JOURNAL OF INDUSTRIAL AND
MANAGEMENT OPTIMIZATION
Volume **13**, Number **4**, October **2017**

doi:10.3934/jimo.2017009

pp. **1601–1623**

A MIXED-INTEGER LINEAR PROGRAMMING MODEL FOR OPTIMAL VESSEL SCHEDULING IN OFFSHORE OIL AND GAS OPERATIONS

Elham Mardaneh, Ryan Loxton and Qun Lin

Curtin University, Perth, 6102, Australia

Phil Schmidli

Woodside Energy Ltd, Perth, 6000, Australia

(Communicated by Chao Xu)

Abstract. This paper introduces a non-standard vehicle routing problem (VRP) arising in the oil and gas industry. The problem involves multiple offshore production facilities, each of which requires regular servicing by support vessels to replenish essential commodities such as food, water, fuel, and chemicals. The support vessels are also required to assist with oil off-takes, in which oil stored at a production facility is transported via hose to a waiting tanker. The problem is to schedule a series of round trips for the support vessels so that all servicing and off-take requirements are fulfilled, and total cost is minimized. Other constraints that must be considered include vessel suitability constraints (not every vessel is suitable for every facility), depot opening constraints (base servicing can only occur during specified opening periods), and off-take equipment constraints (the equipment needed for off-take support can only be deployed after certain commodities have been offloaded). Because of these additional constraints, the scheduling problem under consideration is far more difficult than the standard VRP. We formulate a mixed-integer linear programming (MILP) model for determining the optimal vessel schedule. We then verify the model theoretically and show how to compute the vessel utilization ratios for any feasible schedule. Finally, simulation results are reported for a real case study commissioned by Woodside Energy Ltd, Australia's largest dedicated oil and gas company.

1. **Introduction.** The *vehicle routing problem* (VRP) is a classic problem in operations research. The basic framework of the VRP involves a single depot, multiple customers, and multiple vehicles of identical type. The vehicles are used to transport a certain commodity from the depot to the customers, and both the capacity of each vehicle (in terms of how much commodity it can carry) and the demand of each customer (in terms of how much commodity it requires) are known. The problem is to find a set of closed routes (one for each vehicle) so that total distance is minimized subject to the following constraints:

(a) Each route starts and ends at the depot.
(b) Each customer is visited by precisely one vehicle.

2010 *Mathematics Subject Classification.* Primary: 90B06; Secondary: 90B10, 90C11, 90C27.
Key words and phrases. Optimal scheduling, vehicle routing problem, mixed-integer linear programming, oil and gas industry.

(c) The total demand of customers along each route should not exceed the vehicle capacity.

This paper considers a variation of the standard VRP described above. Specifically, we consider a real scheduling scenario faced by Woodside Energy Ltd (Woodside), one of Australia's largest companies with a market capitalization of around $18 billion US dollars (as at April 2016). The problem involves scheduling a fleet of support vessels to service Woodside's offshore oil and gas facilities. Before describing Woodside's problem in detail, we give a brief review of the relevant literature on the VRP and its variants.

1.1. **Literature review.** Various mathematical formulations for the VRP have been presented in the literature. For a thorough review of these different formulations, we refer the reader to [18]. Common variations of the standard VRP include the site-dependent VRP (there are multiple vehicle types and not all vehicles are suitable for all customers), the multi-trip VRP (vehicles may perform multiple trips), the periodic VRP (vehicles may perform multiple trips and customers must be visited a prescribed number of times), the VRP with limited durations (route durations cannot exceed a maximum time limit), and the VRP with time windows (customers must be visited during specific time periods). The problem to be considered in this paper is a combination of the site-dependent VRP, the multi-trip VRP, and the VRP with time windows.

The multi-trip VRP was first introduced by Fleischmann in 1990 [9]. Various solution algorithms have since been proposed for solving the multi-trip VRP. These include genetic algorithms [19], multi-phase algorithms [16], and algorithms based on adaptive memory programming [15]; see reference [20] for a comprehensive survey. Reference [17] considers an interesting variant of the multi-trip VRP in which cost penalties are imposed for route durations that exceed a given time limit. The problem was solved via a population-based algorithm using the tabu search and bin packing approaches. Incidentally, tabu search is also an effective tool for solving periodic VRPs; see [2] for details.

The multi-trip VRP with time windows is considered in reference [5]. In this reference, a novel solution approach is proposed that involves decomposing the VRP into two subproblems, each of which can be solved heuristically—the heuristic for the first subproblem generates feasible routes; the heuristic for the second subproblem solves a corresponding bin packing problem. These heuristics are called by a master algorithm that implements a self-adaptive guidance strategy to ensure the route heuristic only ever generates improved routes. Reference [7] considers a variant of the multi-trip VRP with time windows in which the following additional features are incorporated: vehicles have heterogeneous capacities, access to customers is restricted by certain rules, loading/unloading times are non-negligible, extra vehicles can be hired when necessary, and the cost function incorporates transport costs (including fuel and maintenance costs) and wages (including fixed wages and overtime charges). The algorithm developed in [7] for solving this problem uses a nearest neighbour and insertion procedure followed by tabu search.

For perishable commodities, route duration constraints are often required in the VRP. Reference [3] considers a multi-trip VRP with time windows and route duration constraints (a combination of the multi-trip VRP, the VRP with time windows, and the VRP with limited durations). This problem involves only a single vehicle and can be solved using a two-phase exact method based on dynamic programming.

In [4], the same problem is considered, but with a homogeneous fleet instead of a single vehicle. Similar two-phase approaches for solving the multi-trip VRP with time windows and route duration constraints are described in [11, 14].

1.2. **Contribution of this paper.** In this paper, we introduce a new variant of the VRP that is based on a real scheduling problem faced by oil and gas company Woodside in the management of its offshore oil and gas assets. The problem setting is described as follows.

Woodside operates various oil and gas facilities situated in the Indian Ocean off the coast of Western Australia. A dedicated fleet of support vessels is available for servicing these facilities, and each support vessel has different characteristics in terms of speed, deckspace, and carrying capacity for various commodities. The offshore facilities require their commodity stocks (e.g., food, water, fuel, chemicals) to be regularly replenished. In addition, the oil facilities require *off-takes* to be performed at scheduled times. This involves moving oil from the facility to a waiting tanker. Before the off-take can begin, a support vessel needs to connect a hose from the oil facility to the tanker and only some support vessels have the equipment needed to perform this connection. Moreover, on some vessels, the deck must be completely clear before the off-take support equipment can be used. This means that any cargo supplies stored on deck must be offloaded prior to assisting with off-takes.

The aim of the problem is to determine an optimal series of trips for the support vessels so that all cargo delivery and off-take support requirements are fulfilled over a given planning horizon. This problem can be viewed as a type of multi-trip VRP with time windows, in which the vehicles are support vessels, the customers are offshore oil and gas facilities, and the depot is a supply base. Compared with other VRPs studied in the literature, the VRP described above involves the following non-standard features:

(a) The vessel fleet is heterogeneous.
(b) There are multiple commodities, not just one.
(c) Transport costs do not necessarily satisfy the triangle inequality.
(d) There are constraints on the types of vessels that can service each facility.
(e) Base servicing can only occur during specified opening times.
(f) Vessels cannot wait at an offshore facility for more than a maximum specified time limit.
(g) Some commodities must be offloaded before off-take support can be provided.

Although some of these non-standard features have already been studied in the literature, we are not aware of any work that considers all of them simultaneously. Our goal in this paper is to present a mixed-integer linear programming (MILP) model for this non-standard VRP. We validate the MILP model numerically using real data provided by Woodside. We also develop a method for determining the vessel utilization ratios corresponding to an optimal schedule. Such ratios are key quantities used by Woodside to measure fleet efficiency in real operations.

2. **Mathematical model.** We consider a directed network in which the nodes represent the facilities and the arcs represent the possible links between the different facilities. Let $\mathcal{N}$ denote the set of nodes in the network and let $\mathcal{A}$ denote the set of arcs. The node set $\mathcal{N}$ can be partitioned into two sets $\mathcal{F}$ and $\mathcal{B}$, where $\mathcal{F}$ contains nodes representing the offshore facilities and $\mathcal{B}$ contains nodes representing the supply base.

The MILP model is based on the following assumptions.

- The time horizon is divided into multiple periods of equal duration.
- Vessel travel times are positive integer multiples of the period duration.
- Vessel travel times are constant throughout the planning horizon (i.e., travel times are period-independent).
- Service durations are integer multiples of the period duration (unlike travel times, service durations can be zero).
- Each vessel starts and ends the time horizon at the supply base.
- Base servicing is only required on intermediate visits to the base (vessels do not require servicing at the start and end of their journeys).
- The service at each facility begins immediately upon vessel arrival (i.e., vessels only wait at a facility after service completion, not before).
- Except for the supply base, each facility is open continuously (24 hours per day) across the planning horizon.

The parameters in the model are defined below:

- T = number of periods in the planning horizon
- Q^{kr} = storage limit (carrying capacity) of commodity r on vessel k
- τ_{ij}^{k} = travel time (in periods) for vessel k to traverse link (i,j)
- d_{base}^{k} = service duration (in periods) for vessel k at the supply base
- α_{ij}^{k} = fixed cost for vessel k across link (i,j)
- β_{ij}^{kr} = variable cost (per unit of commodity r) for vessel k across link (i,j)

Let $\mathcal{K}$ denote the set of vessels, let $\mathcal{R}$ denote the set of commodities, and let $\mathcal{T} = \{1,\ldots,T\}$ denote the set of time periods. Some vessels cannot perform off-take support while storing certain commodities due to the location of the off-take support equipment (e.g., the equipment on some vessels cannot be deployed until the deck is completely clear). Let $\mathcal{R}_k$ denote the set of commodities that must be unloaded from vessel k before the vessel can assist with off-takes.

Recall that $\mathcal{B}$ contains nodes representing the supply base. The nodes in $\mathcal{B}$ serve as start/end points for round trips undertaken by the support vessels. Thus, the number of nodes in $\mathcal{B}$ is equal to the maximum number of round trips that can be performed by a single vessel during the given time horizon. We choose a fixed node $b_0 \in \mathcal{B}$ to act as the designated start point for all vessels. Thus, each vessel begins at b_0 and ends at another node in $\mathcal{B}$.

We are given a set $\mathcal{O}_{\text{base}} \subset \mathcal{T}$ of allowable operating periods for the supply base; the supply base is closed outside of these periods. Let $\delta^k(t)$ denote the total number of full periods (working and non-working) required at the base if vessel k arrives during period t. The value of $\delta^k(t)$ depends on both d_{base}^{k} and $\mathcal{O}_{\text{base}}$ and can be computed using Algorithm 1. For example, suppose $d_{\text{base}}^{k} = 3$, $\mathcal{O}_{\text{base}} = \{8, 10, 11\}$, and vessel k arrives at the base during period 5. Then vessel k must spend at least the next 6 periods at the base to complete 3 working periods, and thus $\delta^k(5) = 6$. Note that $\delta^k(t) = +\infty$ if there are insufficient working periods in the time horizon to conduct a full base service starting at time t. For example, a vessel arriving at the final time $t = T$ obviously has no time to complete a full base service, and thus $\delta^k(T) = +\infty$ for each vessel $k \in \mathcal{K}$.

For each node $i \in \mathcal{F}$, there is an ordered tuple $(q_i^r, t_i^{\min}, t_i^{\max}, d_i^k, w_i^k, o_i, \mathcal{K}_i)$, where

- q_i^r = amount of commodity r to be delivered to node i
- $t_i^{\min}$ = earliest period during which the service at node i can begin
- $t_i^{\max}$ = latest period during which the service at node i can begin

Algorithm 1 Returns the value of $\delta^k(t)$

```
Set t → t'                              ▷ Initialization step; t' is the period counter
Set 0 → d                               ▷ Initialization step; d is the working period counter
while d < d^k_base do                   ▷ Iterate for d^k_base working periods
    Set t' + 1 → t'                     ▷ Increment period counter
    if t' > T then
        Set +∞ → δ^k(t)                 ▷ Insufficient time to conduct base service
        return δ^k(t)
    else if t' ∈ O_base then
        Set d + 1 → d                   ▷ Increment working period counter if base is open
    end if
end while
Set t' − t → δ^k(t)                     ▷ Calculate δ^k(t)
return δ^k(t)
```

- d_i^k = service duration (in periods) for vessel k at node i
- w_i^k = maximum wait time (in periods) for vessel k after service completion at node i
- o_i = binary parameter indicating whether an off-take is required at node i ($o_i = 1$ if off-take required; $o_i = 0$ otherwise)
- $\mathcal{K}_i$ = set of suitable vessels for conducting the service at node i

We define $\mathcal{K}_i = \mathcal{K}$ for each $i \in \mathcal{B}$ since all vessels are suitable for the supply base. Since each non-base node is associated with just a single service visit, if an offshore facility requires multiple service visits, then duplicate nodes must be included for this facility.

The decision variables in the model are defined below:

- x_{ijt}^k = binary variable indicating whether vessel k arrives at node j along link (i,j) during period t ($x_{ijt}^k = 1$ if this occurs; $x_{ijt}^k = 0$ otherwise)
- y_{ijt}^{kr} = flow of commodity r arriving at node j along link (i,j) during period t via vessel k

To build the MILP model, we use the convention that servicing always begins in the first period after vessel arrival; see Figure 1 for an example. Similarly, after service completion, the vessel must wait until the next period to depart. Based on this convention, we define the set of valid arrival periods for node j along link (i,j) via vessel k as follows:

$$\mathcal{T}_{ij}^k = \{\, t \in \mathcal{T} : \max(t_j^{\min} - 1, t_i^{\min} + d_i^k + \tau_{ij}^k) \le t \le t_j^{\max} - 1 \,\},$$

where, for the supply base nodes,

$$t_b^{\min} = 1, \quad t_b^{\max} = T + 1, \quad d_b^k = \begin{cases} 0, & \text{if } b = b_0 \\ d_{\text{base}}^k, & \text{if } b \neq b_0 \end{cases} \quad b \in \mathcal{B}.$$

This definition for $\mathcal{T}_{ij}^k$ imposes the condition that, if necessary, vessel k will wait at node i before departing for node j to ensure that it arrives at node j within the required time window. Recall that waiting at facilities is allowed, provided that the total wait time does not exceed the maximum specified waiting duration. When counting wait times, we only count full periods to be consistent with our convention for arrival/departure times. Hence, for the example in Figure 1, vessel departure

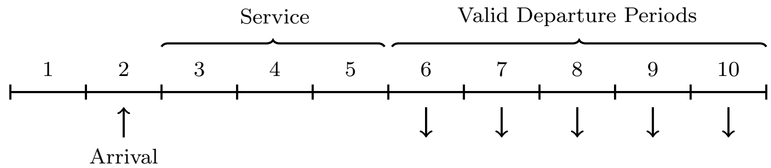


FIGURE 1. An example of our arrival/departure time convention: if $d_i^k = 3$, then a vessel arriving during period 2 and can depart any time from period 6 onwards.

during period 6 will correspond to a wait time of 0, vessel departure during period 7 will correspond to a wait time of 1, and so on. Imposing a bound on the waiting time is necessary to eliminate impractical schedules in which vessels spend an unrealistic amount of time waiting at offshore facilities in an effort to avoid travel costs.

We now list the constraints in the MILP model.

- Each required service visit is performed by a suitable vessel during the given time window:

$$\sum_{j\colon (j,i)\in\mathcal{A}} \sum_{k\in\mathcal{K}_j\cap\mathcal{K}_i} \sum_{t\in\mathcal{T}_{ji}^k} x_{jit}^k = 1, \quad \forall i \in \mathcal{F}. \tag{1}$$

- Any vessel arriving at an offshore facility must leave that facility after service completion:

$$\sum_{j\colon (j,i)\in\mathcal{A}} \sum_{t\in\mathcal{T}_{ji}^k} x_{jit}^k = \sum_{j\colon (i,j)\in\mathcal{A}} \sum_{t\in\mathcal{T}_{ij}^k} x_{ijt}^k, \quad \forall i \in \mathcal{F}, \quad \forall k \in \mathcal{K}_i. \tag{2}$$

- Each vessel starts from the designated start node b_0:

$$\sum_{j\colon (b_0,j)\in\mathcal{A}} \sum_{t\in\mathcal{T}_{b_0 j}^k} x_{b_0 jt}^k \leq 1, \quad \forall k \in \mathcal{K}. \tag{3}$$

- Each vessel never returns to the designated start node b_0:

$$\sum_{j\colon (j,b_0)\in\mathcal{A}} \sum_{t\in\mathcal{T}_{jb_0}^k} x_{jb_0 t}^k = 0, \quad \forall k \in \mathcal{K}. \tag{4}$$

- Each vessel visits each node in $\mathcal{B}\setminus\{b_0\}$ at most once:

$$\sum_{j\colon (j,i)\in\mathcal{A}} \sum_{t\in\mathcal{T}_{ji}^k} x_{jit}^k \leq 1, \quad \forall i \in \mathcal{B}\setminus\{b_0\}, \quad \forall k \in \mathcal{K}. \tag{5}$$

- Each vessel ends at a node in $\mathcal{B}\setminus\{b_0\}$:

$$\sum_{j\colon (j,i)\in\mathcal{A}} \sum_{t\in\mathcal{T}_{ji}^k} x_{jit}^k \geq \sum_{j\colon (i,j)\in\mathcal{A}} \sum_{t\in\mathcal{T}_{ij}^k} x_{ijt}^k, \quad \forall i \in \mathcal{B}\setminus\{b_0\}, \quad \forall k \in \mathcal{K}. \tag{6}$$

- Time sequencing constraints (for offshore facilities):

$$\underbrace{\sum_{j\colon (j,i)\in\mathcal{A}} \sum_{k\in\mathcal{K}_j\cap\mathcal{K}_i} \sum_{t\in\mathcal{T}_{ji}^k} (t + d_i^k + 1) x_{jit}^k}_{\text{Node } i \text{ earliest departure period}}$$

$$\leq \underbrace{\sum_{j\colon (i,j)\in\mathcal{A}} \sum_{k\in\mathcal{K}_i\cap\mathcal{K}_j} \sum_{t\in\mathcal{T}_{ij}^k} (t-\tau_{ij}^k)x_{ijt}^k}_{\text{Node } i \text{ actual departure period}}, \quad \forall i \in \mathcal{F}. \tag{7}$$

- Time sequencing constraints (for base nodes):

$$\begin{aligned}\underbrace{\sum_{j\colon (j,i)\in\mathcal{A}} \sum_{t\in\mathcal{T}_{ji}^k} (t+\delta^k(t)+1)x_{jit}^k}_{\text{Node } i \text{ earliest departure period}} &\leq \underbrace{\sum_{j\colon (i,j)\in\mathcal{A}} \sum_{t\in\mathcal{T}_{ij}^k} (t-\tau_{ij}^k)x_{ijt}^k}_{\text{Node } i \text{ actual departure period}} \\ &+ M\left\{1-\sum_{j\colon (i,j)\in\mathcal{A}} \sum_{t\in\mathcal{T}_{ij}^k} x_{ijt}^k\right\}, \quad \forall i \in \mathcal{B}\setminus\{b_0\}, \quad \forall k\in\mathcal{K},\end{aligned} \tag{8}$$

  where $M > 0$ is a sufficiently large positive number.
- Waiting time constraints:

$$\begin{aligned}&\underbrace{\sum_{j\colon (i,j)\in\mathcal{A}} \sum_{k\in\mathcal{K}_i\cap\mathcal{K}_j} \sum_{t\in\mathcal{T}_{ij}^k} (t-\tau_{ij}^k)x_{ijt}^k}_{\text{Node } i \text{ actual departure period}} - \underbrace{\sum_{j\colon (j,i)\in\mathcal{A}} \sum_{k\in\mathcal{K}_j\cap\mathcal{K}_i} \sum_{t\in\mathcal{T}_{ji}^k} (t+d_i^k+1)x_{jit}^k}_{\text{Node } i \text{ earliest departure period}} \\ &\qquad \leq \sum_{j\colon (i,j)\in\mathcal{A}} \sum_{k\in\mathcal{K}_i\cap\mathcal{K}_j} \sum_{t\in\mathcal{T}_{ij}^k} w_i^k x_{ijt}^k, \quad \forall i\in\mathcal{F}.\end{aligned} \tag{9}$$

- Cargo delivery constraints:

$$\begin{aligned}&\underbrace{\sum_{j\colon (j,i)\in\mathcal{A}} \sum_{k\in\mathcal{K}_j\cap\mathcal{K}_i} \sum_{t\in\mathcal{T}_{ji}^k} y_{jit}^{kr}}_{\text{Flow of commodity } r \text{ entering node } i} \\ &\qquad - \underbrace{\sum_{j\colon (i,j)\in\mathcal{A}} \sum_{k\in\mathcal{K}_i\cap\mathcal{K}_j} \sum_{t\in\mathcal{T}_{ij}^k} y_{ijt}^{kr}}_{\text{Flow of commodity } r \text{ leaving node } i} = q_i^r, \quad \forall i\in\mathcal{F}, \quad \forall r\in\mathcal{R}.\end{aligned} \tag{10}$$

- Commodities in $\mathcal{R}_k$ must be cleared before off-take support can be provided:

$$\sum_{j\colon (i,j)\in\mathcal{A}} \sum_{k\in\mathcal{K}_i\cap\mathcal{K}_j} \sum_{t\in\mathcal{T}_{ij}^k} \sum_{r\in\mathcal{R}_k} o_i y_{ijt}^{kr} = 0, \quad \forall i\in\mathcal{F}. \tag{11}$$

- Vessel capacity constraints:

$$\underbrace{\sum_{j\colon (i,j)\in\mathcal{A}} \sum_{t\in\mathcal{T}_{ij}^k} y_{ijt}^{kr}}_{\text{Outgoing flow}} \leq Q^{kr}, \quad \forall i\in\mathcal{B}, \quad \forall k\in\mathcal{K}, \quad \forall r\in\mathcal{R}. \tag{12}$$

- Commodity flow between two facilities is zero if the corresponding link is not traversed:

$$y_{ijt}^{kr} \leq M x_{ijt}^k, \quad \forall (i,j)\in\mathcal{A}, \quad \forall t\in\mathcal{T}, \quad \forall k\in\mathcal{K}, \quad \forall r\in\mathcal{R}, \tag{13}$$

  where $M > 0$ is a sufficiently large positive number.
- Binary constraints:

$$x_{ijt}^k \in \{0,1\}, \quad \forall (i,j)\in\mathcal{A}, \quad \forall t\in\mathcal{T}, \quad \forall k\in\mathcal{K}. \tag{14}$$

- Non-negativity constraints:

$$y_{ijt}^{kr} \geq 0, \quad \forall (i,j)\in\mathcal{A}, \quad \forall t\in\mathcal{T}, \quad \forall k\in\mathcal{K}, \quad \forall r\in\mathcal{R}. \tag{15}$$

Vessel travel costs can be decomposed into fixed travel costs (independent of commodity flow) and variable travel costs (dependent on commodity flow). Thus, the cost function can be expressed as follows:

$$\text{Total Cost} = \sum_{(i,j)\in\mathcal{A}} \sum_{t\in\mathcal{T}} \sum_{k\in\mathcal{K}} \Big\{ \underbrace{\alpha_{ij}^{k} x_{ijt}^{k}}_{\text{Fixed cost}} + \underbrace{\sum_{r\in\mathcal{R}} \beta_{ij}^{kr} y_{ijt}^{kr}}_{\text{Variable cost}} \Big\}. \tag{16}$$

Our vessel scheduling problem can now be stated formally as the following MILP model: minimize the cost function (16) subject to the constraints (1)-(15).

3. Model analysis and simplification.

The MILP model in Section 2 is usually massive for realistic problem instances. For example, the problems faced by Woodside typically involve well over one million decision variables. To reduce the size of the model, the following variable assignments can be performed in a pre-processing step to eliminate some of the decision variables.

- Vessels should not carry excess cargo back to the base:

$$y_{ijt}^{kr} = 0, \quad \forall i \in \mathcal{F}, \quad \forall j \in \mathcal{B}, \quad \forall t \in \mathcal{T}, \quad \forall k \in \mathcal{K}, \quad \forall r \in \mathcal{R}.$$

- Vessels cannot travel directly between base nodes:

$$x_{ijt}^{k} = 0, \quad \forall i \in \mathcal{B}, \quad \forall j \in \mathcal{B}, \quad \forall t \in \mathcal{T}, \quad \forall k \in \mathcal{K},$$
$$y_{ijt}^{kr} = 0, \quad \forall i \in \mathcal{B}, \quad \forall j \in \mathcal{B}, \quad \forall t \in \mathcal{T}, \quad \forall k \in \mathcal{K}, \quad \forall r \in \mathcal{R}.$$

- Vessels cannot return to the start node b_0:

$$x_{ib_0t}^{k} = 0, \quad \forall i \in \mathcal{N}, \quad \forall t \in \mathcal{T}, \quad \forall k \in \mathcal{K},$$
$$y_{ib_0t}^{kr} = 0, \quad \forall i \in \mathcal{N}, \quad \forall t \in \mathcal{T}, \quad \forall k \in \mathcal{K}, \quad \forall r \in \mathcal{R}.$$

- Vessels cannot visit unsuitable facilities:

$$x_{ijt}^{k} = 0, \quad \forall (i,j) \in \mathcal{A}, \quad \forall t \in \mathcal{T}, \quad \forall k \notin \mathcal{K}_i \cap \mathcal{K}_j,$$
$$y_{ijt}^{kr} = 0, \quad \forall (i,j) \in \mathcal{A}, \quad \forall t \in \mathcal{T}, \quad \forall k \notin \mathcal{K}_i \cap \mathcal{K}_j, \quad \forall r \in \mathcal{R}.$$

- Vessels cannot arrive outside of valid arrival periods:

$$x_{ijt}^{k} = 0, \quad \forall (i,j) \in \mathcal{A}, \quad \forall t \notin \mathcal{T}_{ij}^{k}, \quad \forall k \in \mathcal{K},$$
$$y_{ijt}^{kr} = 0, \quad \forall (i,j) \in \mathcal{A}, \quad \forall t \notin \mathcal{T}_{ij}^{k}, \quad \forall k \in \mathcal{K}, \quad \forall r \in \mathcal{R}.$$

- Vessels cannot visit consecutive facilities where the total cargo demand exceeds vessel capacity:

$$x_{ijt}^{k} = 0, \quad \forall (i,j) \in \mathcal{F}\times\mathcal{F} : \{ r \in \mathcal{R} : q_i^r + q_j^r > Q^{kr} \} \neq \emptyset, \quad \forall t \in \mathcal{T}, \quad \forall k \in \mathcal{K},$$
$$y_{ijt}^{kr} = 0, \quad \forall (i,j) \in \mathcal{F}\times\mathcal{F} : \{ r \in \mathcal{R} : q_i^r + q_j^r > Q^{kr} \} \neq \emptyset, \quad \forall t \in \mathcal{T}, \quad \forall k \in \mathcal{K}, \quad \forall r \in \mathcal{R}.$$

- Vessels cannot deliver commodities in $\mathcal{R}_k$ after off-takes:

$$x_{ijt}^k = 0, \quad \forall (i,j) \in \mathcal{F} \times \mathcal{F} : o_i = 1, \{ r \in \mathcal{R}_k : q_j^r > 0 \} \neq \emptyset, \quad \forall t \in \mathcal{T}, \quad \forall k \in \mathcal{K},$$

$$y_{ijt}^{kr} = 0, \quad \forall (i,j) \in \mathcal{F} \times \mathcal{F} : o_i = 1, \{ r \in \mathcal{R}_k : q_j^r > 0 \} \neq \emptyset, \quad \forall t \in \mathcal{T}, \quad \forall k \in \mathcal{K}, \quad \forall r \in \mathcal{R},$$

$$y_{ijt}^{kr} = 0, \quad \forall i \in \mathcal{F} : o_i = 1, \quad \forall j \in \mathcal{F}, \quad \forall t \in \mathcal{T}, \quad \forall k \in \mathcal{K}, \quad \forall r \in \mathcal{R}_k.$$

Further reductions in the number of decision variables can be obtained by exploiting the following results.

Theorem 3.1. *For any feasible vessel schedule,*

$$x_{ijt}^k = 0, \quad \forall i \in \mathcal{F}, \quad \forall j \in \mathcal{N}, \quad \forall t < \tau_{b_0 i}^k + d_i^k + \tau_{ij}^k + 2, \quad \forall k \in \mathcal{K},$$

$$y_{ijt}^{kr} = 0, \quad \forall i \in \mathcal{F}, \quad \forall j \in \mathcal{N}, \quad \forall t < \tau_{b_0 i}^k + d_i^k + \tau_{ij}^k + 2, \quad \forall k \in \mathcal{K}, \quad \forall r \in \mathcal{R}.$$

Proof. The earliest period during which vessel k can reach node $i \in \mathcal{F}$ is $\tau_{b_0 i}^k + 1$. Thus, the earliest period during which vessel k can leave node $i \in \mathcal{F}$ is $\tau_{b_0 i}^k + d_i^k + 2$. It follows that vessel k cannot arrive at node j before period $\tau_{b_0 i}^k + d_i^k + \tau_{ij}^k + 2$. □

Theorem 3.2. *For any feasible vessel schedule,*

$$x_{ijt}^k = 0, \quad \forall i \in \mathcal{B} \setminus \{b_0\}, \quad \forall j \in \mathcal{F}, \quad \forall t < \min_{s \in \mathcal{S}_i^k} \{ s + \delta^k(s) + \tau_{ij}^k + 1 \}, \quad \forall k \in \mathcal{K},$$

$$y_{ijt}^{kr} = 0, \quad \forall i \in \mathcal{B} \setminus \{b_0\}, \quad \forall j \in \mathcal{F}, \quad \forall t < \min_{s \in \mathcal{S}_i^k} \{ s + \delta^k(s) + \tau_{ij}^k + 1 \}, \quad \forall k \in \mathcal{K}, \quad \forall r \in \mathcal{R},$$

where

$$\mathcal{S}_i^k = \left\{ s \in \mathcal{T} : s \geq \min_{l \in \mathcal{F}} \{ \tau_{b_0 l}^k + d_l^k + \tau_{li}^k + 2 \} \right\}.$$

Proof. Vessel k must have visited at least one offshore node in $\mathcal{F}$ before reaching base node $i \in \mathcal{B} \setminus \{b_0\}$. If vessel k first visits node $l \in \mathcal{F}$, then the earliest period during which it can arrive at node $i \in \mathcal{B} \setminus \{b_0\}$ is $\tau_{b_0 l}^k + d_l^k + \tau_{li}^k + 2$. Hence, $\mathcal{S}_i^k$ contains all potential arrival periods for vessel k at node i. If vessel k arrives at node i during period $s \in \mathcal{S}_i^k$, then the earliest period during which it can arrive at node j is $s + \delta^k(s) + \tau_{ij}^k + 1$. Taking the minimum of this quantity over all potential arrival periods in $\mathcal{S}_i^k$ gives the earliest possible arrival period at node j. □

Theorem 3.3. *For any feasible vessel schedule,*

$$x_{ijt}^k = 0, \quad \forall i \in \mathcal{N}, \quad \forall j \in \mathcal{F}, \quad \forall t > T - d_j^k - \tau_{jb_0}^k - 1, \quad \forall k \in \mathcal{K},$$

$$y_{ijt}^{kr} = 0, \quad \forall i \in \mathcal{N}, \quad \forall j \in \mathcal{F}, \quad \forall t > T - d_j^k - \tau_{jb_0}^k - 1, \quad \forall k \in \mathcal{K}, \quad \forall r \in \mathcal{R},$$

where $\tau_{jb_0}^k$ is the travel time for vessel k from node j to the supply base.

Proof. If vessel k arrives at an offshore node $j \in \mathcal{F}$ during period t, then the earliest period during which the vessel can reach the supply base is $t + d_j^k + \tau_{jb_0}^k + 1$. It follows that any arrival periods t such that $t + d_j^k + \tau_{jb_0}^k + 1 > T$ are invalid, as there is insufficient time for the vessel to return to the base. Thus, the latest period during which vessel k can arrive at node j is $T - d_j^k - \tau_{jb_0}^k - 1$. □

Our final result guarantees that the MILP model in Section 2 is a valid representation of the vessel scheduling problem.

Theorem 3.4. *Constraints (1)-(15) define a set of closed vessel tours, each starting and ending at the supply base.*

Proof. We first show that the time sequencing constraints in the MILP model prevent subtours. Suppose, to the contrary, that vessel k performs a subtour $\{i_0, i_1, \ldots, i_p\}$, where $i_0 = i_p$. Let t_l denote the arrival period of vessel k at node i_l, $l = 0, \ldots, p$ (constraints (1) and (5) ensure that there is only one arrival at each node i_l). Then it follows from the time sequencing constraints (7) and (8) that

$$t_{l+1} - \tau^k_{i_l i_{l+1}} \geq \begin{cases} t_l + d^k_{i_l} + 1, & \text{if } i_l \in \mathcal{F}, \\ t_l + \delta^k(t_l) + 1, & \text{if } i_l \in \mathcal{B}. \end{cases}$$

Therefore,

$$t_{l+1} \geq \begin{cases} t_l + d^k_{i_l} + \tau^k_{i_l i_{l+1}} + 1, & \text{if } i_l \in \mathcal{F}, \\ t_l + \delta^k(t_l) + \tau^k_{i_l i_{l+1}} + 1, & \text{if } i_l \in \mathcal{B}. \end{cases}$$

This implies that $t_{l+1} > t_l$ for each $l = 0, \ldots, p-1$. Thus, $t_0 < t_l$ for each $l = 1, \ldots, p$, and in particular, $t_0 < t_p$. But since $i_p = i_0$, this is a contradiction. Thus, subtours are impossible with the time sequencing constraints (7) and (8).

Now, since there are no subtours, constraints (2)-(6) imply that each vessel starts at b_0 and ends at a node in $\mathcal{B} \setminus \{b_0\}$. This completes the proof. □

4. **Vessel utilization ratios.** One way of assessing the efficiency of a vessel fleet is to measure the utilization rate of each vessel in the fleet. A vessel's utilization rate can be calculated in terms of the time utilization (what proportion of time the vessel is active) or the capacity utilization (what proportion of the vessel's carrying capacity is used). Support vessels in the oil and gas industry are expensive assets and thus high vessel utilization rates are desirable. Low utilization rates are an indication that the fleet contains excess capacity and can potentially be replaced with a smaller, cheaper fleet.

For a given vessel schedule, the capacity utilization ratio for commodity type $r \in \mathcal{R}$ on vessel $k \in \mathcal{K}$ is defined as the ratio of the total outgoing commodity flow to the total available capacity. Mathematically,

$$\rho^{kr}_{\text{capacity}} = \underbrace{\left\{ \sum_{i \in \mathcal{B}} \sum_{j:\,(i,j) \in \mathcal{A}} \sum_{t \in \mathcal{T}^k_{ij}} y^{kr}_{ijt} \right\}}_{\text{Commodity flow}} \div \underbrace{\left\{ \sum_{i \in \mathcal{B}} \sum_{j:\,(i,j) \in \mathcal{A}} \sum_{t \in \mathcal{T}^k_{ij}} Q^{kr} x^k_{ijt} \right\}}_{\text{Available capacity}},$$

where $\rho^{kr}_{\text{capacity}}$ is the capacity utilization ratio for commodity r on vessel k. Note that $\rho^{kr}_{\text{capacity}}$ is undefined if vessel k never leaves the base.

The time utilization ratio of vessel $k \in \mathcal{K}$ is defined as the ratio of the number of active periods to the total number of periods. Mathematically,

$$\rho^k_{\text{time}} = \frac{\omega^k_{\text{travel}} + \omega^k_{\text{offshore}} + \omega^k_{\text{base}}}{T}, \tag{17}$$

where ρ^k_{time} is the time utilization ratio of vessel k, T is the total number of periods, ω^k_{travel} is the number of travel periods for vessel k, $\omega^k_{\text{offshore}}$ is the number of offshore servicing periods for vessel k, and ω^k_{base} is the number of base servicing periods for vessel k.

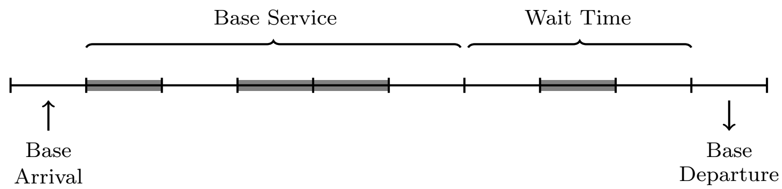


FIGURE 2. An example of base servicing with $d^k_{\text{base}} = 2$. Closed periods are shaded in grey. After arriving at the base, the vessel must stay for at least $\delta^k(t) = 5$ periods to complete the service. For option (i), $\omega^k_{\text{base}} = 2$ since the 3 closed periods during service are not considered active periods. For option (ii), $\omega^k_{\text{base}} = 5$ since the 3 closed periods are considered active periods.

Vessel k's travel and offshore servicing periods can be computed, respectively, using the following equations:

$$\omega^k_{\text{travel}} = \sum_{(i,j)\in\mathcal{A}} \sum_{t\in\mathcal{T}^k_{ij}} \tau^k_{ij} x^k_{ijt} \tag{18}$$

and

$$\omega^k_{\text{offshore}} = \sum_{i\in\mathcal{F}} \sum_{j\colon (j,i)\in\mathcal{A}} \sum_{t\in\mathcal{T}^k_{ji}} d^k_i x^k_{jit}. \tag{19}$$

To compute the number of active periods at the base, there are two options:

(i) Count only open periods in $\mathcal{O}_{\text{base}}$ (each base service counts as d^k_{base} active periods).
(ii) Count both open and closed periods (each base service counts as $\delta^k(t)$ active periods, where t is the arrival period).

The choice between options (i) and (ii) depends on whether vessels are considered to be active during closed periods at the base. See Figure 2 for an example.

For option (i), the number of base servicing periods is given by

$$\omega^k_{\text{base}} = \sum_{i\in\mathcal{B}\setminus\{b_0\}} \sum_{j\colon (i,j)\in\mathcal{A}} \sum_{t\in\mathcal{T}^k_{ij}} d^k_{\text{base}} x^k_{ijt}. \tag{20}$$

For option (ii), the number of base servicing periods is given by

$$\omega^k_{\text{base}} = \sum_{i\in\mathcal{B}\setminus\{b_0\}} \left\{ \sum_{j\colon (i,j)\in\mathcal{A}} \sum_{t\in\mathcal{T}^k_{ij}} x^k_{ijt} \right\} \cdot \left\{ \sum_{j\colon (j,i)\in\mathcal{A}} \sum_{t\in\mathcal{T}^k_{ji}} \delta^k(t) x^k_{jit} \right\}. \tag{21}$$

Combining equations (17)-(20), we obtain the following expression for the time utilization ratio of vessel k when vessels are considered to be inactive during base closure periods:

$$\rho^k_{\text{time}} = \underbrace{\sum_{(i,j)\in\mathcal{A}} \sum_{t\in\mathcal{T}^k_{ij}} \frac{\tau^k_{ij}}{T} x^k_{ijt}}_{\text{Travel}} + \underbrace{\sum_{i\in\mathcal{F}} \sum_{j\colon (j,i)\in\mathcal{A}} \sum_{t\in\mathcal{T}^k_{ji}} \frac{d^k_i}{T} x^k_{jit}}_{\text{Offshore servicing}}$$

$$+ \underbrace{\sum_{i\in\mathcal{B}\setminus\{b_0\}} \sum_{j:\,(i,j)\in\mathcal{A}} \sum_{t\in\mathcal{T}_{ij}^k} \frac{d_{\text{base}}^k}{T} x_{ijt}^k}_{\text{Base servicing}}.$$

Similarly, combining equations (17)-(19) and (21), we obtain the following expression for the time utilization ratio of vessel k when vessels are considered to be active during base closure periods:

$$\rho_{\text{time}}^k = \underbrace{\sum_{(i,j)\in\mathcal{A}} \sum_{t\in\mathcal{T}_{ij}^k} \frac{\tau_{ij}^k}{T} x_{ijt}^k}_{\text{Travel}} + \underbrace{\sum_{i\in\mathcal{F}} \sum_{j:\,(j,i)\in\mathcal{A}} \sum_{t\in\mathcal{T}_{ji}^k} \frac{d_i^k}{T} x_{jit}^k}_{\text{Offshore servicing}}$$
$$+ \underbrace{\sum_{i\in\mathcal{B}\setminus\{b_0\}} \left\{ \sum_{j:\,(i,j)\in\mathcal{A}} \sum_{t\in\mathcal{T}_{ij}^k} x_{ijt}^k \right\} \cdot \left\{ \sum_{j:\,(j,i)\in\mathcal{A}} \sum_{t\in\mathcal{T}_{ji}^k} \frac{\delta^k(t)}{T} x_{jit}^k \right\}}_{\text{Base servicing}}.$$

5. **Case study: Optimal vessel scheduling for Woodside.** In this section, we investigate a real vessel scheduling problem faced by Australian company Woodside in oil and gas operations off the coast of Western Australia. This problem is a special case of the MILP model described in Section 2.

5.1. **Problem setting.** We consider seven Woodside-operated oil and gas facilities in the Indian Ocean off the coast of Western Australia: five in the North West Shelf region (Angel, Goodwyn, North Rankin, Okha, Pluto) and two in the Carnarvon Basin (Nganhurra, Ngujima-Yin). See Figure 3 for a map showing these facilities. Angel, Goodwyn, North Rankin, and Pluto are platforms; the other facilities are floating production, storage, and offloading (FPSO) facilities. Off-takes are only required for the FPSOs.

The offshore facilities in Figure 3 are serviced by a fleet of support vessels based in Karratha. The fleet consists of one platform supply vessel (PSV) and two off-take support vessels (OSVs). The PSV is used for cargo delivery and the OSVs are used for off-take support. The OSVs can also deliver cargo if required.

Both the PSV and the OSVs store cargo on deck, and the OSVs cannot assist with off-take operations unless their decks are completely clear (this is due to the location of the off-take support equipment on these vessels). In these numerical simulations, cargo (measured in m^2 of deck-space) is the only commodity of interest and thus $|\mathcal{R}| = 1$. The deck-space capacity of the PSV is 500 m^2 and the deck-space capacity of each OSV is 300 m^2. Moreover, the fuel consumption rate of the PSV is 54 L/NM and the fuel consumption rate of each OSV is 40 L/NM. All vessels travel at a speed of 10 knots.

We consider a time horizon of three weeks decomposed into 1-hour time periods (giving a total of $T = 504$ periods). Thus,

$$\mathcal{T} = \{1, \ldots, 504\}.$$

The distance between each pair of facilities is given in Table 1. Since each vessel travels at a speed of 10 knots, the travel time between facilities i and j (rounded up to the nearest period) can be computed as follows:

$$\tau_{ij}^k = \left\lceil \frac{\theta_{ij}}{10} \right\rceil,$$

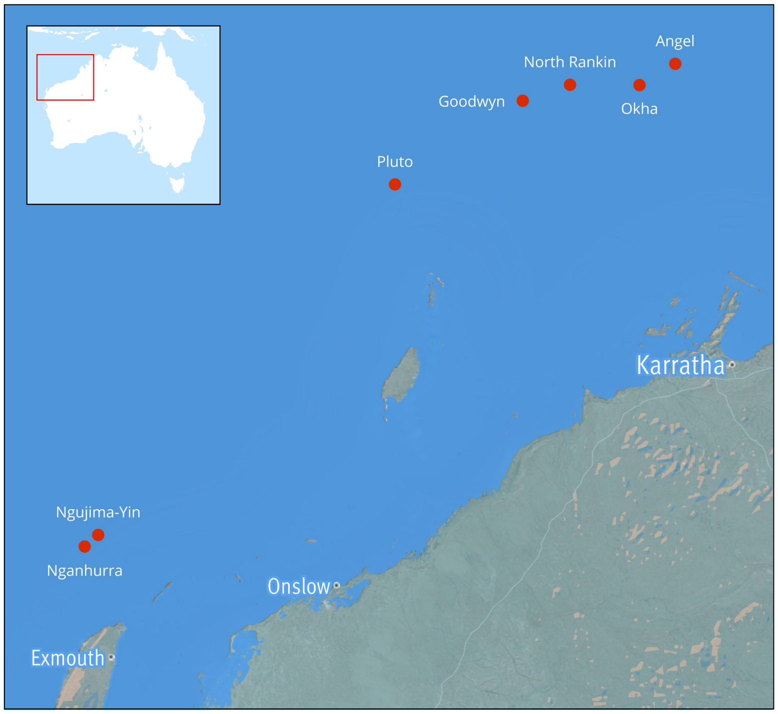


FIGURE 3. Woodside's offshore facilities in the North West Shelf region and Carnarvon Basin.

	Karratha	Angel	Goodwyn	Nganhurra	Ngujima-Yin	North Rankin	Okha	Pluto
Karratha	-	68.4	78.4	180.0	175.0	75.0	65.0	95.9
Angel	68.4	-	38.4	188.4	181.7	27.5	10.0	75.0
Goodwyn	78.4	38.4	-	155.0	155.9	12.5	30.0	38.4
Nganhurra	180.0	188.4	155.0	-	5.0	165.0	170.0	117.5
Ngujima-Yin	175.0	181.7	155.9	5.0	-	160.0	165.0	112.5
North Rankin	75.0	27.5	12.5	165.0	160.0	-	18.4	50.0
Okha	65.0	10.0	30.0	170.0	165.0	18.4	-	65.0
Pluto	95.9	75.0	38.4	117.5	112.5	50.0	65.0	-

TABLE 1. Distances (in nautical miles) between facilities.

where θ_{ij} is the distance (in nautical miles) between facility i and facility j as recorded in Table 1.

The opening hours for the base are 6am to 6pm every day including weekends. Thus,

$$\mathcal{O}_{\text{base}} = \bigcup_{m=0}^{20} \{24m + 7, \ldots, 24m + 18\}.$$

We considered four service scenarios corresponding to historical data provided by Woodside. Each service scenario involves multiple service visits, and each service visit is defined by a given cargo demand, service start time window, service duration, and set of suitable vessels. This information is provided in Tables 2-5. In all scenarios, the service duration for cargo delivery visits is $d_i^k = 3$ hours and the service duration for off-take visits is $d_i^k = 30$ hours. In Scenario 1, the service

Facility	Day	Demand	Time Window	Duration	Suitable Vessels	Off-take?
Ngujima-Yin	1	300 m^2	0:00-24:00	30 hours	OSV	Yes
Goodwyn	2	100 m^2	0:00-24:00	3 hours	PSV, OSV	No
North Rankin	2	200 m^2	0:00-24:00	3 hours	PSV, OSV	No
Okha	3	100 m^2	0:00-24:00	3 hours	PSV, OSV	No
Goodwyn	6	150 m^2	0:00-24:00	3 hours	PSV, OSV	No
North Rankin	6	250 m^2	0:00-24:00	3 hours	PSV, OSV	No
Ngujima-Yin	8	300 m^2	0:00-24:00	3 hours	PSV, OSV	No
Goodwyn	9	100 m^2	0:00-24:00	3 hours	PSV, OSV	No
North Rankin	9	200 m^2	0:00-24:00	3 hours	PSV, OSV	No
Okha	9	100 m^2	0:00-24:00	3 hours	PSV, OSV	No
Okha	10	100 m^2	0:00-24:00	3 hours	PSV, OSV	No
Goodwyn	16	250 m^2	0:00-24:00	3 hours	PSV, OSV	No
Nganhurra	16	300 m^2	0:00-24:00	30 hours	OSV	Yes
North Rankin	16	250 m^2	0:00-24:00	3 hours	PSV, OSV	No
Pluto	17	100 m^2	0:00-24:00	3 hours	PSV, OSV	No
Okha	19	200 m^2	0:00-24:00	30 hours	OSV	Yes
Goodwyn	20	100 m^2	0:00-24:00	3 hours	PSV, OSV	No
North Rankin	20	200 m^2	0:00-24:00	3 hours	PSV, OSV	No

TABLE 2. Service requirements for Scenario 1.

duration at the base is $d^k_{\text{base}} = 21$ hours for the PSV and $d^k_{\text{base}} = 11$ hours for the OSVs; for the other scenarios, $d^k_{\text{base}} = 11$ hours for the PSV and $d^k_{\text{base}} = 8$ hours for the OSVs. The maximum wait time at each offshore facility is unlimited for all scenarios (i.e., $w^k_i = \infty$). Off-take visits can only be performed by OSVs, while cargo delivery visits can be performed by any vessel.

The fixed cost α^k_{ij} is equal to the total fuel consumed while traversing link (i,j) and the corresponding variable cost β^{kr}_{ij} is zero. Thus, based on the speeds and fuel consumption rates of the PSV and OSVs,

$$\alpha^k_{ij} = \begin{cases} 540\tau^k_{ij}, & \text{if vessel } k \text{ is the PSV,} \\ 400\tau^k_{ij}, & \text{if vessel } k \text{ is an OSV.} \end{cases}$$

Hence, the cost function is

$$\text{Cost} = \sum_{(i,j)\in\mathcal{A}} \sum_{t\in\mathcal{T}} \left\{ 540\tau^{\text{PSV}}_{ij} x^{\text{PSV}}_{ijt} + 400\tau^{\text{OSV1}}_{ij} x^{\text{OSV1}}_{ijt} + 400\tau^{\text{OSV2}}_{ij} x^{\text{OSV2}}_{ijt} \right\}, \tag{22}$$

where $\mathcal{A}$ is the arc set of the network defined by the service visits in Tables 2-5 and $\mathcal{T} = \{1, \ldots, 504\}$ is the set of time periods.

The scheduling problem is to minimize (22) subject to constraints (1)-(15) described in Section 2.

Since $w^k_i = \infty$, the waiting time constraints (9) are essentially redundant here. The reason we have ignored these constraints is that the service scenarios defined in Tables 2-5 are extremely tight, and imposing waiting time constraints can lead to infeasibility. We have tested the model on other service scenarios in which the service requirements are less onerous and thus waiting time constraints are valid, but we are unable to publish these results due to commercial confidentiality.

5.2. **Solution procedure.** We first attempted to solve the full MILP model (see Section 2) using the CPLEX Optimization Package [12, 13] embedded within the AIMMS modelling platform [1, 6]. For each scenario, CPLEX failed to find a feasible solution for the full model after more than one day of computation.

We next applied the model reduction techniques described in Section 3. Table 6 shows the model dimensions before and after the model reduction process. As shown in the table, model reduction eliminates around 92-93% of the variables and 86-88%

Facility	Day	Demand	Time Window	Duration	Suitable Vessels	Off-take?
Goodwyn	1	200 m^2	0:00-24:00	3 hours	PSV, OSV	No
Goodwyn	2	200 m^2	0:00-24:00	3 hours	PSV, OSV	No
Ngujima-Yin	3	200 m^2	0:00-24:00	3 hours	PSV, OSV	No
Okha	4	100 m^2	0:00-24:00	3 hours	PSV, OSV	No
Goodwyn	5	150 m^2	0:00-24:00	3 hours	PSV, OSV	No
North Rankin	5	150 m^2	0:00-24:00	3 hours	PSV, OSV	No
Okha	5	100 m^2	0:00-24:00	3 hours	PSV, OSV	No
North Rankin	6	150 m^2	0:00-24:00	3 hours	PSV, OSV	No
Nganhurra	7	100 m^2	0:00-24:00	3 hours	PSV, OSV	No
Ngujima-Yin	7	100 m^2	0:00-24:00	3 hours	PSV, OSV	No
North Rankin	8	200 m^2	0:00-24:00	3 hours	PSV, OSV	No
Goodwyn	9	100 m^2	0:00-24:00	3 hours	PSV, OSV	No
Ngujima-Yin	9	200 m^2	0:00-24:00	30 hours	OSV	Yes
Okha	11	100 m^2	0:00-24:00	3 hours	PSV, OSV	No
Goodwyn	12	150 m^2	0:00-24:00	3 hours	PSV, OSV	No
North Rankin	12	150 m^2	0:00-24:00	3 hours	PSV, OSV	No
Okha	13	200 m^2	0:00-24:00	30 hours	OSV	Yes
Goodwyn	14	250 m^2	0:00-24:00	3 hours	PSV, OSV	No
North Rankin	15	250 m^2	0:00-24:00	3 hours	PSV, OSV	No
Okha	18	100 m^2	0:00-24:00	3 hours	PSV, OSV	No
Goodwyn	19	150 m^2	0:00-24:00	3 hours	PSV, OSV	No
North Rankin	19	150 m^2	0:00-24:00	3 hours	PSV, OSV	No

Table 3. Service requirements for Scenario 2.

Facility	Day	Demand	Time Window	Duration	Suitable Vessels	Off-take?
Angel	1	200 m^2	0:00-24:00	3 hours	PSV, OSV	No
Goodwyn	1	250 m^2	0:00-24:00	3 hours	PSV, OSV	No
North Rankin	1	250 m^2	0:00-24:00	3 hours	PSV, OSV	No
Nganhurra	3	250 m^2	0:00-24:00	3 hours	PSV, OSV	No
Ngujima-Yin	3	250 m^2	0:00-24:00	3 hours	PSV, OSV	No
Angel	4	100 m^2	0:00-24:00	3 hours	PSV, OSV	No
Goodwyn	4	100 m^2	0:00-24:00	3 hours	PSV, OSV	No
Okha	4	100 m^2	0:00-24:00	30 hours	OSV	Yes
North Rankin	5	100 m^2	0:00-24:00	3 hours	PSV, OSV	No
Okha	7	100 m^2	0:00-24:00	3 hours	PSV, OSV	No
Goodwyn	8	150 m^2	0:00-24:00	3 hours	PSV, OSV	No
Ngujima-Yin	8	200 m^2	0:00-24:00	30 hours	OSV	Yes
North Rankin	8	150 m^2	0:00-24:00	3 hours	PSV, OSV	No
North Rankin	10	150 m^2	0:00-24:00	3 hours	PSV, OSV	No
Goodwyn	11	150 m^2	0:00-24:00	3 hours	PSV, OSV	No
North Rankin	12	150 m^2	0:00-24:00	3 hours	PSV, OSV	No
North Rankin	14	150 m^2	0:00-24:00	3 hours	PSV, OSV	No
Okha	14	100 m^2	0:00-24:00	3 hours	PSV, OSV	No
Pluto	14	300 m^2	0:00-24:00	3 hours	PSV, OSV	No
Goodwyn	15	150 m^2	0:00-24:00	3 hours	PSV, OSV	No
Nganhurra	17	100 m^2	0:00-24:00	3 hours	PSV, OSV	No
Ngujima-Yin	17	100 m^2	0:00-24:00	3 hours	PSV, OSV	No
Goodwyn	18	150 m^2	0:00-24:00	3 hours	PSV, OSV	No
North Rankin	19	150 m^2	0:00-24:00	3 hours	PSV, OSV	No
North Rankin	21	250 m^2	0:00-24:00	3 hours	PSV, OSV	No
Okha	21	100 m^2	0:00-24:00	3 hours	PSV, OSV	No

Table 4. Service requirements for Scenario 3.

of the constraints. In the original model, there is always one more continuous-valued variable than binary variable because AIMMS treats the cost function as an additional continuous-valued variable. Despite the massive reduction in model dimension, the simplified MILP model still proved extremely difficult to solve.

To expedite the computation, we applied a greedy scheduling procedure to generate an initial feasible schedule for each problem scenario. The heuristic works by choosing a sequence of suitable service visits for each vessel (starting with the PSV) according to earliest arrival time and subject to the vessel's deck-space capacity and a constraint on the maximum waiting time at each facility. The heuristic repeats

Facility	Day	Demand	Time Window	Duration	Suitable Vessels	Off-take?
Goodwyn	1	250 m^2	0:00-24:00	3 hours	PSV, OSV	No
Nganhurra	1	250 m^2	0:00-24:00	30 hours	OSV	Yes
North Rankin	1	250 m^2	0:00-24:00	3 hours	PSV, OSV	No
Goodwyn	3	150 m^2	0:00-24:00	3 hours	PSV, OSV	No
Ngujima-Yin	3	150 m^2	0:00-24:00	3 hours	PSV, OSV	No
North Rankin	3	150 m^2	0:00-24:00	3 hours	PSV, OSV	No
Nganhurra	4	150 m^2	0:00-24:00	3 hours	PSV, OSV	No
Ngujima-Yin	5	150 m^2	0:00-24:00	30 hours	OSV	Yes
Goodwyn	7	150 m^2	0:00-24:00	3 hours	PSV, OSV	No
North Rankin	7	150 m^2	0:00-24:00	3 hours	PSV, OSV	No
Okha	8	200 m^2	0:00-24:00	3 hours	PSV, OSV	No
Pluto	9	250 m^2	0:00-24:00	3 hours	PSV, OSV	No
Ngujima-Yin	10	150 m^2	0:00-24:00	3 hours	PSV, OSV	No
Goodwyn	11	100 m^2	0:00-24:00	3 hours	PSV, OSV	No
Nganhurra	11	150 m^2	0:00-24:00	3 hours	PSV, OSV	No
North Rankin	11	100 m^2	0:00-24:00	3 hours	PSV, OSV	No
Pluto	12	100 m^2	0:00-24:00	3 hours	PSV, OSV	No
Okha	13	100 m^2	0:00-24:00	30 hours	OSV	Yes
Goodwyn	14	150 m^2	0:00-24:00	3 hours	PSV, OSV	No
North Rankin	14	150 m^2	0:00-24:00	3 hours	PSV, OSV	No
Okha	15	200 m^2	0:00-24:00	3 hours	PSV, OSV	No
Okha	16	200 m^2	0:00-24:00	3 hours	PSV, OSV	No
Ngujima-Yin	17	150 m^2	0:00-24:00	3 hours	PSV, OSV	No
Goodwyn	18	150 m^2	0:00-24:00	3 hours	PSV, OSV	No
Nganhurra	18	150 m^2	0:00-24:00	3 hours	PSV, OSV	No
North Rankin	18	150 m^2	0:00-24:00	3 hours	PSV, OSV	No
Angel	21	300 m^2	0:00-24:00	3 hours	PSV, OSV	No
Okha	21	100 m^2	0:00-24:00	3 hours	PSV, OSV	No

TABLE 5. Service requirements for Scenario 4.

	Original Model			Simplified Model		
	BVs	CVs	Constraints	BVs	CVs	Constraints
Scenario 1	1,646,568	1,646,569	1,646,870	206,868	20,571	207,167
Scenario 2	2,069,928	2,069,929	2,070,258	292,443	31,582	292,771
Scenario 3	2,541,672	2,541,673	2,542,030	322,105	35,540	322,461
Scenario 4	2,795,688	2,795,689	2,796,060	330,484	39,859	330,853

TABLE 6. Model dimensions in terms of binary variables (BVs), continuous-valued variables (CVs), and constraints.

this process for a maximum waiting time of one day, two days, three days, and so on, with the best solution selected as the initial schedule. In addition, any service visits to North Rankin and Goodwyn on the same day are clustered into a single visit, since this is what normally occurs in practice. This heuristic procedure can generate decent feasible schedules in a matter of seconds. Substituting the initial schedule into the linear programming softwares CPLEX [13] and Gurobi [10] allowed us to solve the reduced model and obtain solutions for Scenarios 1-4.

5.3. **Results.** For each test scenario, we first applied the heuristic optimization procedure described in Section 5.2 to generate an initial feasible schedule. We then solved the reduced MILP model by running CPLEX and Gurobi consecutively according to the following sequence: CPLEX for two hours, Gurobi for two hours, CPLEX for two hours, Gurobi for two hours, and CPLEX for two hours. This took a total of ten hours for each schedule.

The performance of our method (in terms of cost) is shown in Table 7, where "Historical" refers to the historical schedule extracted from real data, "Initial" refers to the initial schedule generated by the heuristic optimization procedure, and

	Total Fuel Use (L)			
	Historical	Initial	Optimized	Improvement
Scenario 1	108,620	107,560	97,440	10.29%
Scenario 2	124,460	113,880	96,040	22.83%
Scenario 3	139,500	138,400	125,820	9.81%
Scenario 4	170,680	168,960	148,640	12.91%

TABLE 7. Optimal fuel consumption for Scenarios 1-4.

	Deck-space Utilization			Time Utilization		
	PSV	OSV 1	OSV 2	PSV	OSV 1	OSV 2
Scenario 1	100%	100%	100%	30%	37%	31%
Scenario 2	80%	88%	89%	26%	31%	31%
Scenario 3	100%	78%	83%	51%	33%	27%
Scenario 4	92%	96%	100%	45%	41%	43%

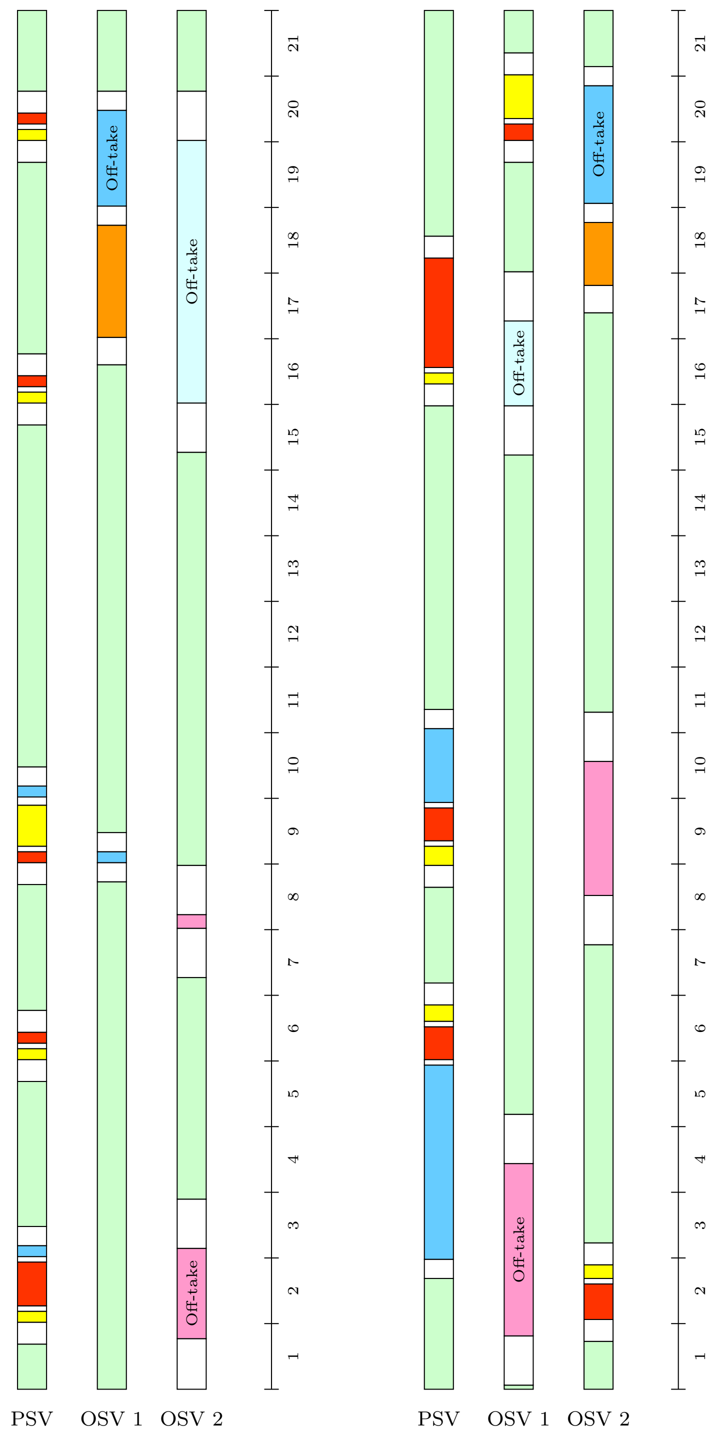
TABLE 8. Optimal vessel utilization for Scenarios 1-4.

"Optimized" refers to the best schedule obtained by CPLEX and Gurobi. As seen from the table, the schedules obtained using our method are significantly better than the historical schedules in terms of fuel cost. Table 8 reports the vessel utilization ratios corresponding to the optimized schedules. Here, the time utilization ratios are computed using the second measure of time utilization, where both open and closed periods at the base are counted as active periods. The optimal schedules are shown in Figures 4-7. The different colours in the figures represent the different facilities, with white representing travel. The colours for the facilities are:

- Supply Base – Light Green;
- Angel – Bright Green;
- Goodwyn – Yellow;
- Nganhurra – Light Blue;
- Ngujima-Yin – Pink;
- North Rankin – Red;
- Okha – Bright Blue; and
- Pluto – Orange.

Note from Figures 4-7 that the optimization process has merged some service visits to save fuel costs. For example, in Scenario 1, the cargo delivery visits to Okha on days 9 and 10 have been combined into a single visit.

6. **Conclusion.** This paper has highlighted the potential of using linear programming techniques for applications in the oil and gas industry. The research described in this paper was initiated by Woodside's Marine Division, which operates the company's supply vessel fleet. As part of a review of its existing fleet, Woodside required an optimization model for assessing different options for future fleet replacements. The model described in this paper is an enhanced version of the original model used by Woodside. Another version of the model is currently being developed for use in a scheduling tool for operational decision support. Our future work will focus on a more detailed comparison between the actual schedules performed in practice and the optimized schedules defined by the MILP model, although direct comparisons



(a) Historical

(b) Optimized

FIGURE 4. Historical and optimized vessel schedules for Scenario 1.

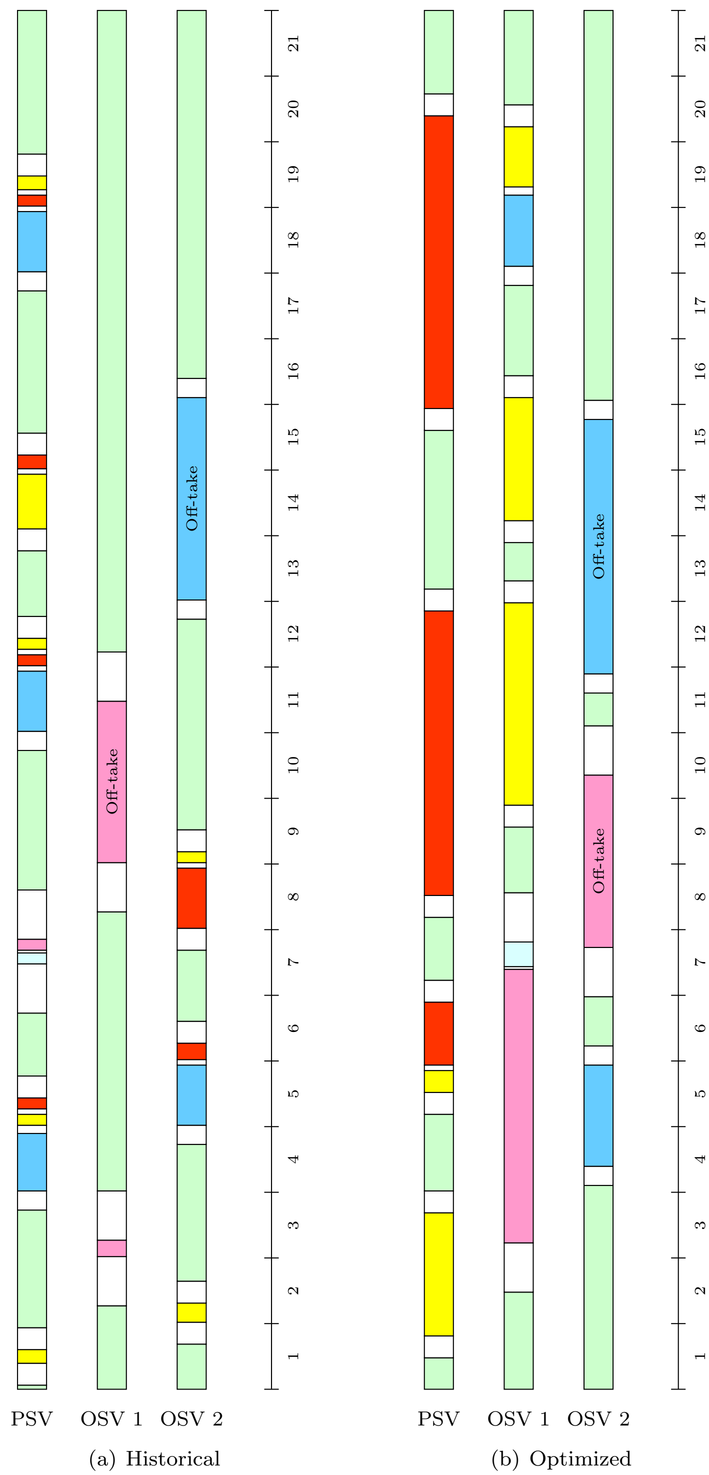
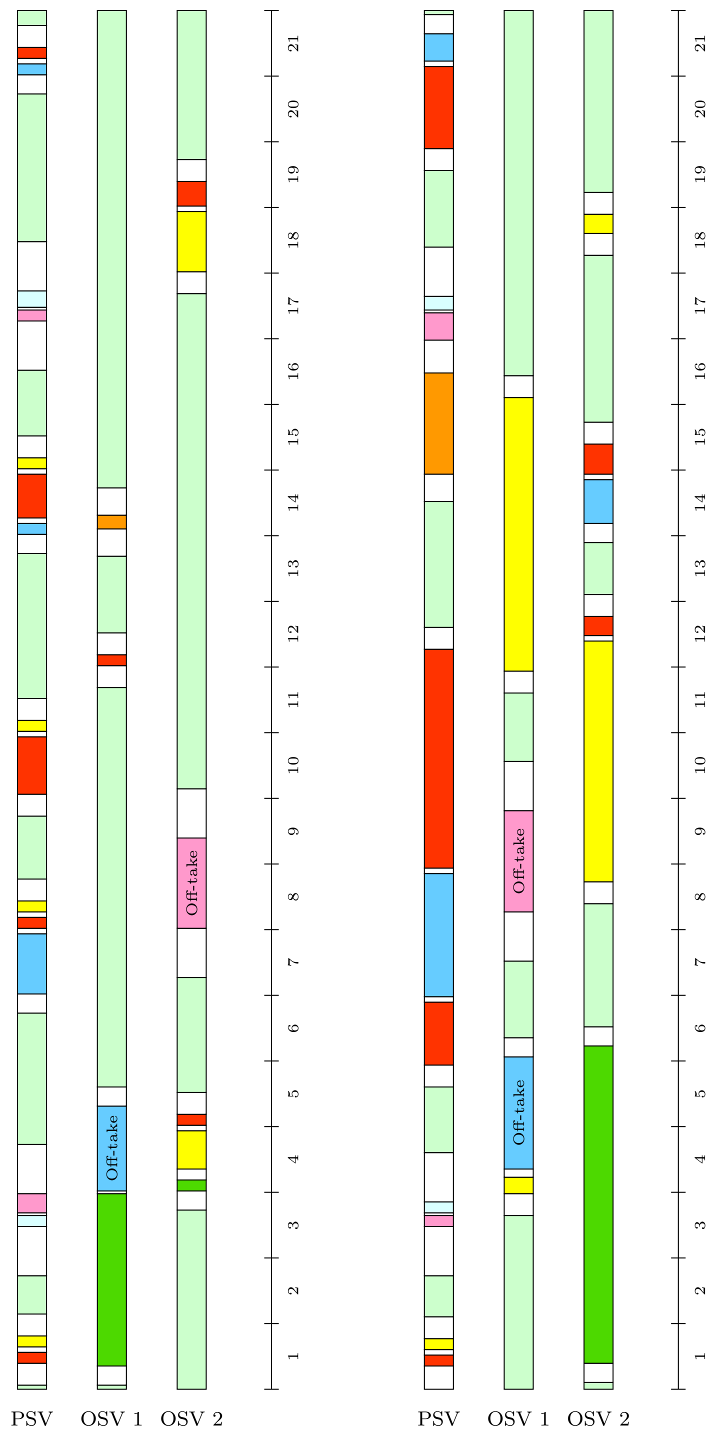


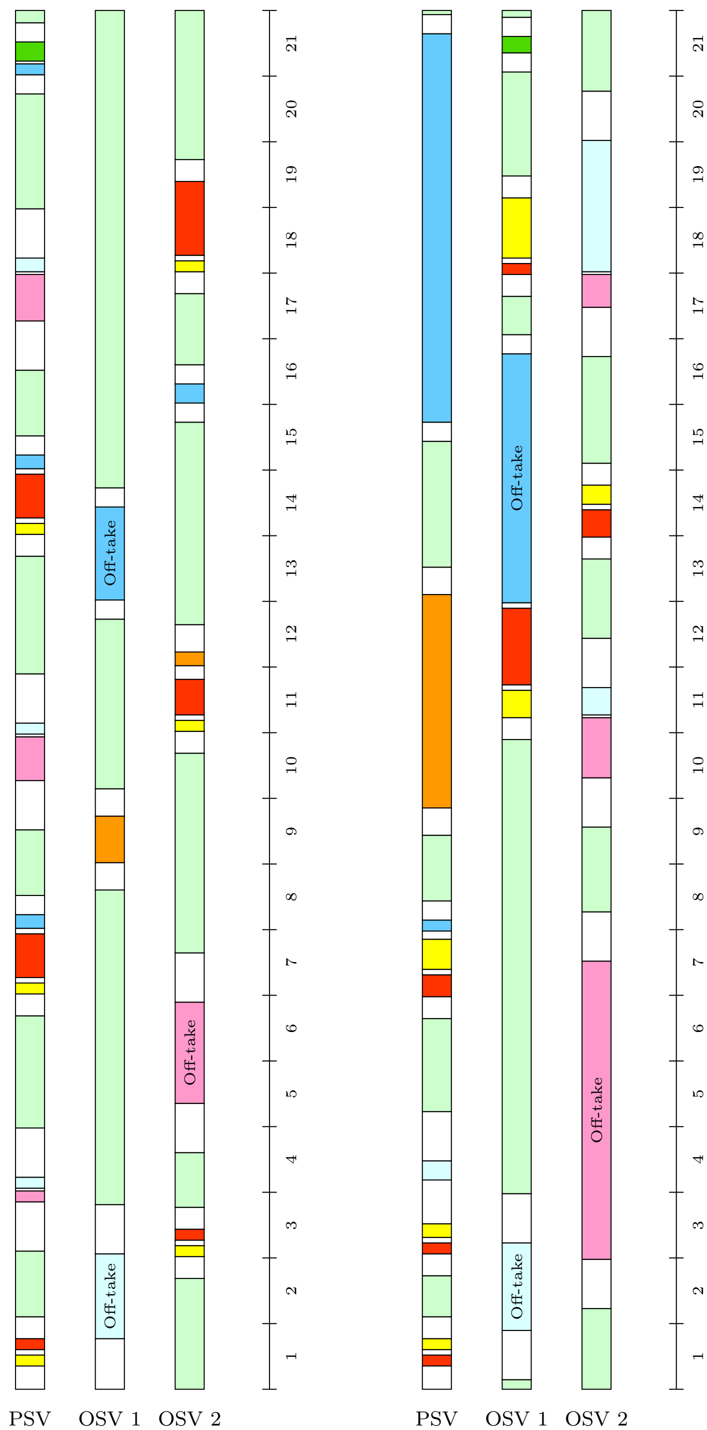
FIGURE 5. Historical and optimized vessel schedules for Scenario 2.



(a) Historical

(b) Optimized

FIGURE 6. Historical and optimized vessel schedules for Scenario 3.



(a) Historical (b) Optimized

FIGURE 7. Historical and optimized vessel schedules for Scenario 4.

are difficult due to limited data. We also plan to investigate methods for generating lower bounds to verify schedule optimality.

Acknowledgments. This work was funded by Woodside under the R2D3 Energy Research Alliance framework, and by the Australian Research Council through the ARC Linkage Scheme. The authors especially thank Rob Duncanson and Nicola Wilson from Woodside for their support of this paper. The authors also thank AIMMS for providing access to the AIMMS software platform.

Disclaimer. The views expressed in this paper are the authors' personal views and are not attributable to Woodside. The paper is intended to be a general discussion and does not constitute advice. Woodside disclaims all liability for any loss or damage incurred by any person or organization as a result of using, acting on, relying on, or disclosing any of the information contained in this paper or otherwise in connection with this paper.

REFERENCES

[1] AIMMS Modelling Platform, AIMMS B. V., Haarlem, Netherlands, accessed 17 April 2016, <http://www.aimms.com>

[2] F. Alonso, M. J. Alvarez and J. E. Beasley, A tabu search algorithm for the periodic vehicle routing problem with multiple vehicle trips and accessibility restrictions, *Journal of the Operational Research Society,* **59** (2008), 963–976.

[3] N. Azi, M. Gendreau and J. Y. Potvin, An exact algorithm for a single-vehicle routing problem with time windows and multiple routes, *European Journal of Operational Research,* **178** (2007), 755–766.

[4] N. Azi, M. Gendreau and J. Y. Potvin, An exact algorithm for a vehicle routing problem with time windows and multiple use of vehicles, *European Journal of Operational Research,* **202** (2010), 756–763.

[5] M. Battarra, M. Monaci and D. Vigo, An adaptive guidance approach for the heuristic solution of a minimum multiple trip vehicle routing problem, *Computers and Operations Research,* **36** (2009), 3041–3050.

[6] J. Bisschop, *AIMMS Optimization Modelling,* Haarlem: AIMMS B.V., 2016.

[7] J. C. S. Brandão and A. Mercer, The multi-trip vehicle routing problem, *Journal of the Operational Research Society,* **49** (1998), 799–805.

[8] D. Feillet, P. Dejax, M. Gendreau and C. Gueguen, An exact algorithm for the elementary shortest path problem with resource constraints: Application to some vehicle routing problems, *Networks,* **44** (2004), 216–229.

[9] B. Fleischmann, The vehicle routing problem with multiple use of vehicles (technical report), Hamburg: Universität Hamburg, 1990.

[10] Gurobi, *Optimizer Reference Manual,* Houston: Gurobi Optimization Inc., 2016.

[11] F. Hernandez, D. Feillet, R. Giroudeau and O. Naud, A new exact algorithm to solve the multi-trip vehicle routing problem with time windows and limited duration, *4OR,* **12** (2014), 235–259.

[12] IBM ILOG CPLEX Optimizer, IBM Corporation, New York, USA, accessed 17 April 2016, <http://www.ibm.com/software/commerce/optimization/cplex-optimizer>

[13] IBM ILOG CPLEX Optimization Studio CPLEX User's Manual, New York, IBM Corporation, 2014.

[14] R. Macedo, C. Alves, J. M. Valério de Carvalho, F. Clautiaux and S. Hanafi, Solving the vehicle routing problem with time windows and multiple routes exactly using a pseudo-polynomial model, *European Journal of Operational Research,* **214** (2011), 536–545.

[15] A. Olivera and O. Viera, Adaptive memory programming for the vehicle routing problem with multiple trips, *Computers and Operations Research,* **34** (2007), 28–47.

[16] R. J. Petch and S. Salhi, A multi-phase constructive heuristic for the vehicle routing problem with multiple trips, *Discrete Applied Mathematics,* **133** (2003), 69–92.

[17] E. D. Taillard, G. Laporte and G. Gendreau, Vehicle routeing with multiple use of vehicles, *Journal of the Operational Research Society,* **47** (1996), 1065–1070.

[18] P. Toth and D. Vigo, editors, *The Vehicle Routing Problem*, Philadelphia: SIAM, 2002.

[19] S. Salhi and R. J. Petch, A GA based heuristic for the vehicle routing problem with multiple trips, *Journal of Mathematical Modelling and Algorithms*, **6** (2007), 591–613.

[20] A. Şen and K. Bülbül, A survey on multi trip vehicle routing problem, In: Proceedings of the International Logistics and Supply Chain Congress 2008, Istanbul, Turkey.

Received November 2015; revised May 2016.

E-mail address: elham.mardaneh@curtin.edu.au
E-mail address: r.loxton@curtin.edu.au
E-mail address: q.lin@curtin.edu.au
E-mail address: phil.schmidli@woodside.com.au