PME 3201 Aula 2 Turma 11 10,09,2020

$$m_1 c \frac{dT_1}{dt} = -\dot{Q} \qquad \dot{Q} = q = \frac{dQ}{dt}$$

$$\begin{cases} m_1 c \dfrac{dT_1}{dt} = -q \\ q = \dfrac{kA}{L}(T_1 - T_2) \end{cases}$$

$T_2 = cte$ //

(2 é um reservatório térmico)

$$m_1 c \frac{dT_1}{dt} = -q = -\frac{kA}{L}(T_1 - T_2)$$

$$\boxed{m_1 c \frac{dT_1}{dt} + \frac{kA}{L} T_1 = \frac{kA}{L} T_2}$$

$m_2 >> m_1$

$$q = m_2 c \frac{dT_2}{dt}$$

q é finito

$m_2 \longrightarrow \infty \Rightarrow \frac{dT_2}{dt} \rightarrow 0 \Rightarrow T_2 = cte$ //

$$\boxed{m_1 c \frac{dT_1}{dt} + \frac{kA}{L} T_1 = \frac{kA}{L} T_2 = cte}$$

Equação Diferencial Ordinária Linear
não homogênea

i) proporcionalidade T_1 é solução $\longrightarrow$ αT_1 também é solução

ii) superposição T_1' também é solução $\rightarrow$ $T_1 + T_1'$ também é solução

$$\boxed{\frac{dx}{dt} + C X = K}$$

$x(t)$ é solução $\alpha x(t)$

$$\frac{d}{dt}(\alpha x(t)) + C\,(\alpha x(t)) = K$$

$$m_1 c \frac{dT_1}{dt} + \frac{kA}{L} T_1 = \frac{kA}{L} T_2$$

$$T_1(t) = T_{1h}(t) + T_{1p}(t)$$

solução particular

solução homogénea

$$\boxed{m_1 c \frac{dT_{1h}}{dt} + \frac{kA}{L} T_{1h} = 0}$$

$$\boxed{\frac{dT_{1h}}{dt} = -\frac{kA}{m_1 c L} T_{1h}} \Rightarrow T_{1h} = A e^{-\alpha t}$$

$$\boxed{\alpha = \frac{kA}{m_1 c L}}$$

$$\frac{dx}{dt} = \alpha x$$ → equação exponencial

$$T_{1h}(t) = A e^{-\alpha t} \quad \Big| \quad \alpha = \frac{hA}{m_1 c L} > 0$$

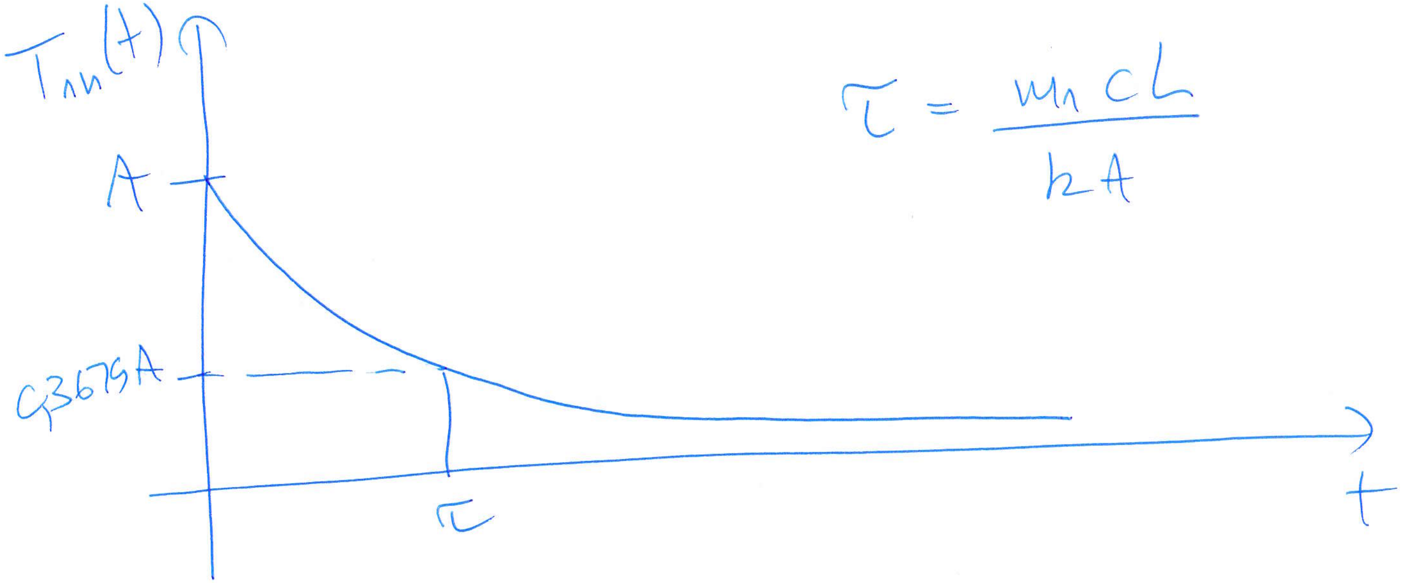
$$\dim \alpha = \frac{1}{T} \qquad [\alpha] = \frac{1}{s}$$

$$[\alpha] = \frac{W \, \cancel{m^2} \, \cancel{kg} \, \cancel{K}}{\cancel{m} \, \cancel{K} \, \cancel{kg} \, J \, \cancel{m}} = \frac{W \cancel{K}}{J} = \frac{\cancel{J}}{s} \cdot \frac{1}{\cancel{J}} = \frac{1}{s}$$

$$\tau = \frac{1}{\alpha} \qquad [\tau] = s$$

τ - constante de tempo do sistema.

$$T_{1h}(t) = A \; e^{-t/\tau}$$



$$\tau = \frac{m_1 c L}{k A}$$

N $\quad t = \tau \Rightarrow T_{1n}(\tau) = ?$

$$T_{1n}(\tau) = A e^{-\tau/\tau} = A e^{-1} = A \cdot 0,3679$$

$$T_{1n}(0) = A$$

$$\frac{T_{1n}(\tau)}{T_{1n}(0)} = \frac{\cancel{A} \cdot 0,3679}{\cancel{A}} = 0,3679 = 36,79\%$$

$$\tau = \frac{200 \cdot 466 \cdot 0,5}{400 \cdot 8000 \cdot 10^{-6}} = 14562,5\,s \;/\!/ = 4h$$

$$m_1 c \frac{dT_{1p}}{dt} + \frac{kA}{L} T_{1p} = \underbrace{\frac{kA}{L} T_2}_{cte}$$

$T_{1p} = ?$

$$T_{1p} = B = cte \Rightarrow \frac{dT_{1p}}{dt} = 0$$

$$\frac{k\cancel{A}}{\cancel{L}} \cdot B = \frac{k\cancel{A}}{\cancel{L}} T_2 \Rightarrow B = T_2 \,//$$

$$T_{1p} = T_2 \,//$$

Solução $\quad T_1 = T_{1h} + T_{1p}$

$$\boxed{T_1(t) = A e^{-t/\tau} + T_2}$$

Solução Geral $\qquad T_1(0) = T_{01}$

$$T_1(t) = A e^{-t/\tau} + T_2$$

$$T_1(0) = T_{01}$$

$$T_{01} = A e^{0} + T_2 \Rightarrow A = (T_{01} - T_2)$$

$$\boxed{T_1(t) = (T_{01} - T_2) e^{-t/\tau} + T_2}$$

$$T_1(t) =$$

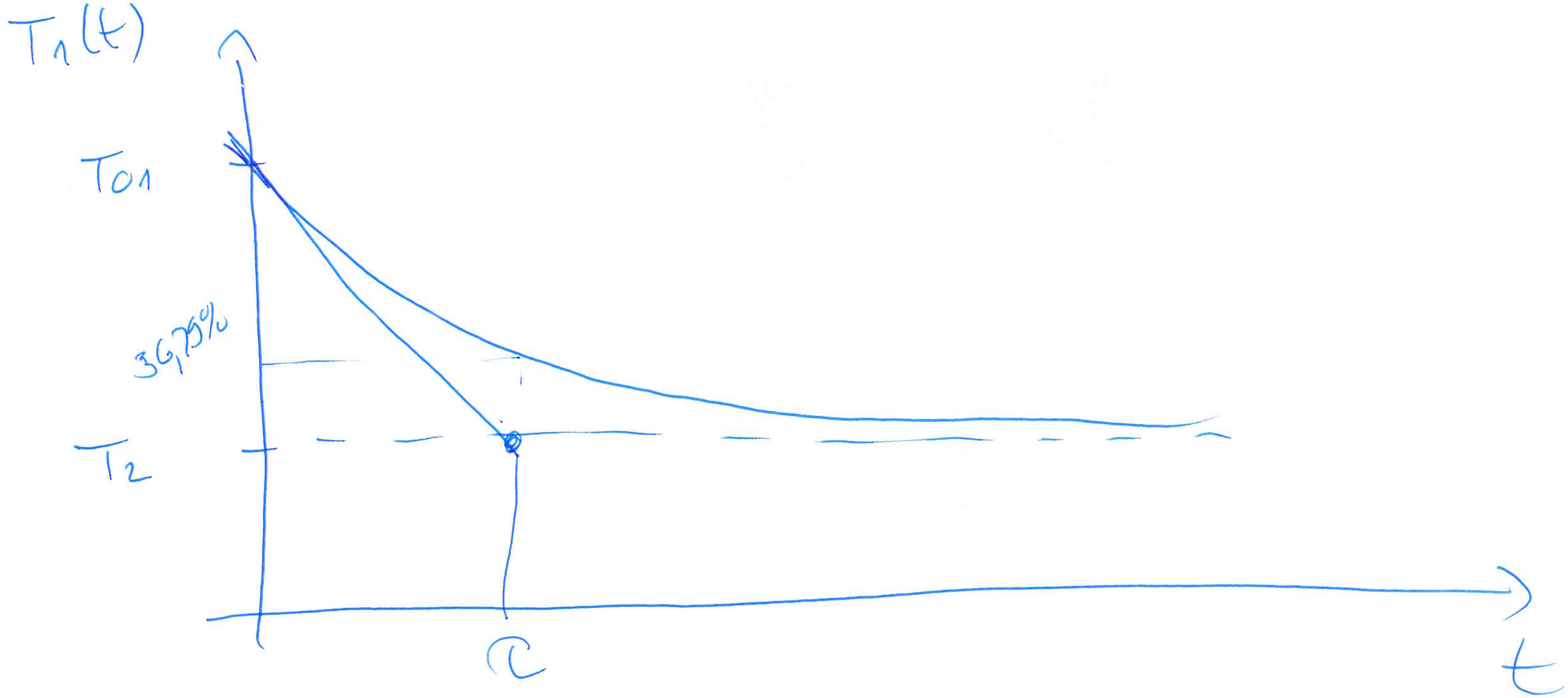
8

$$T_1(t) = (T_{01} - T_2)\, e^{-t/\tau} + T_2$$

$$T_1(0) = T_{01}$$

$$\lim_{t \to +\infty} T_1(t) = \lim_{t \to +\infty} \left[(T_{01} - T_2)\, e^{-t/\tau} + T_2 \right]$$

$$= T_2 \quad //$$



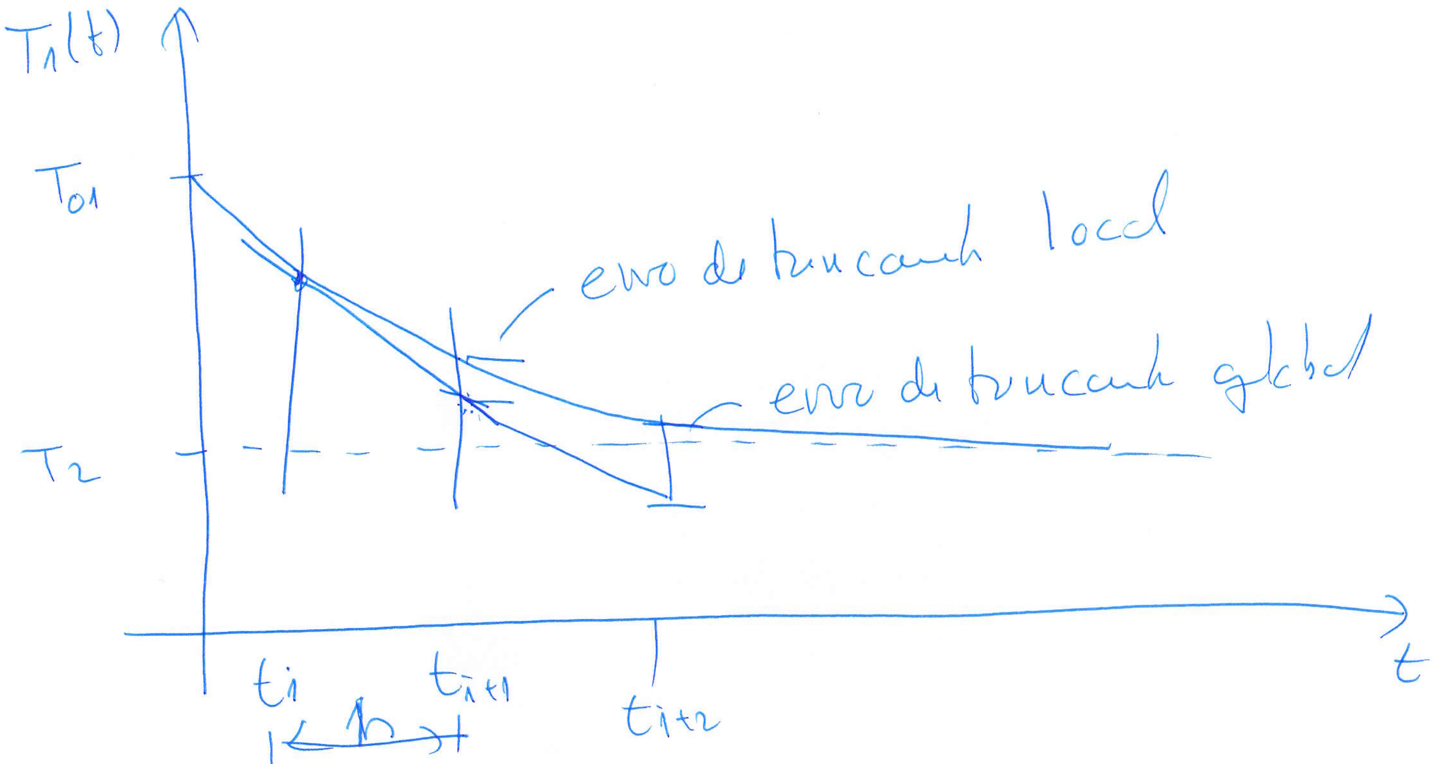
Erros de Truncamento e Arredondamento

Larredondamento no computador

Ltruncamento de Série de Taylor da solução

Real → 64bits //

Erro de Truncamento:



Euler: $X_{i+1} = X_i + \phi(X_i, t_i) \cdot h$

R-K de 1ª ordem

$x(t)$ – solução exata de equação diferencial

$\hat{x}(t)$ – solução numérica.

Série de Taylor:

$$x(t) = x(t_i) + \frac{dx}{dt}\Big|_{t_i} \cdot \underbrace{(t-t_i)}_{h} + \frac{1}{2!}\frac{d^2x}{dt^2}\Big|_{t_i}(t-t_i)^2 + \ldots$$

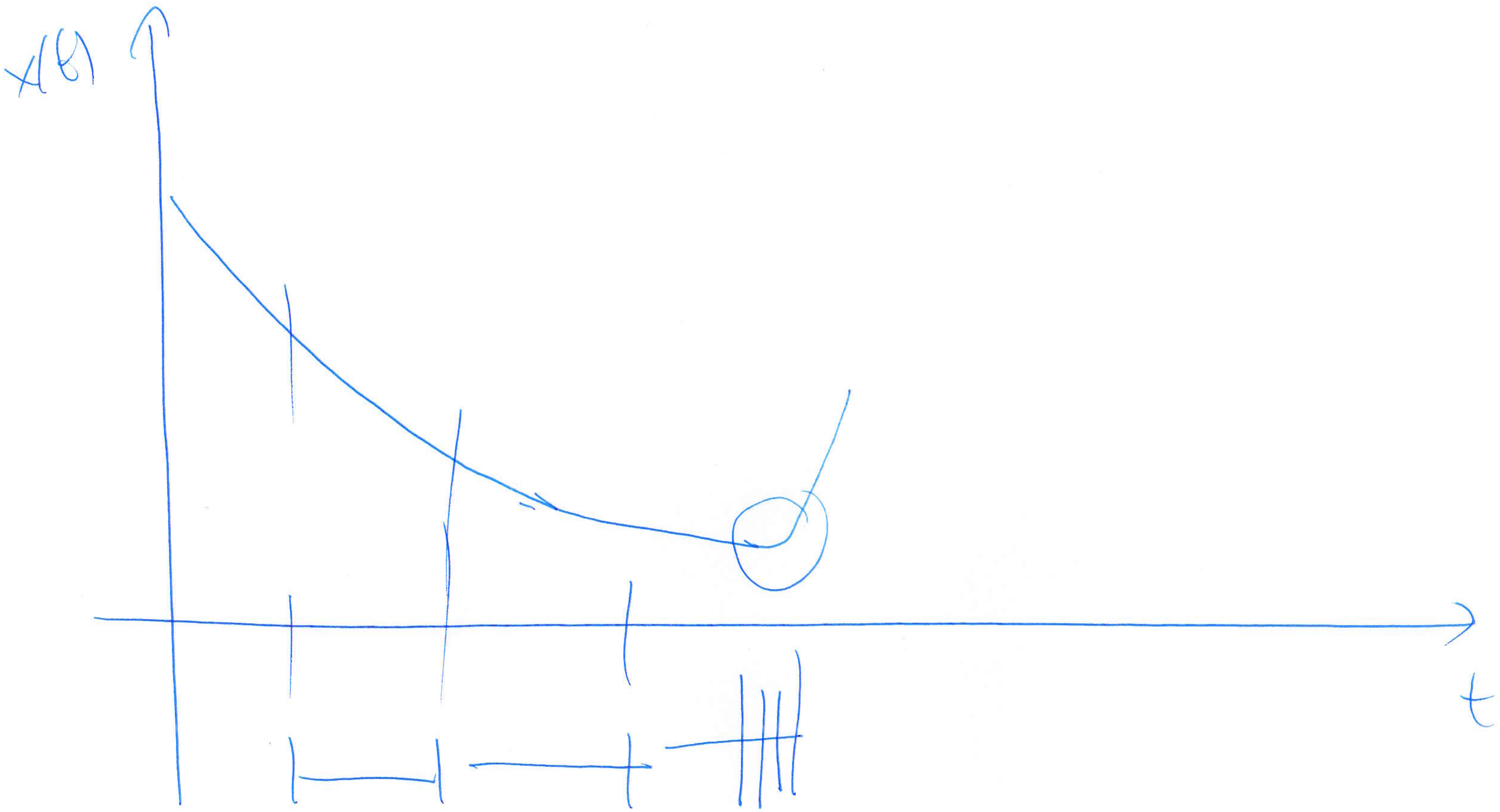
$$x_{i+1} = x_i + \phi_i(t_i, x_i)\cdot h$$ → R-K de 1ª Ordem – Euler

$$\hat{\phi} = \frac{\phi_1+\phi_2}{2} \Leftarrow \qquad \frac{d^2}{dt^2} = \frac{d/dt|_2 - d/dt|_1}{dt}$$

//

Heun
Ponto Médio
R-K 2ª ordem

$$\phi_1 = \frac{dx}{dt}\Big|_{t_i} \qquad \hat{\phi}_2 = \frac{dx}{dt}\Big|_{t_{i+1}}$$


x(t)
t

Euler Explícito! $\quad \phi = \dfrac{dx}{dt} = -\alpha x \quad\quad \alpha > 0$

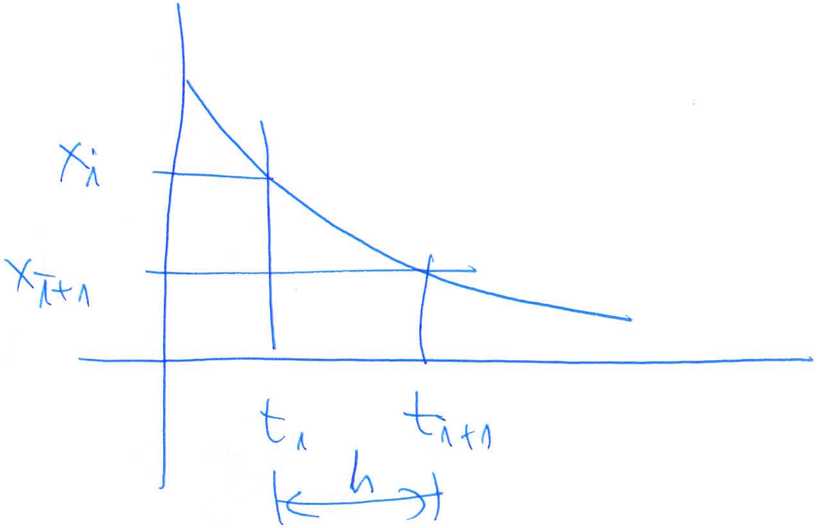
exponencial decreciente

$$x_{i+1} = x_i + \phi(x_i, t_i) \cdot h$$

$$x_{i+1} = x_i - \alpha x_i \cdot h$$

$$x_{i+1} = (1 - \alpha h)\, x_i$$

$$|x_{i+1}| \leq |x_i|$$



$$|1 - \alpha h| \leq 1$$

convergencia

i) $1 - \alpha h < 1 \Rightarrow \alpha h > 0 \Rightarrow h > 0$ //

ii) $1 - \alpha h < -1 \Rightarrow \alpha h < 2 \Rightarrow h < \frac{2}{\alpha} \Rightarrow \boxed{h < 2\tau}$

Euler Implícito: $\phi = \frac{dx}{dt} = -\alpha x$ $\alpha > 0$

expon. decresc.

$$x_{i+1} = x_i + \phi(x_{i+1}, t_{i+1}) \cdot h$$

$$x_{i+1} = x_i - \alpha x_{i+1} \cdot h$$

$$(1 + \alpha h) x_{i+1} = x_i$$

$$x_{i+1} = \frac{1}{1+\alpha h} x_i$$

$$|x_{i+1}| < |x_i|$$

$\alpha h > 0$

$$\left| \frac{1}{1+\alpha h} \right| < 1 \qquad \therefore \left| \frac{1}{1+\alpha h} \right| < 1 \quad \forall h \in \mathbb{R} / h > 0.$$

$$\phi = \frac{dx}{dt} = \alpha x(1-x)$$

$$x_{i+1} = x_i + \phi(x_{i+1}, t_{i+1}) \cdot h$$

$$x_{i+1} = x_i + \alpha x_{i+1}(1 - x_{i+1}) \cdot h$$

$$x_{i+1} = \cdot \; - \; x_1$$

$$(I) \quad m_1 c \frac{dT_1}{dt} = -q$$

$$(II) \quad m_2 c \frac{dT_2}{dt} = +q$$

$$(III) \quad q = \frac{kA}{L}(T_1 - T_2)$$

$T_1(t) = ?$
$T_2(t) = ?$

$$(I) \quad m_1 c \frac{dT_1}{dt} = -\frac{kA}{L}(T_1 - T_2)$$

$$(II) \quad m_2 c \frac{dT_2}{dt} = +\frac{kA}{L}(T_1 - T_2)$$

2 equ. dif. ord. acoplados.

$$\begin{cases} m_1 c \dfrac{dT_1}{dt} + \dfrac{hA}{L} T_1 - \dfrac{hA}{L} T_2 = 0 \\ m_2 c \dfrac{dT_2}{dt} - \dfrac{hA}{L} T_1 + \dfrac{hA}{L} T_2 = 0 \end{cases}$$

$$T_1(t) = C_1 e^{\alpha t} \Rightarrow \frac{dT_1}{dt} = \alpha C_1 e^{\alpha t}$$

$$T_2(t) = C_2 e^{\alpha t} \Rightarrow \frac{dT_2}{dt} = \alpha C_2 e^{\alpha t}$$

$$\begin{cases} m_1 c \alpha C_1 e^{\alpha t} + \dfrac{hA}{L} C_1 e^{\alpha t} - \dfrac{hA}{L} C_2 e^{\alpha t} = 0 \\ m_2 c \alpha C_2 e^{\alpha t} - \dfrac{hA}{L} C_1 e^{\alpha t} + \dfrac{hA}{L} C_2 e^{\alpha t} = 0 \end{cases}$$

$$\left[m_1 c \alpha C_1 + \frac{kA}{L} C_1 - \frac{kA}{L} C_2\right] e^{\alpha t} = 0$$

$$\left[m_2 c \alpha C_2 - \frac{kA}{L} C_1 + \frac{kA}{L} C_2\right] e^{\alpha t} = 0$$

$e^{\alpha t} \neq 0$ solução não trivial

$$\overbrace{\begin{bmatrix} \left(m_1 c \alpha + \frac{kA}{L}\right) & -\frac{kA}{L} \\ -\frac{kA}{L} & \left(m_2 c \alpha + \frac{kA}{L}\right) \end{bmatrix}}^{M} \begin{Bmatrix} C_1 \\ C_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

2 equações e 3 incógnitas.

$\rightarrow \det(M) = 0 \rightarrow$ inf. soluções

$\det M = 0$

$$\det \begin{vmatrix} \left(m_1 c\alpha + \frac{kA}{L}\right) & -\frac{kA}{L} \\ -\frac{kA}{L} & \left(m_2 c\alpha + \frac{kA}{L}\right) \end{vmatrix} = 0 \quad \Rightarrow \alpha \ldots$$

$$\left(m_1 c\alpha + \frac{kA}{L}\right)\left(m_2 c\alpha + \frac{kA}{L}\right) - \left(\frac{kA}{L}\right)^2 = 0$$

$$\alpha_1 \text{ e } \alpha_2 \Leftarrow \quad \tau_1 = \frac{1}{\alpha_1}$$

$$\tau_2 = \frac{1}{\alpha_2}$$

$$\begin{cases} T_1 = C_{11} e^{\alpha_1 t} + C_{21} e^{\alpha_2 t} \\ T_2 = C_{12} e^{\alpha_1 t} + C_{22} e^{\alpha_2 t} \end{cases}$$

$\alpha = \alpha_1$

$$\begin{bmatrix} \left(m_1 c \alpha_1 + \frac{hA}{L}\right) & -\frac{hA}{L} \\ -\frac{hA}{L} & \left(m_2 c \alpha_1 + \frac{hA}{L}\right) \end{bmatrix} \begin{pmatrix} C_{11} \\ C_{12} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\left(m_1 c \alpha + \frac{hA}{L}\right) C_{11} = \frac{hA}{L} C_{12}$$

$\alpha = \alpha_2 \Rightarrow$ C_{21} e C_{22}