

Essentially, the two-echelon VRP is a VRPTT as described in §2, but without trailers and with a fixed assignment of tractors and semi-trailers. However, the VRPTT comprises also the N-echelon VRP and LRP for arbitrary N. This is simply a matter of modeling the fleet and the structure of the network defining a VRPTT instance.

The literature review shows that heuristics for two-stage problems, that is, two-echelon VRPs and LRPs as well as the problem studied in Russell and Morrel (1986), use *decomposition by stage* as their central idea. When no time aspect is present, sequential or iteratively alternating consideration of stages is apparently the adequate strategy to obtain high-quality solutions.

5.4. Operation Synchronization with Precedences

As can be seen Table 2, most papers on operation s. with precedences consider dial-a-ride or pickup-and-delivery problems with transshipments. Persons or goods can be left behind at TLs by an unloading vehicle and be picked up some time later by a reloading vehicle. In other words, operation s. with precedences and fixed load s. are considered.

Table 2 Research on Operation Synchronization with Precedences

Paper	Application(s)	Objects to synchronize	Types of s.	Solution approach(es)	MIP variable type
Grünert and Sebastian (2000)	PDP: Letter mail transport	Lorry-trailer combinations, aircraft	Operation with precedences, continuously split load	—	Arc
Mues and Pickl (2005)	PDP: Long-haul road transport	Lorry-trailer combinations	Operation with precedences, fixed load	Column generation	Path
Cortés, Matamala, and Contardo (2010)	PDP: Passenger transport	Heterogeneous vehicles	Operation with precedences, fixed load	Benders decomposition	Arc
Wen et al. (2009)	PDP with transshipment at a cross-docking centre	Lorries	Operation with precedences, fixed load	Sweep-like construction, unified tabu search (Cordeau, Laporte, and Mercier 2001) embedded in adaptive memory procedure	Arc
Oertel (2000)	General PDP	Abstract, autonomous vehicles	Operation with precedences, fixed load	Two-stage constructive, tabu search	—
Bock (2010)	Dynamic PDP: Intermodal long-haul transport	Lorries, trains	Operation with precedences, fixed load	Sequential insertion	Arc
Jacobsen and Madsen (1980)	2-echelon LRP: Newspaper distribution	Task and support lorries	Operation with precedences, fixed load	Heuristics: Greedy spanning tree, hierarchical decomposition	—
Lin (2008)	PDP: Document courier service	Uncapacitated vans	Operation with precedences, fixed load	Tree search with explicit enumeration	Arc
Aldaihani and Dessouky (2003)	Dial-a-ride	Small taxis and fixed-schedule buses	Operation with precedences, fixed load	Three-stage heuristic construction, local search	—
Shang and Cuff (1996); Thangiah, Fergany, and Awan (2007)	PDP: Parcel transport	Uncapacitated vans	Operation with precedences, fixed load	Parallel best insertion, local search	—
Mitrović-Minić and Laporte (2006)	PDP: Parcel transport	Uncapacitated vans	Operation with precedences, fixed load	Multi-start cheapest insertion, descent improvement	—
Gørtz, Nagarajan, and Ravi (2009)	Dial-a-ride	Abstract, autonomous vehicles	Operation with precedences, fixed load	Route-first-cluster second approximation algorithm	—
Rousseau, Gendreau, and Pesant (2003)	Dynamic VRP: Personnel dispatching	Capacitated service vehicles	Simultaneous and deferred operation, operation with precedences	Heuristic: Construction/insertion by constraint programming, improvement by local search	—
Groër, Golden, and Wasil (2009)	*Consistent* VRP: Multi-period parcel delivery	Parcel vans	Soft operation with precedences	Savings heuristic and local search embedded in record-to-record travel, standard MIP solver	Arc
Fügenschuh (2006, 2009)	VRP: School bus routing	Buses	Operation with precedences	Greedy heuristic and local search, branch-and-cut	Arc

As for exact solution methods, a technique proposed by several authors (Grünert and Sebastian 2000; Mues and Pickl 2005; Crainic, Ricciardi, and Storchi 2009) for column generation approaches is to *explicitly determine routes (paths, flows) for requests and link them to vehicle routes (paths, flows)* by coupling constraints in the master problem. This means that in the master problem, in addition to the path variables representing the possible itineraries a vehicle may take through the network, there are path variables for the possible itineraries a request may take, and there are subproblems for generating vehicle paths as well as subproblems for generating request paths. It must be noted, however, that no implementation of such an approach is reported. The three cited references only present pertinent models and propose to solve them in this way. Apparently, an implementation of this idea is nontrivial and constitutes as yet an open research issue.

The fundamental heuristic technique for addressing operation s. with precedences is to *explicitly determine request leg sequences* (Oertel 2000; Aldaihani and Dessouky 2003; Feige 2003; Mitrović-Minić and Laporte 2006; Bock 2010). The cited references use problem-specific knowledge and employ different methods for the determination of promising sequences. In most papers, the number of possible sequences is limited. In particular, when there are not more than two potential TLs, there are at most five possible request leg sequences for a pickup-and-delivery request (direct transport, two sequences with one transshipment, two sequences with two transshipments). When there are more potential TLs, the number of potential request leg sequences is limited by restricting the number of legs in a sequence, and promising sequences are selected heuristically. In constructive procedures, when inserting a request, most authors try out several different request leg sequences for the request. In improvement procedures, one generic neighborhood consists in replacing the chosen leg sequence for a request by a different sequence.

5.5. Exact Operation Synchronization

As Table 3 shows, literature on exact operation s. considers a wide variety of applications and a broad spectrum of solution approaches.

A recurring modeling technique in MIP approaches for problems with exact operation s. is the use of *one vehicle-independent time variable* t_i *for the beginning of execution of a task or operation requiring more than one vehicle at a vertex* i (Li, Lim, and Rodrigues 2005; Lim, Rodrigues, and Song 2004; Dohn, Kolind, and Clausen 2009; Cortés, Matamala, and Contardo 2010). In arc-variable based formulations, this guarantees implicit temporal synchronization without having to formulate explicit constraints linking the time variables of different vehicles. The drawback is that in path-variable based approaches for solution by column generation or branch-and-price, the time variables remain in the master problem and induce linear costs on the vertices in the subproblem, which makes the solution of the latter with standard labeling algorithms difficult. Ioachim et al. (1999) and Bélanger et al. (2006) use a rather involved extended labeling algorithm presented in Ioachim et al. (1998) to deal with this issue. Dohn, Kolind, and Clausen (2009) obtain a master problem that does not contain the t_i variables by means of an ingenious and quite involved reformulation of their problem.

Another technique several authors propose to deal with exact operation s. in MIP approaches is *branching on time windows*. This branching strategy was introduced by Gélinas et al. (1995) in the context of the VRPTW. It is an incomplete strategy that requires additional strategies to ensure integrality. Mostly, branching on original arc variables is used to this end. Moreover, it requires that the time dimension be discretized, but this is not really restrictive in most applications. The idea is to identify fractional t_i variables or paths visiting the same vertex at different times and to create two new branches by partitioning the remaining time window of the vertex into two subintervals, forbidding the current t_i value. Branching on time windows is successfully used to ensure exact operation s. by Ioachim et al. (1999), Bélanger et al. (2006), and Dohn, Kolind, and Clausen (2009).

The survey shows that interdependencies between vehicles induced by the temporal aspect of operation s. (with precedences as well as exact) are tedious to handle and, above all, to program in heuristics. This is because checking feasibility as well as evaluating an objective function containing time-dependent components may require rescheduling all other routes after even a very simple move such as the relocation of a customer within one route. Doing so after each move may be prohibitively time consuming. Three ways to overcome this situation are (i) *allowing infeasible solutions during the solution process* (Oertel 2000; De Rosa et al. 2002; Wen et al. 2009; Prescott-Gagnon, Desaulniers, and Rousseau 2010), (ii) *evaluating the costs of a move only approximately* (De Rosa et al. 2002; Wen et al. 2009), and (iii) the use of *indirect search* (Feige 2003; Lim, Rodrigues, and Song 2004; Li, Lim, and Rodrigues 2005).

Infeasible solutions are considered by weighted penalty terms in the objective function. The weights are dynamically adjusted in the course of the algorithm; that is, weights are increased/decreased if violations increase/decrease. Solutions are allowed to be infeasible with respect to different constraints; most importantly, time windows may be violated because these are the constraints most difficult to maintain.

An *approximate evaluation of moves* means the determination of the costs of a move by heuristic routines that must be fast but must nevertheless provide a good indication of the exact value. Striking a balance between these two conflicting requirements is nontrivial and is achieved in highly problem-specific ways in the cited references.

Standard heuristic search methods (local search, large neighborhood search) operate on a search space consisting of the set of (feasible) solutions. *Indirect search* (Derigs and Döhmer 2008) operates on a weakly constrained or even unconstrained auxiliary search space whose elements are simple structures that allow using all standard search techniques and move types. To transform an element of the auxiliary space back into an element of the original space, that is, into a solution to the problem under consideration, a *decoder*

Table 3 Research on Exact Operation Synchronization

Paper	Application(s)	Objects to synchronize	Types of s.	Solution approach(es)	MIP variable type
Ioachim et al. (1999); Bélanger et al. (2006)	Aircraft fleet routing and scheduling	Aircraft	Simultaneous operation	Branch-and-price	Arc, path
Rousseau, Gendreau, and Pesant (2003)	Dynamic VRP: Personnel dispatching	Capacitated service vehicles	Simultaneous and deferred operation, operation with precedences	Heuristic: Construction/insertion by constraint programming, improvement by local search	—
Bredström and Rönnqvist (2008)	VRP: Homecare staff scheduling, planning of routes for security guards, forest management	Persons with different qualifications, cranes and forwarding vehicles	Simultaneous and deferred operation, operation with precedences	MIP-based heuristic	Arc
Li, Lim, and Rodrigues (2005); Lim, Rodrigues, and Song (2004)	VRP: Staff scheduling	Workers with different qualifications	Simultaneous operation	Heuristic: Parallel insertion and simulated annealing with indirect search	Arc
Dohn, Kolind, and Clausen (2009)	VRP: Staff scheduling	Workers with different qualifications	Simultaneous operation	Branch-and-price	Arc, path
Feige (2003)	PDP: long-haul swap-body platform road transport	Lorry-trailer combinations with capacity 2	Simultaneous operation, fixed load, resource	Iterative, nested solution of capacitated non-bipartite matching, large neighborhood search	—
Crainic, Ricciardi, and Storchi (2009)	2-echelon VRP: City logistics	Task and support lorries	Simultaneous operation, fixed load	—	Path
De Rosa et al. (2002)	CARP: Garbage collection	Large and small collection vehicles	Simultaneous operation, fixed load	Route-first-cluster-second, local search	—
Del Pia and Filippi (2006)	CARP: Garbage collection	Large and small collection vehicles	Simultaneous operation, fixed load	Construction by VND-CARP (Hertz and Mittaz 2001), improvement by VNS	—
Schmid et al. (2010)	PDP: Concrete delivery to construction sites	Heterogeneous task and support vehicles	Soft simultaneous operation, discretely split load	VNS, VLNS using MIP-based heuristic	Arc
Drexl (2007)	VRP: Raw milk collection	Lorries/tractors and trailers/semi-trailers	Movement en route, simultaneous and deferred operation, continuously split load	Branch-and-cut, Branch-and-price	Arc, turn, path
Bürckert, Fischer, and Vierke (2000)	PDP: Long-distance road transport	Drivers, lorries with loading capacity, lorries without loading capacity, tractors, trailers, semi-trailers, chassis, and swap-bodies	Movement en route, simultaneous and deferred operation, fixed load	Holonic multi-agent system	—
Cheung et al. (2008)	PDP: Seaport container drayage	Tractors, drivers, semi-trailers	Movement en route, simultaneous and deferred operation, fixed load	Attribute-decision model	—

is needed. In the context of VRPMSs, the original search space is the set of vehicle routes. An indirect search approach for a VRPMS might define the auxiliary space as the set of permutations of customers. The search methods used for classical VRPs are well applicable to such a search space. A decoder might then be a greedy construction heuristic that inserts customers into routes in the order in which the customers appear in a permutation. There are, however, some conditions for this approach to be promising. In particular, it should be possible to transform each feasible element of the auxiliary space into a feasible solution to the original problem, and this transformation should be as fast as possible. Moreover, small changes to an element of the auxiliary space should only lead to small changes to the corresponding element of the original space.

A further solution approach used by several authors (Rousseau, Gendreau, and Pesant 2003; Laurent and Hao 2007; El Hachemi, Gendreau, and Rousseau 2011b) is *constraint programming*. This method has its strengths in solving tightly constrained problems. It allows convenient modeling of complex constraints and, at the same time, provides powerful and extensible solution techniques. It is beyond the scope of this survey, though, to go into the details of modeling and solving VRPMSs with constraint programming. Suffice it to say that domain variables are a comfortable means of representing visit times, which are relevant for operation s., and that standard constraint propagation techniques are available to deal with synchronization constraints.

Note that the transition from exact operation s. to operation s. with precedences is a gradual one. Several approaches presented in this section, in particular, heuristic ones, could be modified to consider operation s. with precedences, and also some approaches for operation s. with precedences could be applied to exact operation s.

5.6. Movement Synchronization at the Depot

Problems where two types of nonautonomous objects perform movement s. at a central depot, that is, where the objects perform a complete route together, fall into this category. Table 4 gives an overview.

Although the objects to be synchronized in all papers are vehicles and drivers, the concrete application contexts are all different. Note that it depends on the application whether a certain class of real-world objects is regarded as autonomous or nonautonomous: In the VRPTT, as in most other VRPs, lorries are autonomous objects because it is implicitly assumed that each lorry is manned with a certain driver. In the applications described in the papers in Table 4, both lorries/vehicles and drivers are nonautonomous because there is no fixed assignment of a driver to a vehicle.

Regarding solution approaches, some papers (Chung and Norback 1991; Xiang, Chu, and Chen 2006) *determine abstract routes first and assign concrete vehicles and drivers afterwards*. When determining the routes, some aspects are taken into account that are important to be able to obtain feasible solutions in later stages. For example, driving time limitations for drivers are considered in route construction steps by setting an upper bound on route durations.

Other papers (Laurent and Hao 2007; Prescott-Gagnon, Desaulniers, and Rousseau 2010) take the opposite way and *compute routes for predetermined driver-vehicle pairs*. Initial solutions are computed by assigning tasks to compatible driver-vehicle pairs in a greedy fashion. After that, alternative driver-vehicle pairs for given routes are evaluated in local search steps.

For movement s. at the depot, vehicles and drivers may join and separate only at one location. This suggests to *decompose the problem by object type* (analogous to the decomposition by stage described in §5.3 for pure spatial operation s. but taking into account the temporal aspect present in movement s.). However, only Zäpfel and Bögl (2008) take this approach. These authors first determine vehicle routes by solving a generalized VRPTW and afterward assign vehicle routes to drivers by solving a kind of generalized assignment problem.

In most of the described applications, it is possible that a driver and also a vehicle perform more than one route so that, in addition to finding appropriate pairs of objects, it must be decided *when* these objects

Table 4 Research on Movement Synchronization at the Depot

Paper	Application(s)	Objects to synchronize	Types of s.	Solution approach(es)	MIP variable type
Chung and Norback (1991)	VRP: Food distribution	Lorries and drivers	Movement at depot	Routing via seed points, assignment of vehicles and drivers afterwards	—
Prescott-Gagnon, Desaulniers, and Rousseau (2010)	VRP: Oil delivery	Lorries and drivers	Movement at depot	Construction by greedy best insertion, improvement by hybrid LNS/tabu search	—
Laurent and Hao (2007)	PDP: Limousine rental	Limousines and drivers	Movement at depot	Constraint programming followed by local search using simulated annealing	—
Xiang, Chu, and Chen (2006)	Dial-a-ride	Vehicles and drivers	Movement at depot	Four-stage heuristic with local search, column generation	Path
Zäpfel and Bögl (2008)	VRP: Postal delivery	Vehicles and drivers	Movement at depot	Two-stage heuristic: VRPTW, assignment problem	Arc
Recker (1995)	VRP: Household activities	Cars, drivers, and passengers	Movement at depot	MIP-based heuristic	Arc

join to perform a route. This is mostly achieved by fixing schedules for routes and checking the temporal compatibility of a route with the available time periods of a driver or a vehicle.

5.7. Movement Synchronization En Route

The final type of s. considered is movement s. en route. Composite autonomous objects consisting of two or more types of elementary autonomous and/or nonautonomous objects are required to fulfill tasks. The elementary objects may join and separate at many different locations. Compared to the problems described in the previous section, this adds an additional degree of freedom, and hence complexity, because it needs not only be decided *when* to join and separate but also *where* to do so, and these two decisions are interrelated. Table 5 gives an overview.

The literature search shows that as with movement s. at the depot, the number of publications dealing with movement s. en route is limited, but the applications are again diverse and so are the described solution approaches.

With respect to MIP-based models and methods, four types of formulation are used: (i) Standard three-index arc variables x_{ij}^{k} indicating whether or not vehicle (or object) k uses the arc (i, j); (ii) four-index arc variables $x_{ij}^{kk'}$ indicating whether or not k and k' traverse (i, j) together at the same time (Berning 2009; Kim, Koo, and Park 2010); (iii) so-called *turn variables* x_{hij}^{k} indicating whether or not k traverses arc (h, i) directly before arc (i, j) (this can be regarded as turning from (h, i) into (i, j) at i, hence the name); and (iv) standard path variables λ_{p}^{k} indicating whether or not k performs the route corresponding to path p (Hollis, Forbes, and Douglas 2006). Drexl (2007) uses all four types for modeling the VRPTT.

The $x_{ij}^{kk'}$ variables directly guarantee movement s. and model when and where to join and separate. With the other variable types, constraints are necessary to do so. These constraints link (arc or turn or path) flow variables with time variables at vertices and essentially require that if a vehicle uses an arc, a compatible vehicle use this arc at the same time.

With respect to heuristics, no two papers use the same approach or idea. Kim, Koo, and Park (2010) develop a simple but effective heuristic for synchronizing service teams that must be transported by vehicles between task locations: The teams, the vehicles, and the next tasks are stored in three lists, along with the relevant information on times and locations. In each iteration, a triplet (team, vehicle, task) is selected from the lists, using a best-fit criterion. Then the lists are updated to reflect the situation resulting when the selected vehicle transports the selected team to the location of the selected task.

Hollis, Forbes, and Douglas (2006) essentially extend the approaches by Chung and Norback (1991) and Xiang, Chu, and Chen (2006) described in the previous section to the multiple-depot case: First, abstract vehicle routes are computed by solving a rich pickup-and-delivery problem by heuristic column generation. Then concrete vehicles and drivers are assigned by taking an integrated vehicle and crew *scheduling* approach. This is again done by solving an MIP by heuristic column generation.

Table 5 Research on Movement Synchronization En Route

Paper	Application(s)	Objects to synchronize	Types of s.	Solution approach(es)	MIP variable type
Kim, Koo, and Park (2010)	VRP: Staff scheduling	Vehicles and service teams	Movement en route, operation with precedences	Greedy heuristic, standard MIP solver	Arc
Hollis, Forbes, and Douglas (2006)	PDP: Mail delivery	Vehicles and drivers	Movement en route, operation with precedences	Heuristic column generation	Path
Drexl (2007)	VRP: Raw milk collection	Lorries/tractors and trailers/semi-trailers	Movement en route, exact operation, continuously split load	Branch-and-cut, branch-and-price	Arc, turn, and path
Berning (2009)	PDP: Long-distance road transport	Lorries and drivers	Movement en route	Standard MIP solver	Arc
Bürckert, Fischer, and Vierke (2000)	PDP: Long-distance road transport	Drivers, lorries with loading capacity, lorries without loading capacity, tractors, trailers, semi-trailers, chassis, and swap-bodies	Movement en route, exact operation, fixed load	Holonic multi-agent system	—
Cheung et al. (2008)	PDP: Seaport container drayage	Tractors, drivers, semi-trailers	Movement en route, exact operation, fixed load	Attribute-decision model	—

Bürckert, Fischer, and Vierke (2000) develop a *holonic multi-agent system* for solving a dynamic pickup-and-delivery problem. The authors distinguish no less than eight types of object. Each concrete object is modeled as an agent. Moreover, holonic agents, that is, "agentifications of representations for conglomerates of objects that together form a vehicle able to perform requests," are introduced. Holonic agents (holons) differ from other groups of cooperating agents in that their members act as a unit for a certain time. For each holon, there is a meta-agent that coordinates the formation of the holon and represents it to the outside world. The multi-agent system has three modes of operation: (i) The insertion mode, where solutions are generated by sequential insertion of requests into existing route plans; (ii) the optimization mode, where plans are improved by exchanging requests between vehicles; and (iii) the interactive mode, where a human planner may change existing plans by hand.

Cheung et al. (2008) describe a pickup-and-delivery problem for container transport with three types of objects: drivers, tractors, and semi-trailers. Only drivers are considered autonomous objects. The authors develop an *attribute-decision model* whose basic idea is to represent an elementary or composite object by a set of attributes associated with a set of possible decisions. The attributes are either generic (for example, location and overall time window of availability) or object-type specific (for example, the remaining working time for a driver or the set of compatible semi-trailers for a tractor). There are three types of decisions: couple, uncouple, and modify. Couple decisions represent the act of combining an object with another, such as assigning a driver to a tractor. Uncouple decisions represent the opposite. Modify decisions change the attributes of an object without interaction with other objects. The major steps for solving the model are as follows. The attribute sets for all elementary objects are created. The drivers are sorted in nondecreasing order of earliest available time and added to a candidate list. The first object in the list is selected, the set of possible decisions for this object is determined, and the value of each decision d is computed. This value is composed of three components: direct costs; estimated costs for the use of any other object(s) because of a decision; and estimated future value of $o(d)$, the object resulting from the decision. The decision with the highest value is executed; that is, $o(d)$ is added to the candidate list. After that, the estimated costs for all other objects involved in the decision are updated using the opportunity cost principle. This is repeated until the candidate list is empty or an iteration limit is reached.

Both Bürckert, Fischer, and Vierke (2000) and Cheung et al. (2008) ensure movement s. by allowing only objects comprising of at least a driver and a tractor to move in space. A remarkable feature of both contributions is that the employed procedures are not based on a network, neither explicitly nor implicitly.

It is interesting to note that Imai, Nishimura, and Current (2007) state (p. 88): "A trailer-truck consists of a tractor and a trailer, and normally they can be uncoupled.... However, this is not likely the case especially for intermodal container transportation within Japan, mainly because of the complexity of the tractor assignment to trailer." Scheuerer (personal communication) states that his work on VRPs with trailers for raw milk collection (Scheuerer 2006) was also limited to a fixed lorry-trailer assignment because of the perceived difficulties of developing a heuristic for problems with free lorry-trailer assignment, that is, for the VRPTT. This supports the conjecture that the difficulty of movement s. en route, not the lack of practical applications or importance, is the reason why the number of papers in this field is limited. Nevertheless, Bürckert, Fischer, and Vierke (2000) and Cheung et al. (2008) present approaches that may be modifiable to devise a heuristic for solving real-world VRPTTs.

5.8. Related Fields

In this section, several additional fields and problems requiring some type or other of "synchronization" are presented. Because of the limited scope of this survey, these could not be examined in detail. Nevertheless, their study may yield insights for solving VRPMSs, too. Therefore, pertinent references, mostly surveys, are given in Table 6, and a brief discussion follows.

Network design problems have synchronization aspects, and, regularly, VRPs have to be solved as subproblems. However the vehicle routing component, because of the strategic/tactical nature of network design, is often only addressed by coarse approximations. *Intermodal transport*, by its definition, requires the transshipment of load, but most applications lack a direct vehicle routing component.

With the exception of airline fleet assignment with schedule synchronization, *integrated vehicle and crew scheduling* (as opposed to simultaneous vehicle and crew *routing*) in airline, railway, or public transport applications was not considered, for two reasons: (i) These problems do require synchronization, but experience shows that approaches that work well for scheduling problems are difficult to transfer to a routing context. The presence of given schedules for itineraries, vehicles, or crews changes the nature of the problem. Nevertheless, the existing relationship to VRPMSs deserves further study, but this is clearly beyond the scope of the present work. (ii) There is already a vast body of literature on these problems, and comprehensive and excellent surveys are available. The same holds for the related area of public

Table 6 Fields Related to VRPMSs and Pertinent Literature

Application	References
Network design	Crainic and Kim (2007); Wieberneit (2008); Andersen, Crainic, and Christiansen (2009)
Intermodal transport	Macharis and Bontekoning (2004); Caris, Macharis, and Janssens (2008)
Integrated vehicle and crew scheduling	Freling, Huisman, and Wagelmans (2003); Klabjan (2005); Caprara et al. (2007)
Transit scheduling	Desaulniers and Hickman (2007)
School bus routing	Park and Kim (2010)
Maritime transport	Christiansen et al. (2007); Hennig (2010)
Inventory routing problems	Campbell et al. (1998); Moin and Salhi (2007); Andersson et al. (2010); Oppen, Løkketangen, and Desrosiers (2010)
Periodic VRPs	Cordeau, Gendreau, and Laporte (1997); Angelelli and Speranza (2002)
Stochastic VRPs	Gendreau, Laporte, and Séguin (1996); Rivers (2002); Christiansen and Lysgaard (2007)
Swapping problem	Anily and Hassin (1992); Bordenave, Gendreau, and Laporte (2010)
VRPs with uncapacitated intermediate depots or refill stations	Ghiani, Improta, and Laporte (2001); Tarantilis, Zachariadis, and Kiranoudis (2008)
Single-echelon location-routing problems	Nagy and Salhi (2007)
Truck-and-trailer routing problem (TTRP)	Semet and Taillard (1993); Gerdessen (1996); Chao (2002); Scheuerer (2006)

transit scheduling (again, *scheduling* as opposed to *routing*). In the context of *school bus routing*, Park and Kim (2010, p. 318) state: "The issue of student transshipment should . . . be considered in future research."

Vehicle routing problems also arise in *maritime transport*. A prominent example of exceeding economic importance is oil tanker routing and scheduling.

A very special type of VRP are *inventory routing problems* (IRPs). In IRPs, there are no customer demands. Instead, each customer has a given consumption rate of a good, an initial stock, and a storage capacity. The depot has to perform zero or more deliveries to each customer during a multiperiod planning horizon to ensure that no customer runs out of stock. The objective is to plan delivery routes of minimal cost. Essentially, IRPs fulfill the definition of a VRPMS as given in the introduction. However, because of their special nature, a more detailed treatment of IRPs is beyond the scope of this paper.

In *periodic VRPs*, several visits are required to serve a customer during the planning horizon. These visits must take place in different periods and may be performed by different vehicles. However, changing a route that visits a certain customer in one period does not impact the routes visiting this customer in other periods, unless the time a customer is visited in one period influences the times when he may be visited in other periods. Hence, only the *consistent VRP* (Groër, Golden, and Wasil 2009) was cited in §5.4, whereas other periodic VRPs were excluded.

Further VRPs that do not require multiple synchronization in the sense used here are *stochastic VRPs*, the *swapping problem*, *VRPs with uncapacitated intermediate depots or refill stations*, *single-echelon location-routing problems*, and the *truck-and-trailer routing problem* (TTRP). The TTRP is a VRPTT with a fixed lorry-trailer assignment; that is, each trailer can be pulled by only one lorry, and only this lorry can transfer load into the trailer. Hence, there are no support vehicles, and there is no interdependence problem.

6. Summary, Conclusion, and Outlook

This paper has studied vehicle routing problems with multiple synchronization constraints. A VRPMS has been defined as a VRP where more than one vehicle may or must be used to fulfill a task. The VRP with trailers and transshipments has been presented as an archetypal example of a VRPMS, and a classification of different types of synchronization requirements that appear in real-world vehicle routing problems has been developed. The decisive modeling and solution issues with VRPMSs have been pointed out: The interdependence problem encountered in VRPMSs, that a change in one route may have effects on other routes, considerably complicates the use of standard solution techniques for VRPs, such as column generation and local search. Most importantly, literature on synchronization has been surveyed, focusing on applications and on the techniques for dealing with the synchronization requirements.

With respect to *applications* or rather problem classes, the literature review has shown that there is an increasing number of papers on VRPMSs with transshipments and temporal s. of visits. Research on synchronizing autonomous and nonautonomous objects is still rare. The *most important concrete problem classes* are (i) N-echelon VRPs/LRPs, (ii) PDPTW with transshipments, and (iii) simultaneous vehicle and crew routing and scheduling problems.

With respect to *modeling and design decisions* in papers presenting MIP approaches, it is very interesting to observe the different options for creating an MIP model, even though none of these options constitutes a silver bullet: The spectrum where the information, data, and relationships of a concrete problem (in particular, the synchronization requirements) are represented ranges from "model the underlying logic completely by means of decision variables and constraints" to "create a highly involved network that by itself ensures synchronization." At the former end of the spectrum lie the works of Kim, Koo, and Park (2010); Schmid et al. (2010); Zäpfel and Bögl (2008); and Bock (2010). Moving toward the latter

Table 7 Recurrent Exact Techniques

Technique	References
Using one vehicle-independent time variable for the beginning of execution of a task or operation requiring more than one vehicle	Li, Lim, and Rodrigues (2005); Lim, Rodrigues, and Song (2004); Dohn, Kolind, and Clausen (2009); Cortés, Matamala, and Contardo (2010)
Discretizing time	Ioachim et al. (1999); Bélanger et al. (2006); Dohn, Kolind, and Clausen (2009); Grünert and Sebastian (2000)
Branching on time windows	Ioachim et al. (1999); Bélanger et al. (2006); Dohn, Kolind, and Clausen (2009)
Explicitly determining request paths or flows and linking them to vehicle paths/flows	Crainic, Ricciardi, and Storchi (2009); Grünert and Sebastian (2000); Mues and Pickl (2005)

end, the papers of Recker (1995); Wen et al. (2009); Cortés, Matamala, and Contardo (2010); Mues and Pickl (2005); Grünert and Sebastian (2000); and Lin (2008) can be located. The papers of Fügenschuh (2006, 2009) are close to the latter end, which is defined by Amaya, Langevin, and Trépanier (2007, 2010).

With respect to *solution methods*, there are some *recurring algorithmic techniques* and principles that are used in several (more than two) papers. These were discussed above and are summarized in Tables 7 and 8.

With respect to *applications* of VRPMSs, it is to be expected that *integrated vehicle and crew routing and scheduling in road transport*, *city logistics*, *forest management*, and *agricultural field operations* will be more intensively studied and hence yield more publications in the near future. In particular, the last topic seems to offer a wide range of challenging applications, as the recent surveys by Bochtis and Sørensen (2009, 2010) show. These authors present several problems in agricultural field logistics that can be modeled as VRPs. Some of these problems contain multiple synchronization constraints. Most importantly, many of these problems require the use of task and support vehicles, and, consequently, of task, operation, and load s. Solution approaches or results of practical projects are not reported yet.

Table 8 Recurrent Heuristic Techniques

Technique	References
Decomposition by stage for 2-echelon VRPs/LRPs	Russell and Morrel (1986); Crainic et al. (2010); Nguyen, Prins, and Prodhon (2010); Boccia et al. (2010); Crainic, Ricciardi, and Storchi (2009)
Explicitly determining request leg sequences	Feige (2003); Oertel (2000); Bock (2010); Aldaihani and Dessouky (2003); Mitrović-Minić and Laporte (2006)
Allowing intermediate infeasible solutions and penalizing them in the objective function	De Rosa et al. (2002); Wen et al. (2009); Oertel (2000); Prescott-Gagnon, Desaulniers, and Rousseau (2010)
Indirect search	Li, Lim, and Rodrigues (2005); Lim, Rodrigues, and Song (2004); Feige (2003)
Constraint programming	Rousseau, Gendreau, and Pesant (2003); Laurent and Hao (2007); El Hachemi, Gendreau, and Rousseau (2011a,b)
Determining abstract routes first and assigning concrete objects afterward	Chung and Norback (1991); Xiang, Chu, and Chen (2006); Hollis, Forbes, and Douglas (2006)

With respect to *algorithmic approaches*, two fields not directly connected to transport logistics, contrary to the areas described in §5.8, that nevertheless show great relevance for VRPMSs are *scheduling* and *robotics*.

Some problems, for example, the staff scheduling problems cited in §5.5, bear similarity to *machine or project scheduling* problems. In particular, resource-constrained project scheduling problems are related to VRPMSs with resource s. It was beyond the scope of this paper to survey the considerable body of literature on scheduling problems. A thorough introduction and an up-to-date overview are given by Brucker and Knust (2006) and Brucker (2007). It is to be expected that ideas and principles used for the solution of scheduling problems can also be useful for solving VRPMSs and vice versa.

The same holds true for applications in *robotics and control theory*. The free textbook of Bullo, Cortés, and Martínez (2009) contains a very extensive reference list for further reading in this field. It seems that recently, similar to the case of VRPMSs in the vehicle routing and operational research literature, there is an increasing number of papers addressing problems of coordinating robots for performing interdependent tasks in space and time. Two particularly interesting journal papers are Shima et al. (2006) and Jones, Dias, and Stentz (2010). The latter paper also contains a substantial list of pertinent references.

A closer look at interfaces and commonalities of these three fields, VRPMSs, resource-constrained scheduling, and robot coordination, constitutes an interesting and promising research perspective from which all three fields may benefit.

All in all, this survey has demonstrated that multiple synchronization constraints are a challenging extension of VRPs and that they are of practical relevance in many different application areas. Consequently, VRPMSs require and deserve to be studied further.

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Appendix

Glossary of Terms

Active vehicle	The vehicle transferring load in a transshipment.
Autonomous vehicle	A vehicle able to move in time and space on its own.
Collection vehicle	Task vehicles in problems where supply has to be collected at the customers.
Delivery vehicle	Task vehicles in problems where demand has to be delivered to customers.
Dynamic time window	Time window for execution of a task or an operation; determined in the course of a solution procedure; depends on the execution of another task or operation.
Interdependence problem	Fact that in VRPMSs, vehicle routes may depend on one another so that changes in one route affect other routes.
Lead vehicle	The vehicle determining the location and the time for a transshipment.
Leg	Ordered pair of locations or vertices a request or vehicle must visit in the specified order. Other locations or vertices may be visited in between. In problems with transshipments, a request may only be transshipped between legs; on one leg, a request is always transported by the same vehicle.
Nonautonomous vehicle	A vehicle that must be accompanied by an autonomous vehicle or must join with one or more other nonautonomous vehicle(s) to be able to move in space (but that is able to move in time, that is, wait at a location, on its own).
Operation	Something that may or must be performed by a vehicle at a location or vertex to allow or facilitate the execution of one or more tasks (a transshipment, coupling or uncoupling a trailer, etc.).
Passive vehicle	The vehicle receiving load in a transshipment.
Path, itinerary	Sequence of locations or vertices visited by a vehicle or a request on its way from its origin (depot) to its destination (depot).
Request leg sequence	Sequence of legs where the first location or vertex of the first leg is the origin of the request, the first location or vertex of all other legs is the second location or vertex of the previous leg, and the second location or vertex of the last leg is the destination of the request.
Support vehicle	Support vehicles are not allowed to visit customers and only act as mobile depots for the task vehicles. The term was independently coined by Rivers (2002) and Drexl (2007).
Task, request	A mandatory duty, something that must be done and requires zero or more units of some capacity (collecting supply or delivering demand at one location, picking up load at one location and delivering this load to another location, visiting a location to render a service, etc.).
Task vehicle	Task vehicles are allowed to visit customers.

Summary of Abbreviations

AGV	automated guide vehicle
BCP	branch-and-cut-and-price
CARP	(capacitated) arc routing problem
ESPPRC	(elementary) shortest path problem with resource constraints
IRP	inventory routing problem
LRP	location-routing problem
MIP	mixed integer program(ming)
PDPTW	pickup-and-delivery problem (with time windows)
s.	synchronization
TL	transshipment location
TTRP	truck-and trailer routing problem
VRPMS	VRP with multiple synchronization constraints
VRPTT	VRP with trailers and transshipments
VRPTW	vehicle routing problem (with time windows)

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