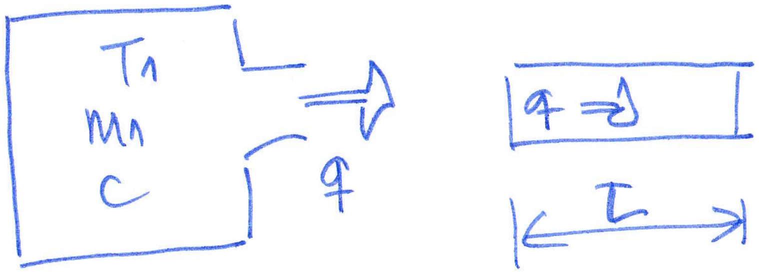
PME3201 Aula 2 Turma 12 17.09.2020

$$q = \mathring{Q} = \frac{dQ}{dt}$$



$T_2 = T_{02}$ pois $m_2 >> m_1$

$$\begin{cases} m_1 c \dfrac{dT_1}{dt} = -q \\ q = \dfrac{kA}{L}(T_1 - T_2) \end{cases}$$

$m_1 = 200$ kg

$c = 0{,}466 \frac{J}{g\,K}$

$T_{01} = 800$ K

$T_{02} = 300$ K $\quad m_2 >> m_1$

$$m_1 c \frac{dT_1}{dt} = -\frac{kA}{L}(T_1 - T_{02})$$

$k = 400 \frac{W}{m\,K}$

$A = 8000$ mm^2

$$\frac{dT_1}{dt} = -\frac{kA}{m_1 c L}(T_1 - T_{02}) \quad //$$

$L = 0{,}5$ m

$$m_1 c \frac{dT_1}{dt} = -q$$

$$q = \frac{kA}{L}(T_1 - T_{02})$$

$$m_1 c \frac{dT_1}{dt} = -\frac{kA}{L}(T_1 - T_{02})$$

$$m_1 c \frac{dT_1}{dt} + \frac{kA}{L} T_1 = \frac{kA}{L} T_{02}$$

Equação Diferencial Ordinária Linear não homogênea

$$T_1(t) = T_{1h}(t) + T_{1p}(t)$$

solução particular (forçada)

solução homogênea (transitório)

$$m_1 c \frac{dT_{1h}}{dt} + \frac{kA}{L} T_{1h} = 0$$

$$T_{1h} = C_1 e^{-\lambda t} \Rightarrow \frac{dT_{1h}}{dt} = -\lambda C_1 e^{-\lambda t}$$

$$\left[-m_1 c \lambda + \frac{kA}{L}\right] C_1 e^{-\lambda t} = 0$$

$$C_1 \neq 0 \qquad e^{-\lambda t} \neq 0$$

$$\frac{kA}{L} - m_1 c \lambda = 0 \Rightarrow \lambda = \frac{kA}{m_1 c L} \;//$$

$$\tau = \frac{1}{\lambda} = \frac{m_1 c L}{kA} \geqslant 0$$

$$\boxed{T_{1h} = C_1 e^{-t/\tau}} \quad \Bigg| \quad \tau = \frac{m_1 c L}{kA} \quad \text{constante de tempo}$$

$$\forall C_1 \in \mathbb{R}$$

Constante de tempo

$$\tau = \frac{m_1 c L}{k A}$$

$$[\tau] = \frac{\cancel{kg}\, J}{\cancel{kg}\, \cancel{K}} \frac{\cancel{m}}{W} \frac{\cancel{m}\,\cancel{K}}{\cancel{m^2}} = \frac{J}{W} = \frac{s}{\cancel{J}} \cdot \cancel{J} = s$$

$$T_{1h}(t) = C_1 e^{-t/\tau} \quad \Big| \quad \tau = \frac{m_1 c L}{k A}$$

Solução homogénea.

$$m_1 c \frac{dT_{1p}}{dt} + \frac{kA}{L} \cdot T_{1p} = \frac{kA}{L} T_{02} = cte$$

$$T_{1p} = C_2 \Rightarrow \frac{dT_{1p}}{dt} = 0$$

$$\frac{kA}{\cancel{L}} \cdot C_2 = \frac{\cancel{kA}}{\cancel{L}} T_{02} \Rightarrow C_2 = T_{02} \, //$$

$$\therefore \quad T_{1p}(t) = T_{02} \, //$$

Solução particular.

Solução Geral:

$$T_1(t) = T_{1p} + T_{1h}$$

$$\boxed{T_1(t) = C_1 e^{-t/\tau} + T_{02}}$$

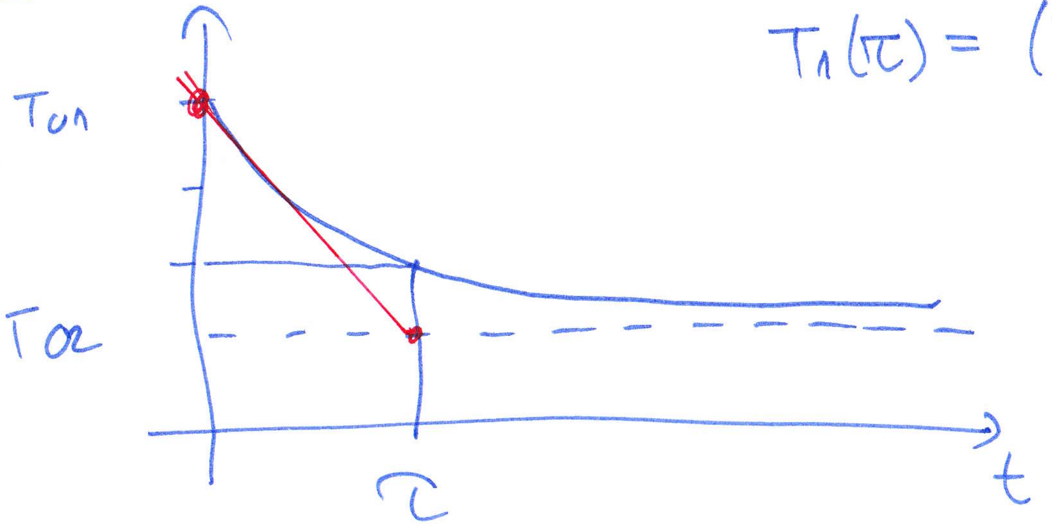
Solução Geral da E.D.O.

$T_1(0) = T_{01}$

$T_1(0) = C_1 e^{0} + T_{02} \Rightarrow C_1 = T_{01} - T_{02}$ //

$$\therefore \boxed{T_1(t) = (T_{01} - T_{02})\, e^{-t/\tau} + T_{02}}$$

$T_1(\tau) = (T_{01} - T_{02}) \cdot 0{,}3679 + T_{02}$



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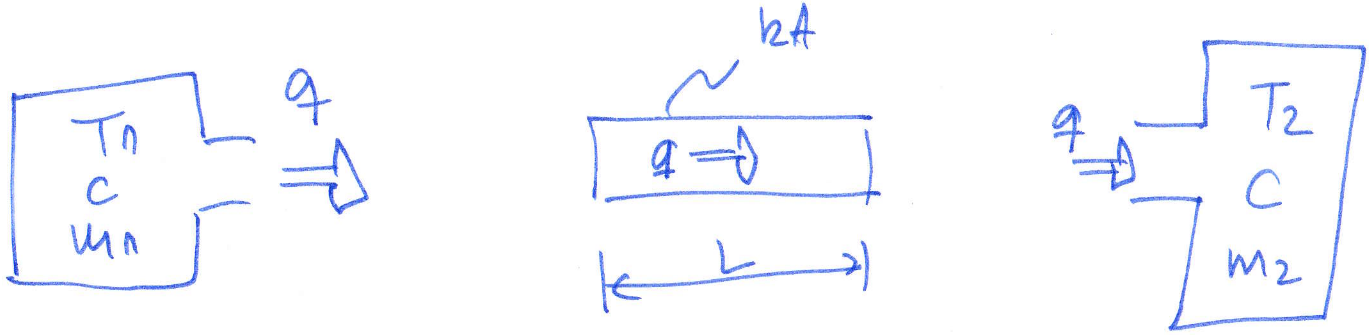


$$m_1 c \frac{dT_1}{dt} = -q$$

$$q = \frac{kA}{L}(T_1 - T_2)$$

$$m_2 c \frac{dT_2}{dt} = +q$$

$$\begin{cases} m_1 c \dfrac{dT_1}{dt} + \dfrac{kA}{L} T_1 - \dfrac{kA}{L} T_2 = 0 \\ m_2 c \dfrac{dT_2}{dt} - \dfrac{kA}{L} T_1 + \dfrac{kA}{L} T_2 = 0 \end{cases}$$

Sistema de 2 E.D.O homogéneas.

$T_1(t) = ?$

$T_2(t) = ?$

$$T_1(t) = C_1 e^{-\lambda t}$$

$$T_2(t) = C_2 e^{-\lambda t}$$

$$m_1 c \frac{dT_1}{dt} + \frac{kA}{L} T_1 - \frac{kA}{L} T_2 = 0$$

$$m_2 c \frac{dT_2}{dt} - \frac{kA}{L} T_1 + \frac{kA}{L} T_2 = 0$$

$$\left[\frac{kA}{L} - m_1 c \lambda\right] C_1 e^{-\lambda t} - \frac{kA}{L} C_2 e^{-\lambda t} = 0$$

$$-\frac{kA}{L} C_1 e^{-\lambda t} + \left[\frac{kA}{L} - m_2 c \lambda\right] C_2 e^{-\lambda t} = 0$$

$$e^{-\lambda t} \neq 0 \mid \forall t \neq \infty$$

$$\begin{bmatrix} \left(\frac{kA}{L} - m_1 c \lambda\right) & -\frac{kA}{L} \\ -\frac{kA}{L} & \left(\frac{kA}{L} - m_2 c \lambda\right) \end{bmatrix} \begin{Bmatrix} C_1 \\ C_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\det \begin{pmatrix} \frac{kA}{L} - m_1 c \lambda & -\frac{kA}{L} \\ -\frac{kA}{L} & \frac{kA}{L} - m_2 c \lambda \end{pmatrix} = 0$$

polinómio característico 2º grau $\Rightarrow \lambda_1, \lambda_2$

$$\Downarrow$$

$$\tau_1, \tau_2$$

$\lambda = \lambda_1$

$$\begin{bmatrix} (kA/L - m_1 c \lambda_1) & -\frac{kA}{L} \\ -\frac{kA}{L} & \left(\frac{kA}{L} - m_2 c \lambda_1\right) \end{bmatrix} \begin{Bmatrix} C_1 \\ C_2 \end{Bmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\left(\frac{kA}{L} - m_1 c \lambda_1\right) C_{11} = \frac{kA}{L} C_{21}$$

$\lambda = \lambda_2 \Rightarrow$ $\left(\frac{kA}{L} - m_1 c \lambda_2\right) C_{12} = \frac{kA}{L} C_{22}$

Solução Geral:

$$\boxed{\begin{aligned} T_1(t) &= C_{11}\, e^{-t/\tau_1} + C_{12}\, e^{-t/\tau_2} \\ T_2(t) &= C_{21}\, e^{-t/\tau_1} + C_{22}\, e^{-t/\tau_2} \end{aligned}}$$

$$\tau_1 = \frac{1}{\lambda_1} \qquad \tau_2 = \frac{1}{\lambda_2}$$

$$\left(\frac{hA}{L} - m_1 c \lambda_1\right) C_{11} = \frac{hA}{L}\, C_{21}$$

$$\left(\frac{hA}{L} - m_1 c \lambda_2\right) C_{12} = \frac{hA}{L}\, C_{22}$$

$$T_1(0) = T_{01}, \quad T_2(0) = T_{02} \quad \rightarrow \quad \begin{aligned} T_{01} &= C_{11} + C_{12} \\ T_{02} &= C_{21} + C_{22} \end{aligned}$$

Erros de Truncamento e Arredondamento:

- Erro de Arredondamento:

  resolução finita do computador digital

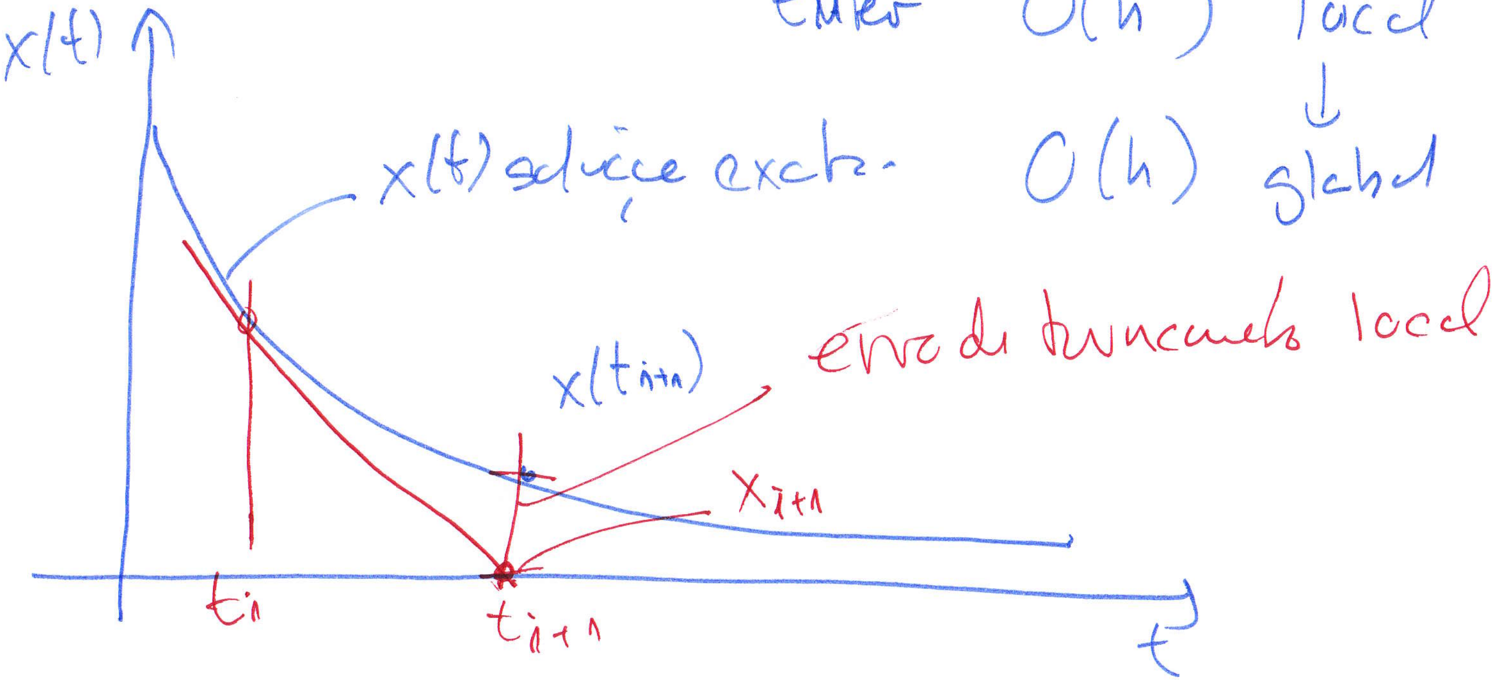
  64 bits – 14 bits → 14 alg. sign.

- Erro de Truncamento:

Euler $O(h^2)$ local

$\downarrow$

$O(h)$ global



Serie de Taylor da Solução!

$$x(t) = x(t_i) + \left.\frac{dx}{dt}\right|_{t_i} \underbrace{(t-t_i)}_{h} + \frac{1}{2!}\left.\frac{d^2x}{dt^2}\right|_{t_i}(t-t_i)^2 + \dots$$

Metodo de Euler:

$$x_{i+1} = x_i + \left.\frac{dx}{dt}\right|_{t_i} \cdot h + O(h^2)$$

erro de truncamento local $O(h^2)$

$\Downarrow$

erro de truncamento global $\to$ $O(h)$

$$e_{i+1} = x_{i+1} - x(t_{i+1})$$

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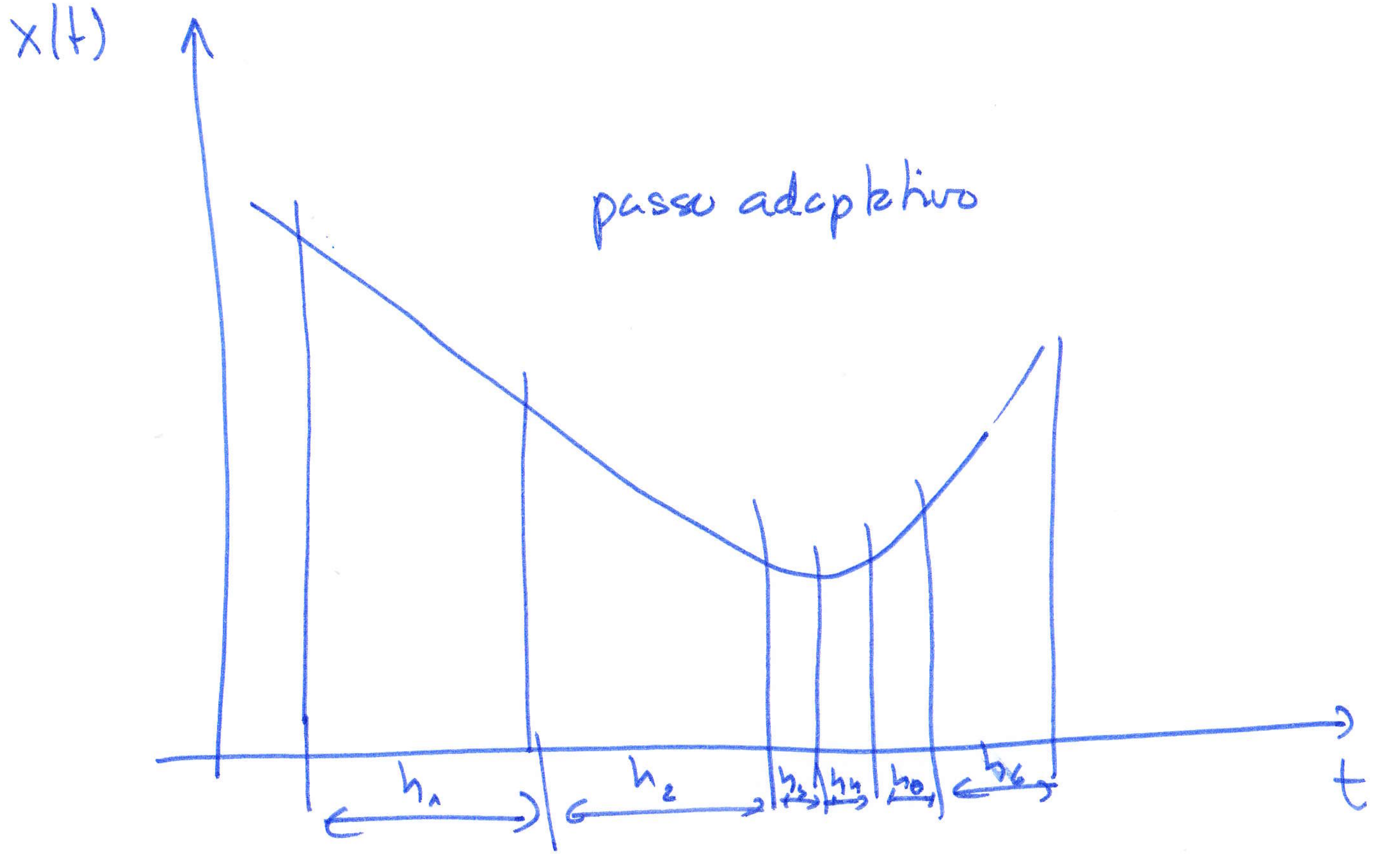
$$\frac{d^2x}{dt^2} = \frac{d}{dt}\left(\frac{dx}{dt}\right)$$

$$\simeq \frac{\Delta}{\Delta t}\left(\frac{dx}{dt}\right) = \frac{\left.\frac{dx}{dt}\right|_{t_{i+1}} - \left.\frac{dx}{dt}\right|_{t_i}}{\Delta t}$$

$$\frac{d^2x}{dt^2} \simeq \frac{\left.\frac{dx}{dt}\right|_{t_{i+1}} - \left.\frac{dx}{dt}\right|_{t_i}}{\Delta t}$$

$$\frac{d^2x}{dt^2} \doteq \frac{\phi(x_{i+1}, t_{i+1}) - \phi(x_i, t_i)}{h}$$



i) queda livre

ii) contato.

$\Downarrow$

Sistemas Rígidos
(Stiff)

τ_1, τ_2 muito diferentes!

Problema p/ integração!

Estabilidade da solução!

$$\frac{dx}{dt} = -\alpha x$$

↳ exp. decrescente:

Euler explícito:

$$x_{i+1} = x_i + \left.\frac{dx}{dt}\right|_{t_i} \cdot h$$

$$x_{i+1} = x_i + (-\alpha x_i) \cdot h$$

$$x_{i+1} = (1 - \alpha h) x_i$$

$$|x_{i+1}| < |x_i| \Rightarrow |1 - \alpha h| < 1$$

$$1 - \alpha h < +1 \Rightarrow \alpha h > 0 \Rightarrow h > 0 \,//$$

$$1 - \alpha h < -1 \Rightarrow \alpha h < 2 \Rightarrow h < \frac{2}{\alpha} \Rightarrow h < 2\tau \,/$$

Euler implicito: $\frac{dx}{dt} = -\alpha x \quad | \ \alpha > 0$

↳ exp. descr.

$$x_{i+1} = x_i + \left.\frac{dx}{dt}\right|_{t_{i+1}} h$$

$$x_{i+1} = x_i - \alpha x_{i+1} \cdot h$$

$$(1 + \alpha h)\, x_{i+1} = x_i$$

$$|x_{i+1}| < |x_i|$$

$$|1 + \alpha h| > 1 \implies \forall h \in \mathbb{R} \mid h > 0$$

//

- Verificação de Modelos

modelo matemático $\longrightarrow$ solução numérica

(S.E.D.O.)

- Validação de Modelos:

mundo real $\longrightarrow$ solução numérica

experimento

&

observação