LOM3202 –CIRCUITOS ELÉTRICOS
AULA 2

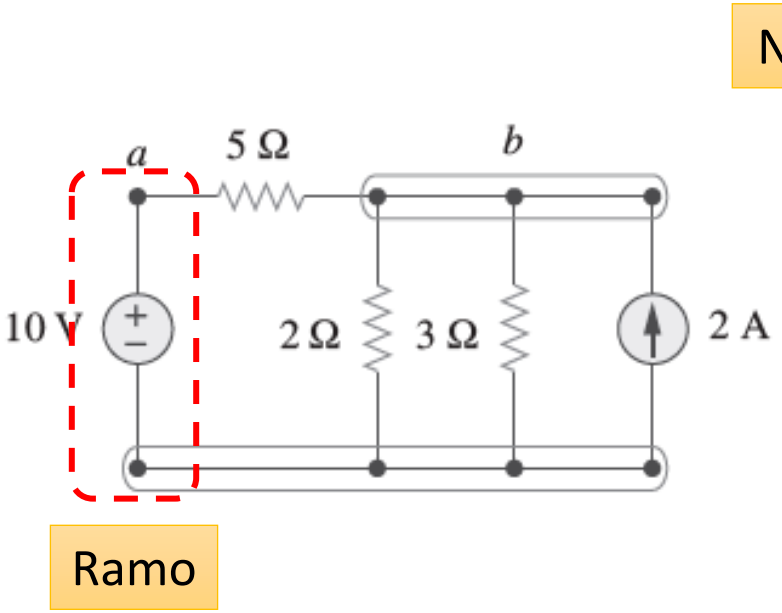
Prof. Dr. Emerson G. Melo

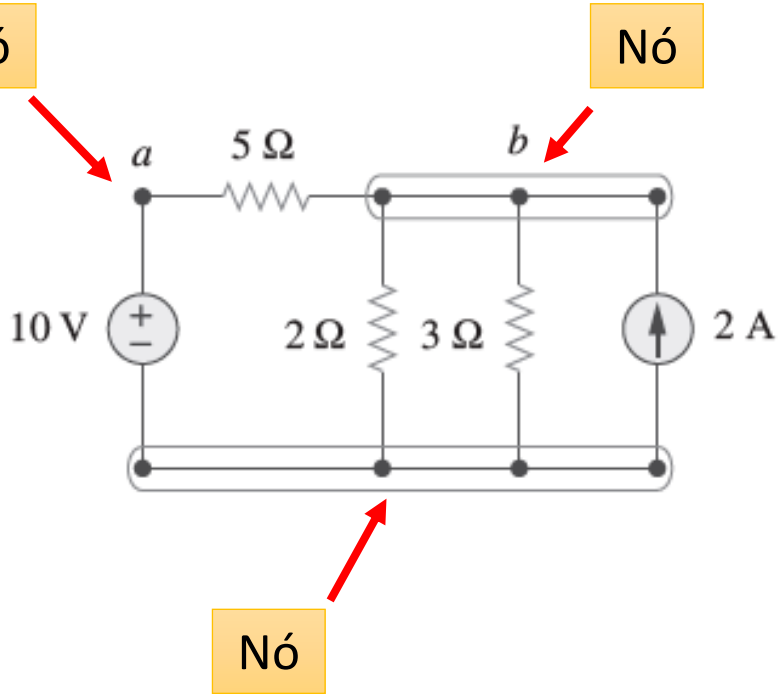
Sumário

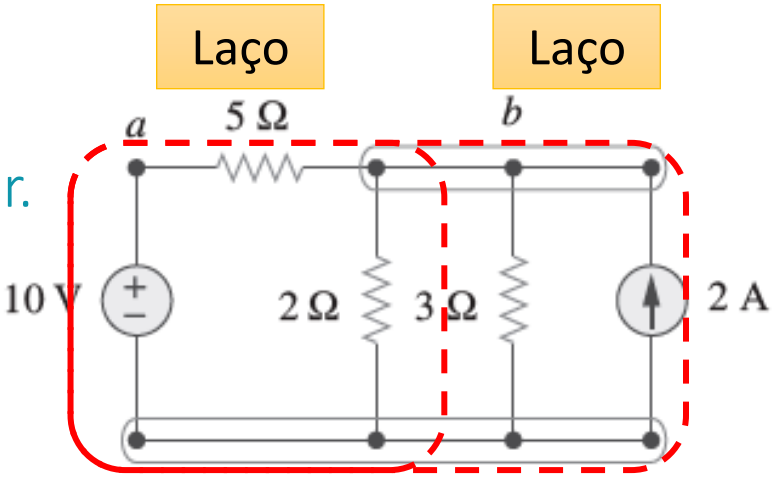
- Leis de Kirchhoff.
- Divisão de Tensão.
- Divisão de Corrente.
- Resistência Equivalente.
  - Circuito Série;
  - Circuito Paralelo;
  - Circuito Misto.
- Conversão Y-Delta.

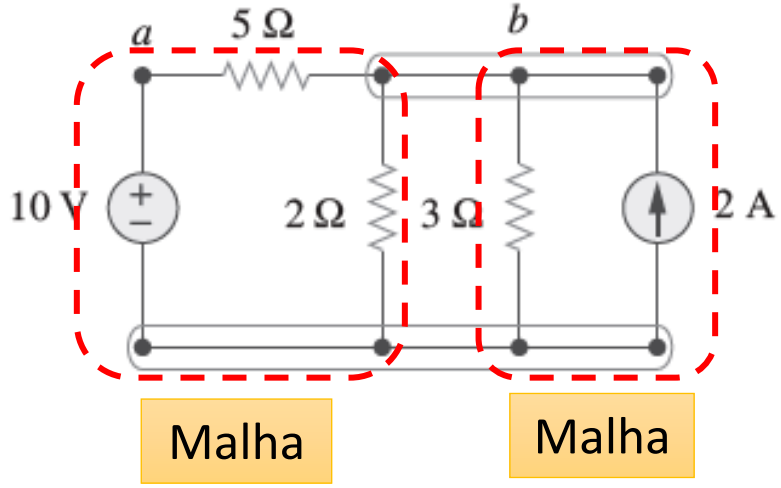
Leis de Kirchhoff: Nós, Ramos, Laços e Malhas

- Ramos representa um elemento único como fontes ou resistores;
- Nó é o ponto de conexão entre dois ou mais ramos;
- Laço é qualquer caminho fechado de um circuito;
- Malha é um laço que não apresenta outros laços em seu interior.









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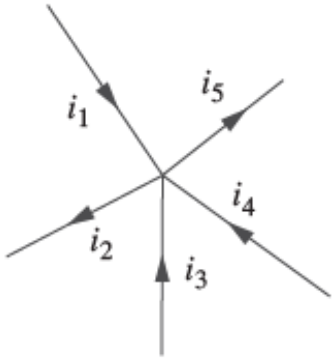
Leis de Kirchhoff

Leis derivadas dos princípios de conservação de carga e energia.

LKC – Lei de Kirchhoff para Corrente

Lei dos nós

$$\sum_{n=1}^{N} i_n = 0$$



$$i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0$$

$$i_1 + i_3 + i_4 = i_2 + i_5$$

$$i_{in} = i_{out}$$

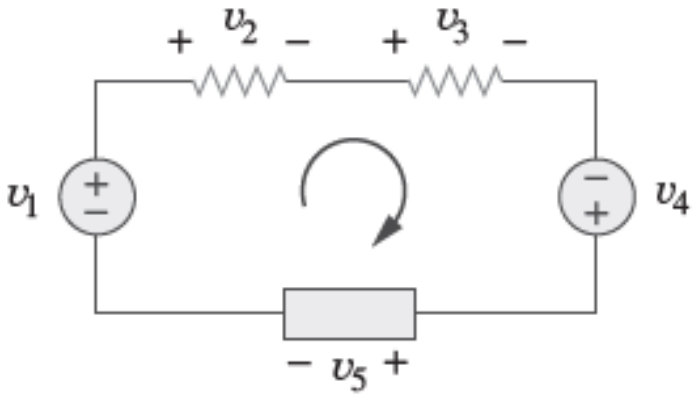
$$\int i_{in}(t)dt = \int i_{out}(t)dt$$

$$Q_{in} = Q_{out}$$

LKT – Lei de Kirchhoff para Tensão

Lei das malhas

$$\sum_{n=1}^{N} v_n = 0$$

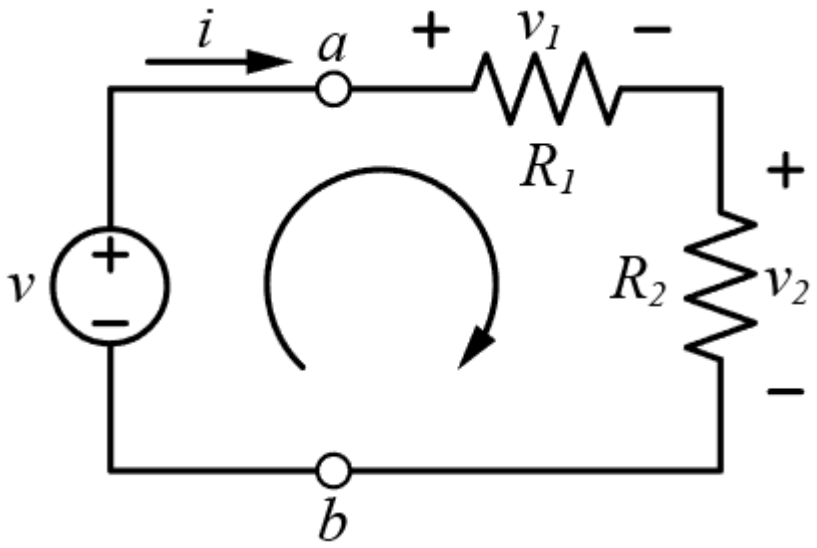


$$-Qv_1 + Qv_2 + Qv_3 - Qv_4 + Qv_5 = 0$$

$$Q(v_2 + v_3 + v_5) = Q(v_1 + v_4)$$

$$v_2 + v_3 + v_5 = v_1 + v_4$$

Divisão de Tensão



LKT

$$\sum_{n=1}^{N} v_n = 0$$

$$-v + v_1 + v_2 = 0$$

$$v = v_1 + v_2$$

Lei de Ohm $v = iR$

$$v = iR_1 + iR_2$$

$$v = i(R_1 + R_2)$$

$$i = \frac{v}{R_1 + R_2}$$

$$v_1 = iR_1 \qquad v_2 = iR_2$$

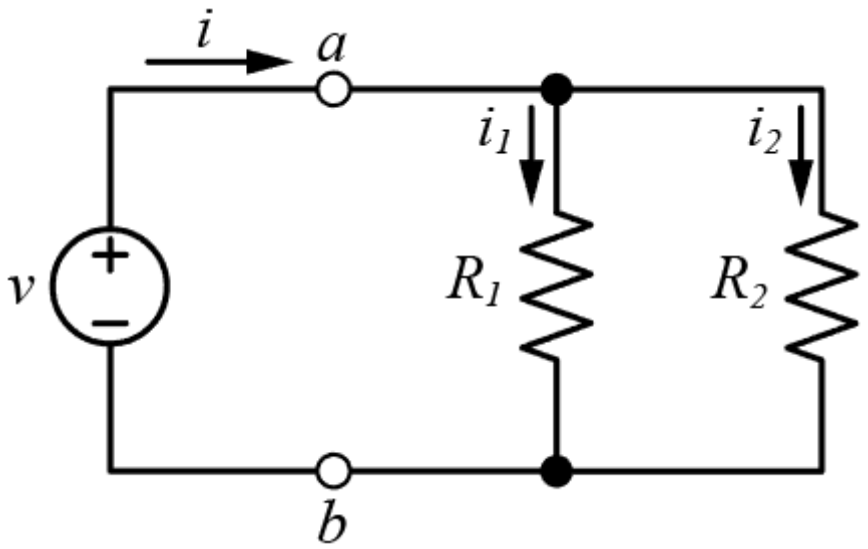
$$v_1 = \frac{R_1}{R_1 + R_2} v \qquad v_2 = \frac{R_2}{R_1 + R_2} v$$

Princípio da Divisão de Tensão

$$v_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N} v$$

A tensão se distribui de forma proporcional através das resistências.

Divisão de Corrente



LKC

$$\sum_{n=1}^{N} i_n = 0$$

$$i - i_1 - i_2 = 0$$

$$i = i_1 + i_2$$

Lei de Ohm

$$i = \frac{v}{R}$$

$$i = \frac{v}{R_1} + \frac{v}{R_2} \qquad i = v\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$v = i\frac{R_1 R_2}{R_1 + R_2}$$

$$i_1 = \frac{v}{R_1} \qquad i_1 = i\frac{R_2}{R_1 + R_2} \qquad i_1 = i\frac{\frac{R_2}{R_1 R_2}}{\frac{R_1 + R_2}{R_1 R_2}}$$

$$i_1 = i\frac{\frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2}} \qquad i_1 = i\frac{G_1}{G_1 + G_2}$$

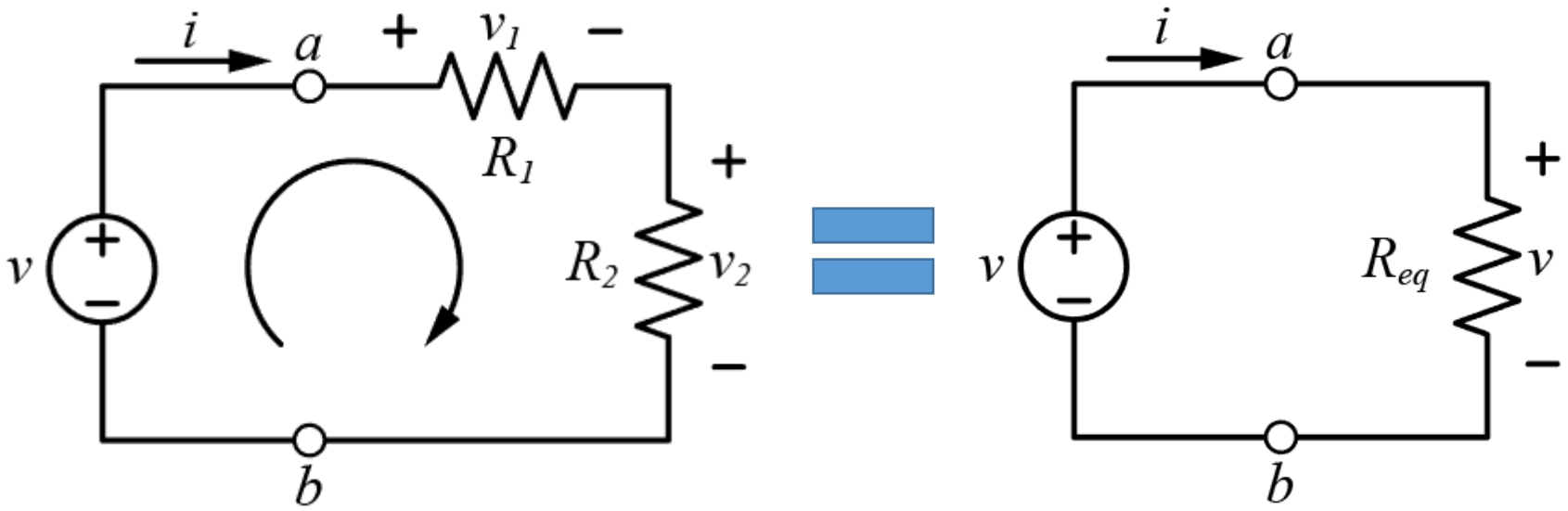
$$i_2 = \frac{v}{R_2} \qquad i_2 = i\frac{G_2}{G_1 + G_2}$$

Princípio da Divisão de Corrente

$$i_n = \frac{G_n}{G_1 + G_2 + \cdots + G_N} i$$

A corrente se distribui de forma proporcional através das condutâncias.

Resistência Equivalente: Circuito Série



Lei de Ohm $v = iR$

$$v = iR_1 + iR_2$$

$$v = i(R_1 + R_2)$$

$$R_1 + R_2 = R_{eq}$$

$$v = iR_{eq}$$

Resistência Equivalente Série

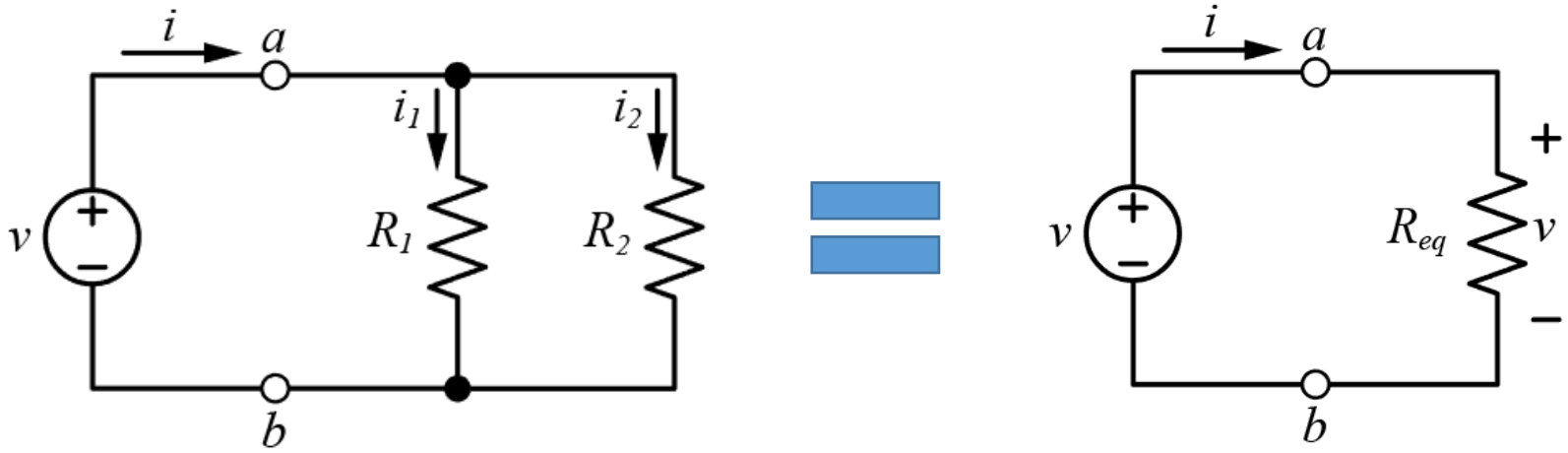
$$R_{eq} = R_1 + R_2 + \cdots + R_N$$

Princípio da Divisão de Tensão

$$v_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N} v$$

$$v_n = \frac{R_n}{R_{eq}} v$$

Resistência Equivalente: Circuito Paralelo



Lei de Ohm

$$i = \frac{v}{R}$$

$$i = \frac{v}{R_1} + \frac{v}{R_2} \qquad i = v\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R_{eq}} \qquad R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$i = \frac{v}{R_{eq}}$$

Resistência Equivalente Paralela

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

$$G_{eq} = G_1 + G_2 + \cdots + G_N$$

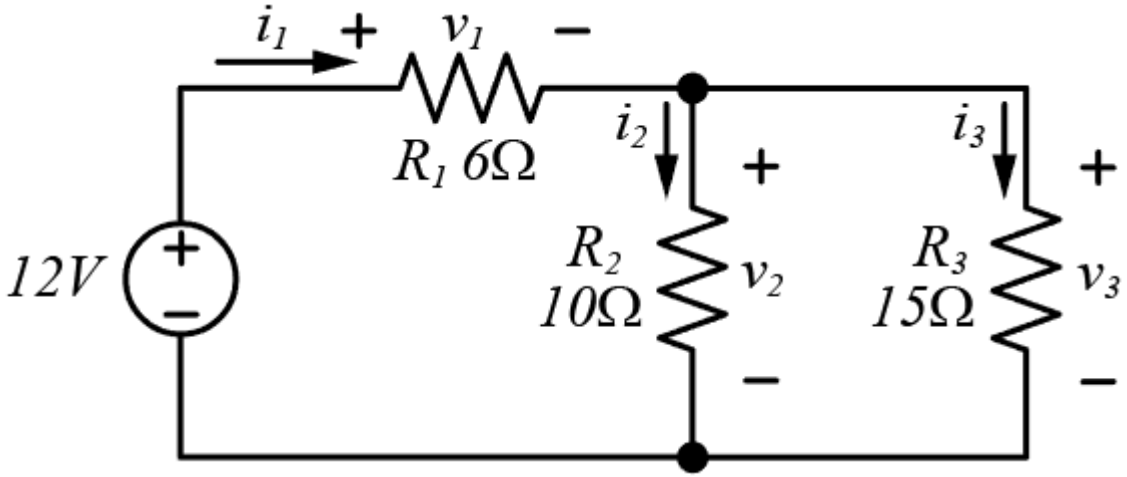
Princípio da Divisão de Corrente

$$i_n = \frac{G_n}{G_1 + G_2 + \cdots + G_N} i$$

$$i_n = \frac{G_n}{G_{eq}} i$$

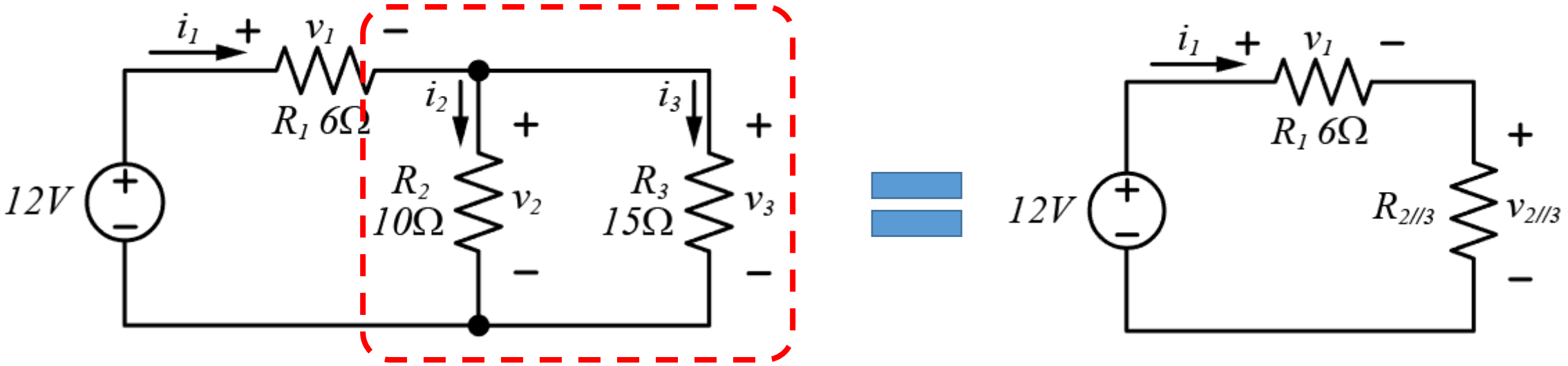
Aplicação

- Calcule os valores de Tensão, Corrente e Potência totais e individuais;



Aplicação

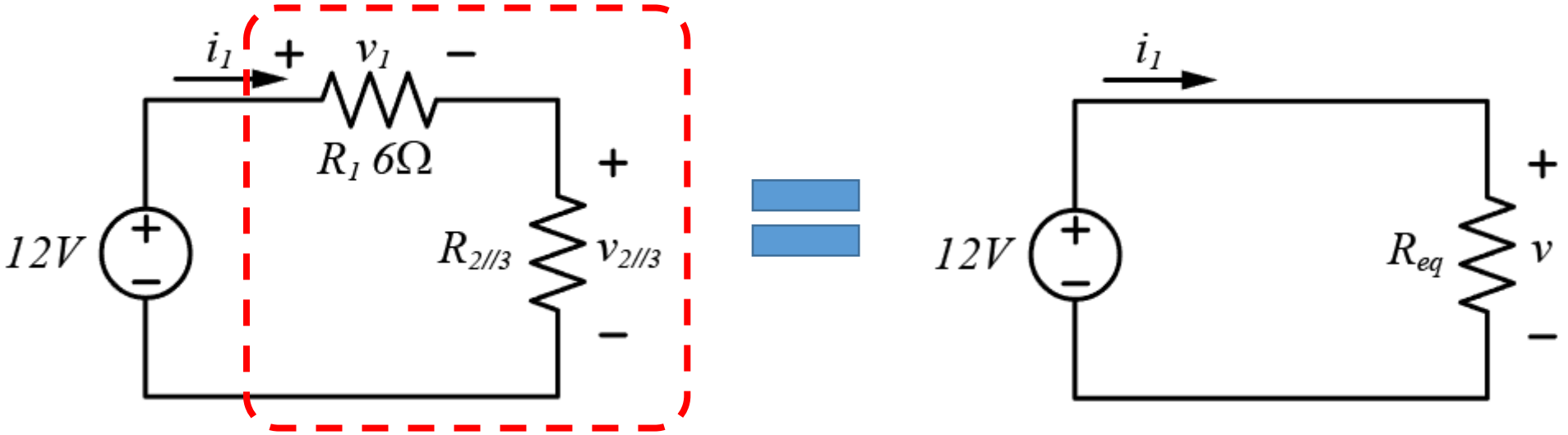
- Calcule os valores de Tensão, Corrente e Potência totais e individuais;



$$\frac{1}{R_{2\|3}} = \frac{1}{R_2} + \frac{1}{R_3} \qquad R_{2\|3} = \frac{R_2 R_3}{R_2 + R_3} \qquad R_{2\|3} = \frac{10\Omega \times 15\Omega}{10\Omega + 15\Omega} \qquad R_{2\|3} = 6\ \Omega$$

Aplicação

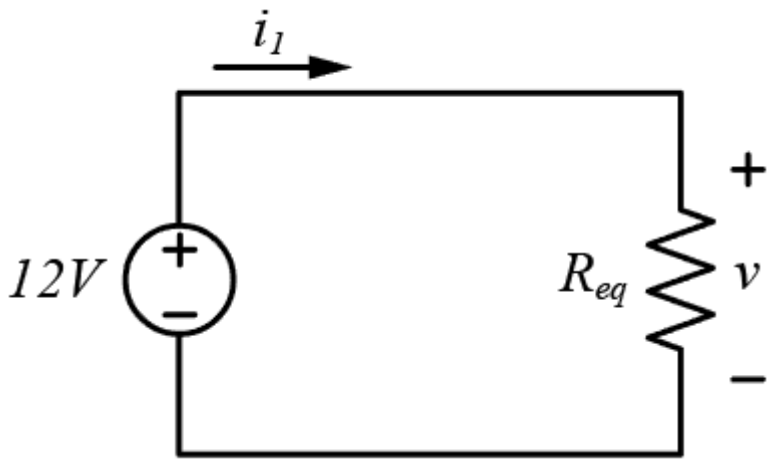
- Calcule os valores de Tensão, Corrente e Potência totais e individuais;



$$R_{eq} = R_1 + R_{2\|3} \qquad R_{eq} = 6\Omega + 6\Omega \qquad R_{eq} = 12\ \Omega$$

Aplicação

- Calcule os valores de Tensão, Corrente e Potência totais e individuais;



$$R_{eq} = 12\ \Omega$$

$$i_1 = \frac{v}{R_{eq}}$$

$$i_1 = \frac{12\ V}{12\ \Omega}$$

$$i_1 = 1\ A$$

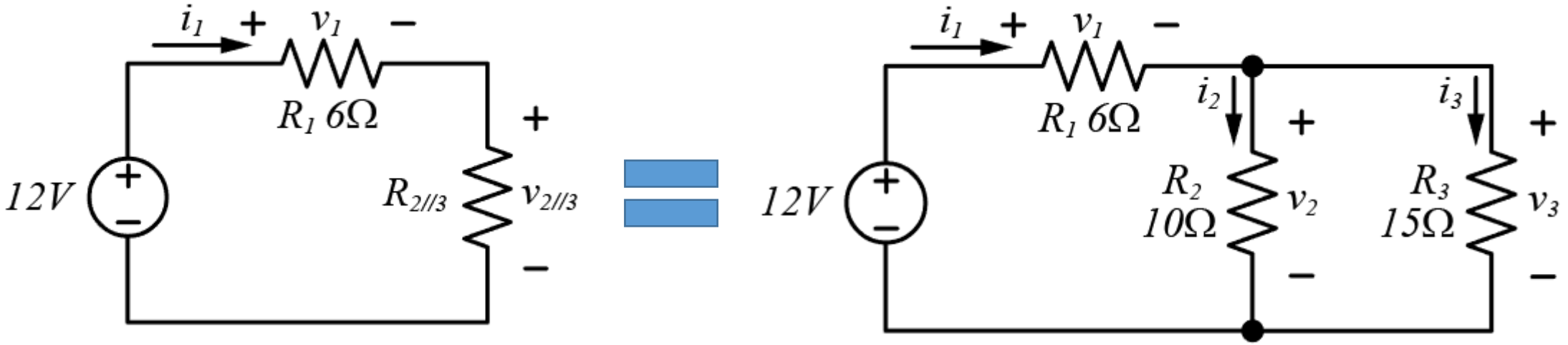
$$P = i_1 v$$

$$P = 1\ A \times 12\ V$$

$$P = 12\ W$$

Aplicação

- Calcule os valores de Tensão, Corrente e Potência totais e individuais;



Princípio da Divisão de Tensão

$$v_n = \frac{R_n}{R_{eq}} v$$

$$v_1 = \frac{R_1}{R_{eq}} v$$

$$v_1 = \frac{6\ \Omega}{12\ \Omega} 12\ V$$

$$v_1 = 6\ V$$

$$v_{2\|3} = \frac{R_{2\|3}}{R_{eq}} v$$

$$v_{2\|3} = \frac{6\ \Omega}{12\ \Omega} 12\ V$$

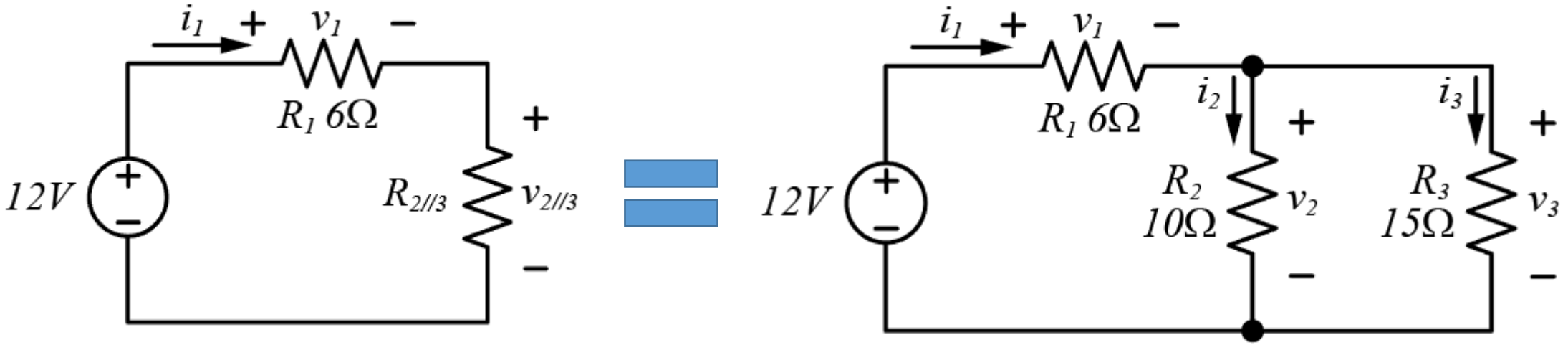
$$v_{2\|3} = 6\ V$$

$$v_2 = 6\ V$$

$$v_3 = 6\ V$$

Aplicação

- Calcule os valores de Tensão, Corrente e Potência totais e individuais;



Princípio da Divisão de Corrente

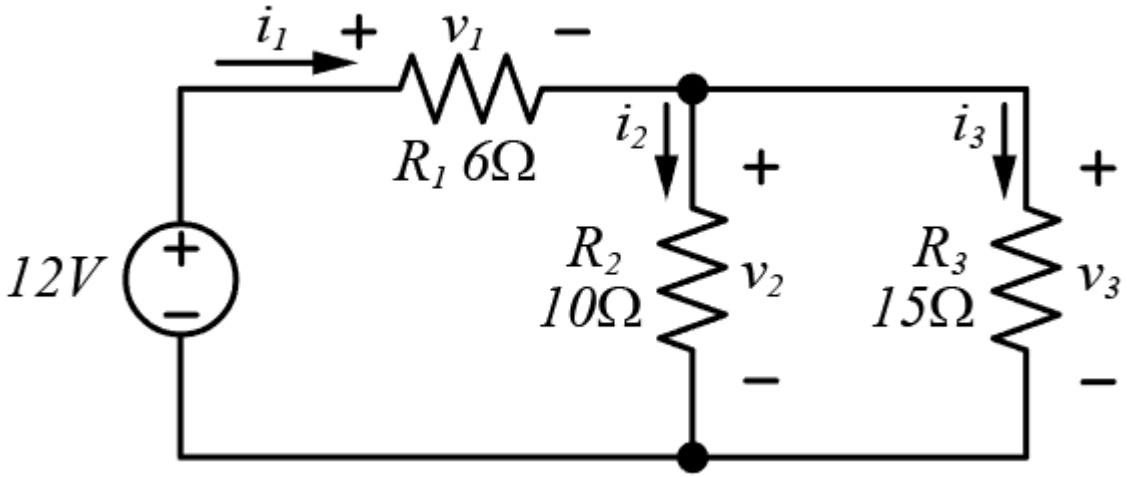
$$i_n = \frac{G_n}{G_{eq}} i$$

$$i_2 = \frac{\frac{1}{R_2}}{\frac{1}{R_{2\|3}}} i_1 \qquad i_2 = \frac{\frac{1}{10\ \Omega}}{\frac{1}{6\ \Omega}} 1\ A \qquad i_2 = \frac{6\ \Omega}{10\ \Omega} 1\ A \qquad \boxed{i_2 = 0{,}6\ A}$$

$$i_3 = \frac{\frac{1}{R_3}}{\frac{1}{R_{2\|3}}} i_1 \qquad i_3 = \frac{\frac{1}{15\ \Omega}}{\frac{1}{6\ \Omega}} 1\ A \qquad i_3 = \frac{6\ \Omega}{15\ \Omega} 1\ A \qquad \boxed{i_3 = 0{,}4\ A}$$

Aplicação

- Calcule os valores de Tensão, Corrente e Potência totais e individuais;



$v = 12\ V$ $\quad$ $i = i_1 = 1\ A$ $\quad$ $P = 12\ W$

$v_1 = 6\ V$ $\quad$ $i_1 = 1\ A$

$v_2 = 6\ V$ $\quad$ $i_2 = 0{,}6\ A$

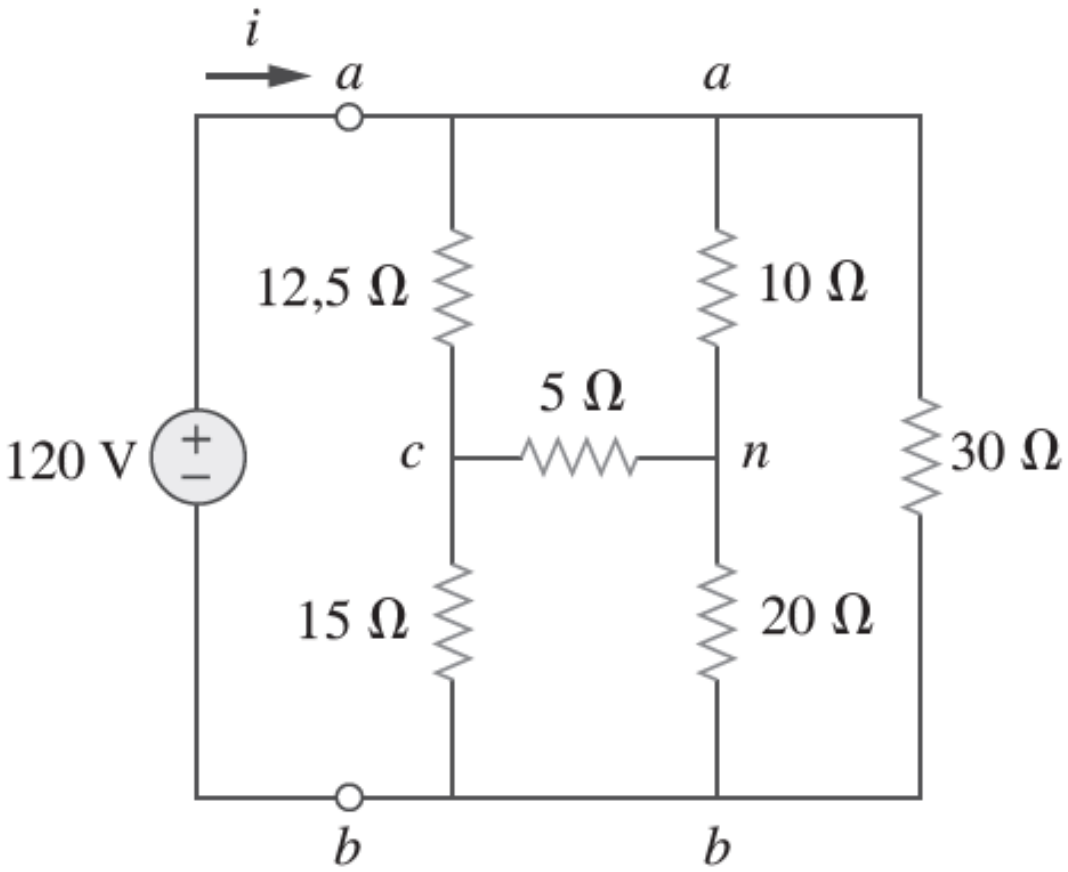
$v_3 = 6\ V$ $\quad$ $i_3 = 0{,}4\ A$

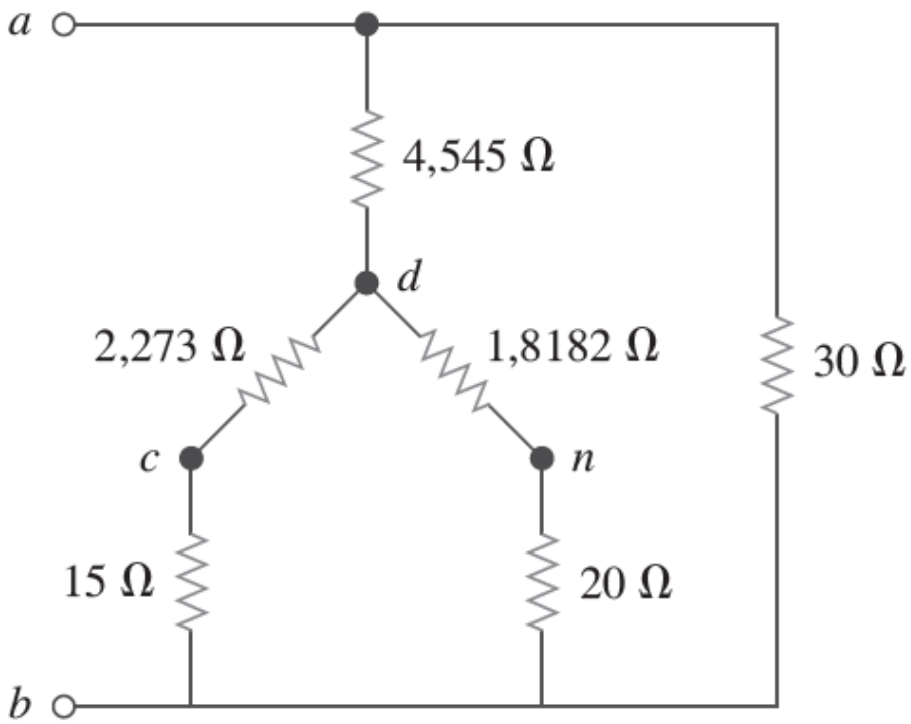
$P_1 = i_1 v_1$ $\quad$ $P_2 = i_2 v_2$ $\quad$ $P_3 = i_3 v_3$

$P_T = 1\ A \times 6\ V$ $\quad$ $P_2 = 0{,}6\ A \times 6\ V$ $\quad$ $P_3 = 0{,}4\ A \times 6\ V$

$P_1 = 6\ W$ $\quad$ $P_1 = 3{,}6\ W$ $\quad$ $P_3 = 2{,}4\ W$

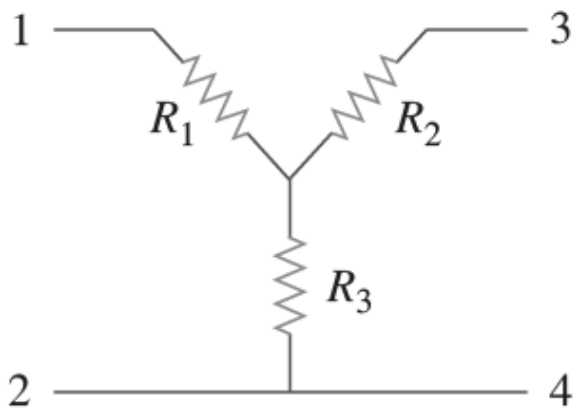
Conversões Y-Delta (Estrela-Triângulo)





Conversões Y-Delta (Estrela-Triângulo)

Y



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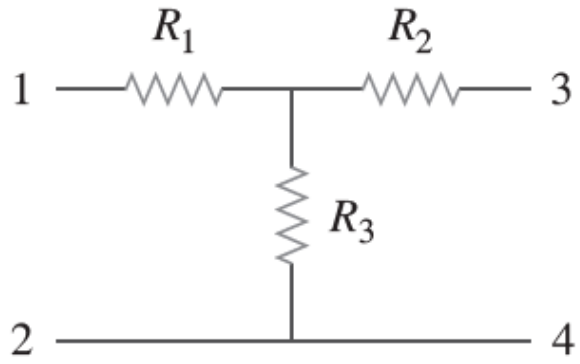
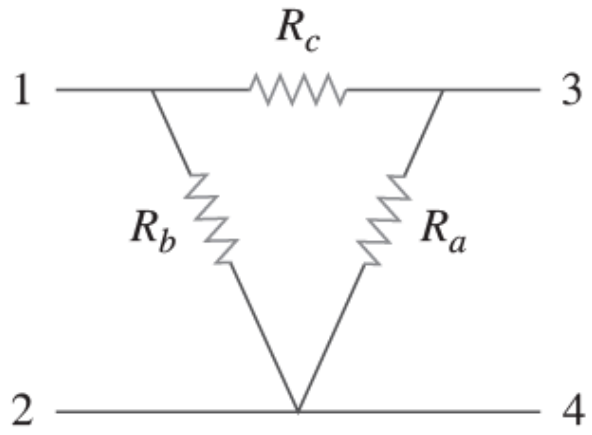


Δ



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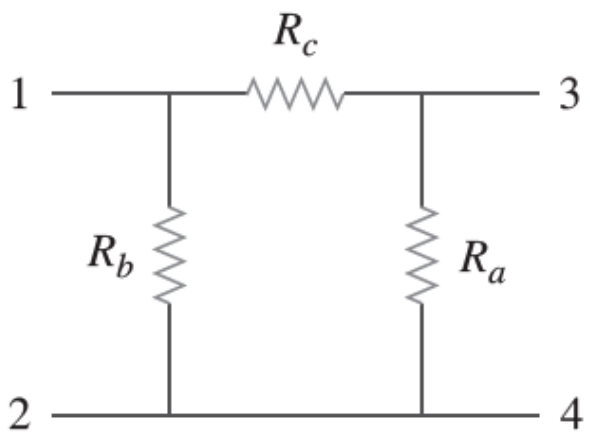
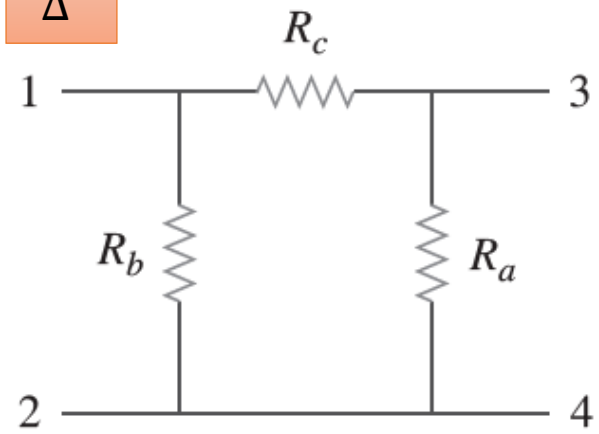


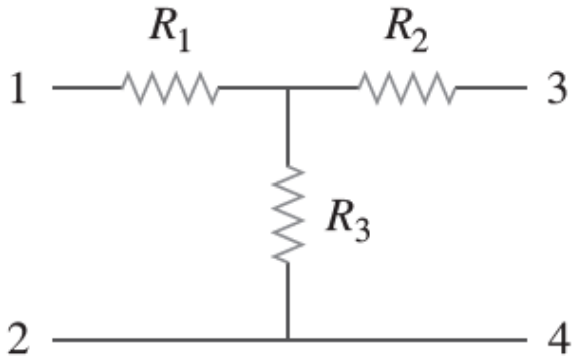
Conversão Delta-Y

Δ



$$R_{12}(\Delta) = R_b \parallel (R_a + R_c)$$

Y



$$R_{12}(Y) = R_1 + R_3$$

$$R_{12} = R_1 + R_3 = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c} \qquad R_3 = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c} - R_1$$

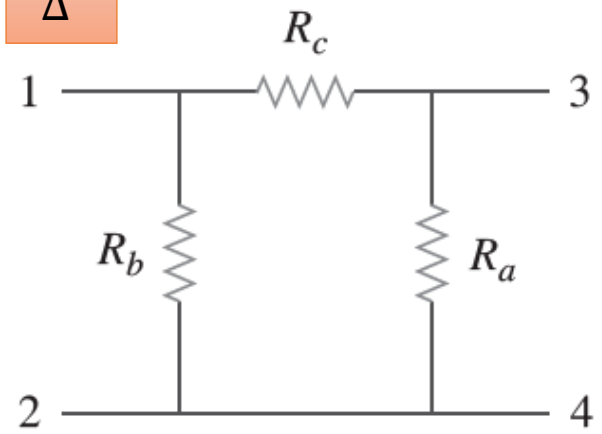
$$R_{13} = R_1 + R_2 = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c}$$

$$R_1 - R_2 = \frac{R_b R_a + R_b R_c - R_a R_b - R_a R_c}{R_a + R_b + R_c} \qquad R_1 - R_2 = \frac{R_c(R_b - R_a)}{R_a + R_b + R_c}$$

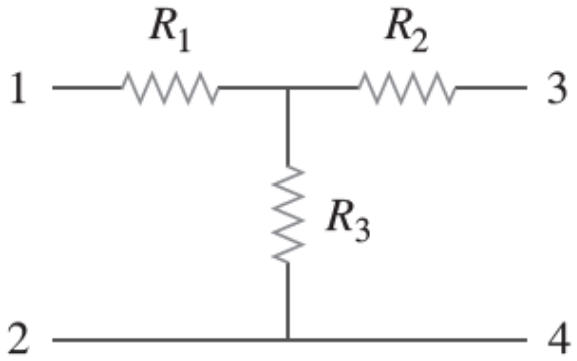
$$R_{34} = R_2 + R_3 = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} \qquad R_3 = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} - R_2$$

Conversão Delta-Y

Δ



Y



$$R_1 + R_2 = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c}$$

$$R_1 - R_2 = \frac{R_c(R_b - R_a)}{R_a + R_b + R_c}$$

$$2R_1 = \frac{R_c R_a + R_c R_b + R_c R_b - R_c R_a}{R_a + R_b + R_c}$$

$$2R_1 = \frac{2R_c R_b}{R_a + R_b + R_c}$$

$$R_1 = \frac{R_c R_b}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

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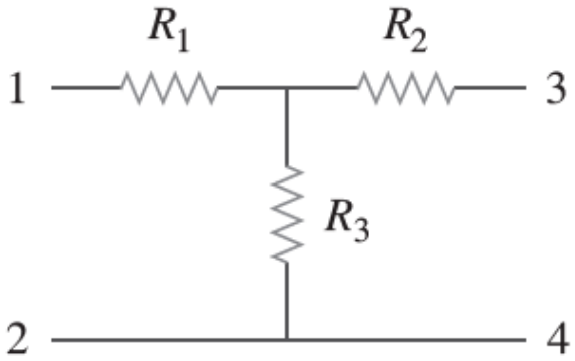
Conversão Y-Delta

Y



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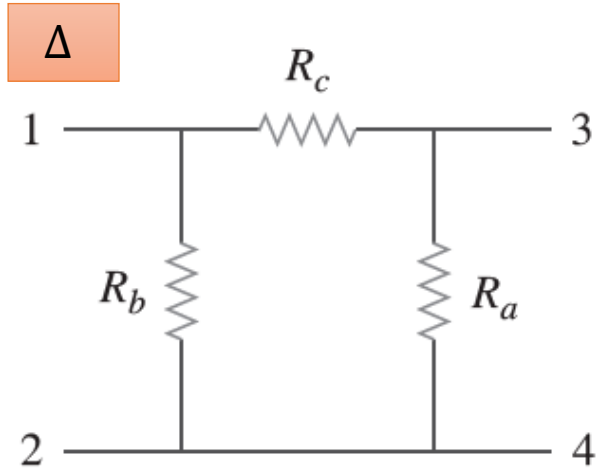


$$R_1 = \frac{R_c R_b}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

$$R_1 R_2 = \frac{R_c R_b R_c R_a}{(R_a + R_b + R_c)^2}$$

$$R_2 R_3 = \frac{R_c R_a R_a R_b}{(R_a + R_b + R_c)^2}$$

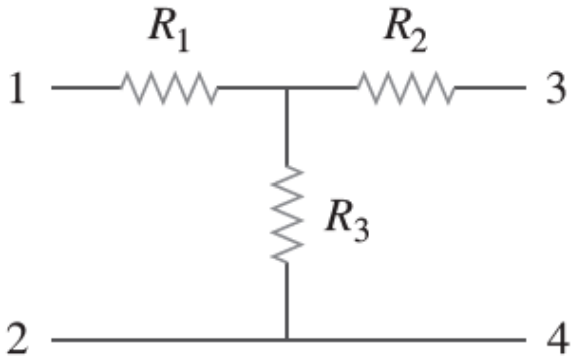
$$R_3 R_1 = \frac{R_a R_b R_c R_b}{(R_a + R_b + R_c)^2}$$

$$R_1 R_2 + R_2 R_3 + R_3 R_1 = \frac{R_c R_b R_c R_a}{(R_a + R_b + R_c)^2} + \frac{R_c R_a R_a R_b}{(R_a + R_b + R_c)^2} + \frac{R_a R_b R_c R_b}{(R_a + R_b + R_c)^2}$$

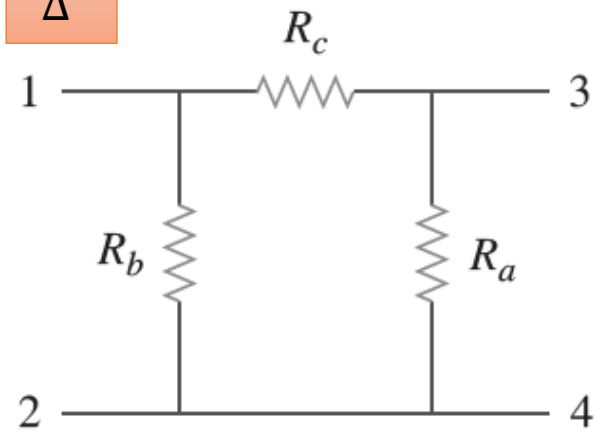
$$R_1 R_2 + R_2 R_3 + R_3 R_1 = \frac{R_a R_b R_c (R_a + R_b + R_c)}{(R_a + R_b + R_c)^2} = \frac{R_a R_b R_c}{R_a + R_b + R_c}$$

Conversão Y-Delta

Y



Δ



$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_aR_bR_c}{R_a + R_b + R_c}$$

$$R_1 = \frac{R_cR_b}{R_a + R_b + R_c}$$

$$\frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} = R_a$$

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

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Conversões Delta-Y: Resumo

Δ-Y

$$R_1 = \frac{R_c R_b}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

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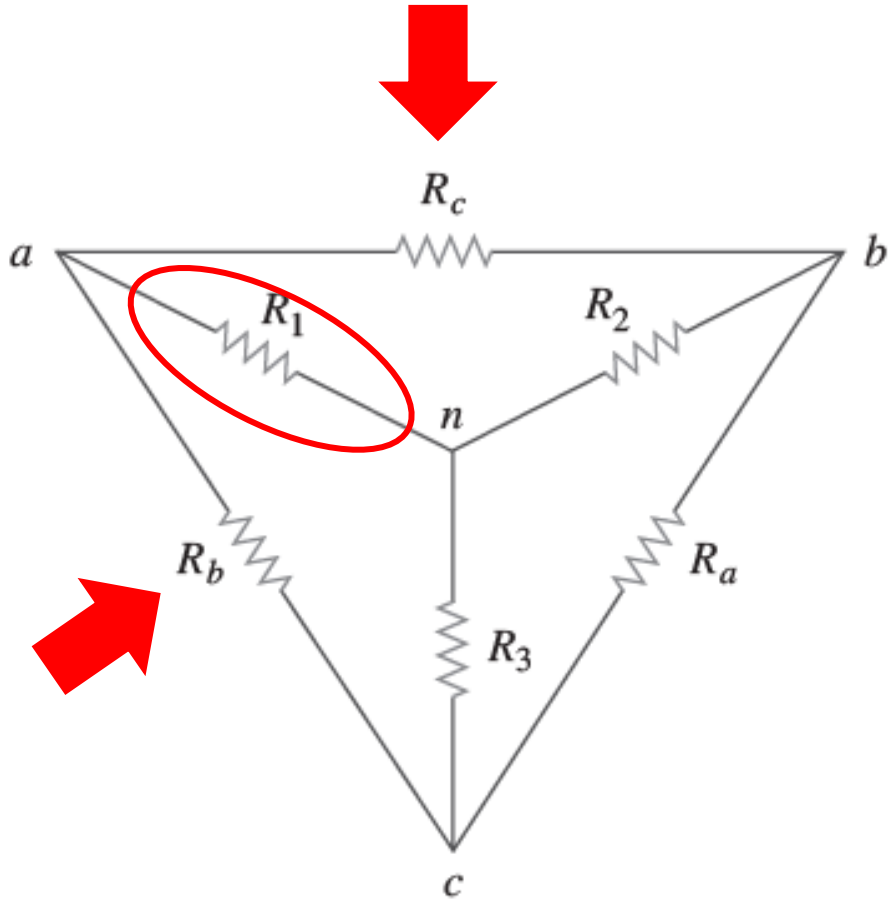

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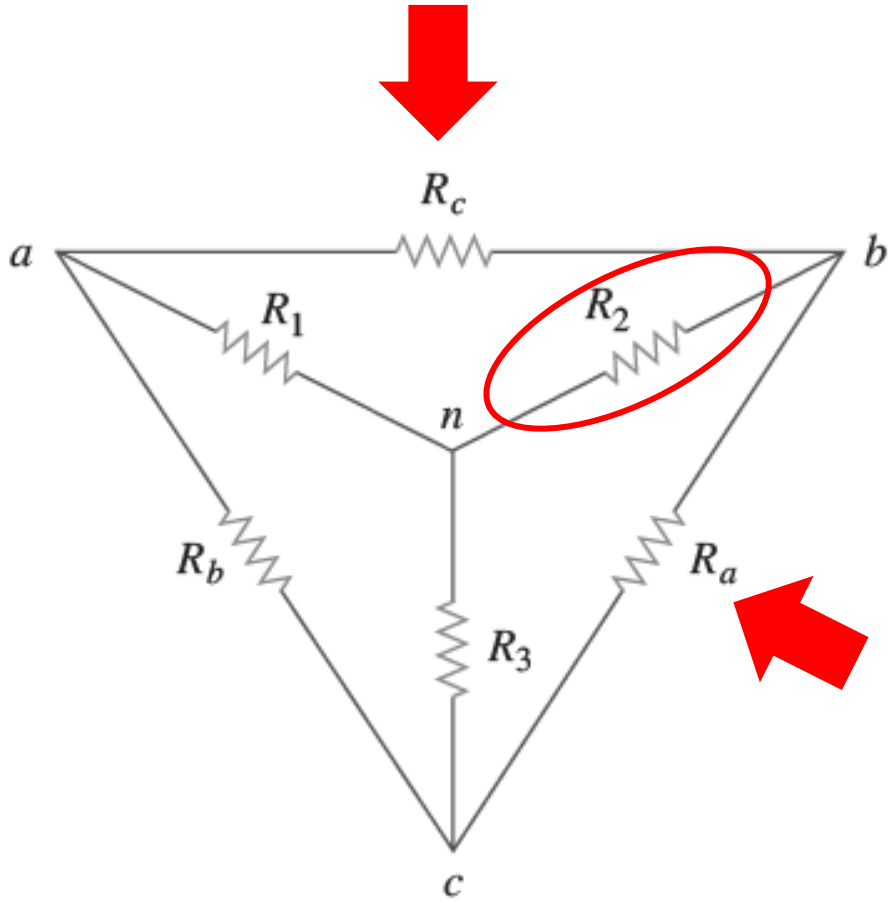

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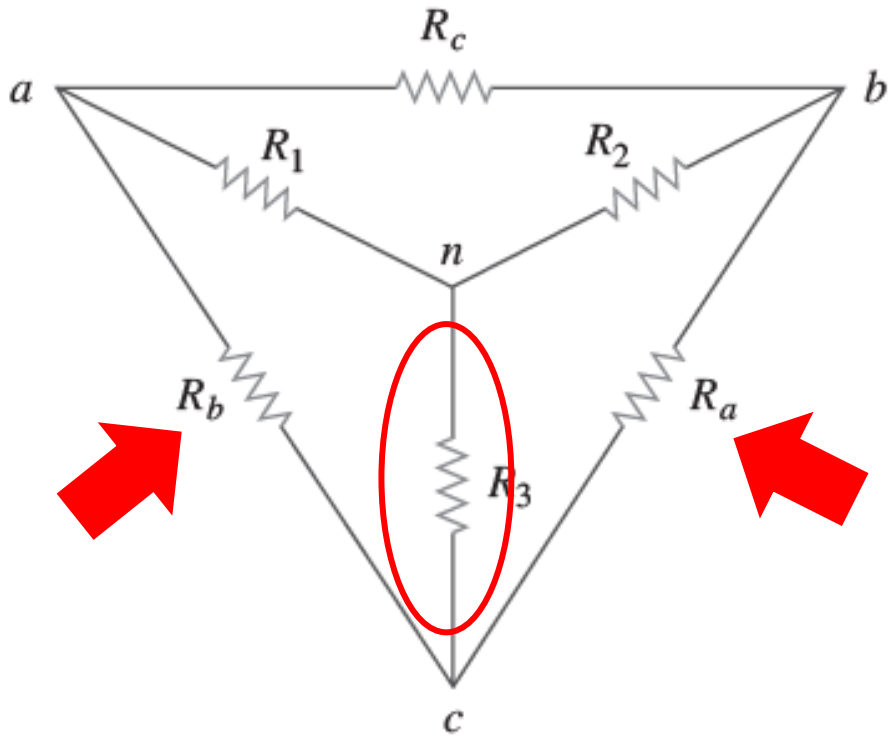
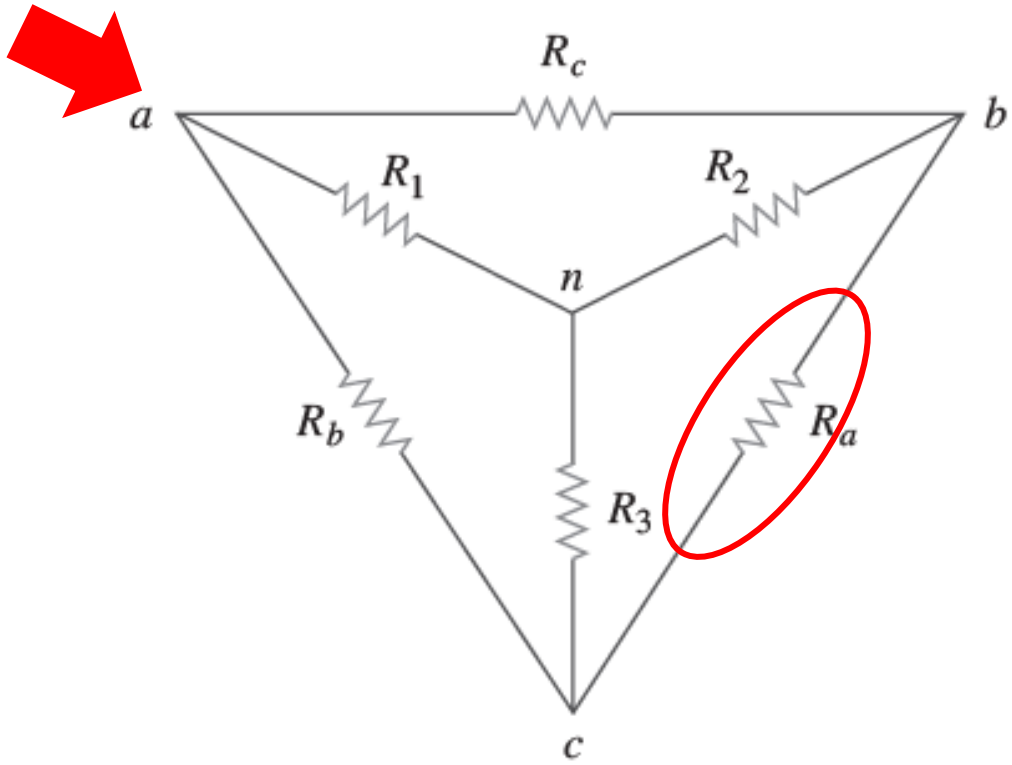


Conversões Y-Delta: Resumo



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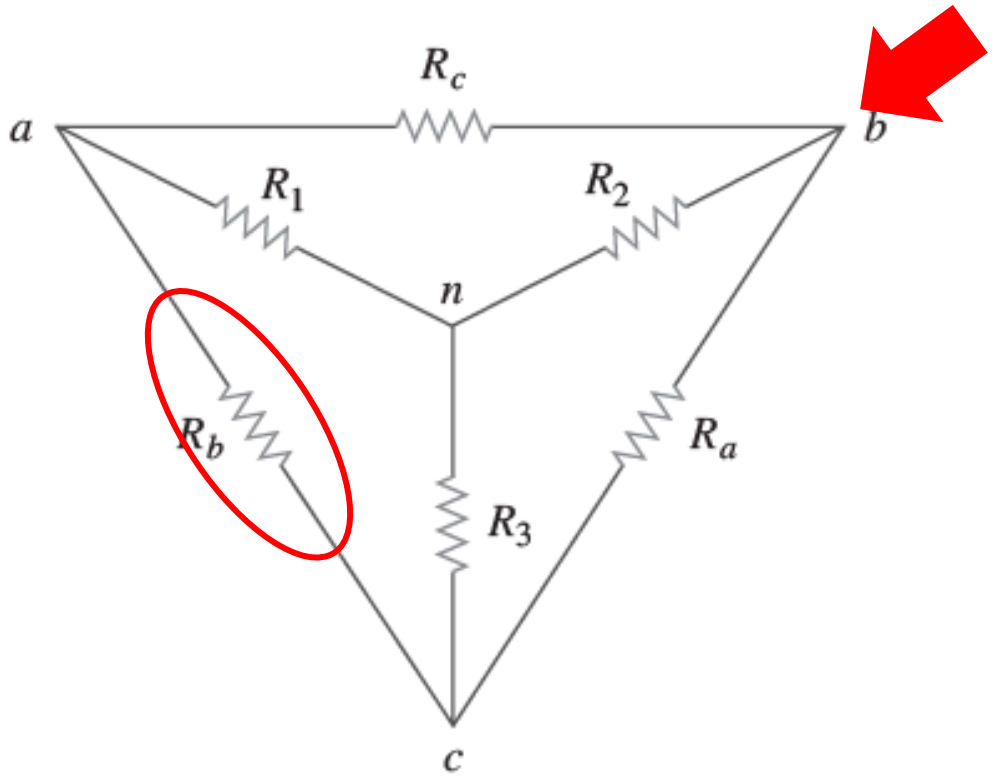

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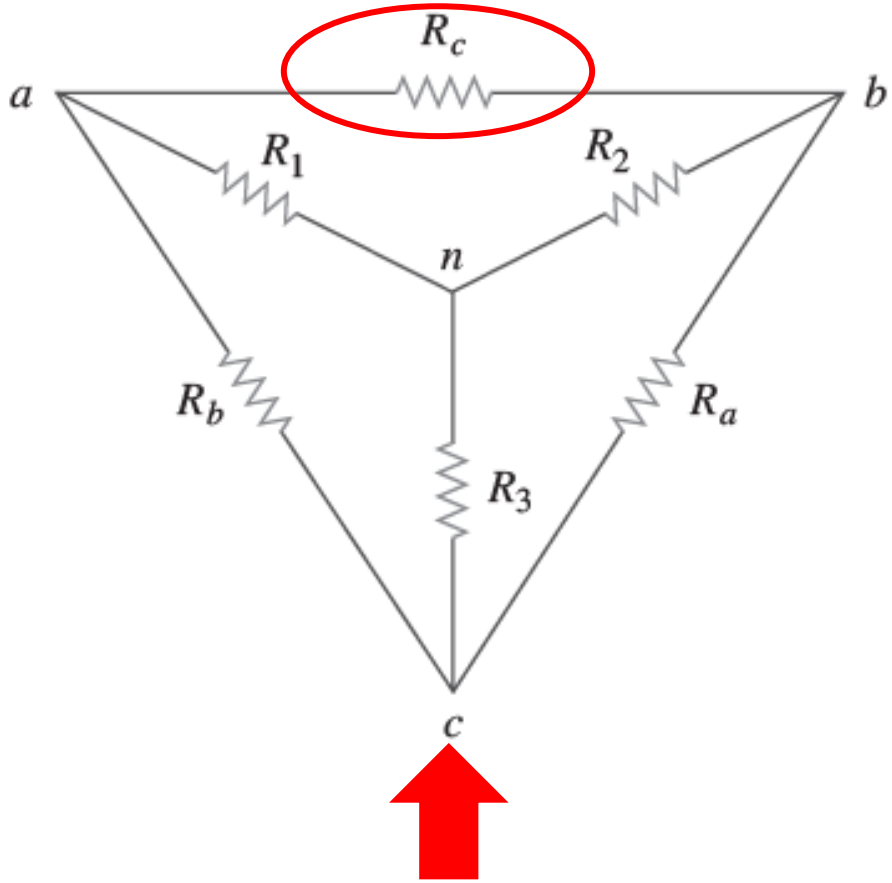

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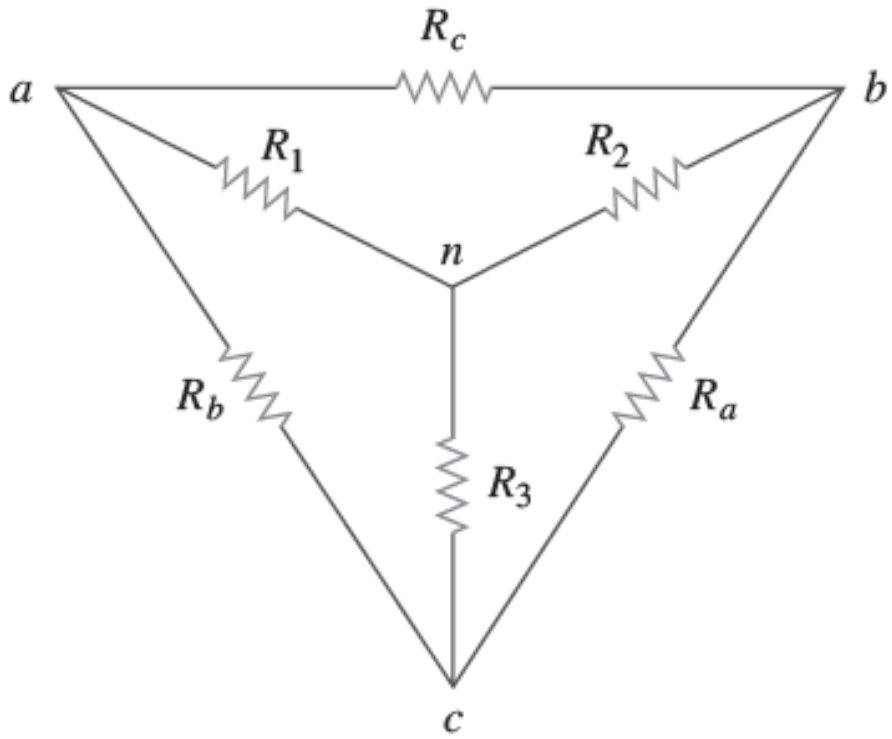

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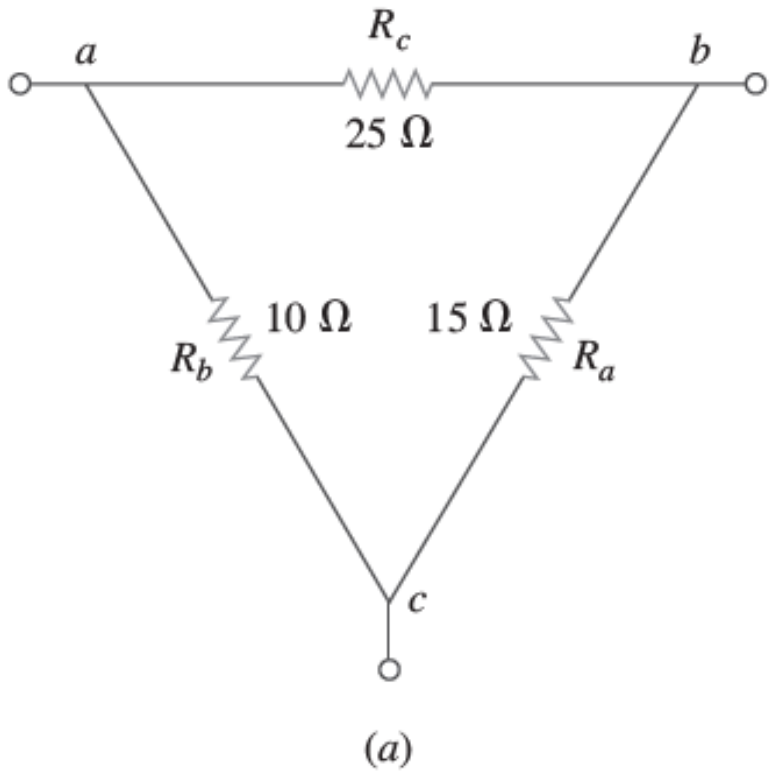
Conversões Y-Delta: Exemplo

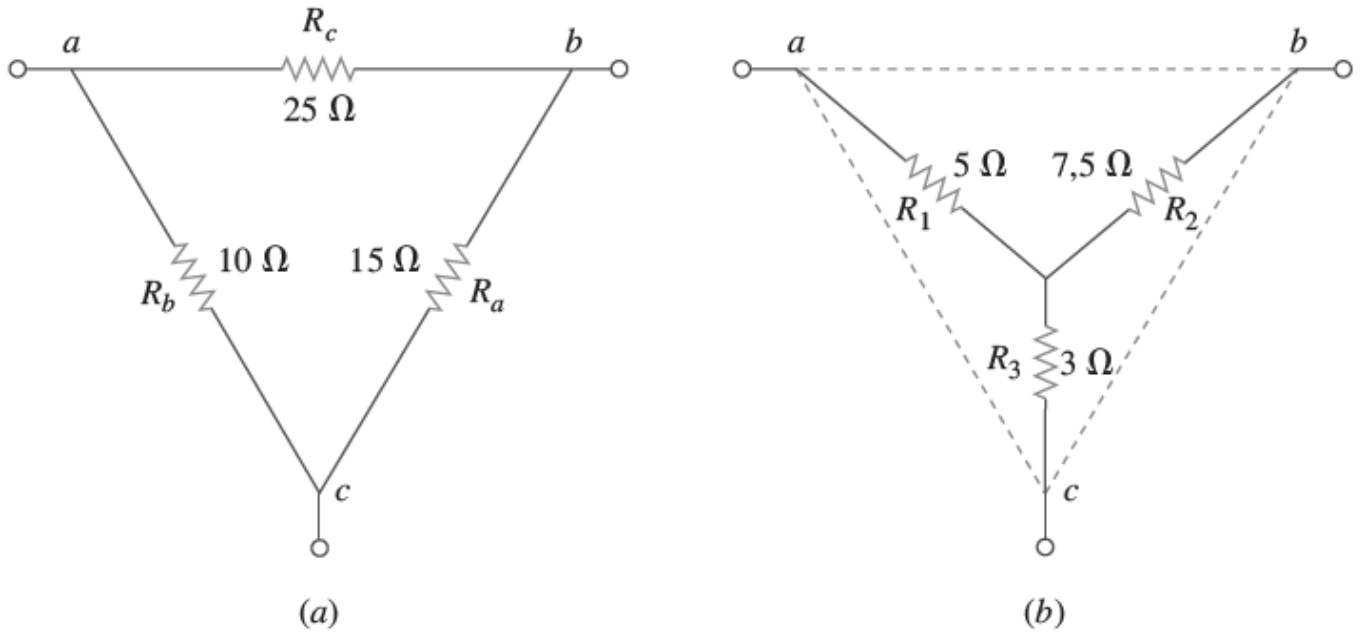
- Converta a rede Δ em uma rede Y equivalente.



(a)

Conversões Y-Delta: Exemplo

- Converta a rede Δ em uma rede Y equivalente.



Δ-Y

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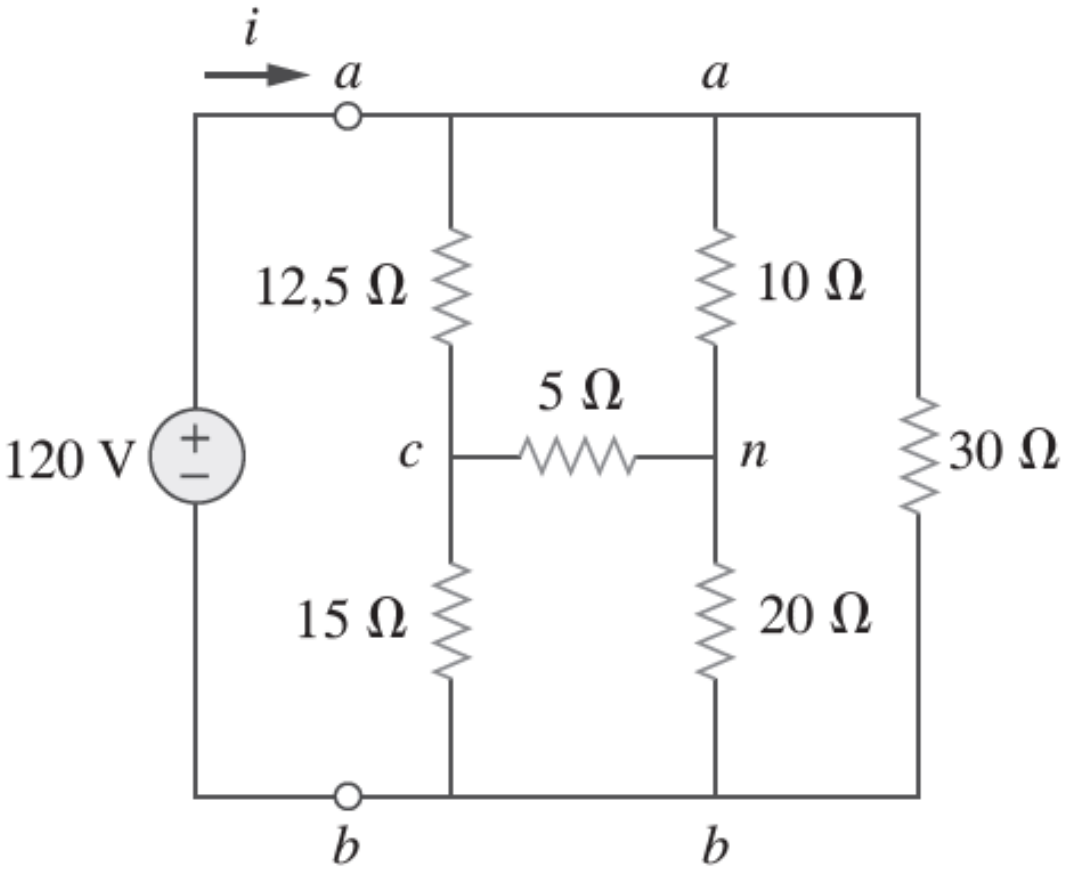
$$R_1 = \frac{25 \times 10}{15 + 10 + 25} = 5\Omega \qquad R_2 = \frac{25 \times 15}{15 + 10 + 25} = 7{,}5\Omega \qquad R_3 = \frac{15 \times 10}{15 + 10 + 25} = 3\Omega$$

Conversões Y-Delta: Exemplo

- Calcular a corrente i e a resistência equivalente do circuito R_{ab}.



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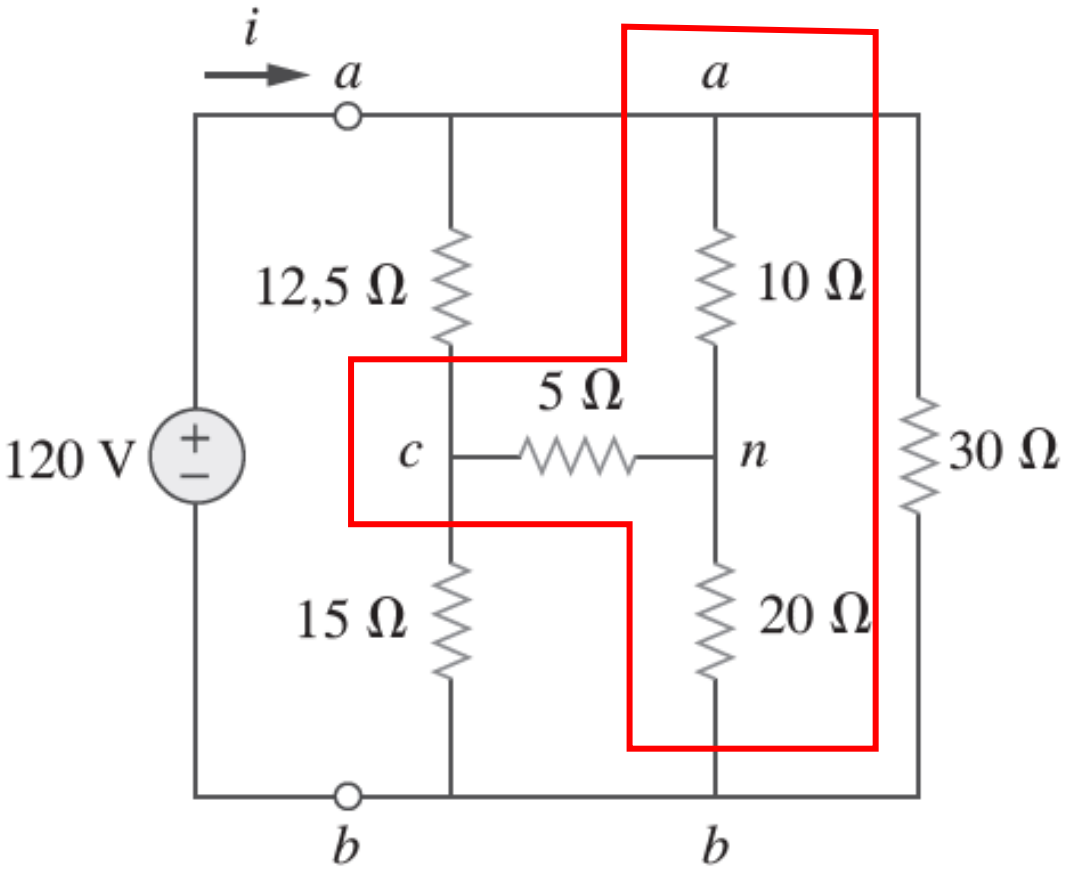

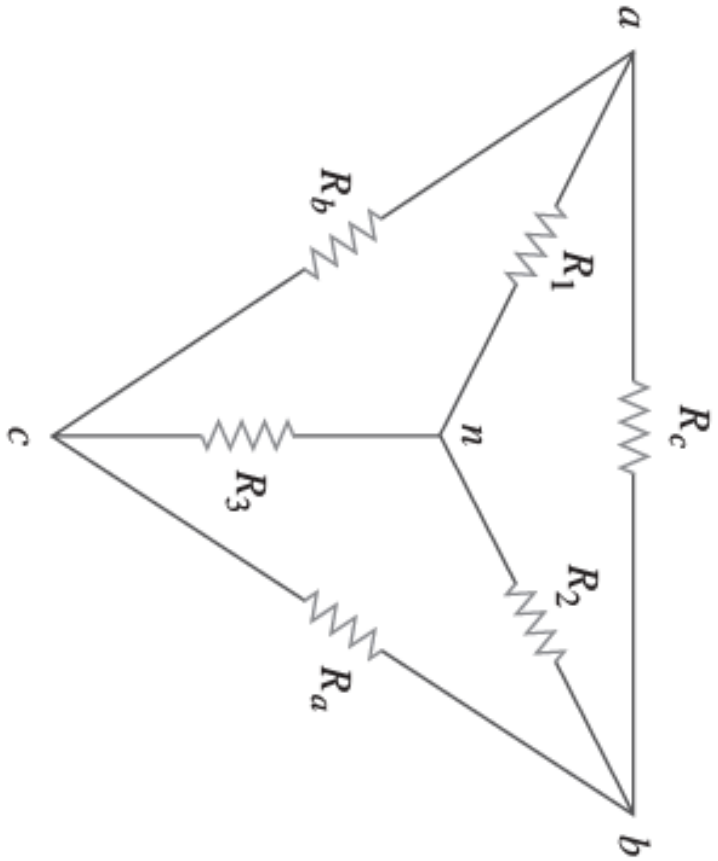

Y-Δ

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

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Conversões Y-Delta: Exemplo

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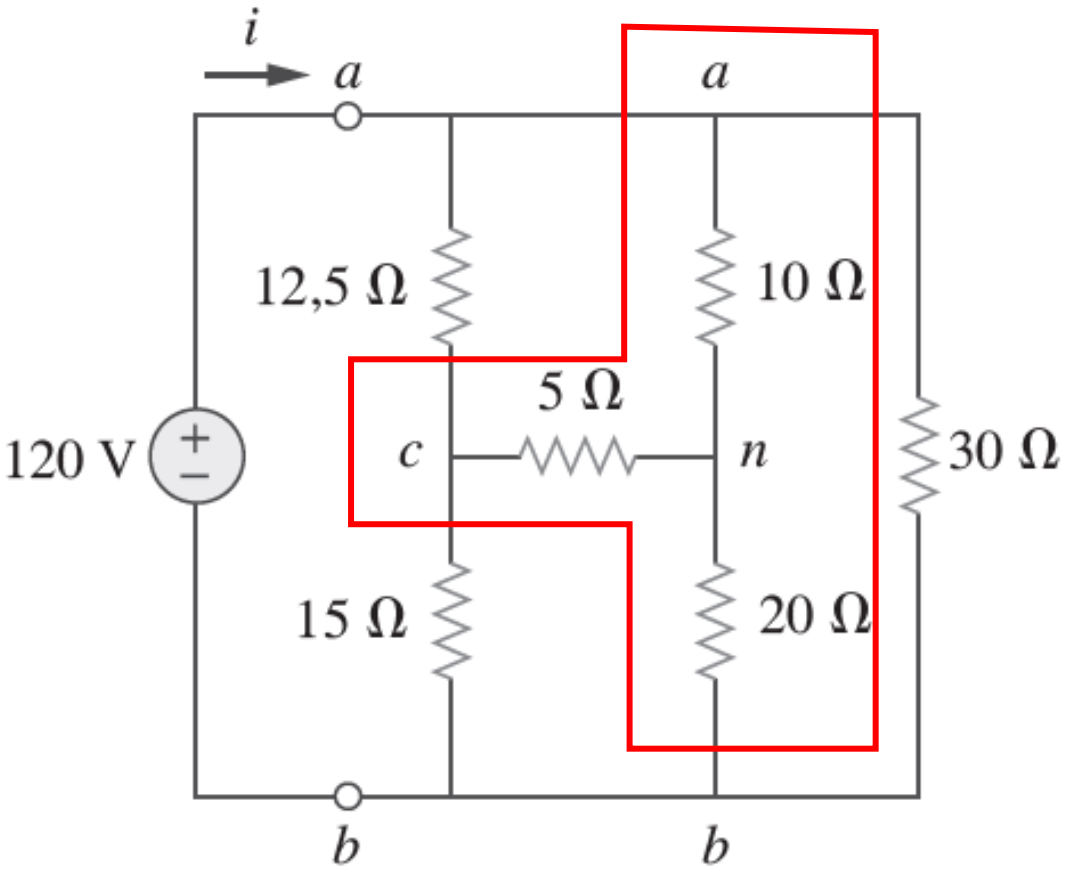
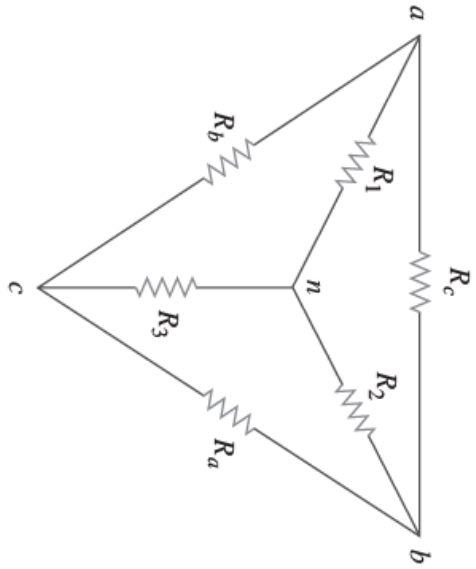




Y-Δ

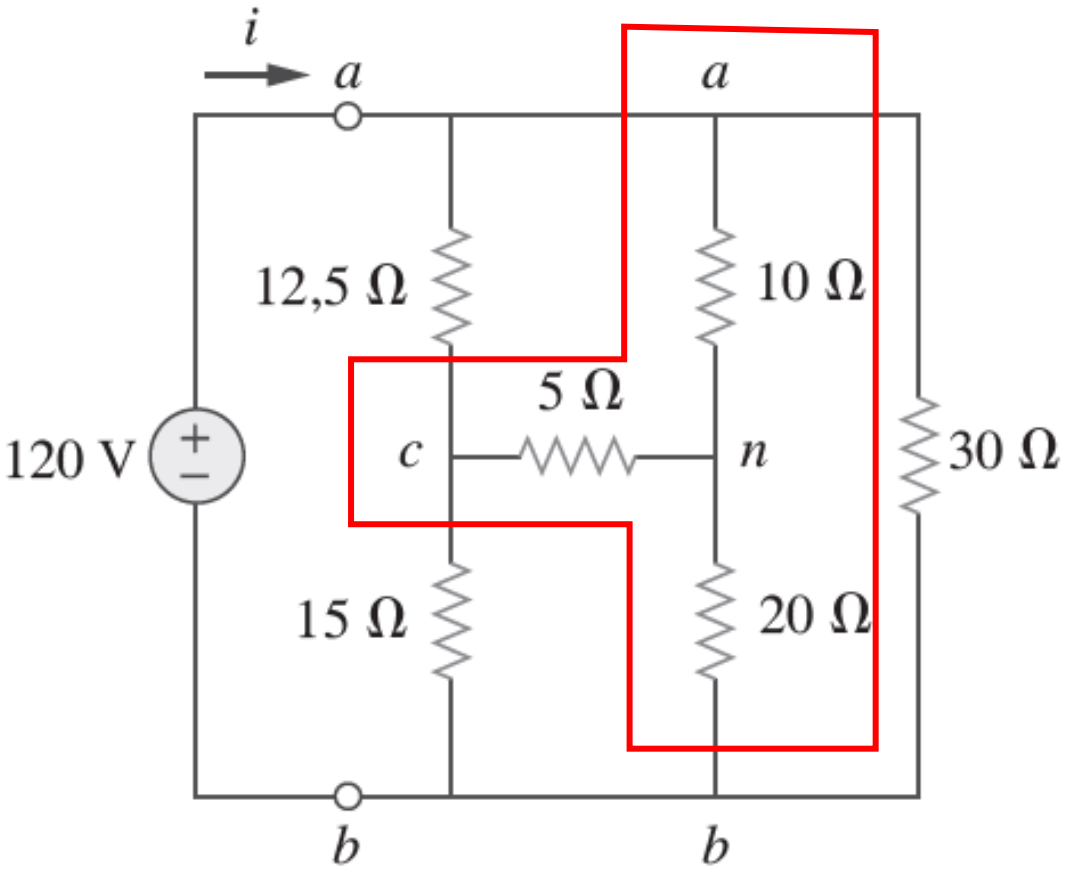
$$R_a = \frac{10 \times 20 + 20 \times 5 + 5 \times 10}{10} = 35\Omega$$

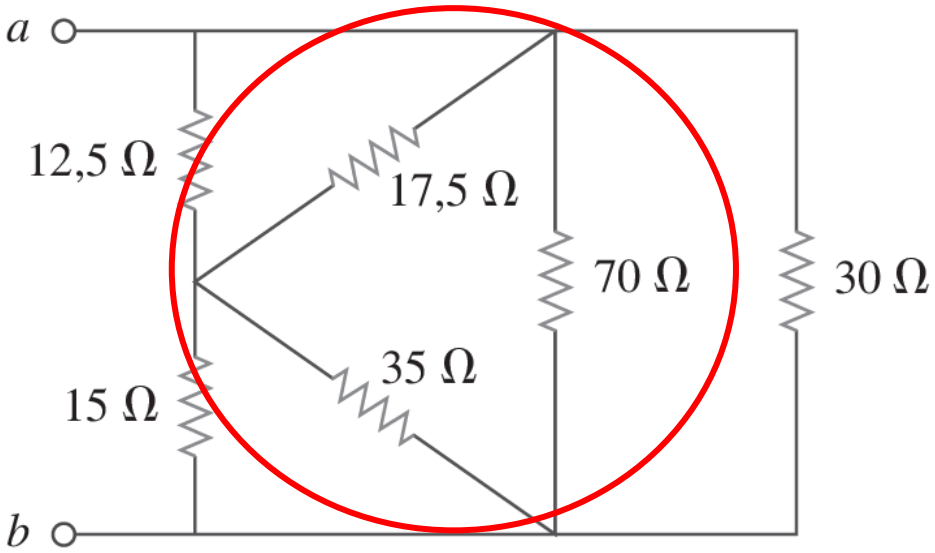
$$R_b = \frac{10 \times 20 + 20 \times 5 + 5 \times 10}{20} = 17{,}5\Omega$$

$$R_c = \frac{10 \times 20 + 20 \times 5 + 5 \times 10}{5} = 70\Omega$$

Conversões Y-Delta: Exemplo

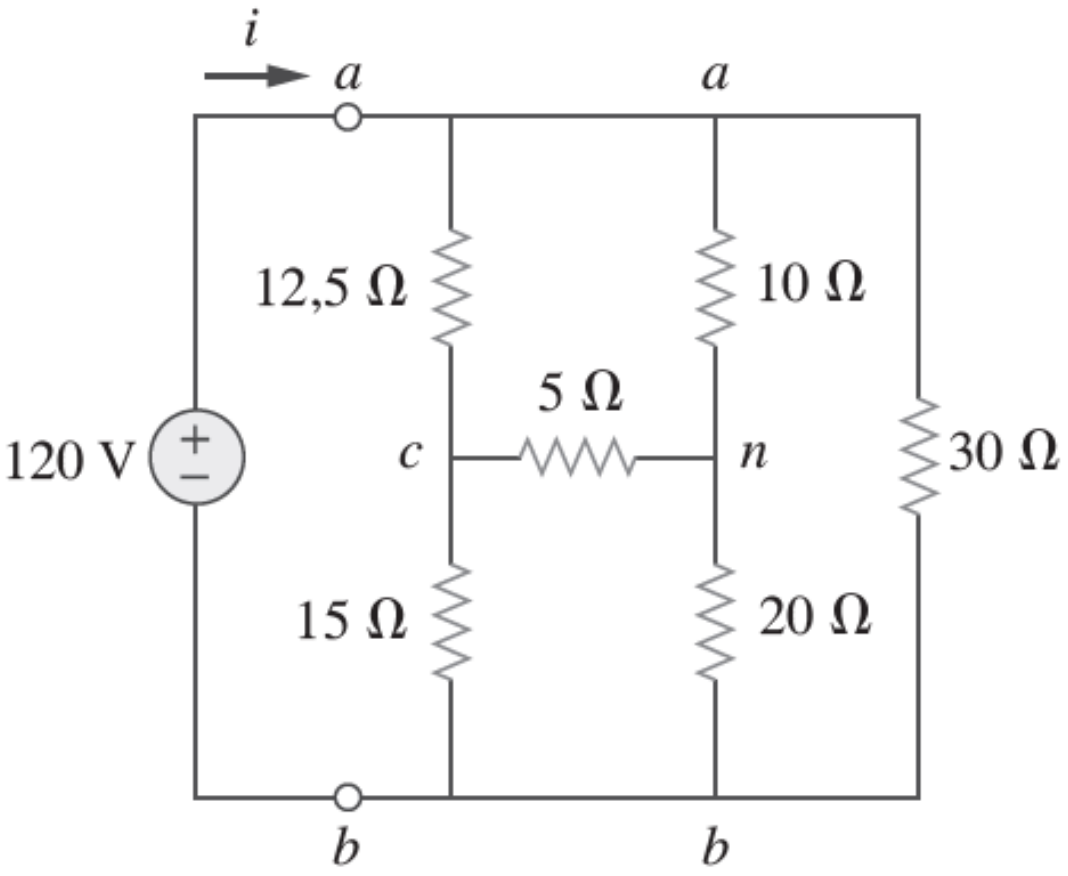
- Calcular a corrente i e a resistência equivalente do circuito R_{ab}.

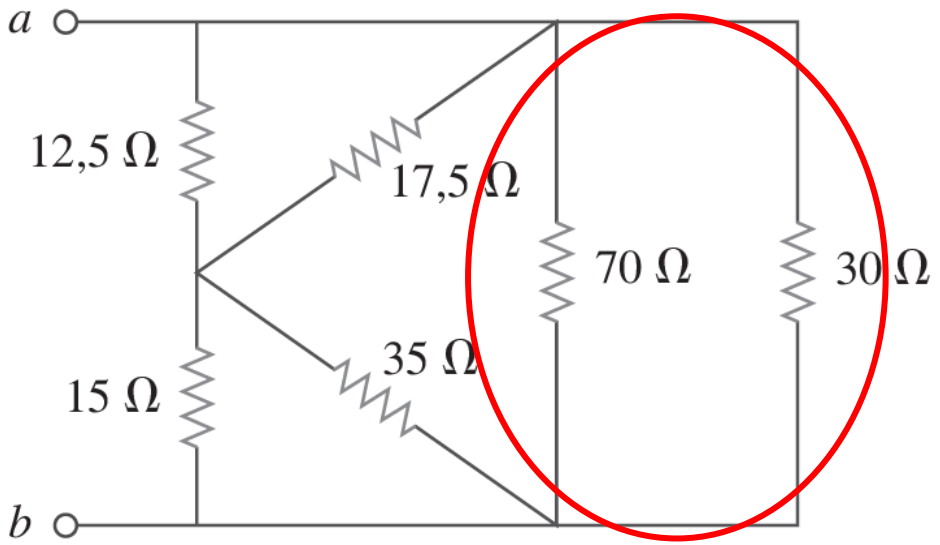




Conversões Y-Delta: Exemplo

- Calcular a corrente i e a resistência equivalente do circuito R_{ab}.

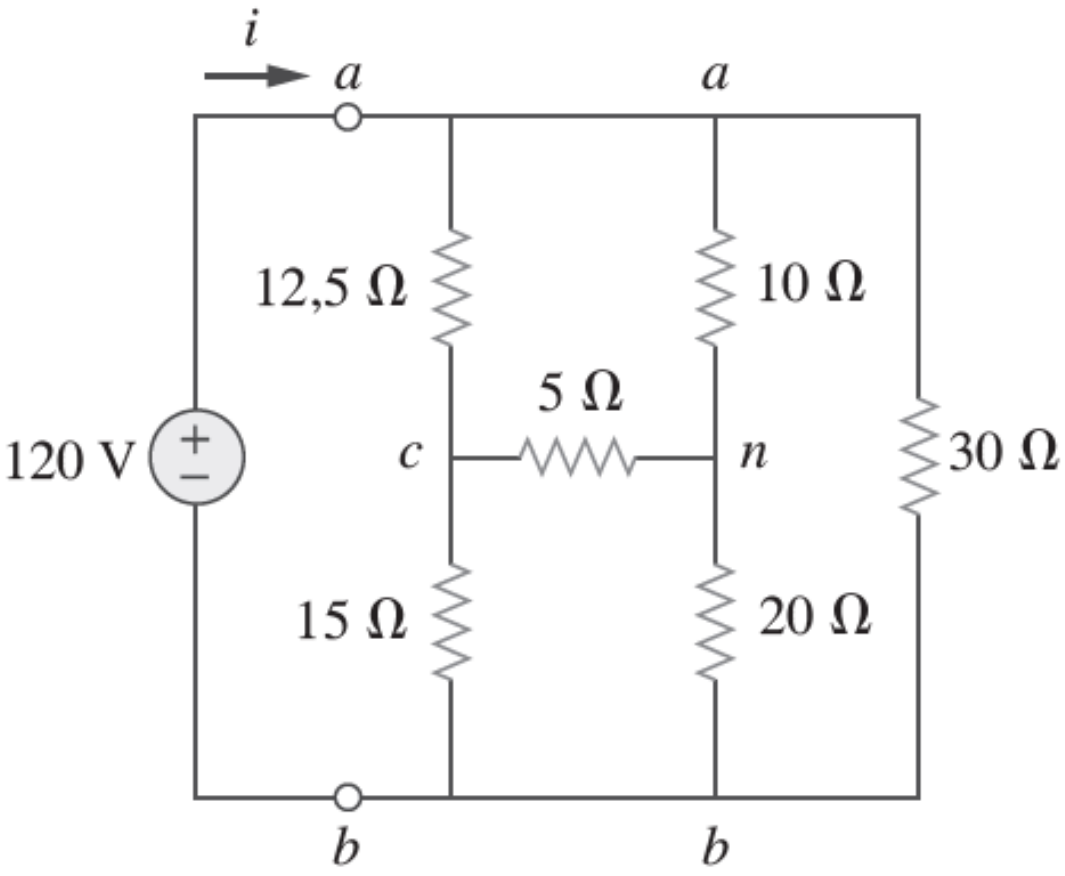


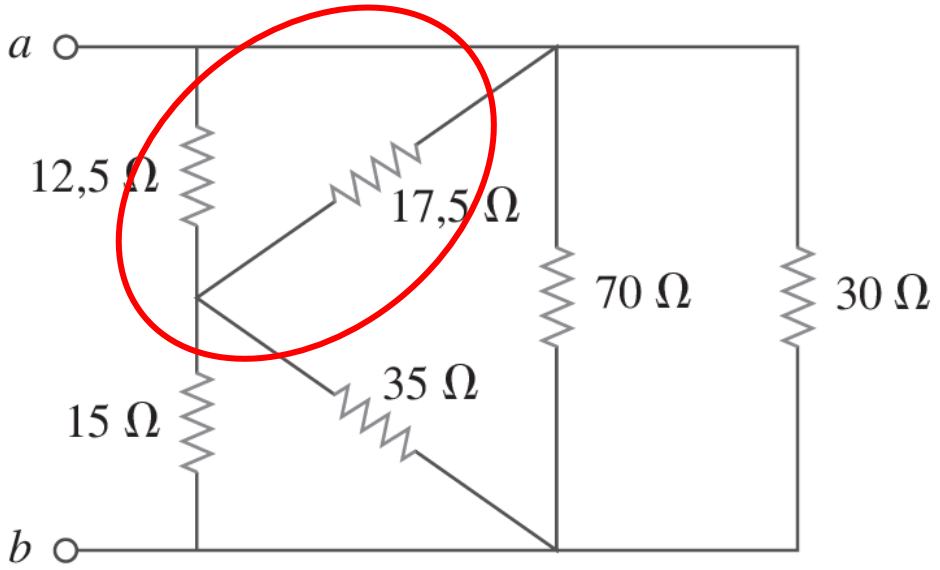


$$R_{70\|30} = \frac{70 \times 30}{70 + 30} = 21\Omega$$

Conversões Y-Delta: Exemplo

- Calcular a corrente i e a resistência equivalente do circuito R_{ab}.

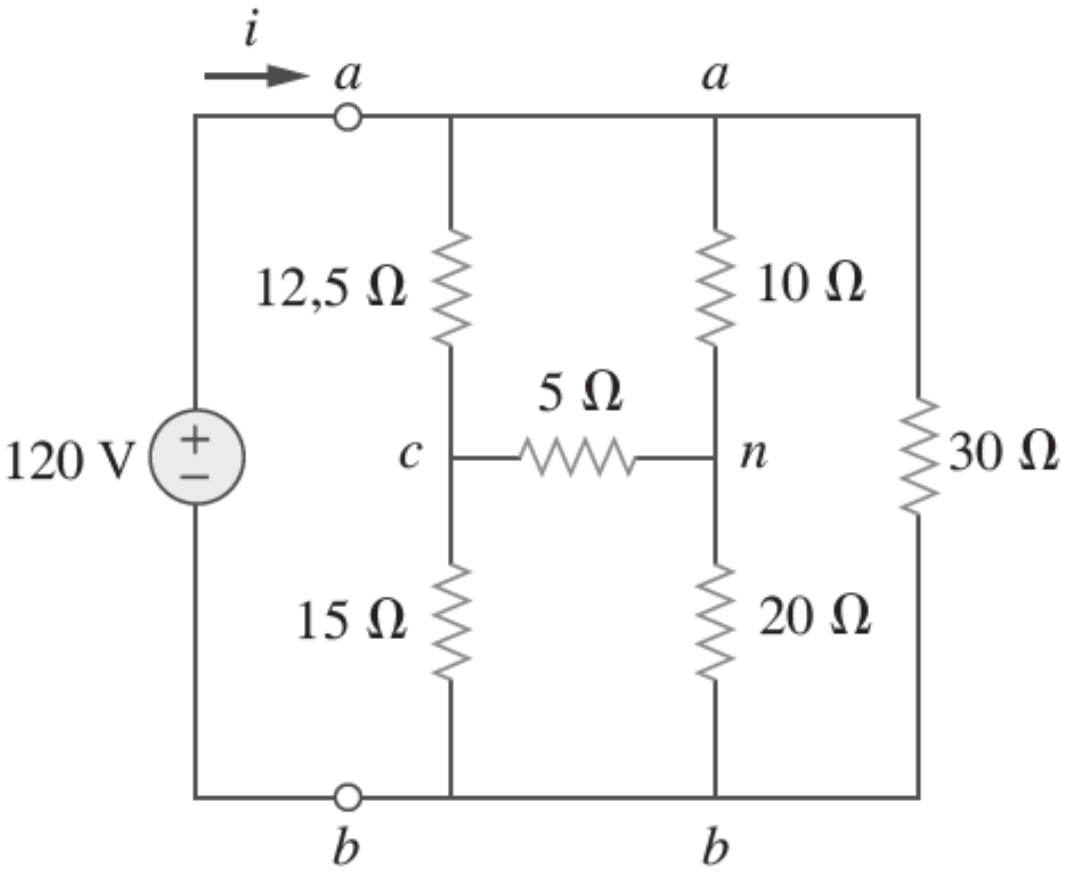


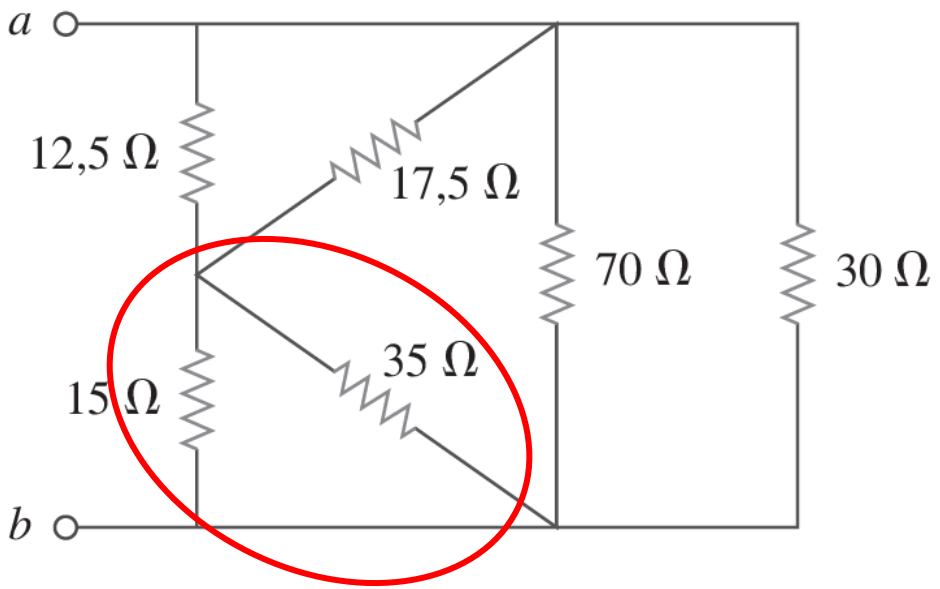


$$R_{12,5\|17,5} = \frac{12,5 \times 17,5}{12,5 + 17,5} = 7,292\Omega$$

Conversões Y-Delta: Exemplo

- Calcular a corrente i e a resistência equivalente do circuito R_{ab}.

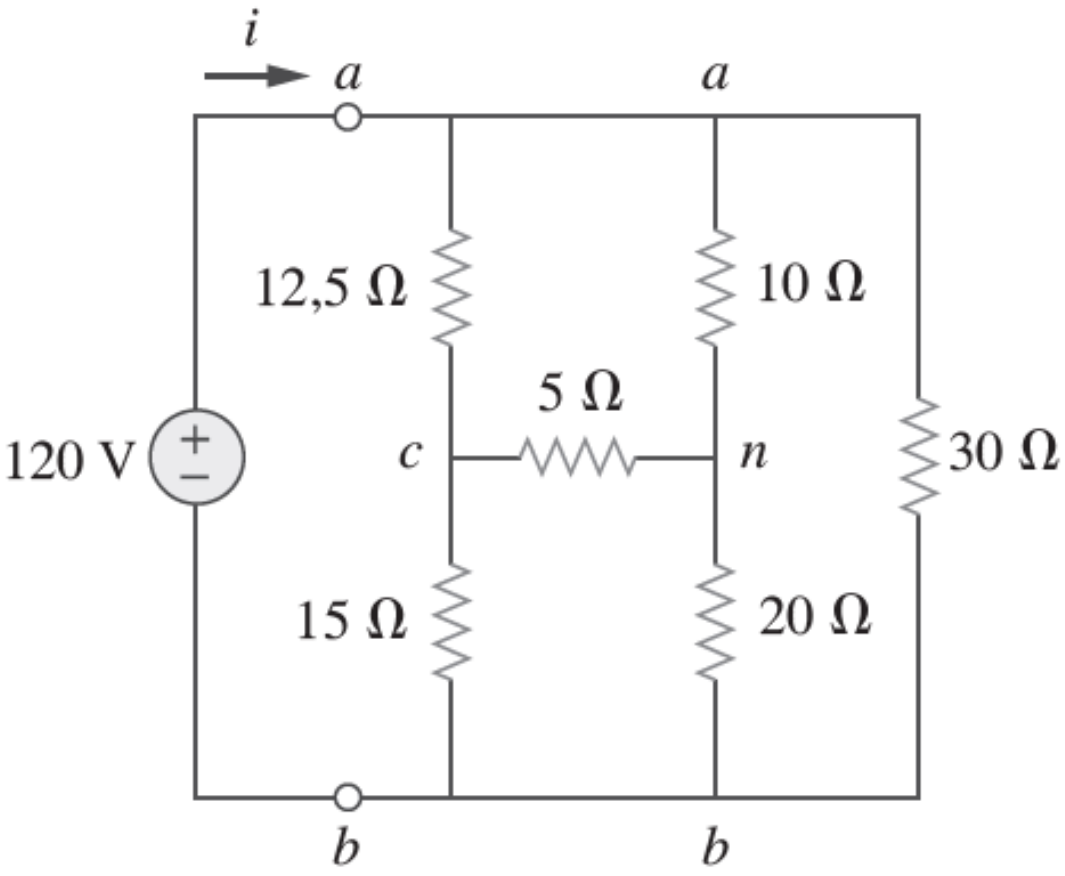


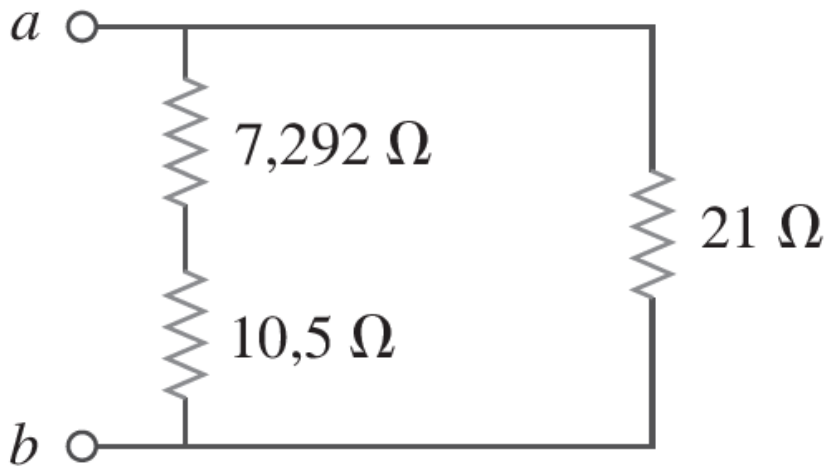


$$R_{15\|35} = \frac{15 \times 35}{15 + 35} = 10{,}5\Omega$$

Conversões Y-Delta: Exemplo

- Calcular a corrente i e a resistência equivalente do circuito R_{ab}.



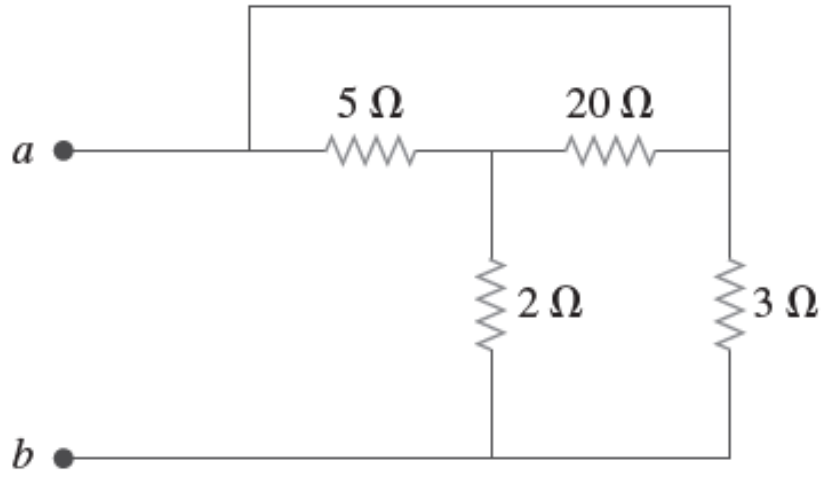


$$R_{ab} = \frac{(7{,}292 + 10{,}5) \times 21}{7{,}292 + 10{,}5 + 21} = 9{,}632\Omega$$

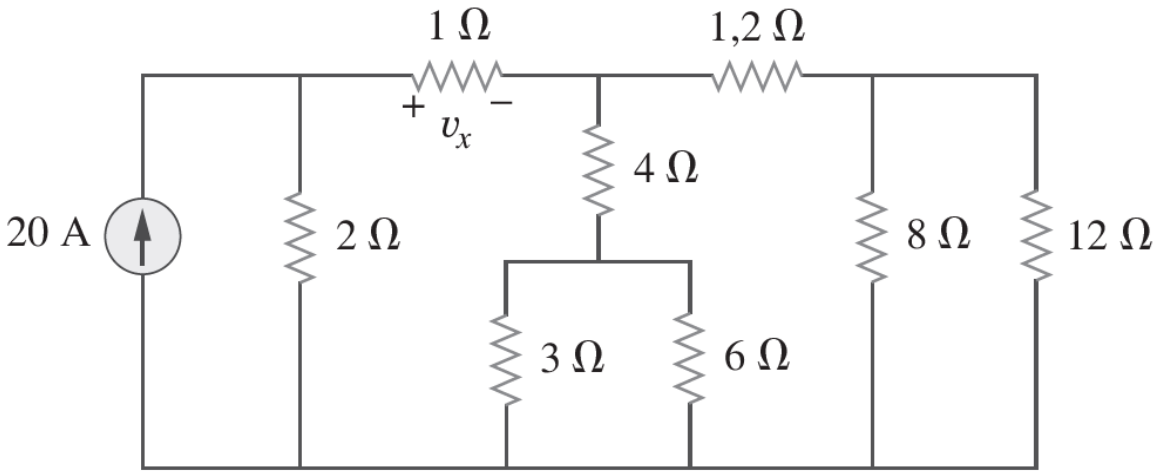
$$i = \frac{v}{R_{ab}} = \frac{120}{9{,}632} = 12{,}458\ A$$

Exercícios Propostos

❑1 – Determine a resistência equivalente entre os pontos a e b.

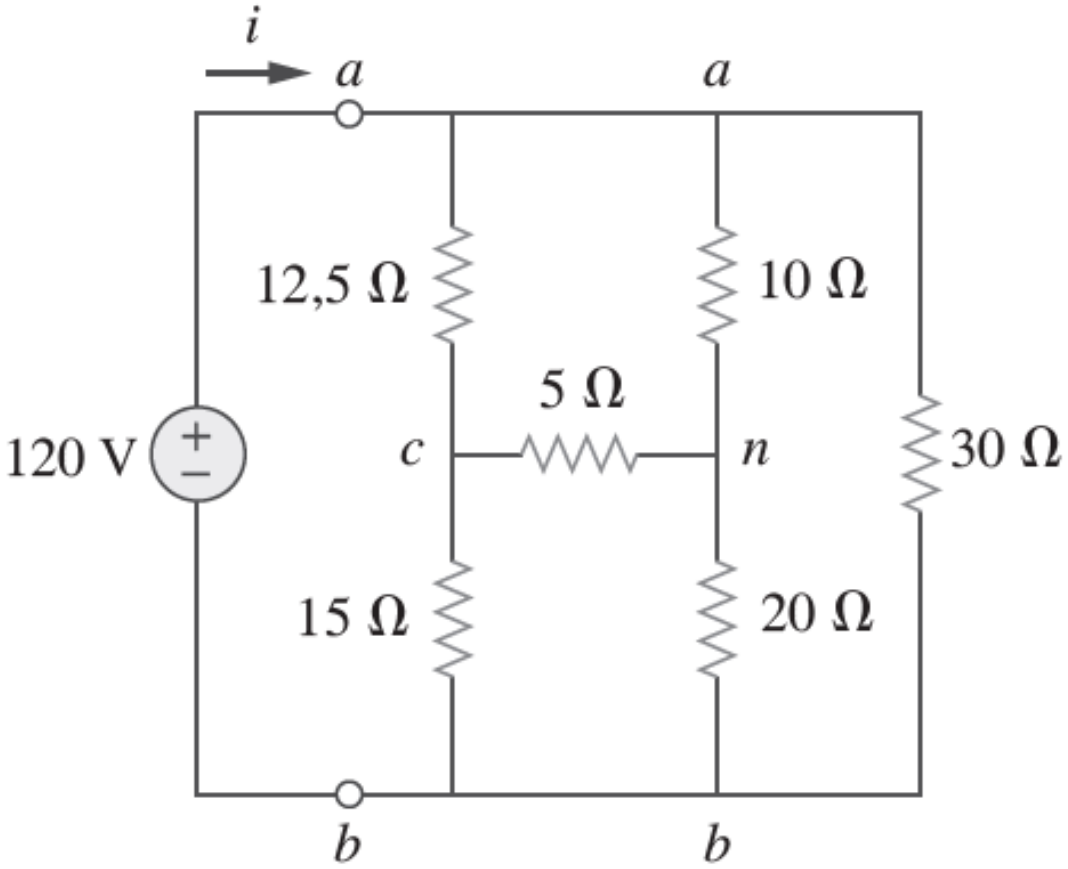


❑2 – Calcule v_x e a potência dissipada pelo resistor de 12 Ω.



Exercícios Propostos

❑3 – Demonstre!!!



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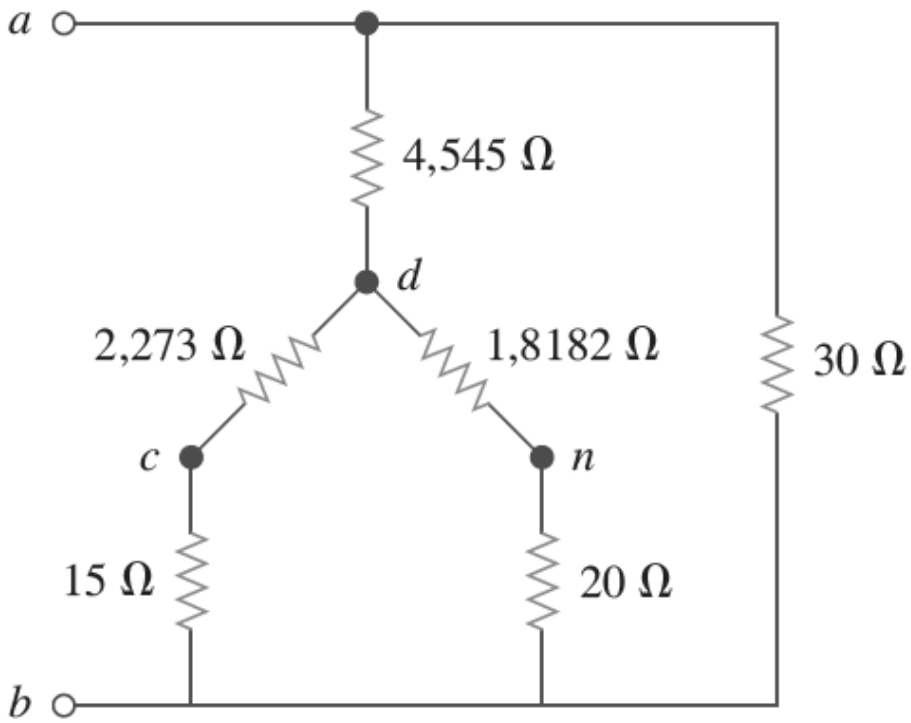

Respostas dos Exercícios Propostos

- 1
  - $R_{eq} = 2\ \Omega$
- 2
  - $v_x = 6{,}66\ V\ e\ P_x = 44{,}44\ W;\ v_{12\Omega} = 16\ V\ e\ P_{12\Omega} = 21{,}33\ W$

Referências Bibliográficas

- J. W. Nilsson, e S. A. Riedel, “Electric Circuits”, 9 ed., New York, Prentice Hall (2011).
- W. H. Hyat, J. E. Kemmerly, e S. M Durbin, “Análise de Circuitos em Engenharia”, 7 ed., São Paulo, McGraw-Hill (2008).
- C. K. Alexander, e M. N. O. Sadiku, “Fundamentos de Circuitos Elétricos”, 5 ed., Porto Alegre, AMGH (2013).