Transformada Inversa Z

quinta-feira, 27 de agosto de 2020

Obter x(kT) a partir de X(z)

MÉTODO FRAÇÕES PARCIAIS

Ex) $X(z) = \dfrac{(1-e^{-aT})z}{(z-1)(z-e^{-aT})}$

1) SE EXISTIR UM ZERO NA ORIGEM, TRABALHAR C/ $\dfrac{X(z)}{z}$

$$\frac{X(z)}{z} = \frac{1-e^{-aT}}{(z-1)(z-e^{-aT})}$$

2 polos: 1, e^{-aT}

$$\frac{X(z)}{z} = \frac{C_1}{z-1} + \frac{C_2}{z-e^{-aT}} = \frac{1}{z-1} - \frac{1}{z-e^{-aT}}$$

$$C_1 = \left[\frac{X(z)}{z}\cdot(z-1)\right]_{z=1} = 1$$

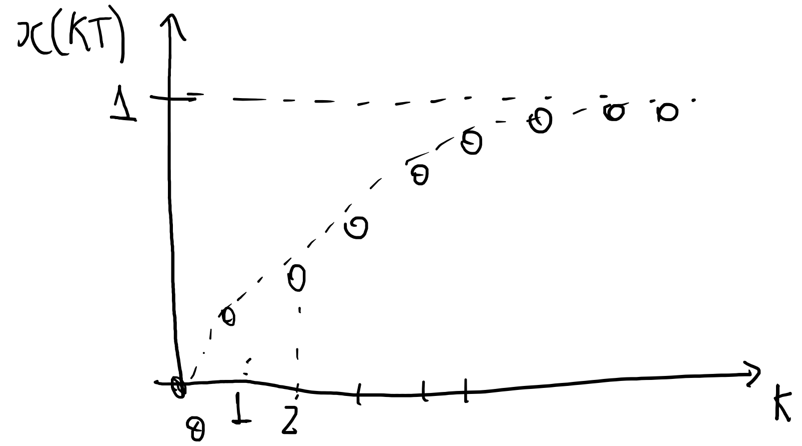
$$C_2 = \left[\frac{X(z)}{z}\cdot(z-e^{-at})\right]_{z=e^{-at}} = \frac{1-e^{-at}}{e^{-at}-1} = -1$$

$$X(z) = \frac{z}{z-1} - \frac{z}{z-e^{-at}} = \frac{1}{1-z^{-1}} - \frac{1}{1-e^{-aT}\cdot z^{-1}}$$

e^{-akT}	$\dfrac{1}{1-e^{-aT}z^{-1}}$
$1(k)$	$\dfrac{1}{1-z^{-1}}$

$$\boxed{x(kT) = 1(k) - e^{-akT}}$$

x(kT) ↑



FORMALIZANDO

$$X(z) = \frac{b_0 z^m + b_1 z^{m-1} \ldots + b_{m-1} z + b_m}{(z-p_1) \ldots (z-p_n)}$$

1) SE HOUVER UM ZERO NA ORIGEM ($b_m = 0$) USAR $\frac{X(z)}{z}$

2) $X(z)$ ou $\frac{X(z)}{z} = \frac{a_1}{s-p_1} + \ldots + \frac{a_n}{s-p_n}$

$$a_i = \left[\frac{X(z)}{z} (z-p_i) \right]_{z=p_i}$$

ou p/ POLO MÚLTIPLOS

$$X(z) \text{ ou } \frac{X(z)}{z} = \ldots \frac{c_1}{(z-p_1)^2} + \frac{c_2}{(z-p_1)}$$

$$c_1 = \left[(z-p_1)^2 \cdot \frac{X(z)}{z} \right]_{z=p_1}$$

$$c_2 = \left[\frac{d}{dz} \left((z-p_1)^2 \cdot \frac{X(z)}{z} \right) \right]_{z=p_1}$$

3) CONSULTAR TABELAS

LEMBRANDO QUE:

$$\boxed{\mathcal{Z}\left[x(t-nT)\right] = z^{-n} X(z)}$$

exemplos

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1) $X(z) = \dfrac{2z^3 + z}{(z-2)^2 (z-1)}$

$$\frac{X(z)}{z} = \frac{2z^2+1}{(z-2)^2 \cdot (z-1)} = \frac{2z^2+1}{z^3 - 5z^2 + 8z - 4}$$

$$\frac{X(z)}{z} = \frac{3}{(z-1)} + \frac{-1}{(z-2)} + \frac{9}{(z-2)^2}$$

FAZER USANDO AS DEFINIÇÕES DOS RESÍDUOS

scipy.signal.residue

scipy.signal.**residue**(b, a, tol=0.001, rtype='avg') [source]

Compute partial-fraction expansion of b(s) / a(s).

If M = len(b) and N = len(a), then the partial-fraction expansion H(s) is defined as

```
        b(s)     b[0] s**(M-1) + b[1] s**(M-2) + ... + b[M-1]
H(s) = ------ = ------------------------------------------------
        a(s)     a[0] s**(N-1) + a[1] s**(N-2) + ... + a[N-1]

         r[0]       r[1]             r[-1]
     = -------- + -------- + ... + --------- + k(s)
       (s-p[0])   (s-p[1])         (s-p[-1])
```

If there are any repeated roots (closer together than tol), then H(s) has terms like:

```
  r[i]      r[i+1]              r[i+n-1]
-------- + ----------- + ... + -----------
(s-p[i])  (s-p[i])**2          (s-p[i])**n
```

Returns: r : ndarray
Residues.
p : ndarray
Poles.
k : ndarray
Coefficients of the direct polynomial term.

$$X(z) = \frac{3z}{z-1} + \frac{-z}{z-2} + \frac{9z}{(z-2)^2}\frac{\times z^{-2}}{\times z^{-2}}$$

$$X(z) = \frac{3}{1-z^{-1}} + \frac{-1}{1-2z^{-1}} + \frac{9z^{-1}}{(1-2z^{-1})^2}$$

COMO $\mathcal{Z}^{-1}\left[\dfrac{1}{1-az^{-1}}\right] = a^k$

$$\mathcal{Z}^{-1}\left[\frac{z^{-1}}{(1-az^{-1})^2}\right] = k\,a^{k-1}$$

$$x(k) = 3 - 1.2^k + 9k\,2^{k-1}$$

k	$x(k)$
0	$2 = 3 - 1 + 0$
1	$10 = 3 - 2 + 9$
2	$35 = 3 - 4 + 36$
⋮	⋮

TRABALHANDO COM $X(z)$

```
num = [2,0,1,0]
den = [1,-5,8,-4]

[r,p,n] = sig.residue(num,den)

r
Out[59]: array([ 3., 7., 18.])

p
Out[60]: array([1., 2., 2.])

n
Out[61]: array([2.])
```

$$X(z) = \frac{3}{z-1} + \frac{7}{z-2} + \frac{18}{(z-2)^2} + 2$$

$$X(z) = \frac{3z^{-1}}{1-z^{-1}} + \frac{7z^{-1}}{(1-2z^{-1})} + \frac{18z^{-2}}{(1-2z^{-1})^2} + 2$$

ERRO TABELA OGATA

$a^{k-1} \cdot 1(k-1)$, $k = 1, 2, 3, \ldots$	$\dfrac{z^{-1}}{1-az^{-1}}$
$ka^{k-1} \cdot 1(k-1)$	$\dfrac{z^{-1}}{(1-az^{-1})^2}$

$$\underbrace{\frac{18z^{-1}}{(1-2z^{-1})^2}}_{18.k.2^{k-1}} \cdot z^{-1}$$

$18.(k-1).2^{k-2}.1(k-2)$

$$x(kT) = 3.\underbrace{1(k-1)}_{\text{ATRASO 1T}} + 7.\underbrace{1(k-1)}_{\text{ATRASO 1T}}.2^{k-1} + 18(k-1).2^{k-2}.\underbrace{1(k-2)}_{\text{ATRASO 2T}} + 2.\delta(k)$$

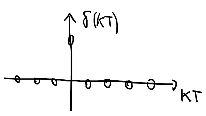
$k=0 \rightarrow 0 + 0 + 0 + 2 = 2$

$k=1 \rightarrow 3 + 7 + 0 + 0 = 10$

$k=2 \rightarrow 3 + 7.2 + 18. + 0 = 35$

SEJA $\delta(kT)$ O SINAL IMPULSO DISCRETO

$$\delta(kT) = \begin{cases} 1 & \text{se } k = 0 \\ 0 & \text{se } k > 0 \end{cases}$$



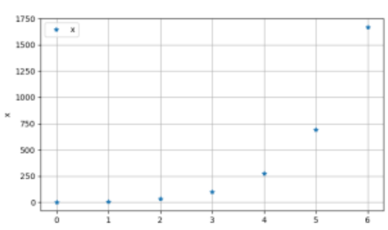
$x(kT) = \delta(kT) \Rightarrow X(z) = 1$

$X(z) \longrightarrow \boxed{G(z)} \xrightarrow{Y(z)}$

SENDO $G(z)$ UMA RELAÇÃO ENTRE SINAIS DE ENTRADA E SAÍDA. SE $X(z) = 1$

$$\boxed{Y(z) = G(z)}$$

A SAÍDA NO TEMPO DE UM SISTEMA C/ ENTRADA IGUAL AO δ É A PRÓPRIA INVERSA DE SUA F. TRANSF. DISCRETA




```
yout , t = impulse(X)

# plot result
plt.figure(1)
plt.plot(t,yout,'*',linewidth=2,label='x')
plt.xlabel('time')
plt.ylabel('x')
plt.legend()
plt.show()

yout
Out[56]: array([ 2., 10., 35., 103., 275., 691., 1667.])
```

Exemplo

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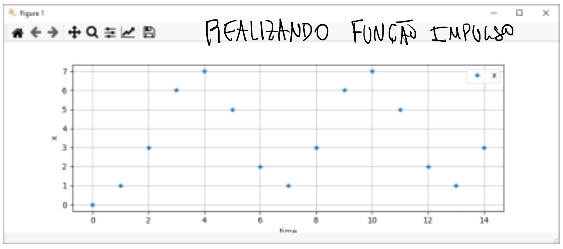
EX2) $X(z) = \dfrac{z^2+z+2}{(z-1)(z^2-z+1)} = \begin{cases} \text{Polos } 1 \\ 2 \text{ conjugados} \end{cases}$



REALIZANDO FUNÇÃO IMPULSO

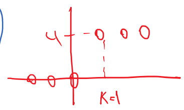
$$X(z) = \frac{c_1 = 4}{z-1} + \frac{az+b}{z^2-z+1} = \frac{4z^2-4z+4+az^2-az+bz-b}{(z-1)(z^2-z+1)} = \frac{z^2+z+2}{(z-1)(z^2-z+1)}$$

$a = -3$

$b = 2 \Rightarrow \boxed{X(z) = \dfrac{4}{z-1} + \dfrac{-3z+2}{z^2-z+1}} = \boxed{\dfrac{4z^{-1}}{1-z^{-1}} + \underbrace{\dfrac{-3z^{-1}+2z^{-2}}{1-z^{-1}+z^{-2}}}_{(A)}}$

$4 \cdot 1(k-1)$

$e^{-akT}\sin \omega kT$	$\dfrac{e^{-aT}z^{-1}\sin\omega T}{1-2e^{-aT}z^{-1}\cos\omega T + e^{-2aT}z^{-2}}$
$e^{-akT}\cos \omega kT$	$\dfrac{1-e^{-aT}z^{-1}\cos\omega T}{1-2e^{-aT}z^{-1}\cos\omega T + e^{-2aT}z^{-2}}$



$$x(k) = \left[4 - 3\cos\frac{\pi}{3}(k-1) + \frac{1}{\sqrt{3}}\sin\frac{\pi}{3}(k-1)\right] \cdot 1(k-1)$$

$$(A) = z^{-1} \cdot (-3) \cdot \left(\frac{1-0{,}5z^{-1}}{1-z^{-1}+z^{-2}}\right) + z^{-1} \cdot \frac{0{,}5z^{-1}}{1-z^{-1}+z^{-2}}$$

$x(0) = 0$

$x(1) = 1$

$x(2) = 3$

6

7

$$1-2e^{-aT}z^{-1}\cos\omega T + e^{-2aT}z^{-2} = 1 - z^{-1} + z^{-2} \Rightarrow \begin{cases} \omega T = \pi/3 \\ a = 0 \end{cases}$$

$$\left[-3 \cdot \cos\frac{\pi}{3}(k-1) + \frac{1}{\sqrt{3}} \cdot \sin\frac{\pi}{3}(k-1)\right] \cdot 1(k-1)$$

EX3) $X(z) = \dfrac{z+2}{(z-2)z^2}$

```
In [72]: r
Out[72]: array([-1., -1.,  1.])

In [73]: p
Out[73]: array([0., 0., 2.])

In [74]: n
Out[74]: array([], dtype=float64)
```

$X(z) = \dfrac{-1}{z} - \dfrac{1}{z^2} + \dfrac{1}{z-2}$

$X(z) = -z^{-1} - z^{-2} + \dfrac{z^{-1}}{1-2z^{-1}}$

	$\delta_0(n-k)$	
1	$n = k$	z^{-k}
0	$n \neq k$	

$$\boxed{x(k) = -\delta(k-1) - \delta(k-2) + 2^{k-1} \cdot 1(k-1)}$$

$k=0 \to +0 + 0 + 0 = 0$

$k=1 \to -1 + 0 + 1 = 0$

$k=2 \to 0 - 1 + 2 = 1$

$\vdots$