PROVINHA 1

O ESTADO DE TENSÃO NO PONTO

$$\sigma_{ij} = \begin{bmatrix} \sigma & a\sigma & b\sigma \\ a\sigma & \sigma & c\sigma \\ b\sigma & c\sigma & \sigma \end{bmatrix}$$

$$\underbrace{\vec{t}^{(\hat{n})}_{3\times 1} = \sigma_{ij} \cdot \hat{n} = \vec{0}}$$

CONHECIDO

$\hookrightarrow$ 3 Eq.
3 Incog.

a, b, c constantes

σ = valor tensão

Determine a, b, c / $\vec{t}^{(\hat{n})} = \vec{0}$

$$\hat{n} = (1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$$

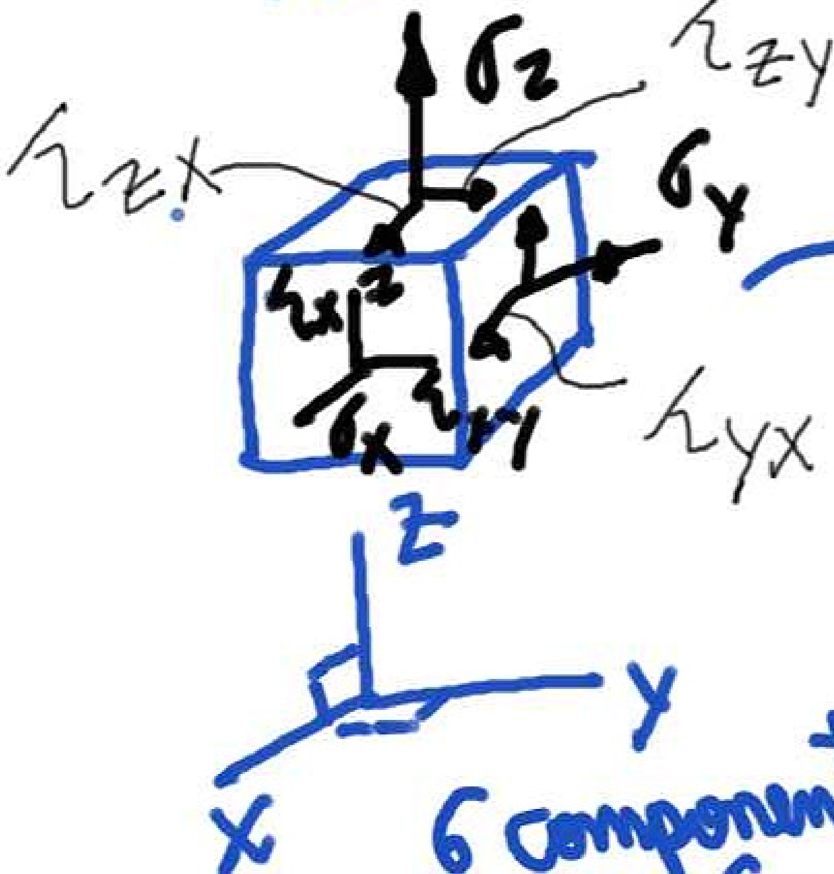
$$\begin{bmatrix} \sigma & a\sigma & b\sigma \\ a\sigma & \sigma & c\sigma \\ b\sigma & c\sigma & \sigma \end{bmatrix} \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

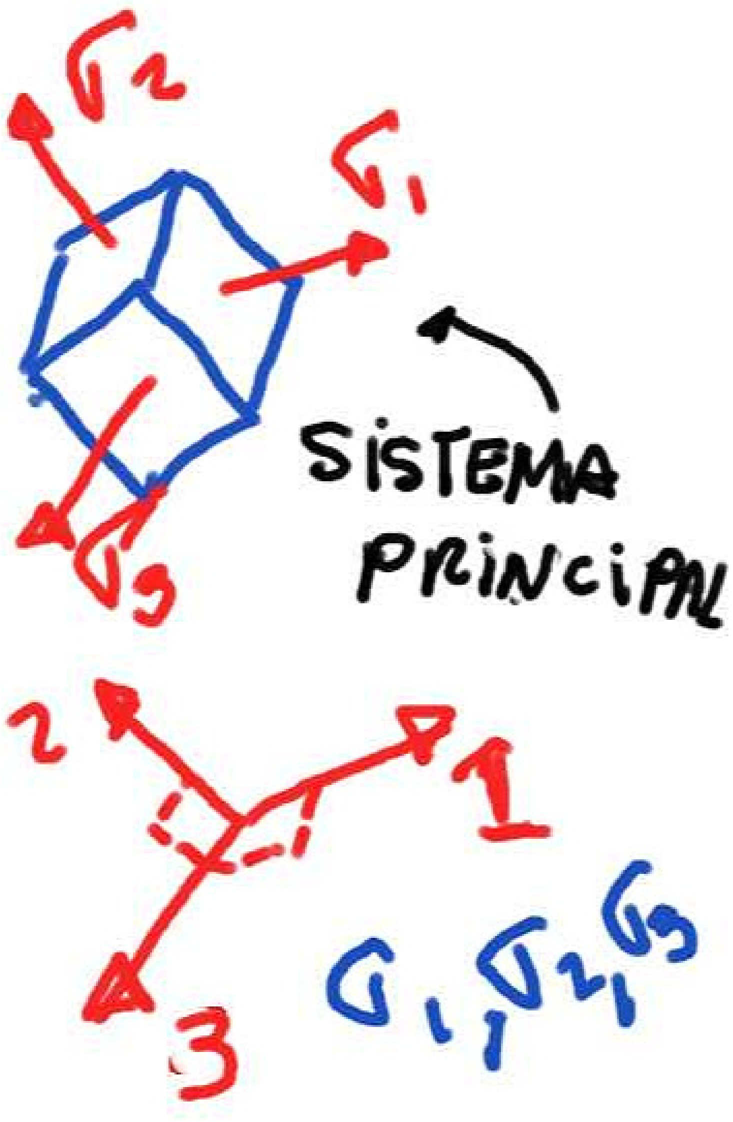
$$\Downarrow$$

$$\begin{aligned} a+b &= -1 \\ a+c &= -1 \\ b+c &= -1 \end{aligned} \quad \Leftrightarrow \quad \begin{aligned} a &= b \\ &= c \\ &= -1/2 \end{aligned}$$

CIRCULO DE MOHR (3D)

REPRESENTAÇÃO GRÁFICA DO ESTADO DE TENSÃO NO PONTO



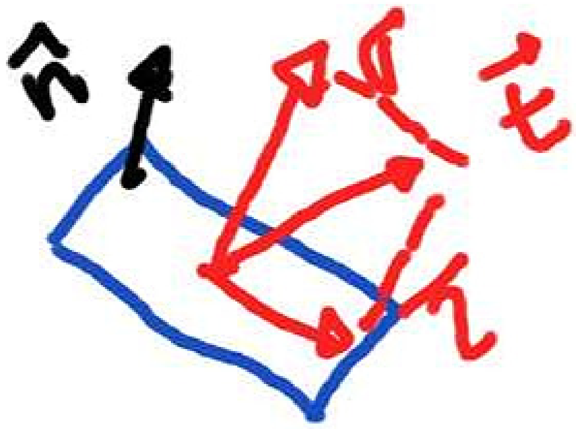


$\sigma_1, \sigma_2, \sigma_3$

6 componentes

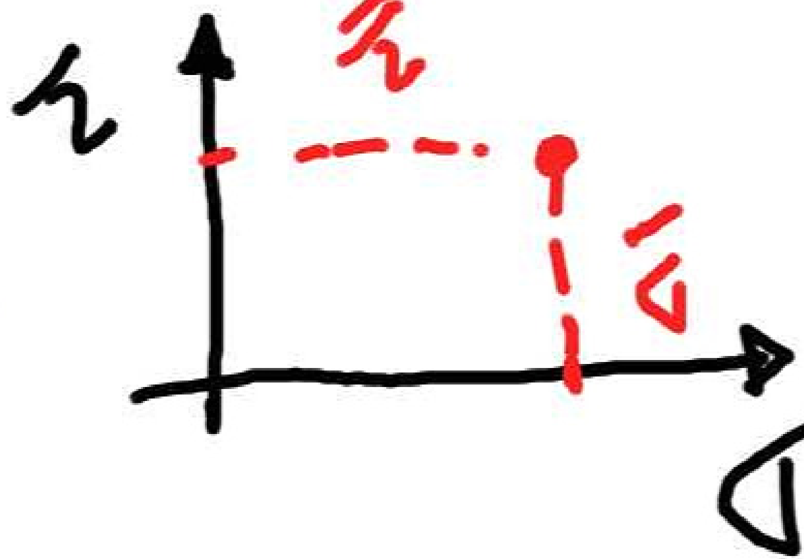
$\sigma_x, \sigma_y, \sigma_z$

$\tau_{xy}, \tau_{xz}, \tau_{yz}$



(σ, τ)

↳ ponto repres. no plano cartes.

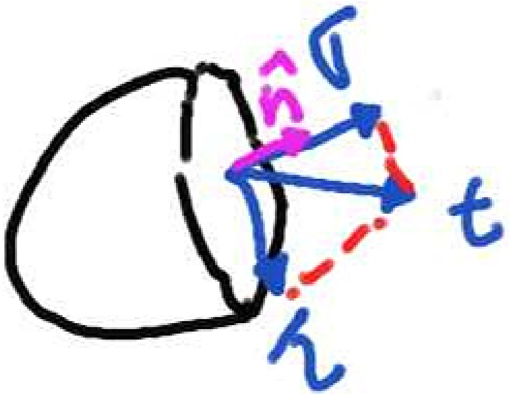


o conjunto (σ, τ)

↓

Fig. Conhecida

. DESCOMPOSIÇÃO DO VETOR TRAÇÃO $\vec{t}^{(\vec{n})}$



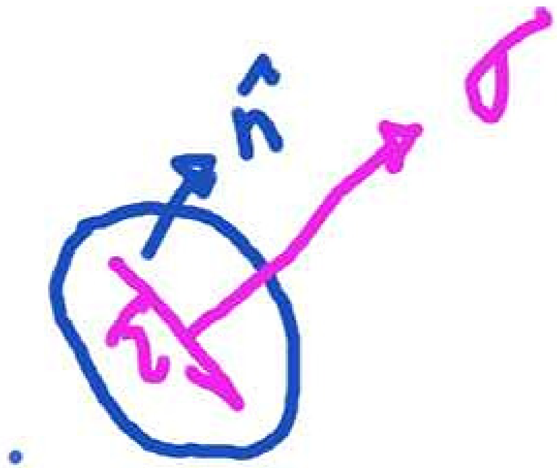
$\hat{n} \equiv$ versor

$\left.\begin{array}{l}\sigma = ?\\ \tau = ?\end{array}\right\}$ Magnitudes

$\vec{t}^{(\vec{n})}$ ✓

$\hat{n} = (n_x, n_y, n_z)$ ✓

$\sigma = \vec{t}^{(\hat{n})} \cdot \hat{n}$ (produto escalar)



$$\sigma = \vec{t} \cdot \hat{n}$$

$$\vec{t} = \sigma_{ij} \cdot \hat{n}$$

$$\sigma_{ij} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$

$$\downarrow$$

$$\sigma = \begin{bmatrix} \sigma_1 & 0 & 0 \\ & \sigma_2 & 0 \\ & & \sigma_3 \end{bmatrix} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} \cdot \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}$$

$$\sigma = (\sigma_1 n_x, \sigma_2 n_y, \sigma_3 \sigma_z) \cdot$$

$$(n_x, n_y, n_z)$$

$$\sigma = \sigma_1 n_x^2 + \sigma_2 n_y^2 + \sigma_3 n_z^2$$

$$|\vec{t}| = ? \qquad \vec{t}^{(\hat{n})} = \sigma_{ij} \cdot \hat{n}$$

$$|\vec{t}|^2 = (\sigma_{ij} \cdot \hat{n}) \cdot (\sigma_{ij} \cdot \hat{n})$$

$$|\vec{t}|^2 = \begin{bmatrix} \sigma_1 & & \\ & \sigma_2 & \\ & & \sigma_3 \end{bmatrix} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} \cdot \begin{bmatrix} \sigma_1 & & \\ & \sigma_2 & \\ & & \sigma_3 \end{bmatrix} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}$$

$$|\vec{t}|^2 = \sigma_1^2 n_x^2 + \sigma_2^2 n_y^2 + \sigma_3^2 n_z^2$$

$$\begin{cases} \sigma = \sigma_1 n_x^2 + \sigma_2 n_y^2 + \sigma_3 n_z^2 \\ 1 = n_x^2 + n_y^2 + n_z^2 \\ \tau^2 + \sigma^2 = \sigma_1^2 n_x^2 + \sigma_2^2 n_y^2 + \sigma_3^2 n_z^2 \end{cases}$$

$\left.n_x^2, n_y^2, n_z^2\right\}$ INCÓGNITAS

$$\begin{bmatrix} \sigma_1 & \sigma_2 & \sigma_3 \\ \sigma_1^2 & \sigma_2^2 & \sigma_3^2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} n_x^2 \\ n_y^2 \\ n_z^2 \end{bmatrix} = \begin{bmatrix} \sigma \\ \sigma^2 + \tau^2 \\ 1 \end{bmatrix}$$

Resolvendo

$\sigma_1 \geq \sigma_2 \geq \sigma_3 \qquad n_i^2 \geq 0$

$$n_1^2 = \frac{\tau^2 + (\sigma - \sigma_2)(\sigma - \sigma_3)}{(\sigma_1 - \sigma_2)(\sigma_1 - \sigma_3)} \geq 0$$

$$n_2^2 = \frac{\tau^2 + (\sigma - \sigma_3)(\sigma - \sigma_1)}{(\sigma_2 - \sigma_3)(\sigma_2 - \sigma_1)} \leq 0$$

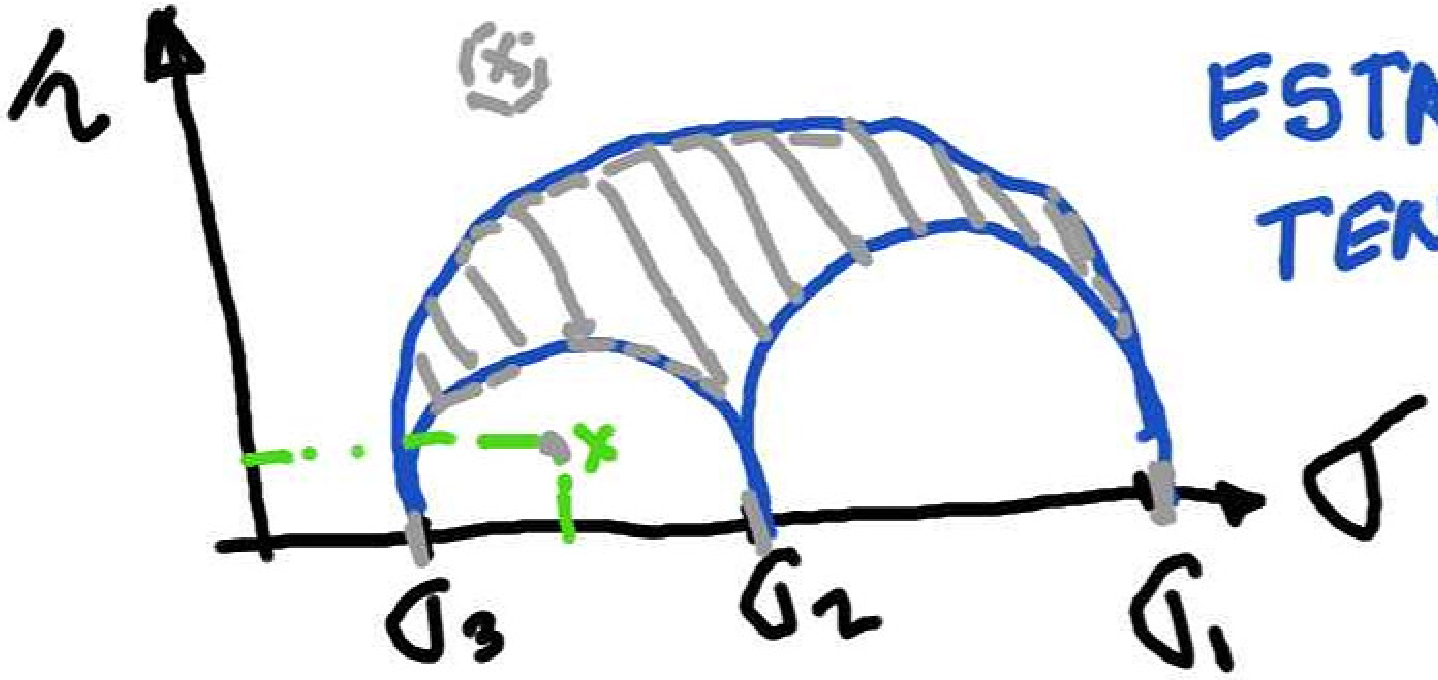
$$n_3^2 = \frac{\tau^2 + (\sigma - \sigma_1)(\sigma - \sigma_2)}{(\sigma_3 - \sigma_1)(\sigma_3 - \sigma_2)} \geq 0$$

IGUAL ZERO

$$\tau^2 + (\sigma - \sigma_3)(\sigma - \sigma_1) = 0$$

$$\tau^2 + (\sigma - \sigma_2)(\sigma - \sigma_3) = 0$$

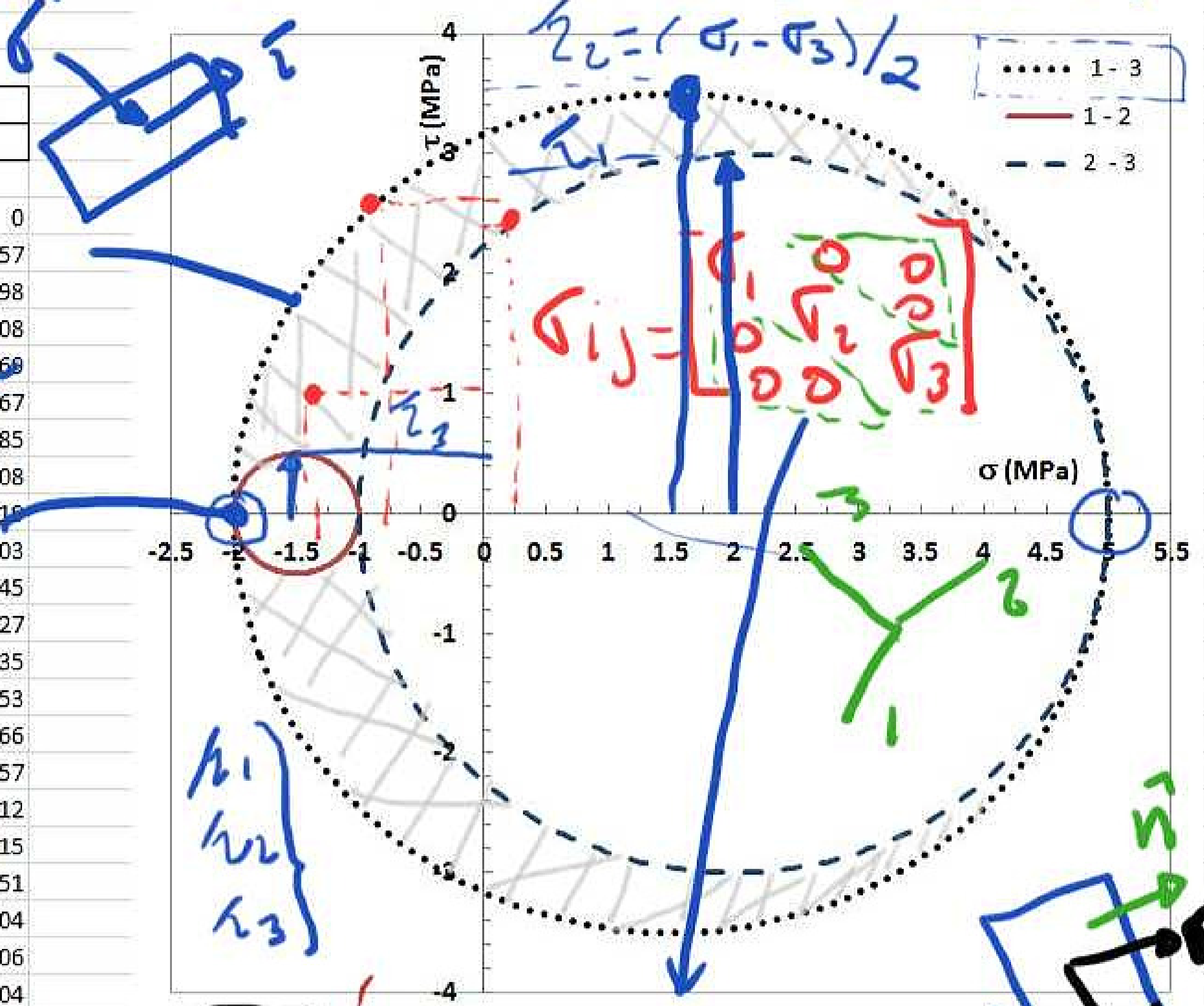
$$\tau^2 + (\sigma - \sigma_1)(\sigma - \sigma_2) = 0$$

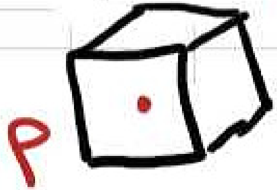


ESTADO TENSÃO

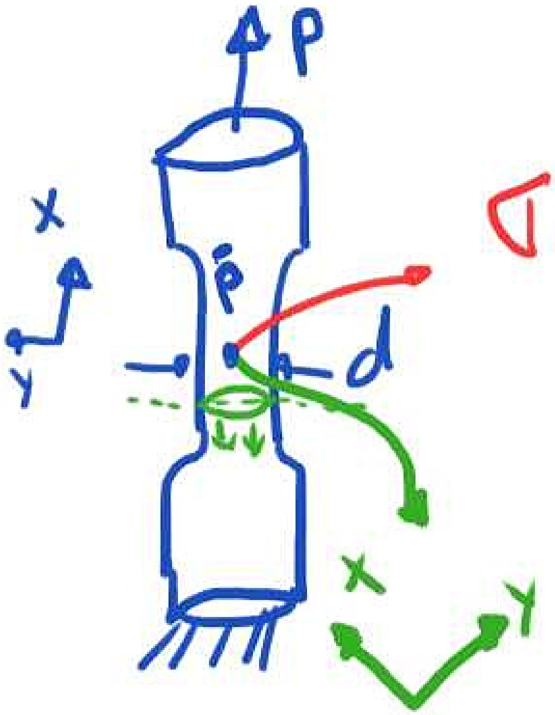
• C. MOHR (REPRESENTAÇÃO GRÁFICA DO E.T)

σ1	2	MPa	1 - 3				
σ2	1	MPa	1 - 2				
σ3	-5	MPa	2 - 3				
centro2 =	-1.5		centro3 =	1.5		centro1 =	-2
r2=	3.5		r3=	0.5		r1=	3
	theta	x2	y2	x3	y3	x1	y1
	0	5	0	-1	0	5	0
	1	4.999467	0.061083	-1.00008	0.008726	4.999543	0.052357
	2	4.997868	0.122148	-1.0003	0.01745	4.998172	0.104698
	3	4.995203	0.183176	-1.00069	0.026168	4.995889	0.157008
	4	4.991474	0.244148	-1.00122	0.034878	4.992692	[illegible]
	5	4.986681	0.305045	-1.0019	0.043578	4.988584	0.261467
	6	4.980827	0.36585	-1.00274	[illegible]	[illegible]	0.313585
	7	4.973912	0.426543	-1.00373	0.060935	[illegible]	0.365608
	8	4.965938	0.487106	-1.00487	[illegible]	4.970804	[illegible]
	9	4.956909	0.547521	-1.00616	[illegible]	4.963065	0.469303
	10	4.946827	0.607769	-1.0076	0.086824	4.954423	0.520945
	11	4.935695	0.667831	[illegible]	[illegible]	[illegible]	0.572427
	12	4.923517	0.727691	-1.01093	0.103956	[illegible]	0.623735
	13	4.910295	0.787329	-1.01281	[illegible]	[illegible]	0.674853
	14	4.896035	0.846727	-1.01485	0.120961	[illegible]	0.725766
	15	4.88074	0.905867	-1.01704	0.12941	4.897777	0.776457
	16	4.864416	0.964731	-1.01937	0.137819	4.883785	0.826912
	17	4.847067	1.023301	-1.02185	0.146186	4.868914	0.877115
	18	4.828698	1.081559	-1.02447	0.154508	4.85317	0.927051
	19	4.809315	1.139489	-1.02724	0.162784	4.836556	0.976704
	20	4.788924	1.197071	-1.03015	0.17101	4.819078	1.02606
	21	4.767531	1.254288	-1.03321	0.179184	4.800741	1.075104
	22	4.745143	1.311123	-1.03641	0.187303	4.781552	1.12382
	23	4.721767	1.367559	-1.03975	0.195366	4.761515	1.172193





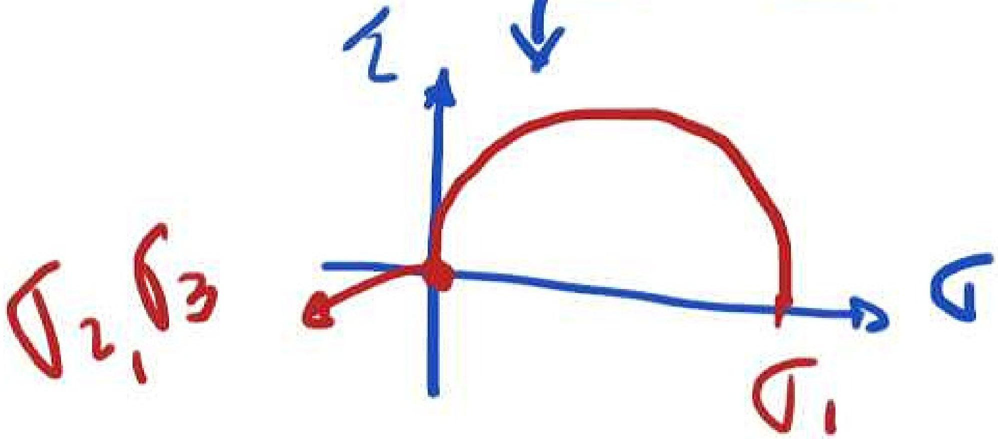
$$\sigma_{ij} = \begin{bmatrix} -1 & 2\sqrt{2} & 0 \\ 2\sqrt{2} & -3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$



1D

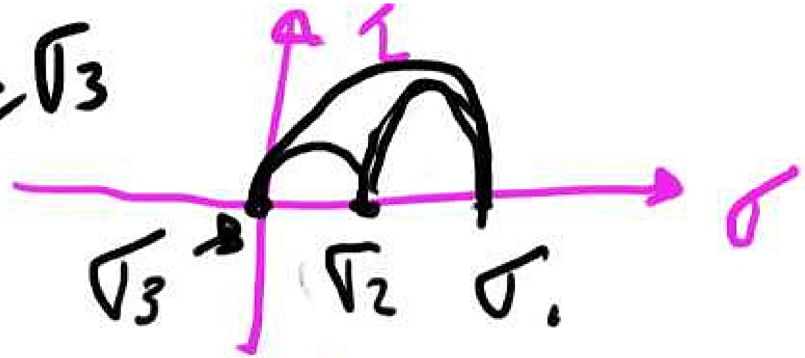
$$\sigma_{ij} = \begin{bmatrix} \sigma_x & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{matrix} \sigma_x = \sigma_1 \\ \sigma_2 = \sigma_3 = 0 \end{matrix}$$

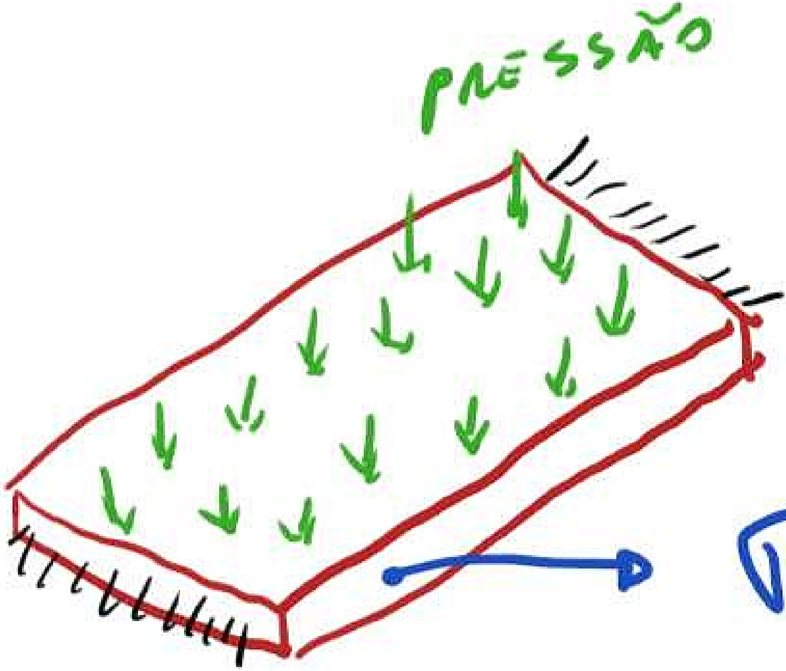
$$\sigma_{ij}' = \begin{bmatrix} \sigma_x' & \tau_{xy} & 0 \\ \tau_{xy} & \sigma_y' & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



2D (3D)

$\sigma_1 \geq \sigma_2 \geq \sigma_3$





$$\sigma_{ij} = \begin{bmatrix} \sigma_x & & \\ & \sigma_y & \\ & & \sigma_z \end{bmatrix}$$

$2D \rightarrow \sigma_z = 0$

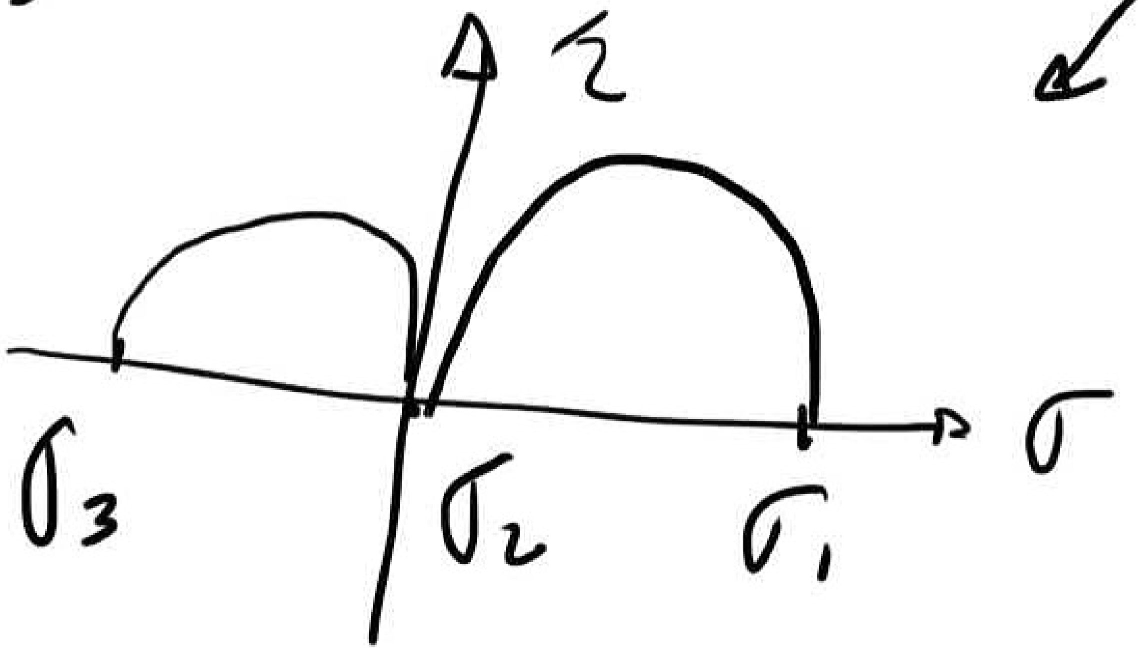
$\sigma_3 = 0 \rightarrow \sigma_1, \sigma_2 \neq 0$

$\sigma_1 > 0$

$\sigma_2 < 0 \rightarrow$

$\sigma_3 = 0$

$$\left.\begin{aligned} \sigma_1 &> 0 \\ \sigma_2 &= 0 \\ \sigma_3 &< 0 \end{aligned}\right\} \underline{\sigma_1 \geq \sigma_2 \geq \sigma_3}$$



* ESTADO PLANO DE TENSÃO

$$\sigma_{ij} = \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix}$$

E.P.T

↳ $\sigma_z = 0$

↓

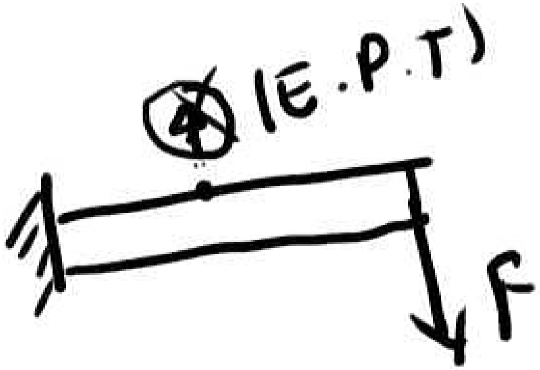
CÍRCULO MOHR ? → $r = ?$

$(a,b) = ?$

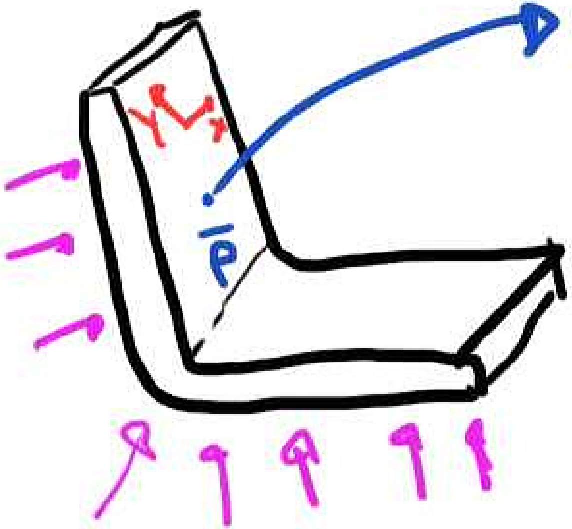
c. centro

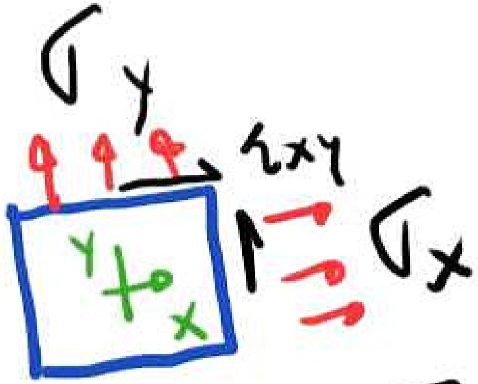
$$r^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$

$$(a, b) = \left(\frac{\sigma_x + \sigma_y}{2}, 0\right)$$



• EQUILÍBRIO SÓLIDO NO PONTO $\bar{P}$





✓ ✓ ✓

$\sigma_x, \sigma_y, \tau_{xy} \neq 0$

$\downarrow$

⊢ ✓

(a, b)

$\downarrow$

$\sigma_1, \sigma_2, \sigma_3 =$

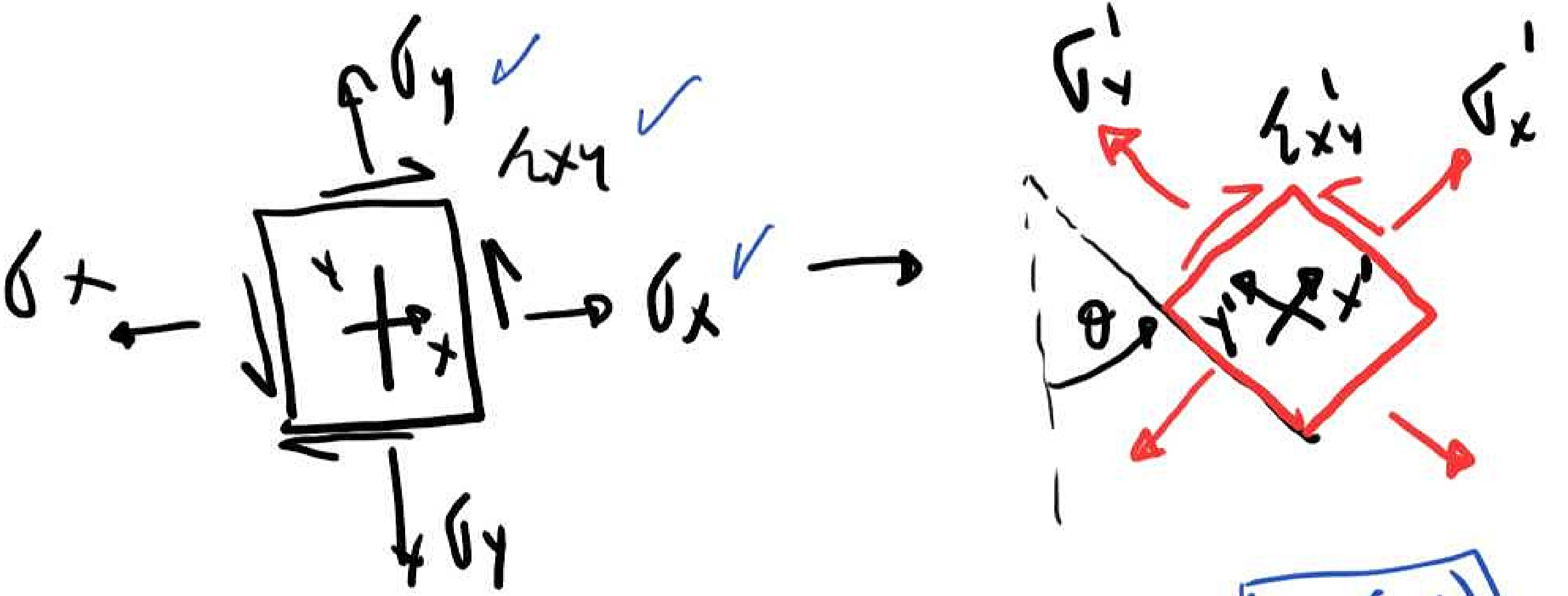
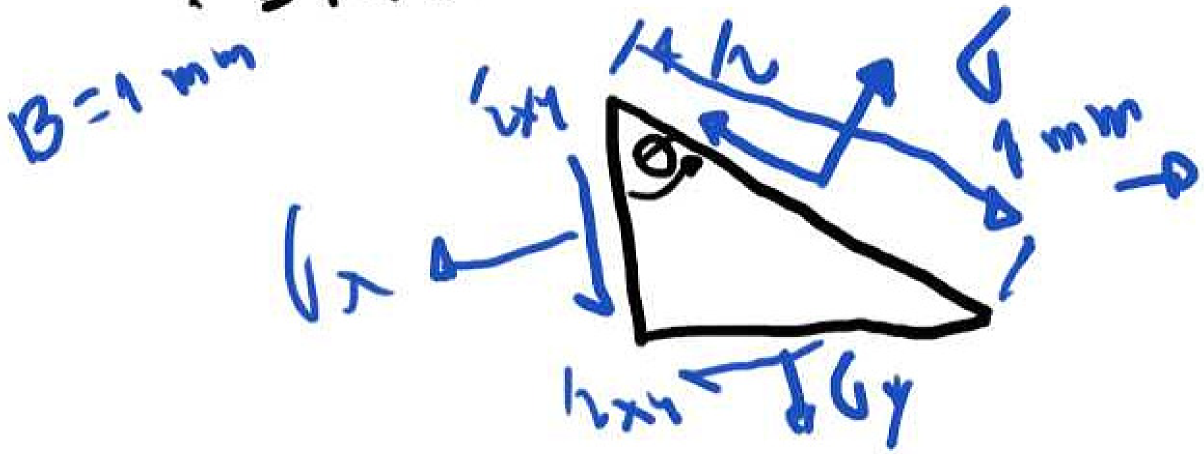


DIAGRAMA DE CORPO LIVRE

$\boxed{(\theta, \sigma, \tau)}$



$\rightarrow \sum F_x = 0$

$\sum F_y = 0$