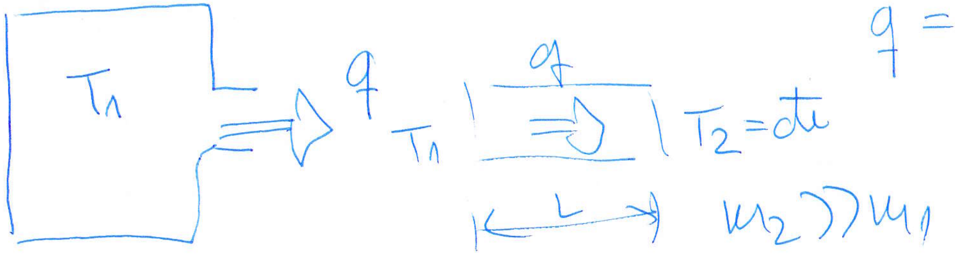
PME 3201 Aula 2 Turma 21 11.09.2020



$$q = \dot{Q} = \frac{dQ}{dt}$$

$$m_1 c \frac{dT_1}{dt} = -q$$

$$q = \frac{kA}{L}(T_1 - T_2) \geq 0$$

$$m_1 c \frac{dT_1}{dt} = -\frac{kA}{L}(T_1 - T_2)$$

$$\frac{dT_1}{dt} = -\frac{kA}{m_1 c L}(T_1 - T_2)$$

$$\boxed{T_1' = T_1 - T_2}$$

$$\frac{dT_1'}{dt} = \frac{dT_1}{dt}$$

$$\boxed{\frac{dT_1'}{dt} = -\frac{kA}{m_1 c L} T_1'}$$

$$m_1 c \frac{dT_1}{dt} = -\frac{kA}{L}(T_1 - T_2) \quad \Big| \quad T_2 = T_{02} = cte$$

$$\boxed{m_1 c \frac{dT_1}{dt} + \frac{kA}{L} T_1 = \frac{kA}{L} T_2}$$

Equ. Dif. Ord. Linear não homogénea.

$$T_1(t) = T_{1h}(t) + T_{1p}(t)$$

T_{1p}: solução particular

T_{1h}: solução homogénea

$$\underbrace{m_1 c \frac{dT_{1h}}{dt} + \frac{kA}{L} T_{1h}}_{0} + m_1 c \frac{dT_{1p}}{dt} + \frac{kA}{L} T_{1p} = \frac{kA}{L} T_2$$

$$m_1 c \frac{dT_{1h}}{dt} + \frac{kA}{L} T_{1h} = 0$$

$$T_{1h} = C_1 e^{-\alpha t} \Rightarrow \frac{dT_{1h}}{dt} = -C_1 \alpha e^{-\alpha t}$$

$$-m_1 c\, C_1 \alpha e^{-\alpha t} + \frac{kA}{L} C_1 e^{-\alpha t} = 0$$

$$e^{-\alpha t} \neq 0$$

$$\left[\frac{kA}{L} - m_1 c \alpha\right] C_1 = 0$$

$C_1 \neq 0$ $\quad \alpha = \dfrac{kA}{m_1 c L}$ $\quad \left| C_1 \in \mathbb{R} \right| C_1 \neq 0$ $\quad \tau = \dfrac{m_1 c L}{kA} = \dfrac{1}{\alpha}$

$$\boxed{T_{1h} = C_1 e^{-t/\tau}}$$ Solução Homogénea.

3.

$$\boxed{\tau = \frac{m_1 c L}{h A}}$$

$$[\tau] = \frac{\cancel{kg}\ J\ \cancel{m}\ \cancel{m}\ \cancel{K}}{\cancel{kg}\ \cancel{K}\ W\ \cancel{m^2}} = \frac{J}{W} = \frac{\cancel{J}\ s}{\cancel{J}} = s$$

Costante di Tempo

$$\tau = \frac{200 \cdot 466 \cdot 0{,}5}{400 \cdot 8000 \cdot 10^{-6}} = 14.562{,}5\ s = 4{,}05\ h$$

$$m_1 c \frac{dT_{1p}}{dt} + \frac{kA}{L} T_{1p} = \frac{kA}{L} T_{02}$$

cte

$$T_{1p} = C_2 = cte \Rightarrow \frac{dT_{1p}}{dt} = 0$$

$$\frac{k\cancel{A}}{\cancel{L}} C_2 = \frac{k\cancel{A}}{\cancel{L}} T_{02} \Rightarrow \boxed{C_2 = T_{02}}$$ Solução particular

Solução Geral:

$$T_1(t) = T_{1h}(t) + T_{1p}(t)$$

$$\boxed{T_1(t) = C_1 e^{-t/\tau} + T_{02}}$$

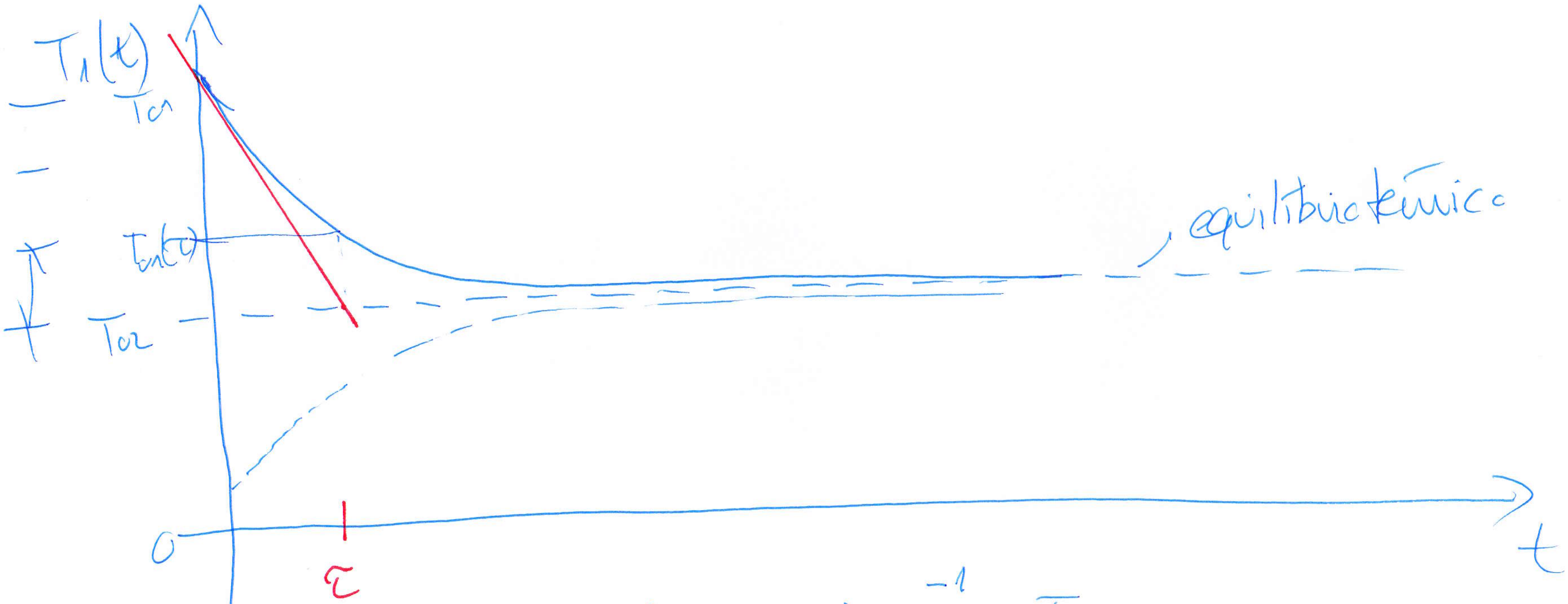
$C_1 = ?$ $\quad T_1(0) = T_{01} = C_1 e^0 + T_{02}$

$$C_1 = (T_{01} - T_{02})$$

$$\boxed{T_1(t) = (T_{01} - T_{02})\, e^{-t/\tau} + T_{02}}$$

$$\tau = \frac{m_1 c L}{k A}$$

Solução Geral
em termos dos parâmetros físicos.

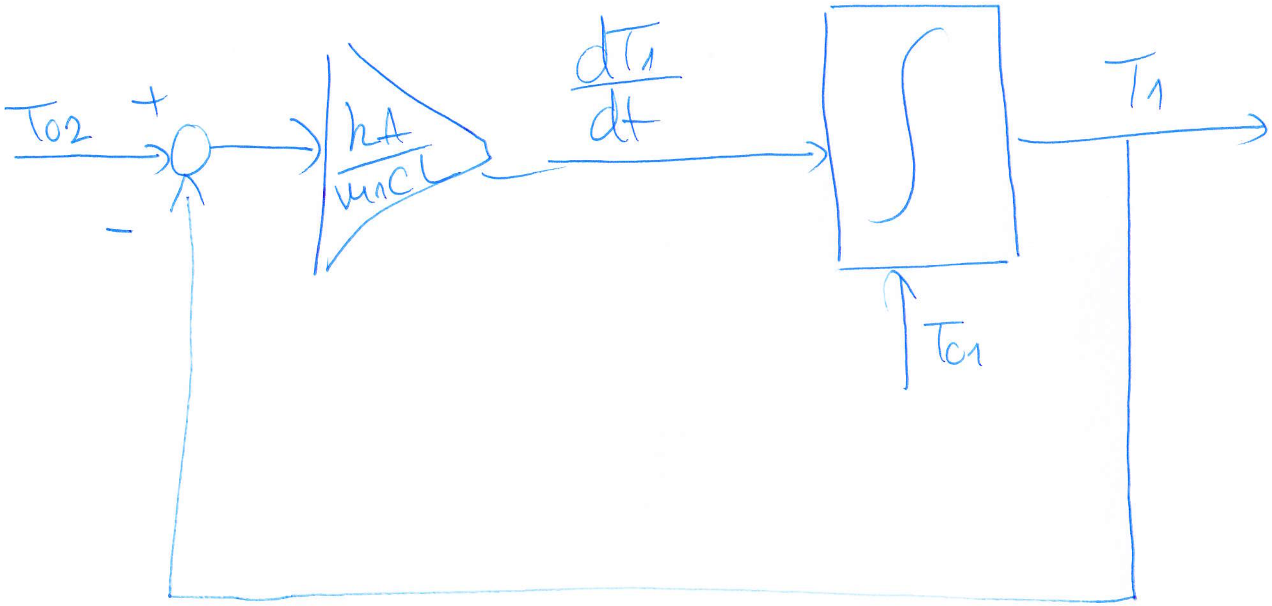


$$T_1(\tau) = (T_{01} - T_{02})\, e^{-1} + T_{02}$$

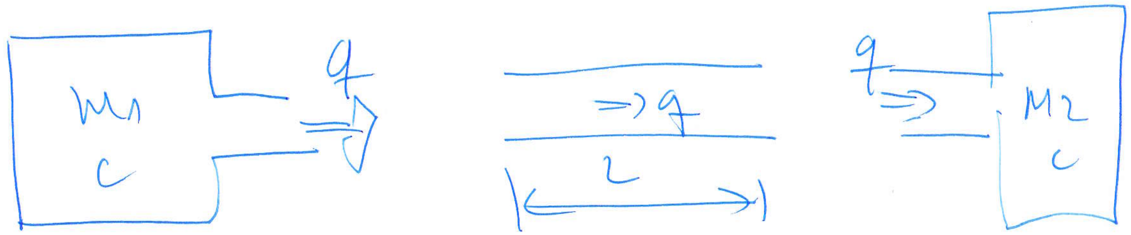
$$= (T_{01} - T_{02})\, 0{,}3679 + T_{02}$$

$$\frac{dT_1}{dt} = \frac{-hA}{m_1 c L}\left(T_1 - T_{o2}\right)$$

Espaço de Estados.



Sistema c/ realimentação negativa.



$$m_1 c \frac{dT_1}{dt} = -q$$

$$q = \frac{kA}{L}(T_1 - T_2)$$

$$m_2 c \frac{dT_2}{dt} = q$$

$$\begin{cases} m_1 c \dfrac{dT_1}{dt} = -\dfrac{kA}{L}(T_1 - T_2) \\ m_2 c \dfrac{dT_2}{dt} = +\dfrac{kA}{L}(T_1 - T_2) \end{cases}$$

Sistema de duas Equ. D.f. Ord. Lineares acopladas.

$$m_1 c \frac{dT_1}{dt} + \frac{hA}{L} T_1 - \frac{hA}{L} T_2 = 0$$

$$m_2 c \frac{dT_2}{dt} - \frac{hA}{L} T_1 + \frac{hA}{L} T_2 = 0$$

$$T_1(t) = C_1 e^{-\alpha t} \Longrightarrow \frac{dT_1}{dt} = -\alpha C_1 e^{-\alpha t}$$

$$T_2(t) = C_2 e^{-\alpha t} \Longrightarrow \frac{dT_2}{dt} = -\alpha C_2 e^{-\alpha t}$$

$$\begin{cases} \left[\left(\frac{hA}{L} - m_1 c \alpha\right) C_1 - \frac{hA}{L} C_2\right] e^{-\alpha t} = 0 \\ \left[-\frac{hA}{L} C_1 + \left(\frac{hA}{L} - m_2 c \alpha\right) C_2\right] e^{-\alpha t} = 0 \end{cases}$$

$$e^{-\alpha t} \neq 0 \qquad \alpha > 0$$

$$\begin{bmatrix} \left(\frac{kA}{L} - m_1 c\alpha\right) & -\frac{kA}{L} \\ -\frac{kA}{L} & \left(\frac{kA}{L} - m_2 c\alpha\right) \end{bmatrix} \begin{Bmatrix} C_1 \\ C_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Solução Trivial : $C_1 = C_2 = 0 \mid \det(M) \neq 0$

$\det(M) = 0 \implies$ sistema fica indeterminado $\to$ infinitas soluções

$$\det(M) = \left(\frac{kA}{L} - m_1 c\alpha\right)\left(\frac{kA}{L} - m_2 c\alpha\right) - \left(\frac{kA}{L}\right)^2 = 0$$

α_1, α_2 como solução

$$\downarrow \qquad \downarrow$$

$$\tau_1 = \frac{1}{\alpha_1} \qquad \tau_2 = \frac{1}{\alpha_2}$$

$\alpha = \alpha_1$

$$\begin{bmatrix} \left(\frac{kA}{L} - m_1 c \alpha_1\right) & -\frac{kA}{L} \\ -\frac{kA}{L} & \left(\frac{kA}{L} - m_1 c \alpha_1\right) \end{bmatrix} \begin{Bmatrix} C_{11} \\ C_{21} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\left(\frac{kA}{L} - m_1 c \alpha_1\right) C_{11} = \frac{kA}{L} C_{21}$$

$\alpha = \alpha_2$

$$\left(\frac{kA}{L} - m_2 c \alpha_2\right) C_{12} = \frac{kA}{L} C_{22}$$

$$\boxed{\begin{aligned} T_1(t) &= C_{11}\, e^{-t/\tau_1} + C_{12}\, e^{-t/\tau_2} \\ T_2(t) &= C_{21}\, e^{-t/\tau_1} + C_{22}\, e^{-t/\tau_2} \end{aligned}}$$

Solução geral do sistema

$T_1(0) = T_{01}$

$T_2(0) = T_{02}$

$$\frac{dT_1}{dt} = -\frac{hA}{m_1 c_L}(T_1 - T_2)$$

$$\frac{dT_2}{dt} = +\frac{hA}{m_2 c_L}(T_1 - T_2)$$

$$Y = \begin{Bmatrix} T_1 \\ T_2 \end{Bmatrix} \Rightarrow \quad \dot{Y} = \frac{dY}{dt} = \begin{Bmatrix} \frac{dT_1}{dt} \\ \frac{dT_2}{dt} \end{Bmatrix}$$

$$\begin{Bmatrix} \frac{dT_1}{dt} \\ \frac{dT_2}{dt} \end{Bmatrix} = \overbrace{\begin{bmatrix} -\frac{hA}{m_1 c_L} & +\frac{hA}{m_1 c_L} \\ +\frac{hA}{m_2 c_L} & -\frac{hA}{m_2 c_L} \end{bmatrix}}^{A} \begin{Bmatrix} T_1 \\ T_2 \end{Bmatrix}$$

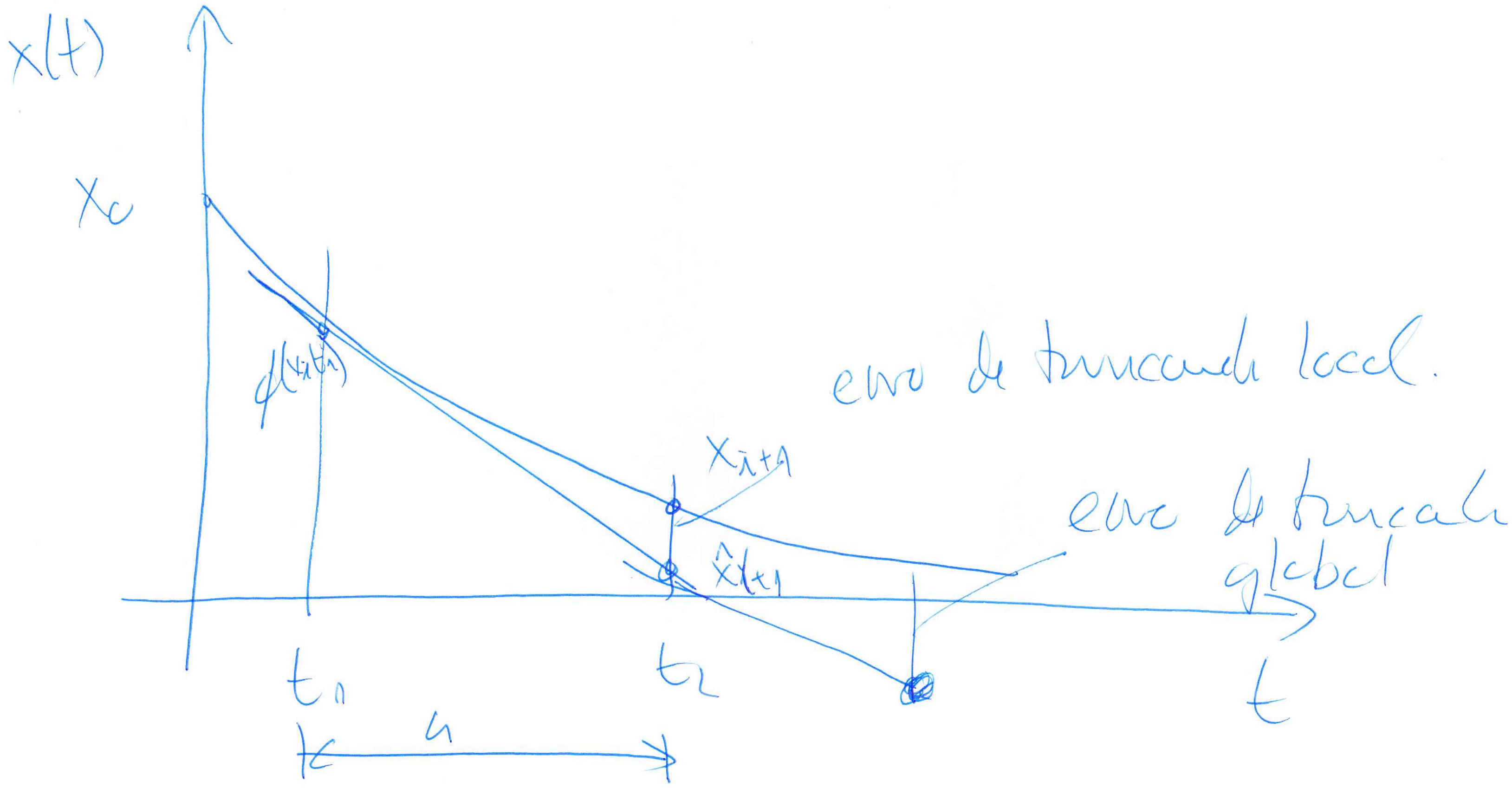
$$\boxed{\dot{Y} = A\,Y} \qquad A = \frac{hA}{c_L}\begin{bmatrix} -1/m_1 & +1/m_1 \\ +1/m_2 & -1/m_2 \end{bmatrix}$$

Erros de Truncamento e Arredondamento:

Arredondamento:

Real → 64 bits

↳ nº de operações.



Método de Euler é um método de 1ª ordem

$x(t)$ - solução exata da equação diferencial

$\hat{x}(t)$ - solução numérica

Série de Taylor:

$$x(t) = x(t_1) + \left.\frac{dx}{dt}\right|_{t_1}(t-t_1) + \frac{1}{2!}\left.\frac{d^2x}{dt^2}\right|_{t_1}(t-t_1)^2 + \ldots$$

$$x_{i+1} = x_i + \phi_i \cdot h$$

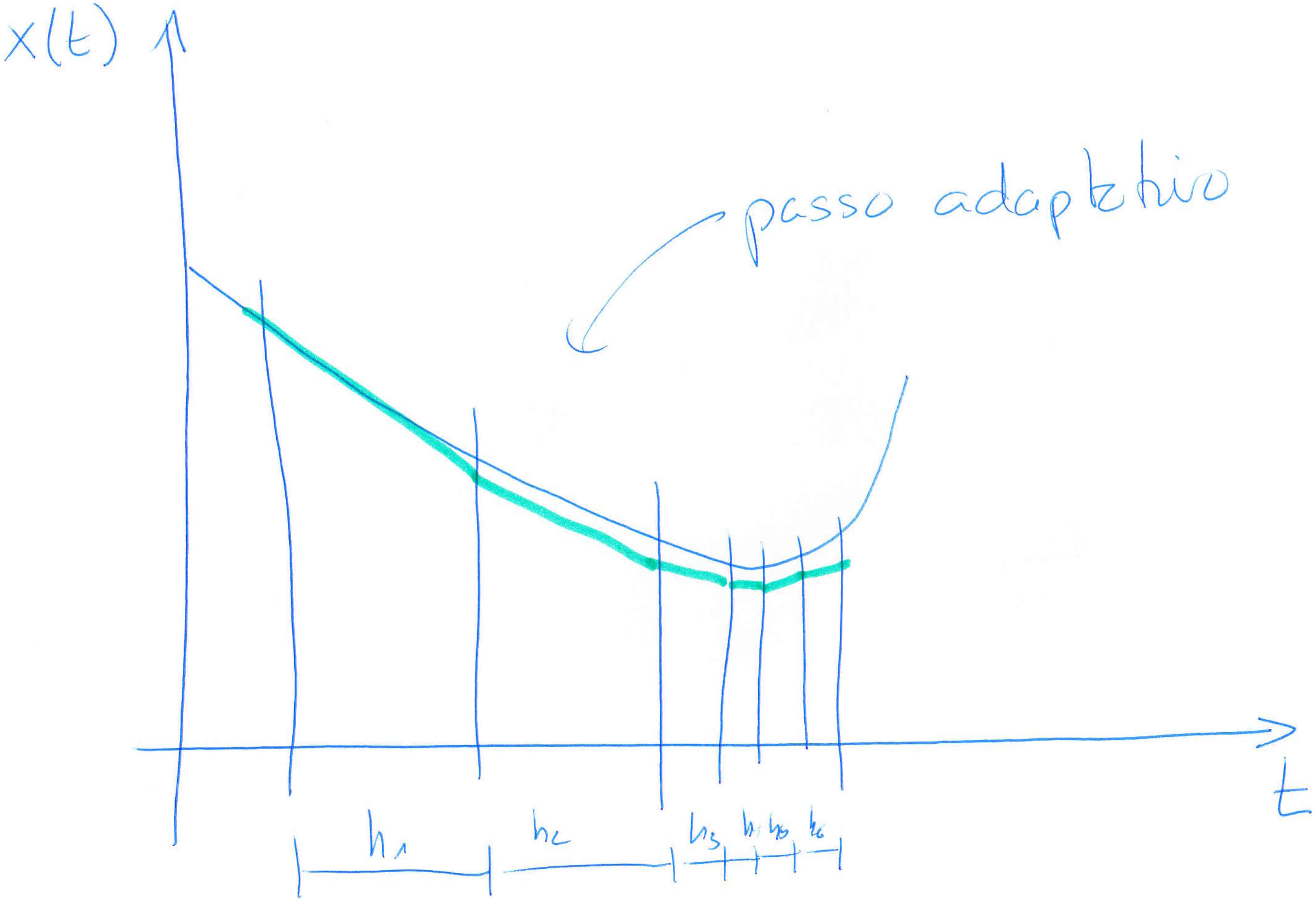
Euler $\phi_i(x_i, t_i)$

Adam

R-K $\phi_i = \frac{\phi_1 + \phi_2}{2}$

$$\frac{d^2}{dt^2} \simeq \frac{d/dt|_2 - d/dt|_1}{2}$$

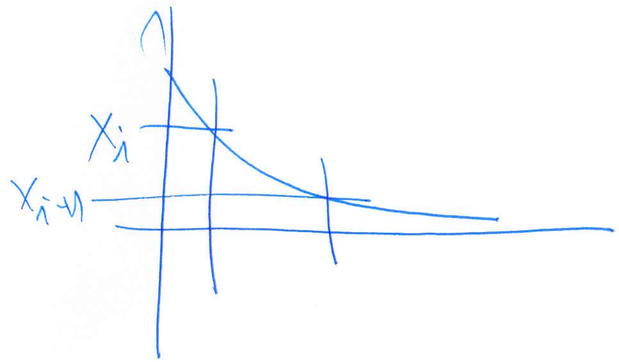
$$\phi_1 = \left.\frac{dx}{dt}\right|_{t_i} \quad \phi_2 = \left.\frac{dx}{dt}\right|_{t_{i+1}}$$



sistemas rígidos:
constantes de tempo muito diferentes!

Euler explícito: $\phi(x,t) = \dfrac{dx}{dt} = -\alpha x \quad | \ \alpha > 0$

$$x_{i+1} = x_i + \phi(x_i, t_i) \cdot h$$



$$x_{i+1} = x_i - \alpha x_i \cdot h$$

$$x_{i+1} = (1 - \alpha h)\, x_i$$

$$|x_{i+1}| < |x_i| \Rightarrow |1 - \alpha h| < 1$$

convergência da solução numér.

$$\begin{cases} 1 - \alpha h < +1 \Rightarrow \alpha h > 0 \Rightarrow h > 0 \\ 1 - \alpha h < -1 \Rightarrow \alpha h < 2 \Rightarrow h < \dfrac{2}{\alpha} \Rightarrow h < 2\tau \end{cases}$$

Euler Implícito!

$$\phi = \frac{dx}{dt} = -\alpha x \quad \Big| \quad \alpha > 0$$

$$x_{i+1} = x_i + \phi(x_{i+1}, t_{i+1}) \cdot h$$

$$x_{i+1} = x_i - \alpha h \, x_{i+1}$$

$$(1 + \alpha h)\, x_{i+1} = x_i$$

$$|x_{i+1}| < |x_i|$$

siempre estable.

$$|1 + \alpha h| > 1$$

$$\forall h \in \mathbb{R} \,|\, h > 0 \Rightarrow \underline{|1 + \alpha h| > 1}$$

$$f(x,t) = \frac{dx}{dt} = -\alpha x$$

$\cancel{f'(x,t)}$ $$x(t) = x_0 e^{-\alpha t} \Big|_{t_i}$$

$$\frac{dx}{dt} = -\alpha x_0 e^{-\alpha t} \Big|_{t_i}$$

$$\frac{d^2x}{dt^2} = +\alpha^2 x_0 e^{-\alpha t} \Big|_{t_i}$$

$$\frac{d^3x}{dt^3} = -\alpha^3 x_0 e^{-\alpha t} \Big|_{t_i}$$

$$x(t) = x(t_i) + \frac{dx}{dt}\Big|_{t_i} (t-t_i) + \frac{1}{2!} \frac{d^2x}{dt^2}\Big|_{t_i} (t-t_i)^2 + \frac{1}{3!} \frac{d^3x}{dt^3}\Big|_{t_i} (t-t_i)^3$$

$$= x_i - \alpha x_i (t-t_i) + \frac{1}{2!} \alpha^2 x_i (t-t_i)^2 + \frac{1}{3!} \alpha^3 x_i (t-t_i)^3$$

$$x_{i+1} = x_i - \alpha x_i h + \frac{1}{2} \alpha^2 x_i h^2 + \frac{1}{6} \alpha^3 x_i h^3 + \dots$$

$$x_{i+1} = x_i + (-\alpha x_i) h$$ 1: order → Euler

$$x_{i+1} = x_i + \frac{(-\alpha x_i) + (x_i - \alpha x_i h)(-\alpha)}{2} h$$

$\cancel{\frac{-\alpha x_i - \alpha x_i + \alpha^2 x_i}{2}}$ $\wedge$ $$\frac{-\alpha x_i - \alpha x_i + \alpha^2 x_i h}{2}$$

$$-\alpha x_i + \frac{\alpha^2 x_i h}{2}$$