PME3201 Turma 21 28.08.2020

$$\frac{dx}{dt} = \alpha x$$

$$\alpha = \frac{75}{1000} \frac{1}{d}$$

$$N = 200 \cdot 10^6$$

$$X_0 = 1$$

$$\frac{dx}{dt} = \alpha x \qquad \alpha \in \mathbb{R}$$

$x(t)$

p/ separação de variáveis !

$$\frac{dx}{dt} = \alpha x \implies \frac{1}{x}\,dx = \alpha\, dt \implies \int_{x_0}^{x} \frac{1}{x}\,dx = \int_{t_0=0}^{t} \alpha\, dt$$

$$\ln x \Big|_{x_0}^{x} = \alpha t \Big|_{0}^{t} \implies \ln\left(\frac{x}{x_0}\right) = \alpha t$$

$$\boxed{x(t) = x_0\, e^{\alpha t}} \qquad x(0) = x_0$$

Solução Geral. condição inicial

u(t) outra possível solução

$\frac{du}{dt} = \alpha u$ //

$u(t) e^{-\alpha t}$

$$\frac{d}{dt}\left(u(t) e^{-\alpha t}\right) = \frac{du}{dt} \cdot e^{-\alpha t} - u \cdot \alpha e^{-\alpha t}$$

$$= \alpha u e^{-\alpha t} - u \alpha e^{-\alpha t}$$

$$= 0$$

$$u(t) e^{-\alpha t} = k \Rightarrow u(t) = k e^{\alpha t}$$ //

3

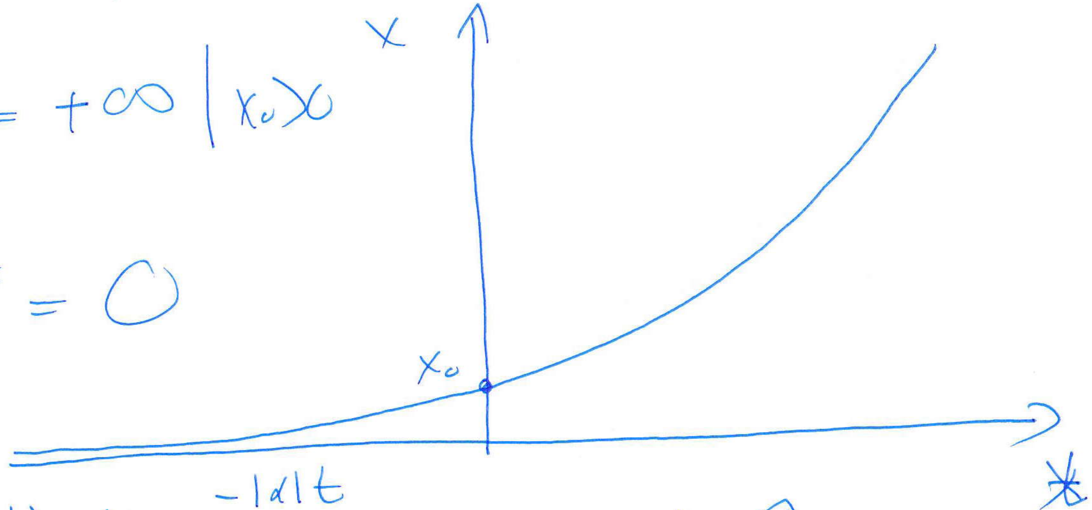
$$\frac{dx}{dt} = \alpha x \implies x(t) = x_0 e^{\alpha t} \quad // \qquad \alpha \in \mathbb{R}$$

i) $\alpha = 0 \implies x(t) = x_0$ // solução constante (inv. no tempo)

ii) $\alpha > 0 \implies x(t) = x_0 e^{\alpha t}$ crescimento exponencial.

$$\lim_{t \to +\infty} x_0 e^{\alpha t} = +\infty \mid x_0 > 0$$

$$\lim_{t \to -\infty} x_0 e^{\alpha t} = 0$$

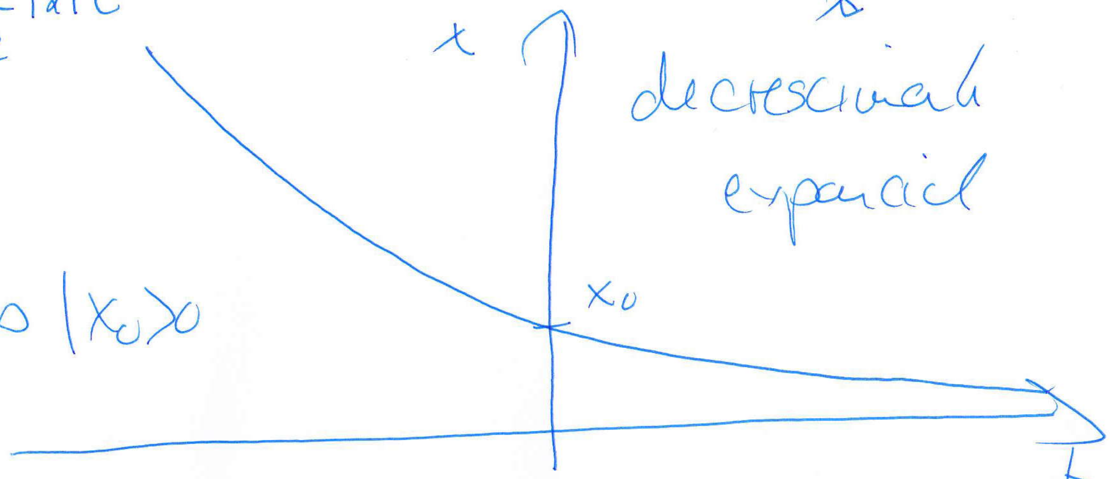


iii) $\alpha < 0 \implies x(t) = x_0 e^{-|\alpha| t}$

$$\lim_{t \to \infty} x_0 e^{-|\alpha| t} = 0$$

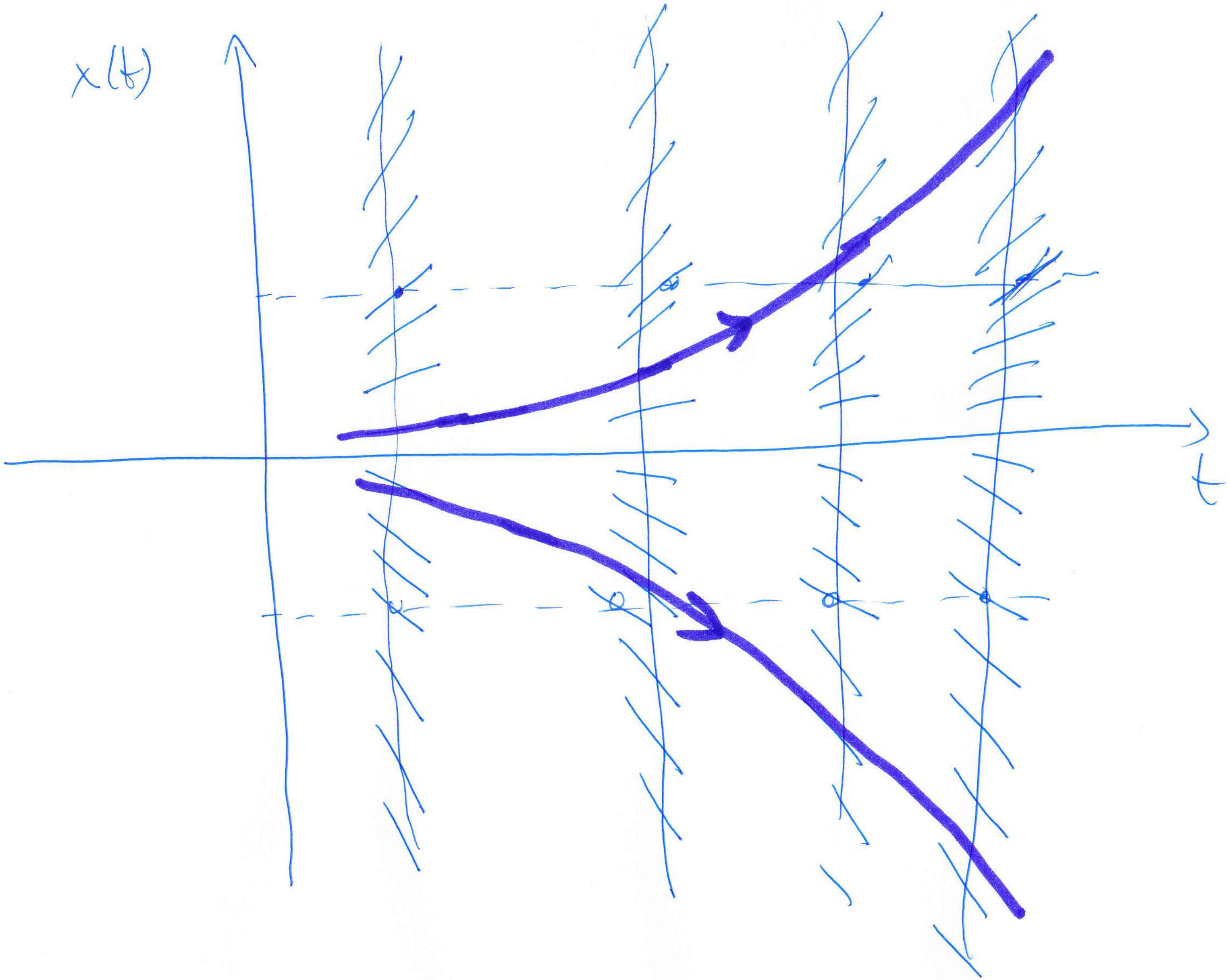
$$\lim_{t \to \infty} x_0 e^{-|\alpha| t} = +\infty \mid x_0 > 0$$

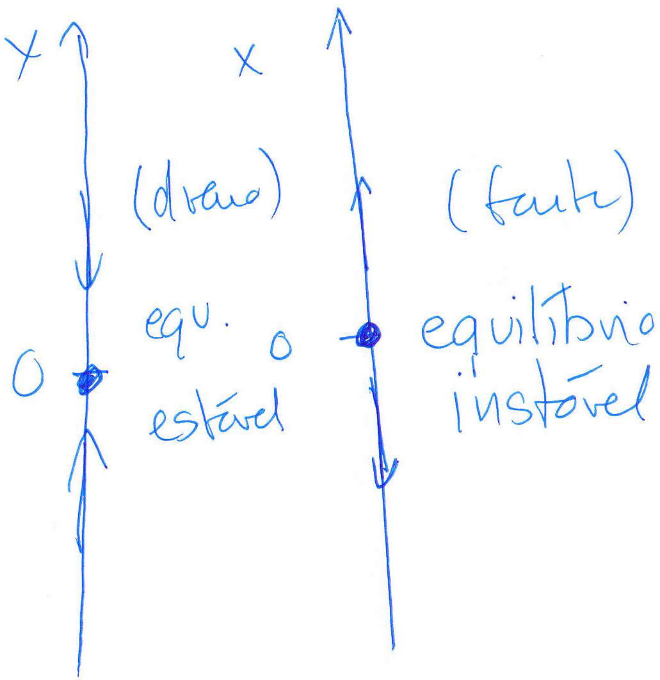
decrescimento exponencial



$$\frac{dx}{dt} = f(x,t) = \alpha x$$ sistema autonomo

$\alpha > 0$





$\alpha < 0$ $\alpha > 0$

linha de fase.

$$\frac{dx}{dt} = \alpha x \Rightarrow f(x,\cancel{t}) = \frac{dx}{dt} = \alpha x$$

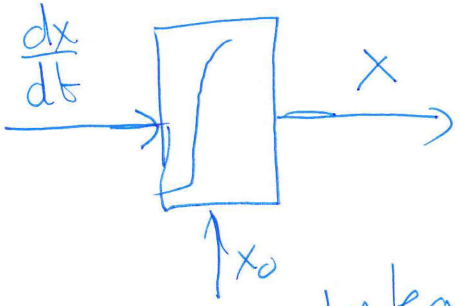
equilíbrio:

$$\frac{dx}{dt} = 0 \Rightarrow \alpha x = 0 \Rightarrow x = 0 \,//$$

$$\frac{d}{dx} f(x) = \frac{d}{dx}\left(\frac{dx}{dt}\right) = \frac{d(\alpha x)}{dx} = \alpha$$

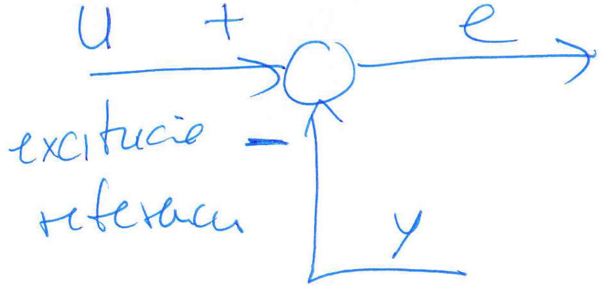
$$\boxed{\frac{dx}{dt} = \alpha x}$$

ODE

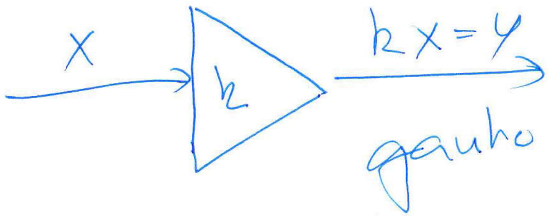


Integrador

$$x = x_0 + \int \frac{dx}{dt}\, dt$$



realimentação

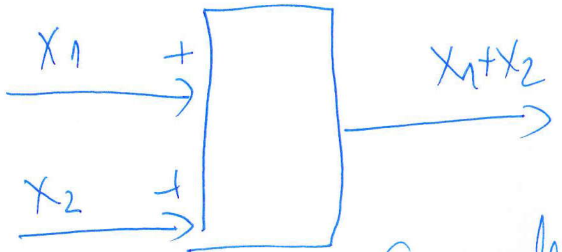


ganho

$$y = kx$$

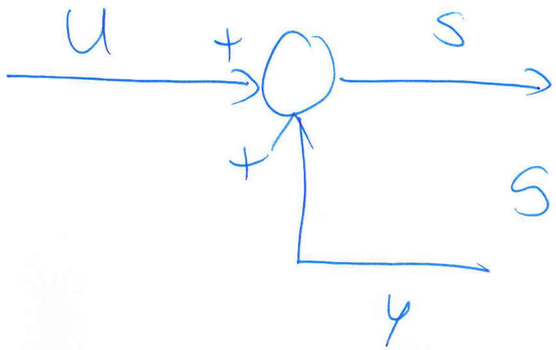
$$e = u - y$$

realimentação negativa



Somador

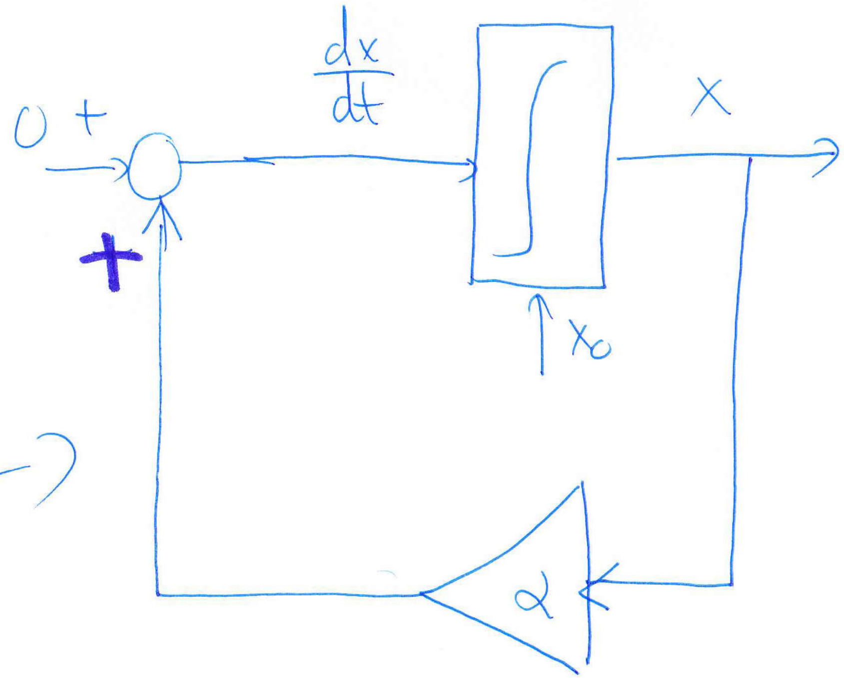
$$y = x_1 + x_2$$



$$s = u + y$$

realimentação positiva

$$\frac{dx}{dt} = \alpha x$$ ← representação algébrica



representação gráfica p/ diagrama de blocos

sistema de realimentação positiva

Integração Numérica:

Método de Euler:

$\Delta t = h$ passo de integração

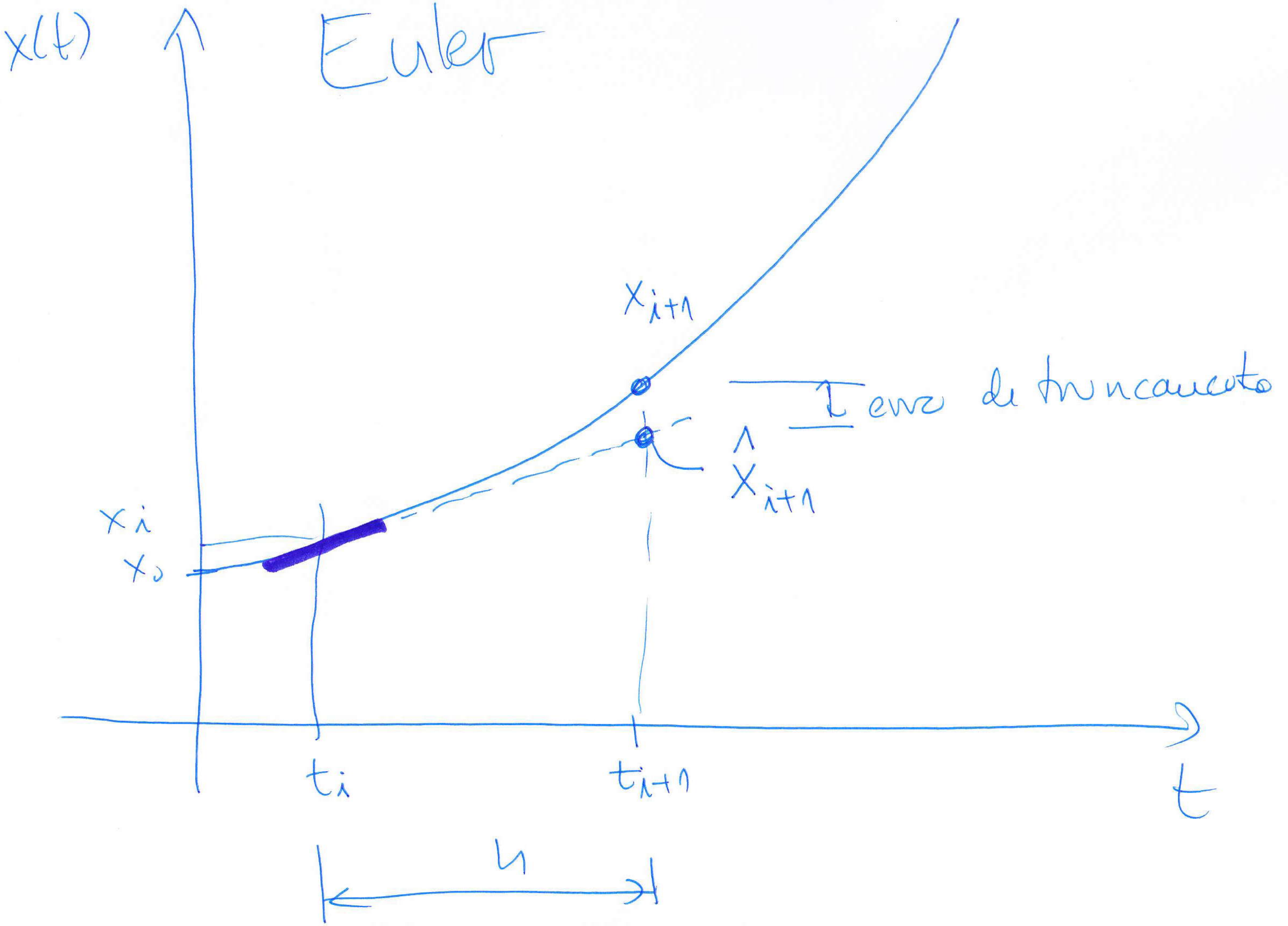
$$\frac{dx}{dt} = f(x,t)$$

$$\frac{\Delta x}{\Delta t} \simeq f(x,t) \Rightarrow x_{i+1} - x_i = f(x,t) \cdot \Delta t$$

$$\boxed{x_{i+1} = x_i + f(x,t) \cdot h}$$

$$\boxed{x_{i+1} = x_i + \phi(x_i, t_i) \cdot h} \leftarrow \text{Método de Euler}$$

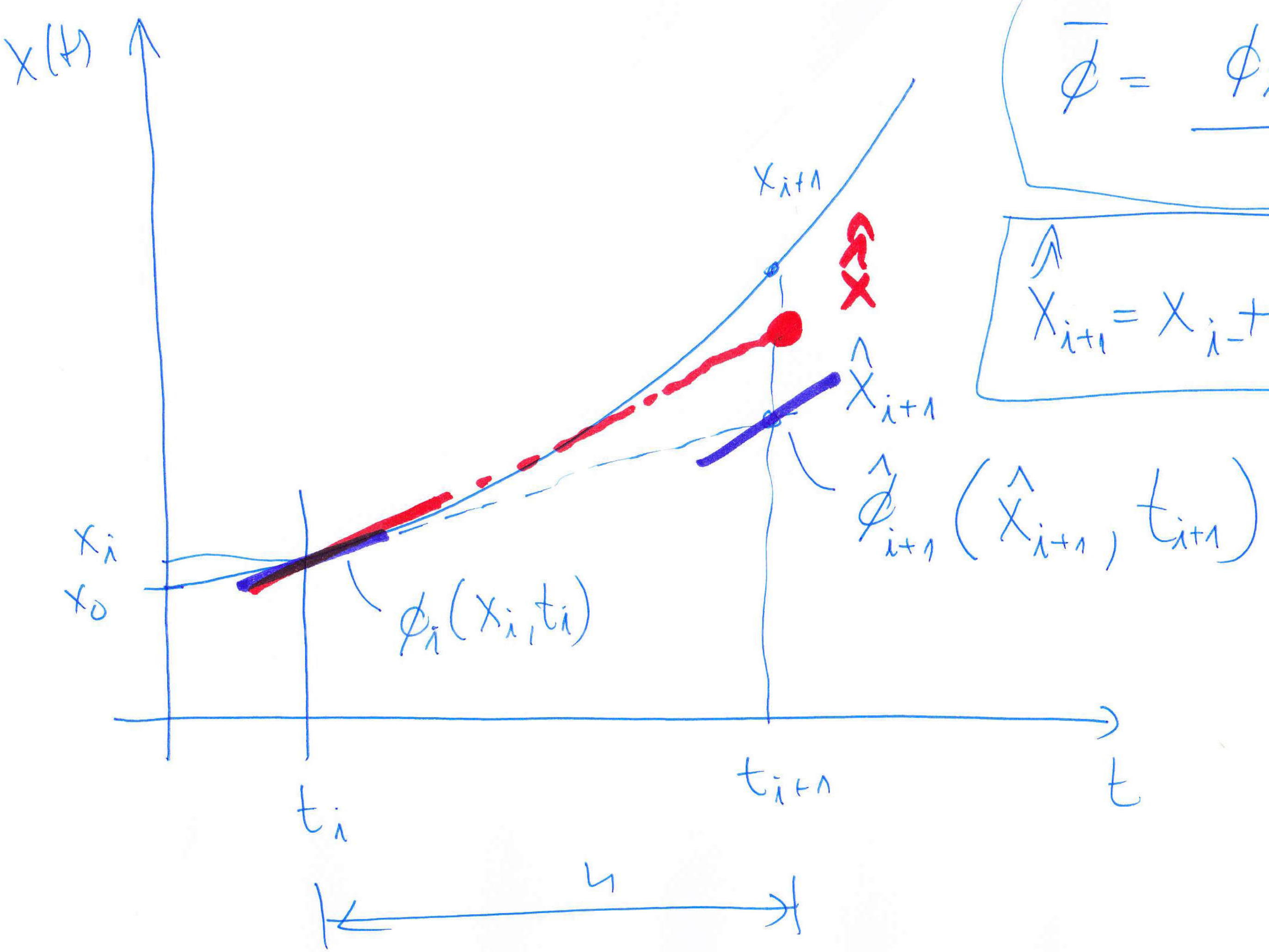
$$t_{i+1} = t_i + h$$



x(t)
Euler
x_{i+1}
errore di troncamento
$\hat{x}_{i+1}$
x_i
x_0
t_i
t_{i+1}
t
h

Heun

$$\phi(x,t) = f(x,t)$$



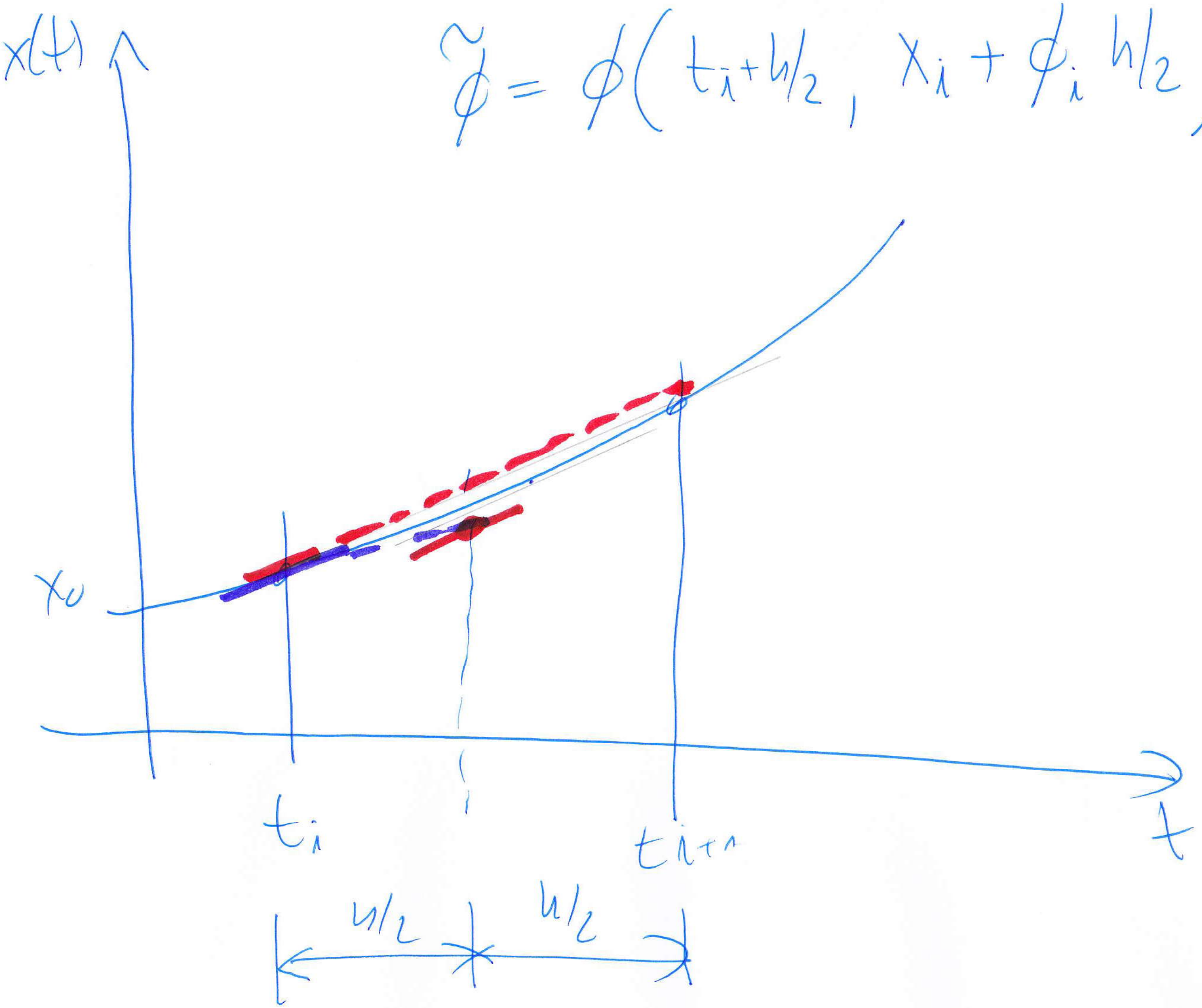
$$\boxed{\bar{\phi} = \frac{\phi_i + \hat{\phi}_{i+1}}{2}}$$

$$\boxed{\hat{x}_{i+1} = x_i + \bar{\phi}\, h}$$

Método do Ponto Médio:

$$\hat{x}_{i+1} = x_i + \tilde{\phi}\, h$$

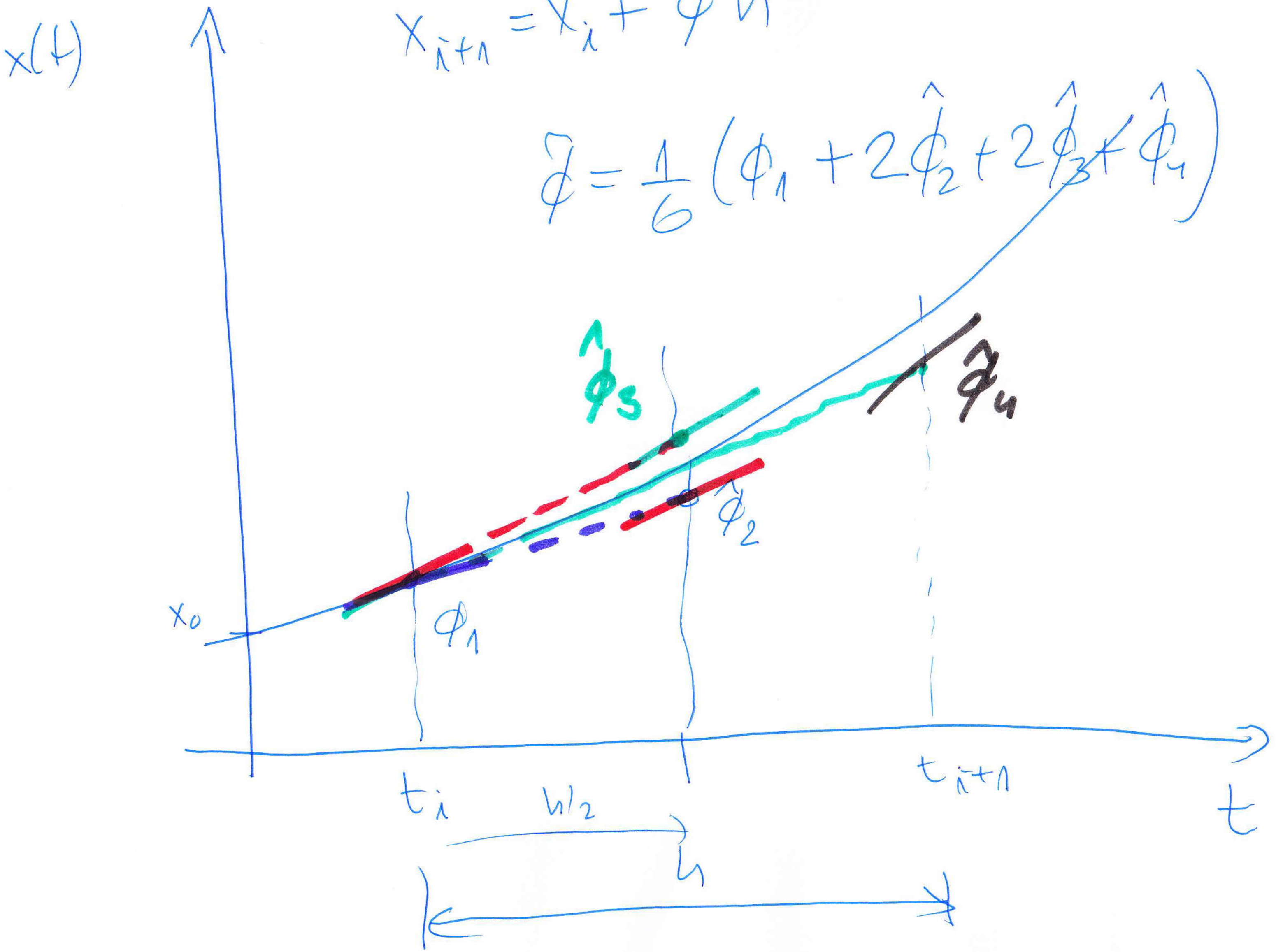
$$\tilde{\phi} = \phi\left(t_i + h/2,\ x_i + \phi_i\, h/2\right)$$



Runge-Kutta:

$$x_{i+1} = x_i + \tilde{\phi} h$$

$$\tilde{\phi} = \frac{1}{6}\left(\phi_1 + 2\hat{\phi}_2 + 2\hat{\phi}_3 + \hat{\phi}_4\right)$$



$$\frac{dx}{dt} = \alpha x \left(1 - \frac{x}{N}\right)$$

$$\boxed{x' = \frac{x}{N}} \Rightarrow \frac{dx'}{dt} = \frac{1}{N} \cdot \frac{dx}{dt} = \frac{1}{\cancel{N}} \alpha \cancel{N} x' \left(1 - x'\right)$$

$$\therefore \boxed{\frac{dx'}{dt} = \alpha x' (1 - x')}$$

$$\frac{dx}{dt} = \alpha x (1-x)$$

$$\frac{1}{x(1-x)} dx = \alpha dt$$

$$\int_{x_0}^{x} \frac{1}{x(1-x)} dx = \alpha t = \int_{x_0}^{x} \frac{1}{x} dx + \int_{x_0}^{x} \frac{1}{1-x} dx$$

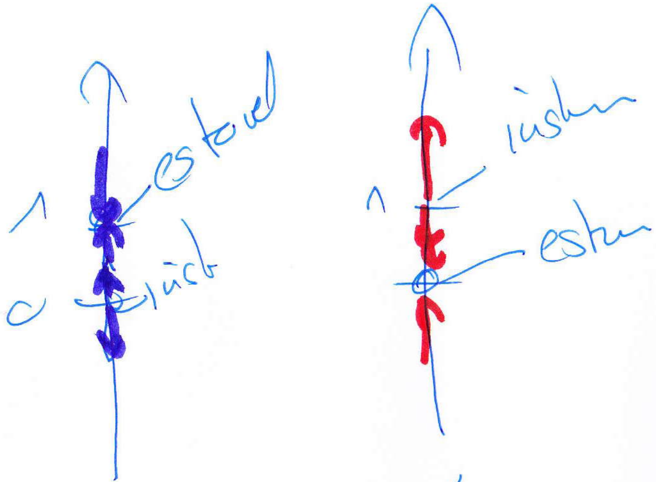
$$\frac{1}{x(1-x)} = \frac{A}{x} + \frac{B}{1-x} = \frac{A(1-x) + Bx}{x(1-x)}$$

$$(B-A)x + A = 1 \Rightarrow A = 1 \Rightarrow B = 1$$

$$\frac{dx}{dt} = \alpha x(1-x)$$

$$\frac{dx}{dt} = 0 \Rightarrow \alpha x (1-x) = 0 \quad \begin{cases} \cancel{x} = 0 \\ x = 1 \end{cases}$$

$$\frac{d}{dx}\left(\frac{dx}{dt}\right) = \alpha(1-2x) \quad \begin{cases} \alpha(1-0) = +\alpha & x=0 \\ \alpha(1-2) = -\alpha & x=1 \end{cases}$$



$\alpha > 0$ $\alpha < 0$

$$\frac{dx}{dt} = \alpha x(1-x)$$