Física IV

1º setembro 2020

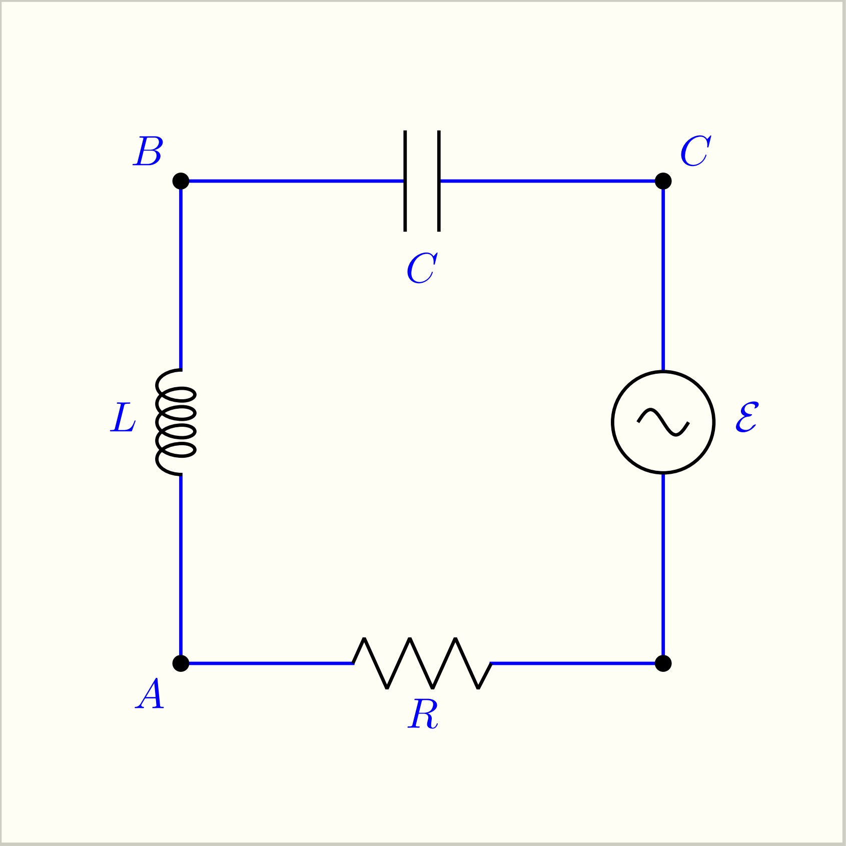
Circuitos de Corrente Alternada

Circuitos de corrente alternada

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = \mathscr{E}$$

$$Q(0) = Q_0$$

$$I(0) = I_0$$



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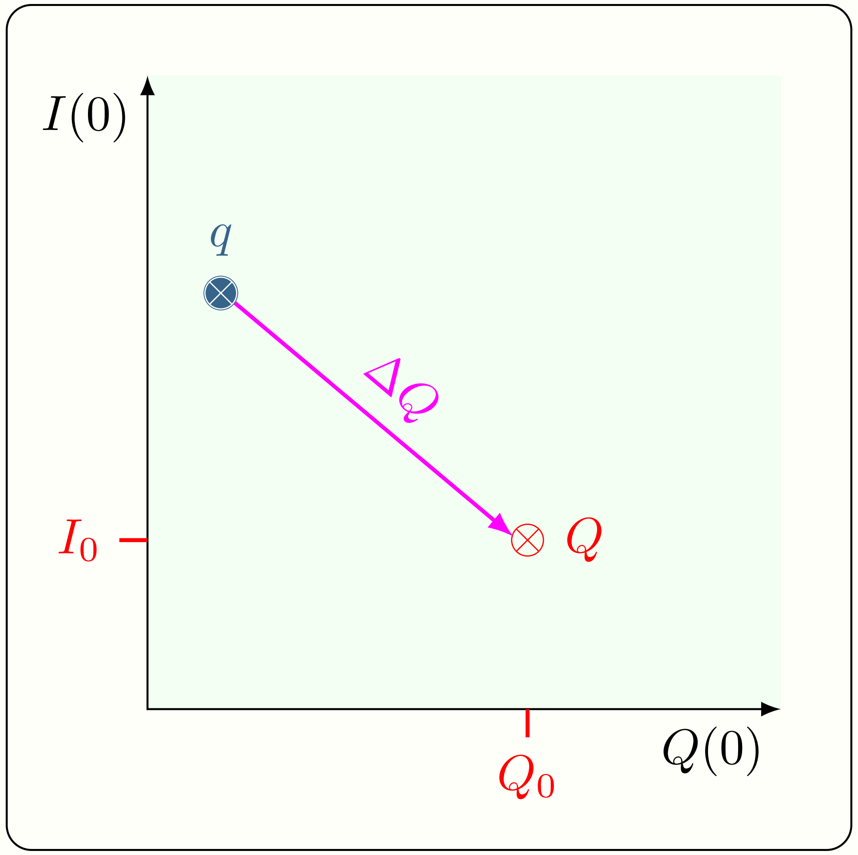
$$L\frac{d^2\Delta Q}{dt^2} + R\frac{d\Delta Q}{dt} + \frac{\Delta Q}{C} = 0$$

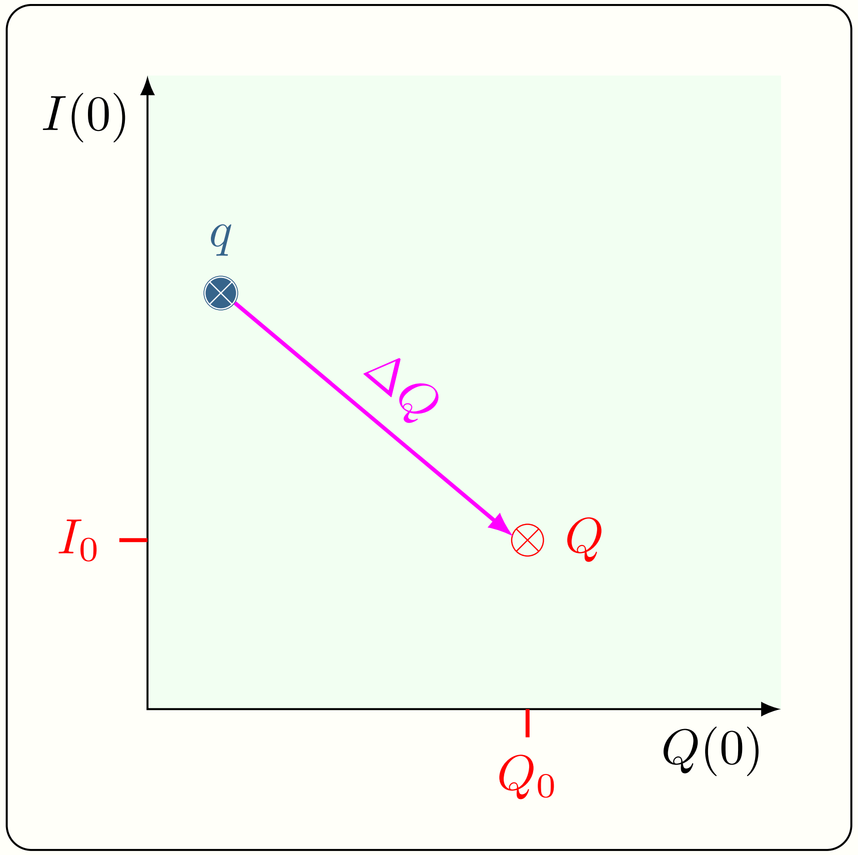


Circuitos de corrente alternada

$$L\frac{d^2\Delta Q}{dt^2} + R\frac{d\Delta Q}{dt} + \frac{\Delta Q}{C} = 0$$

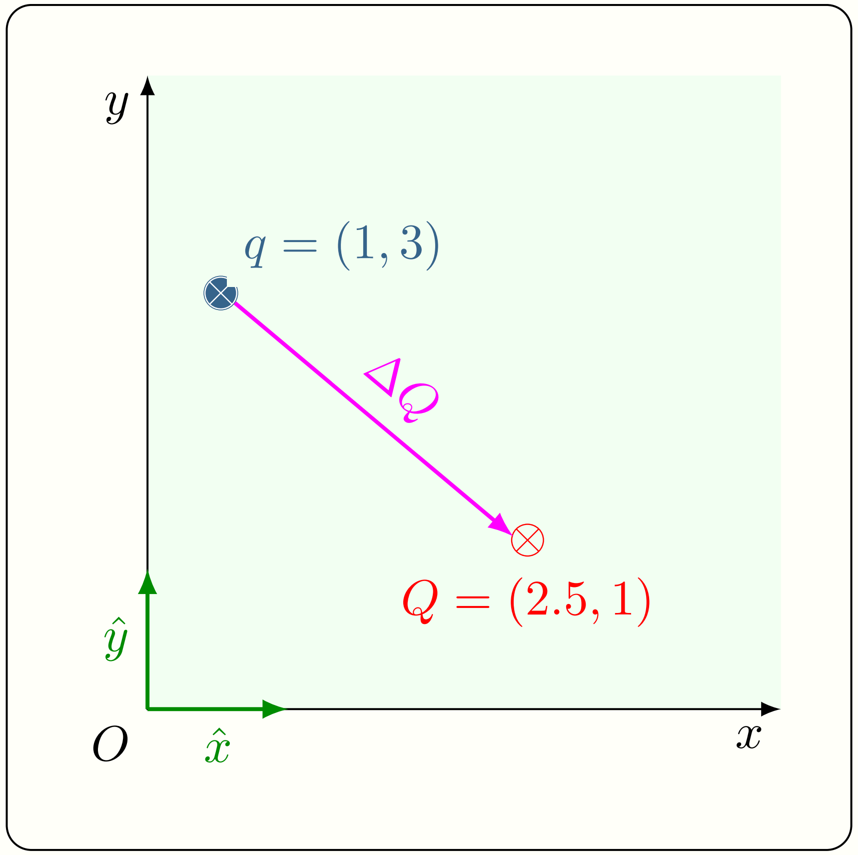
$$\frac{d^2\Delta Q}{dt^2} + \frac{2}{\tau}\frac{d\Delta Q}{dt} + \omega_0^2\Delta Q = 0$$

$$\tau = \frac{2L}{R} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$



Analogia com GA

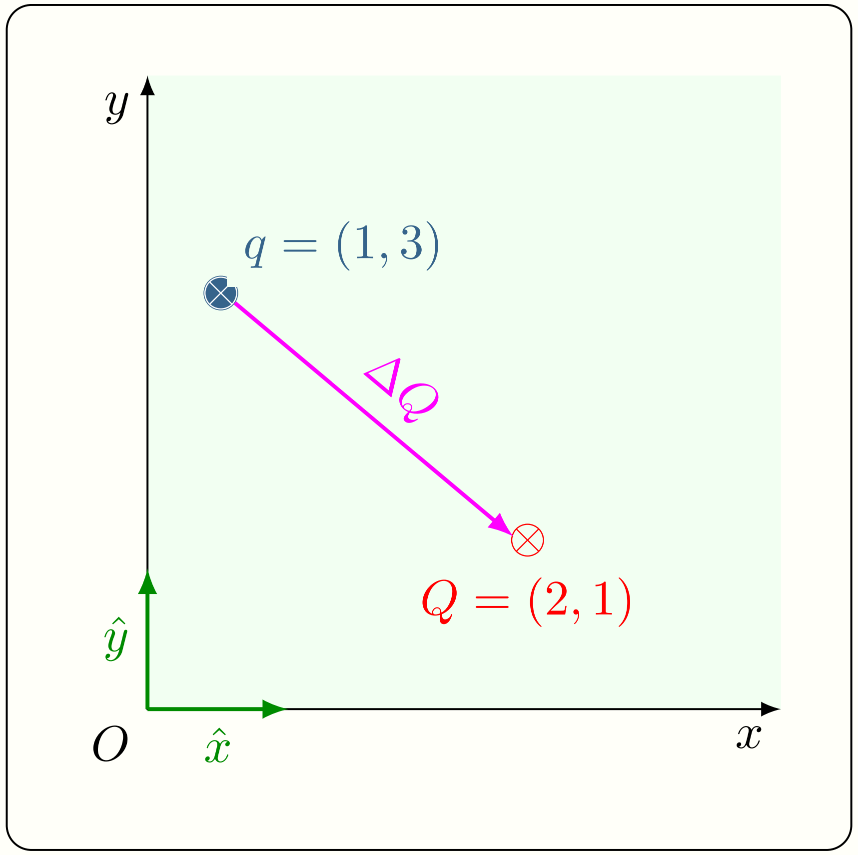
$\Delta Q = ?$



Analogia com GA

$$\Delta Q = ?$$

$$\vec{r}_Q = \vec{r}_q + \alpha\hat{x} + \beta\hat{y}$$



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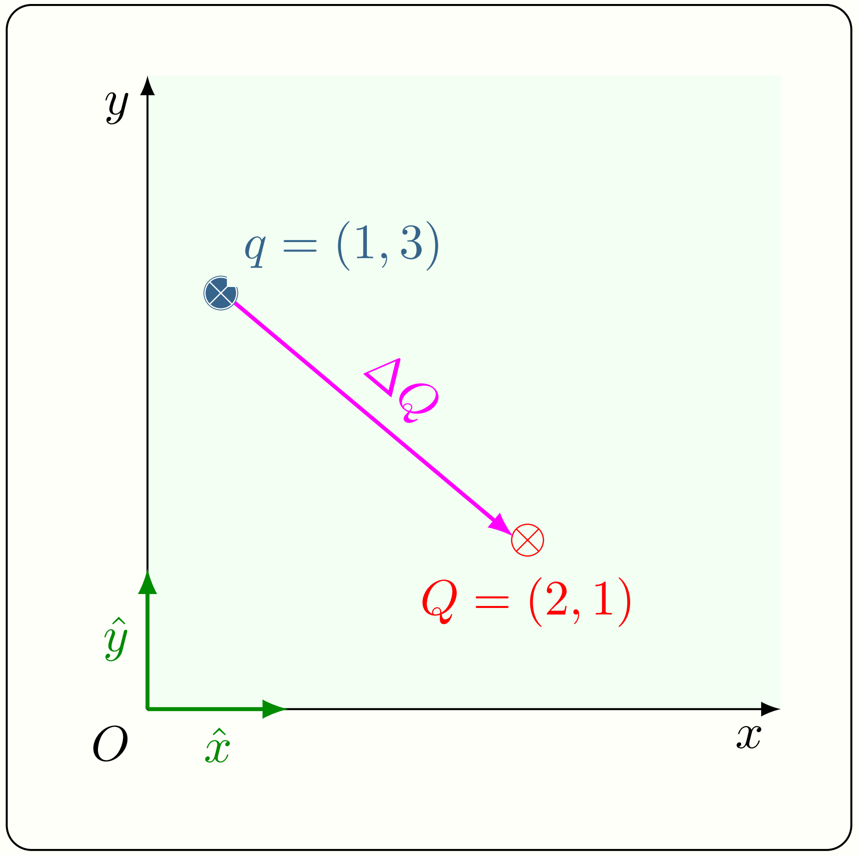

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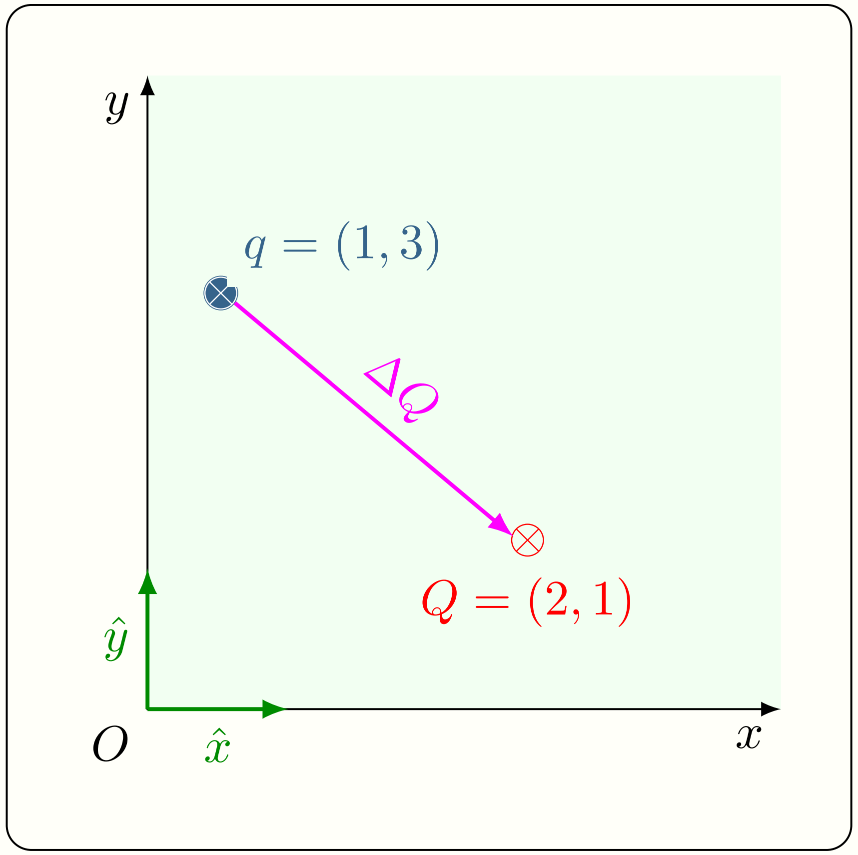

$$2\hat{x} + 1\hat{y} = 1\hat{x} + 3\hat{y} + \alpha\hat{x} + \beta\hat{y}$$

Circuitos de corrente alternada

$$L\frac{d^2\Delta Q}{dt^2} + R\frac{d\Delta Q}{dt} + \frac{\Delta Q}{C} = 0$$

$$\frac{d^2\Delta Q}{dt^2} + \frac{2}{\tau}\frac{d\Delta Q}{dt} + \omega_0^2\Delta Q = 0$$

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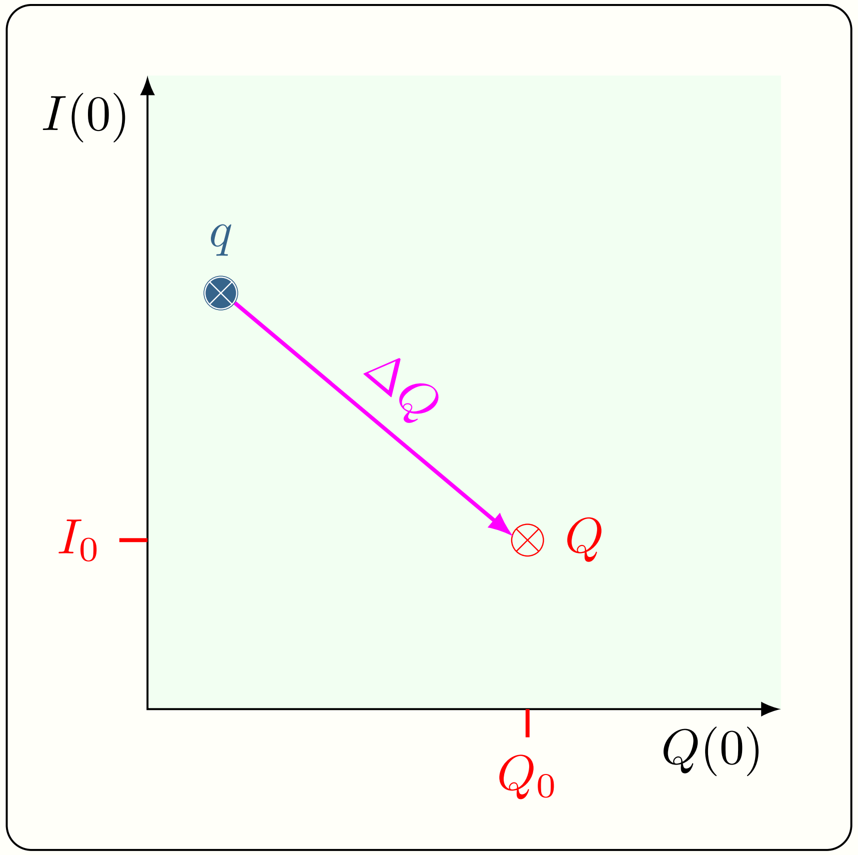


Solução da equação homogênea

$$\Delta Q(t) = \alpha Q_x(t) + \beta Q_y(t)$$

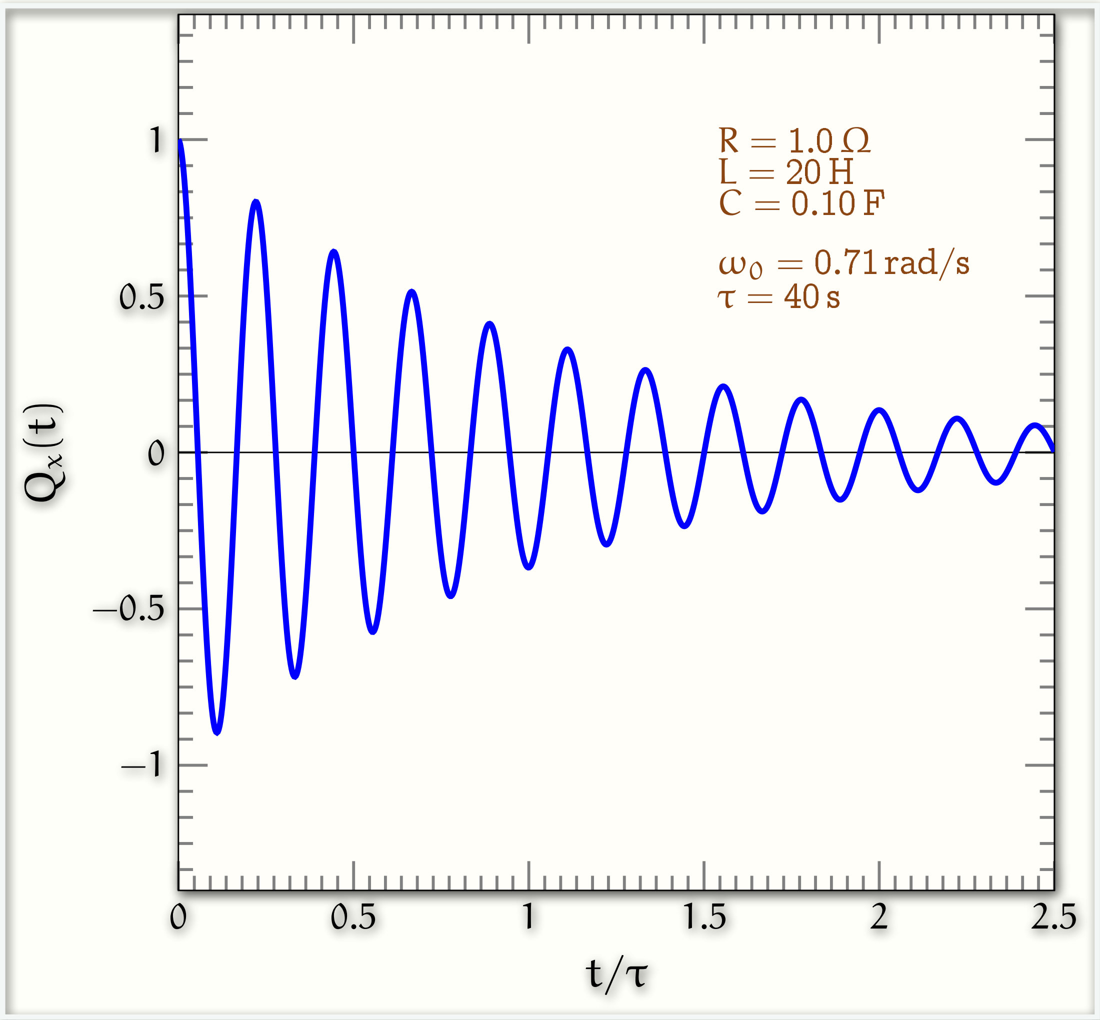
Regime	$\omega_0 \tau$	$\exp(t/\tau) Q_x$	$\exp(t/\tau) Q_y$
Subamortecido	> 1	$\cos(\omega_1 t)$	$\dfrac{\sin(\omega_1 t)}{\omega_1}$
Criticamente amortecido	$= 1$	1	t
Superamortecido	< 1	$\cosh(t/\tau_1)$	$\tau_1 \sinh(t/\tau_1)$

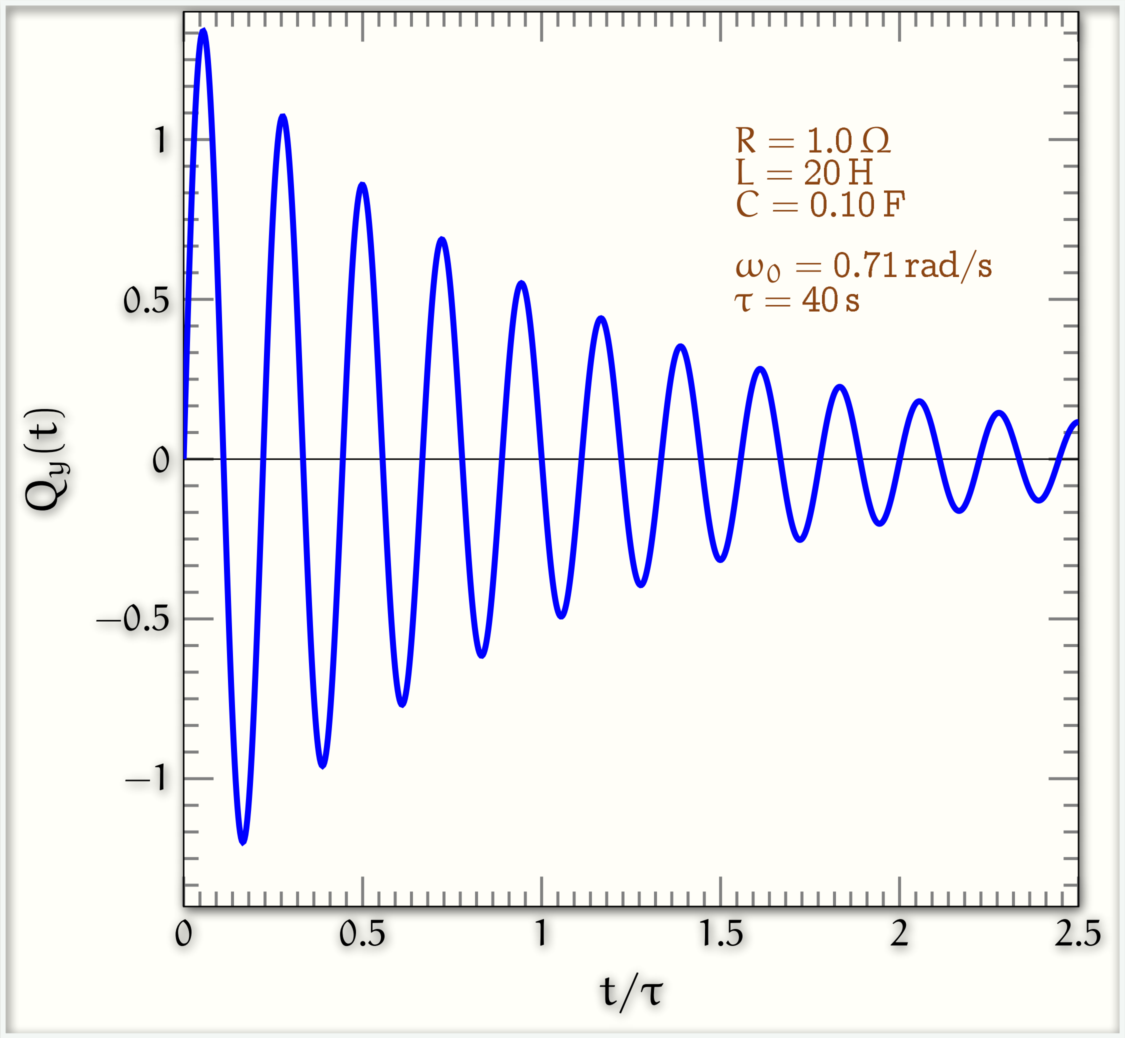
$$\omega_1 = \sqrt{\omega_0^2 - \frac{1}{\tau^2}} \qquad \frac{1}{\tau_1} = \sqrt{\frac{1}{\tau^2} - \omega_0^2}$$

Solução da equação homogênea

$$\Delta Q(t) = \alpha Q_x(t) + \beta Q_y(t)$$

$$\omega_o \tau > 1$$





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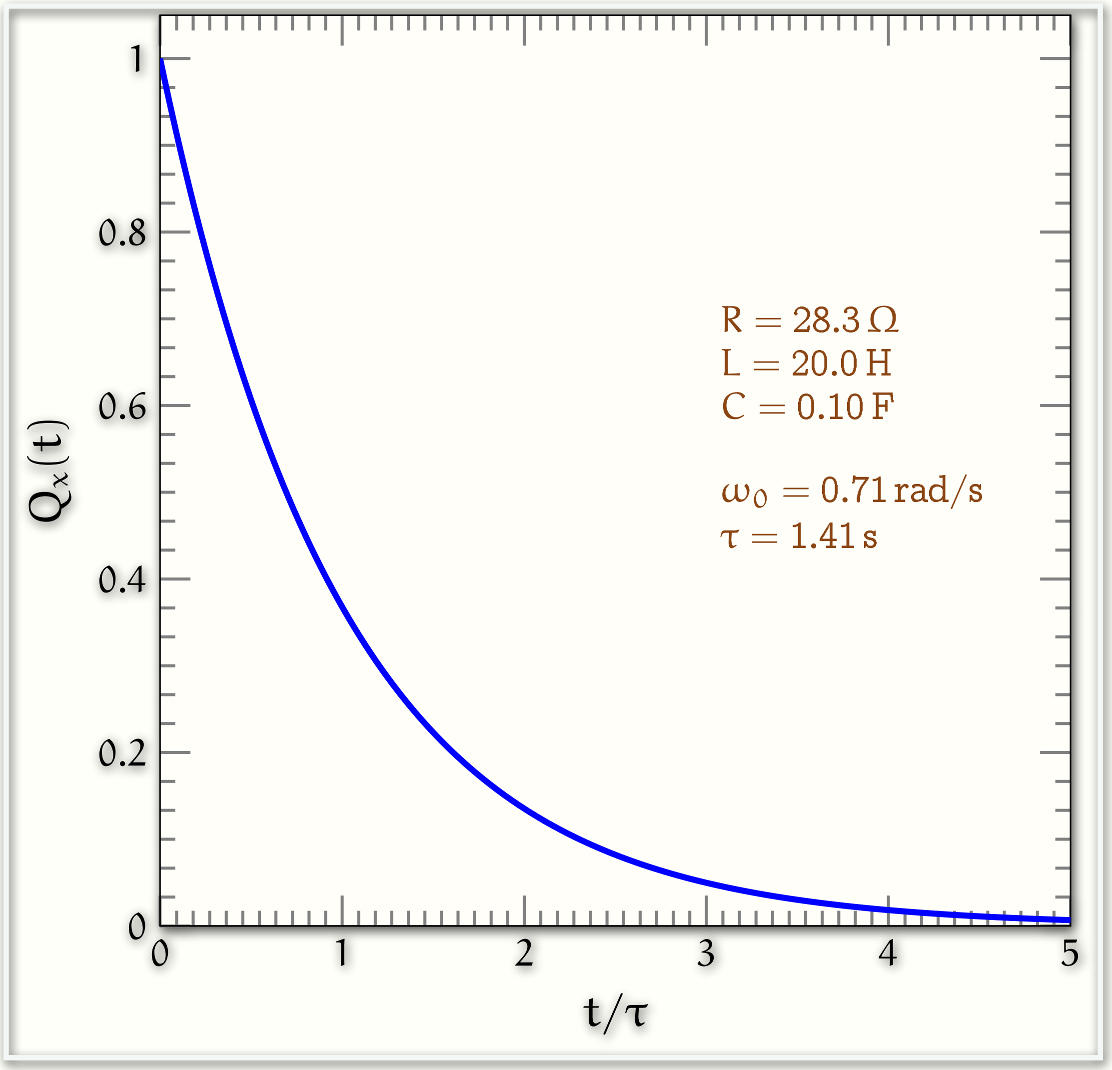
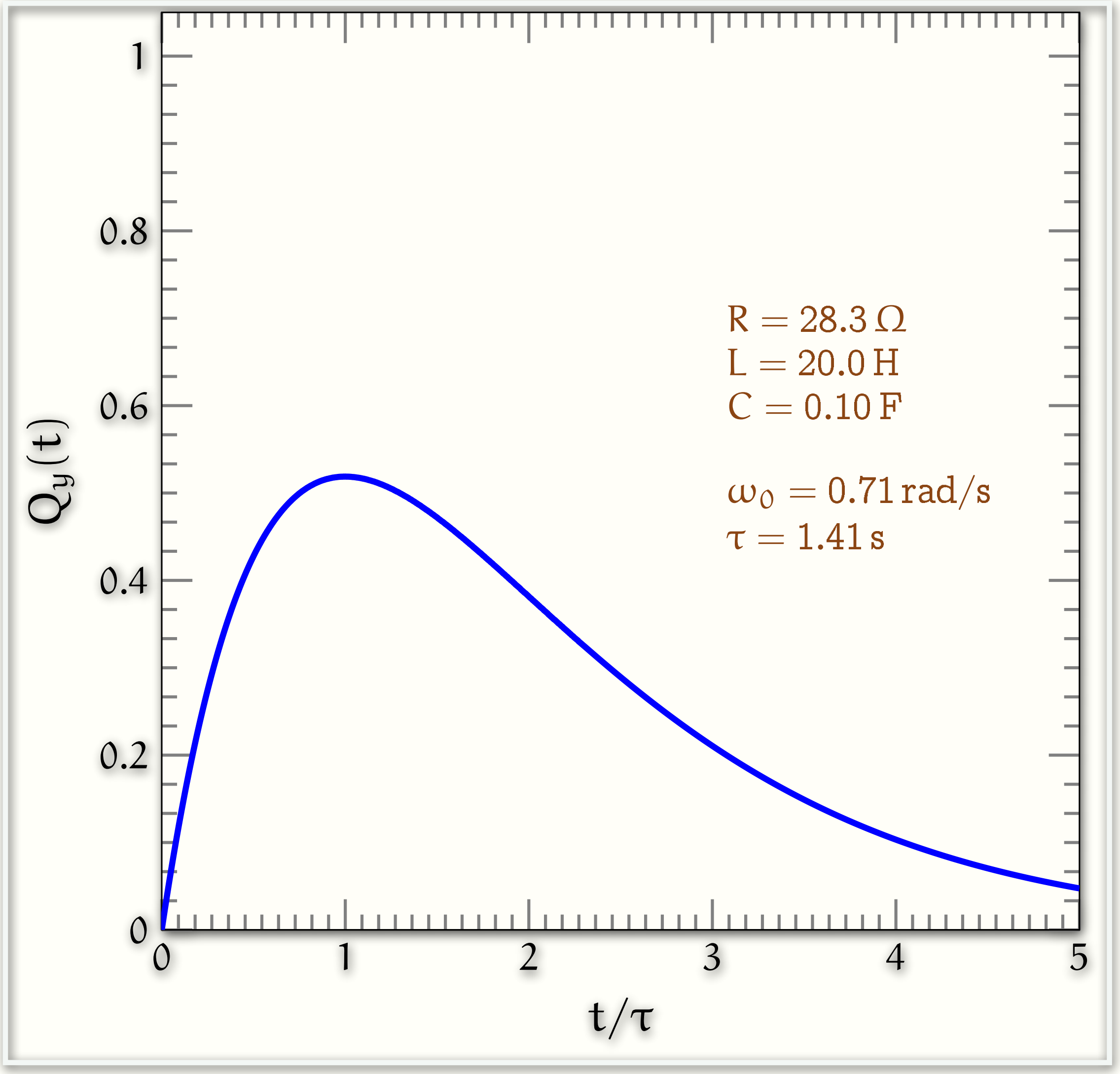
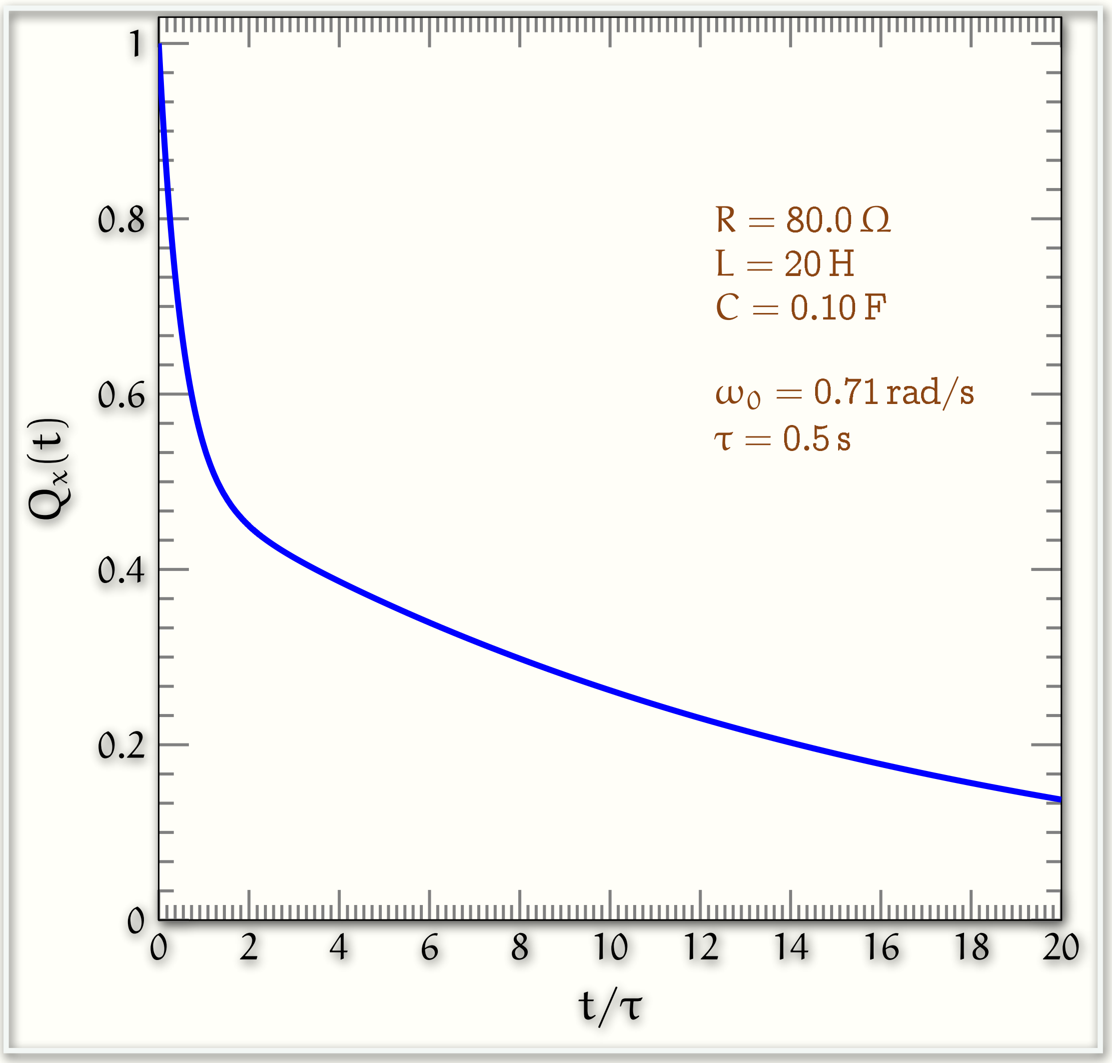


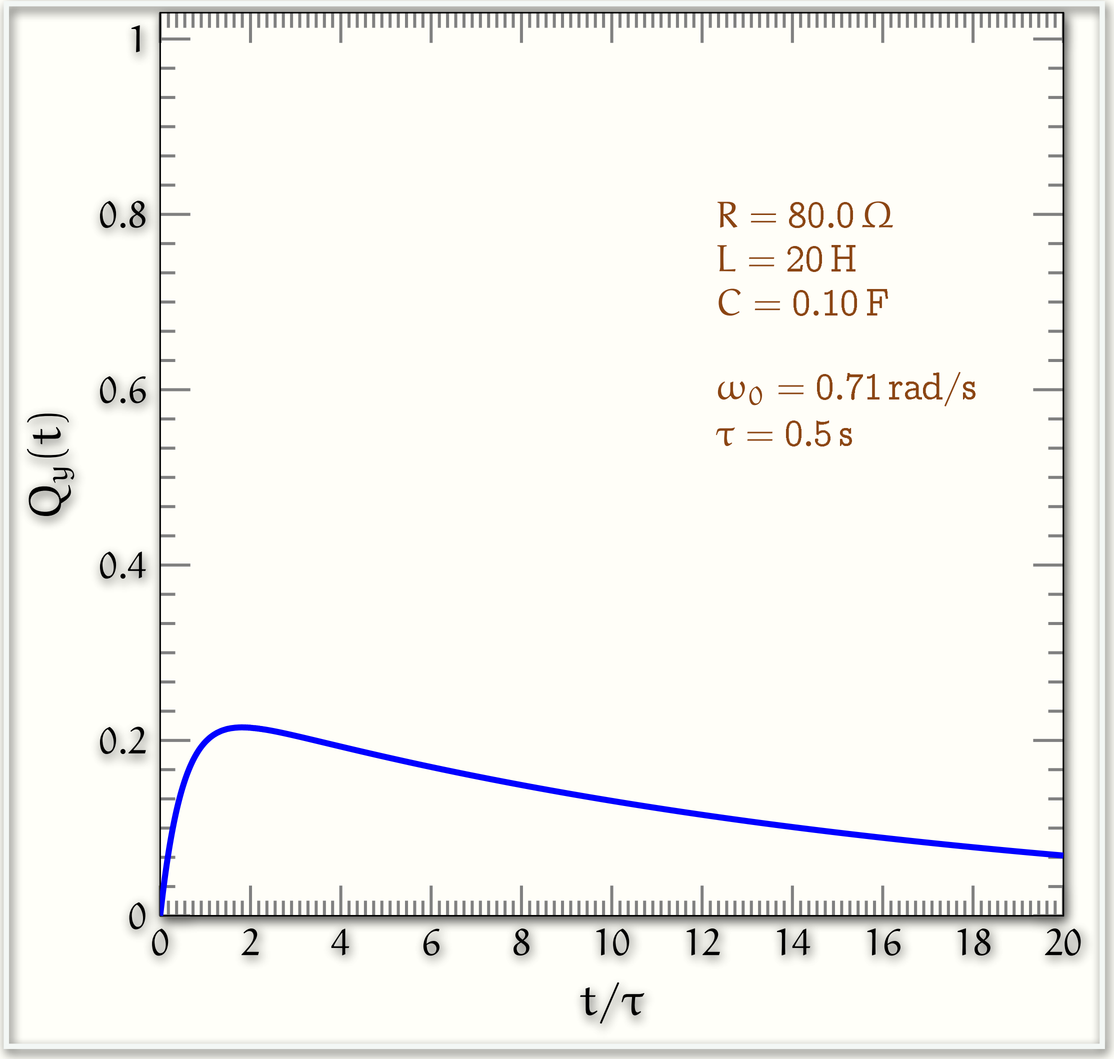


Solução da equação homogênea

$$\Delta Q(t) = \alpha Q_x(t) + \beta Q_y(t)$$

$$\omega_o \tau < 1$$





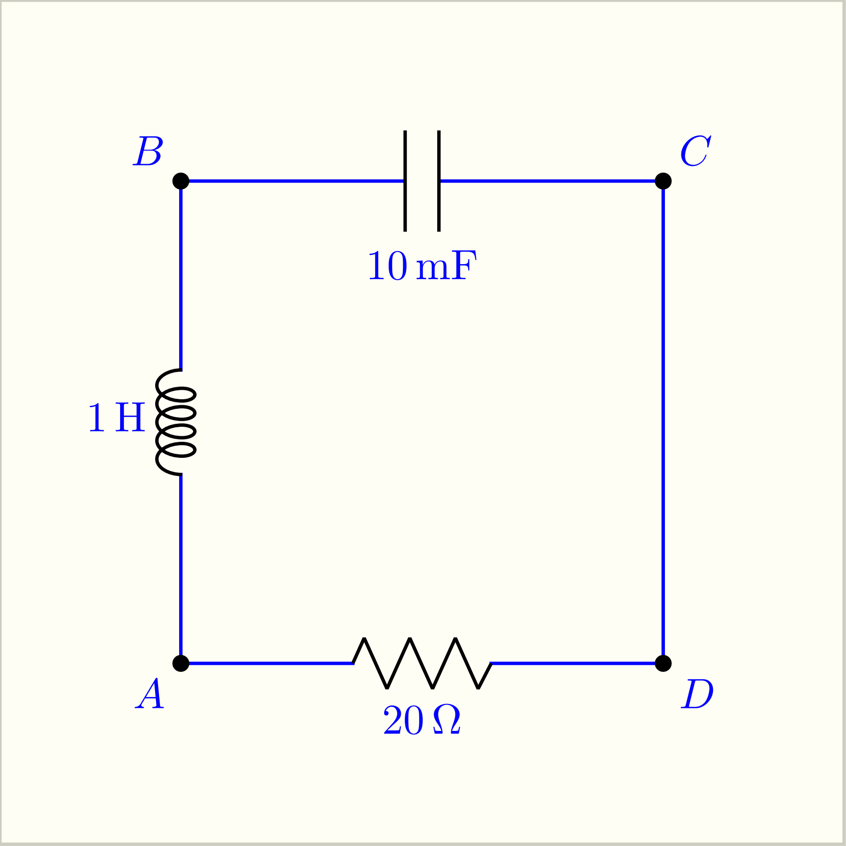
$$\frac{d^2\Delta Q}{dt^2} + \frac{2}{\tau}\frac{d\Delta Q}{dt} + \omega_0^2 \Delta Q = 0$$

Pratique o que aprendeu

$Q(0) = 0$

$I(0) = 1\text{A}$

$Q(t) = ?$



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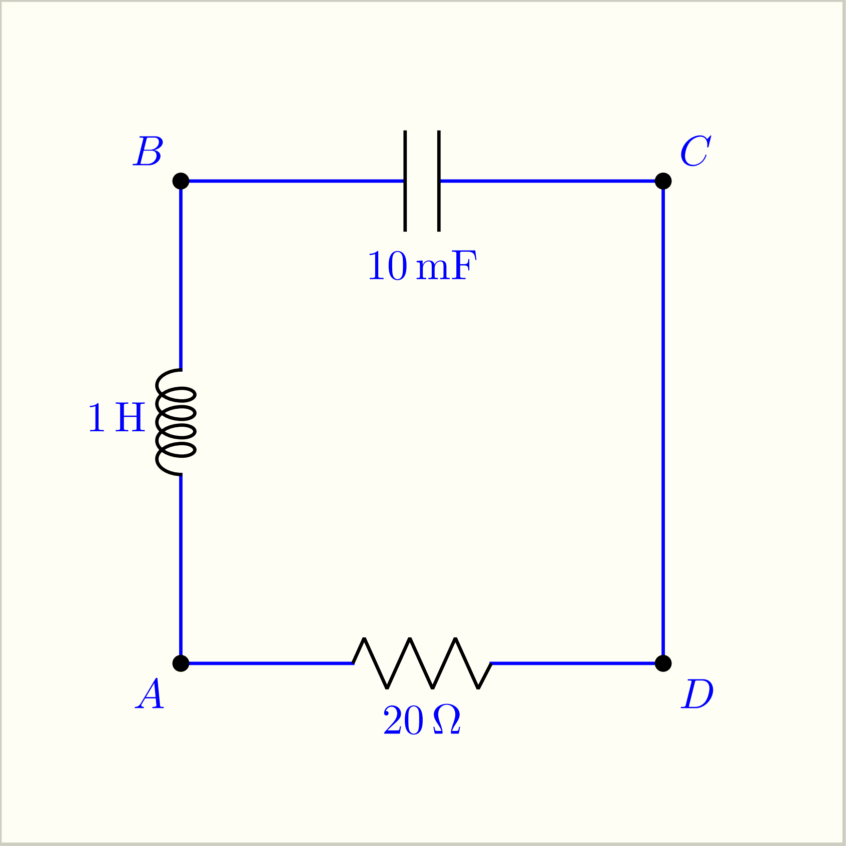

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0$$

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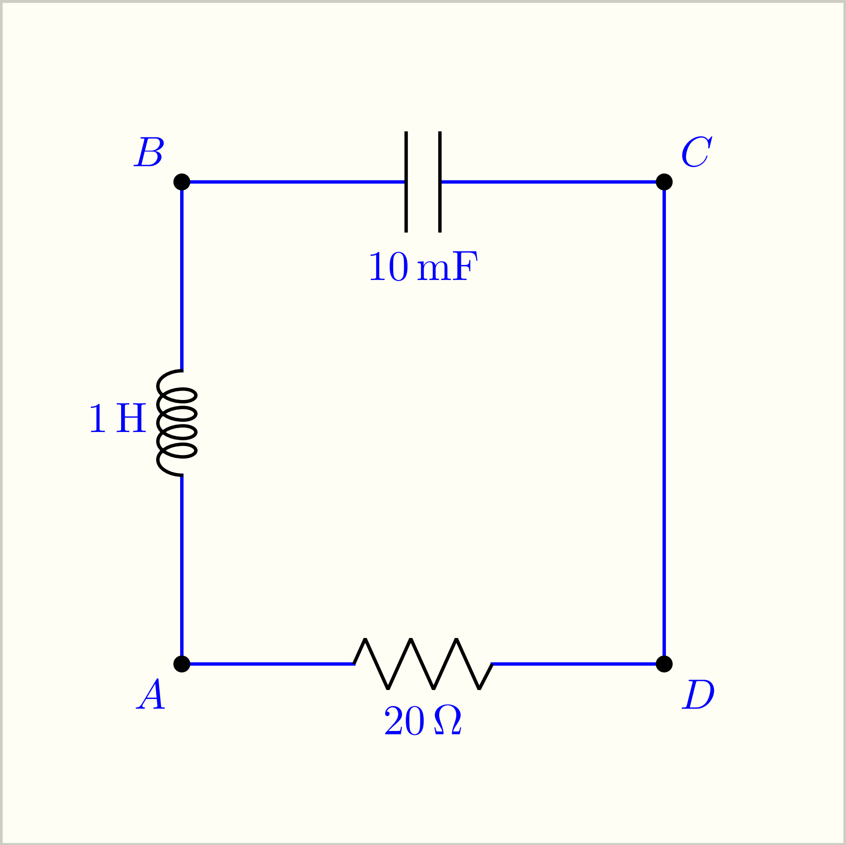

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0$$

$$\omega_0 = \frac{1}{\sqrt{1\text{H} * 0.01\text{F}}} = 10\,\text{rad/s}$$

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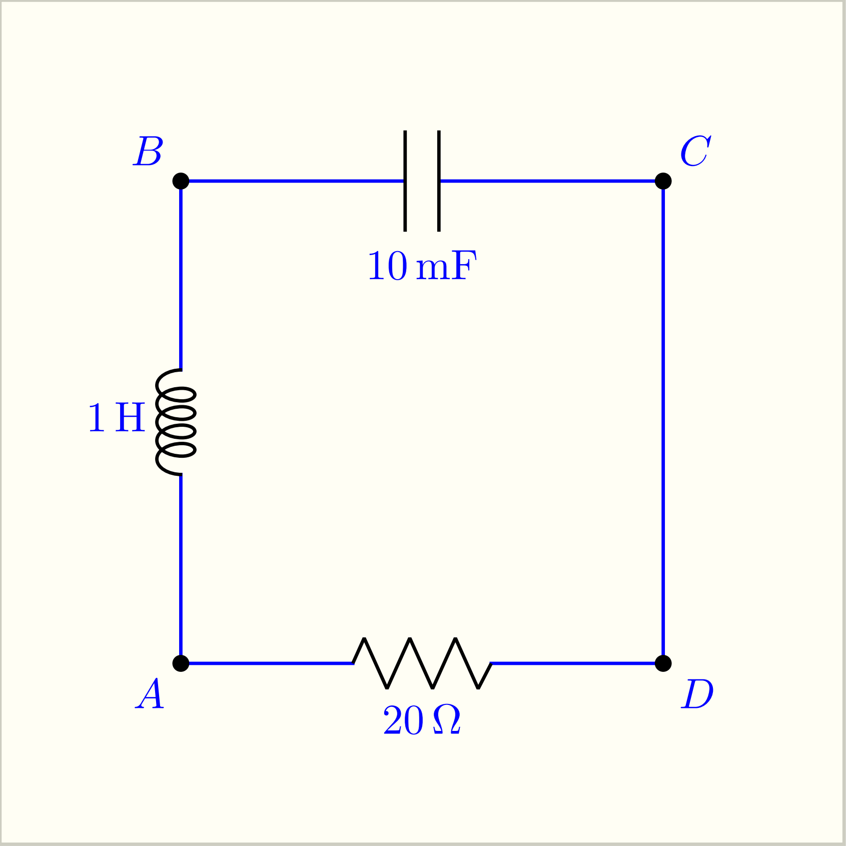

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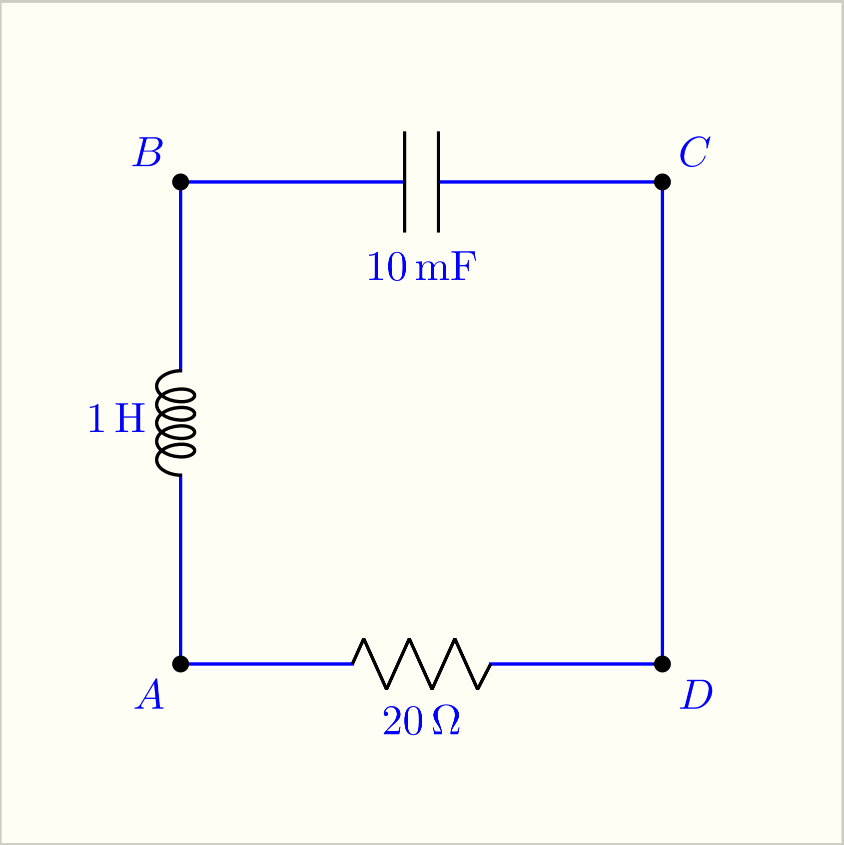

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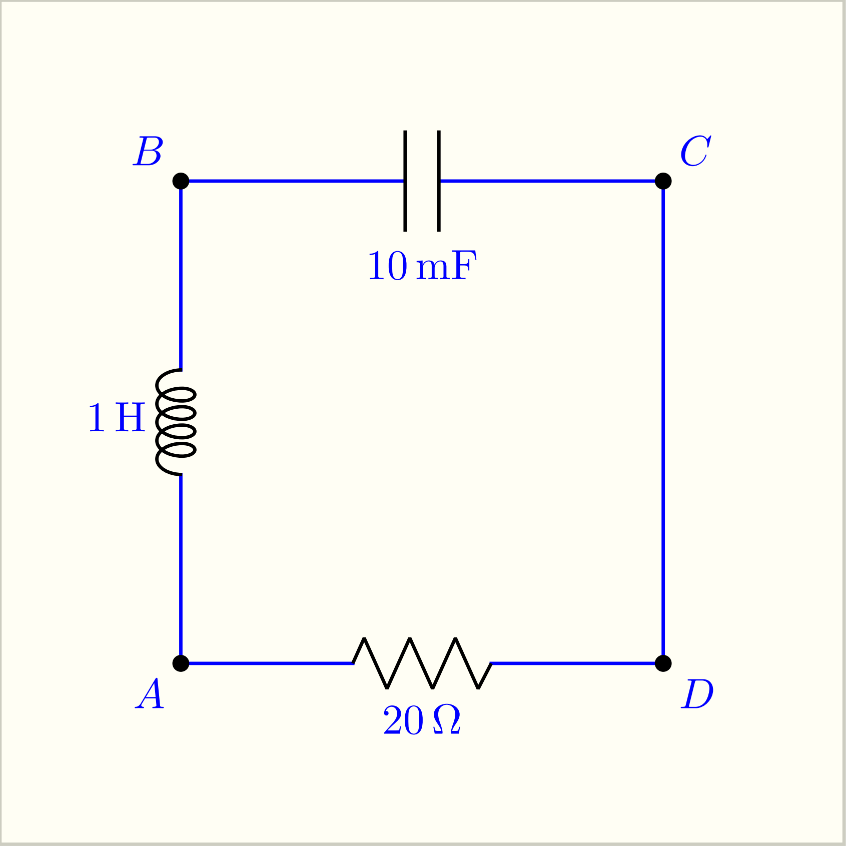

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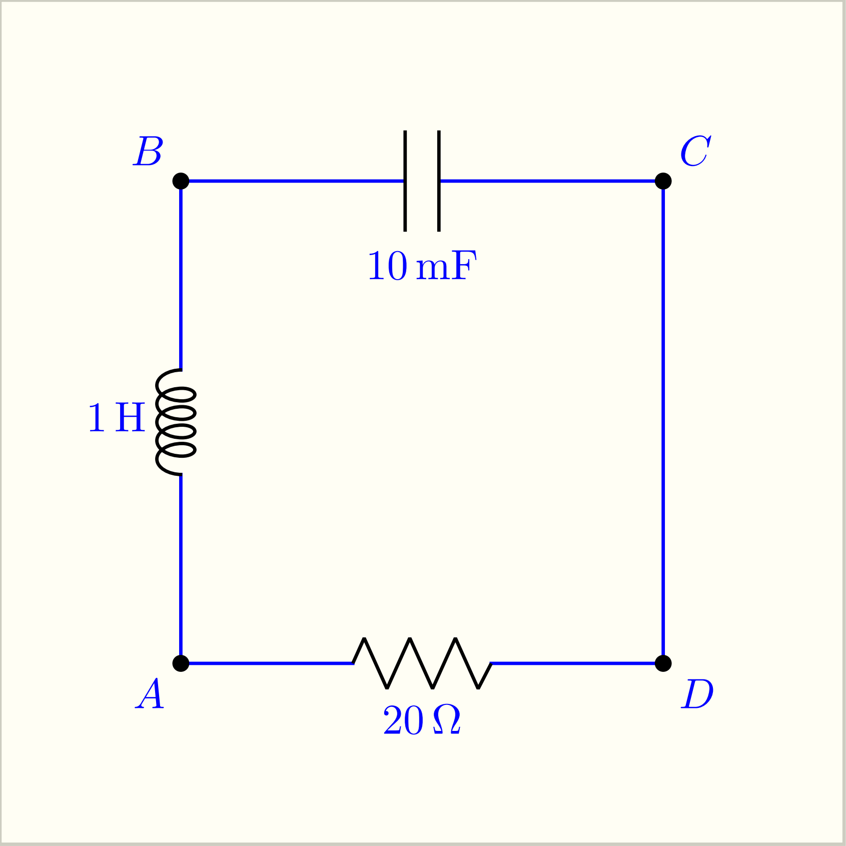

$$Q_x(t) = \exp(-t/\tau)$$

$$Q_y(t) = t\exp(-t/\tau)$$

$$\omega_0 \tau = 1$$

$$\tau = 0.1\ \text{s}$$

$$\frac{d^2\Delta Q}{dt^2} + \frac{2}{\tau}\frac{d\Delta Q}{dt} + \omega_0^2 \Delta Q = 0$$

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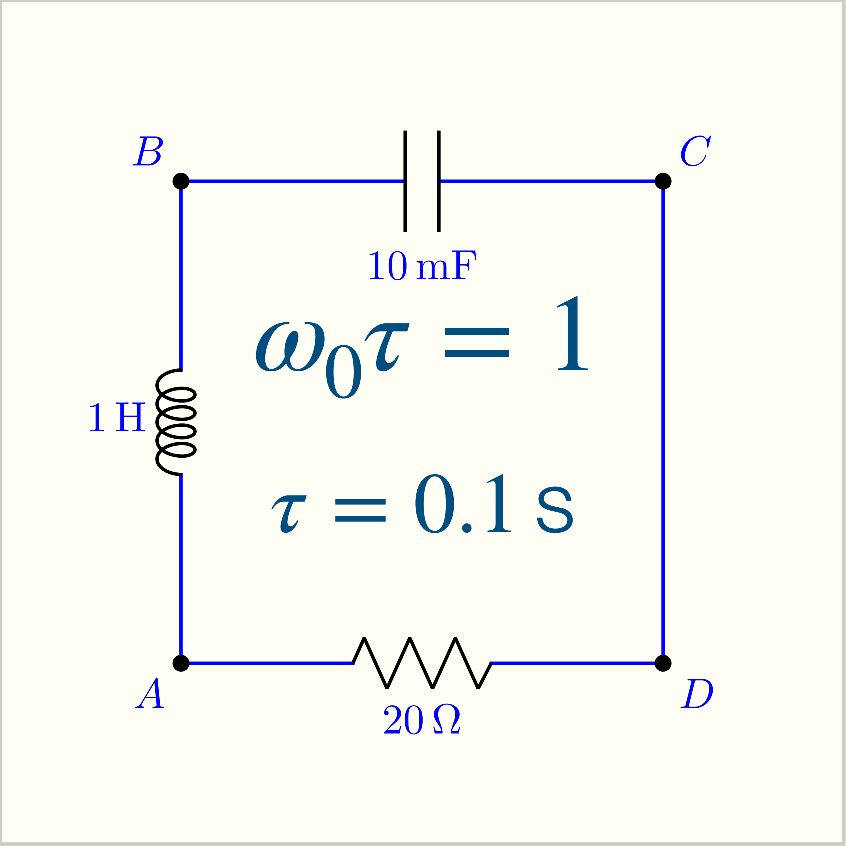

$$Q_x(t) = \exp(-t/\tau)$$

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$$Q(t) = \alpha Q_x + \beta Q_y$$

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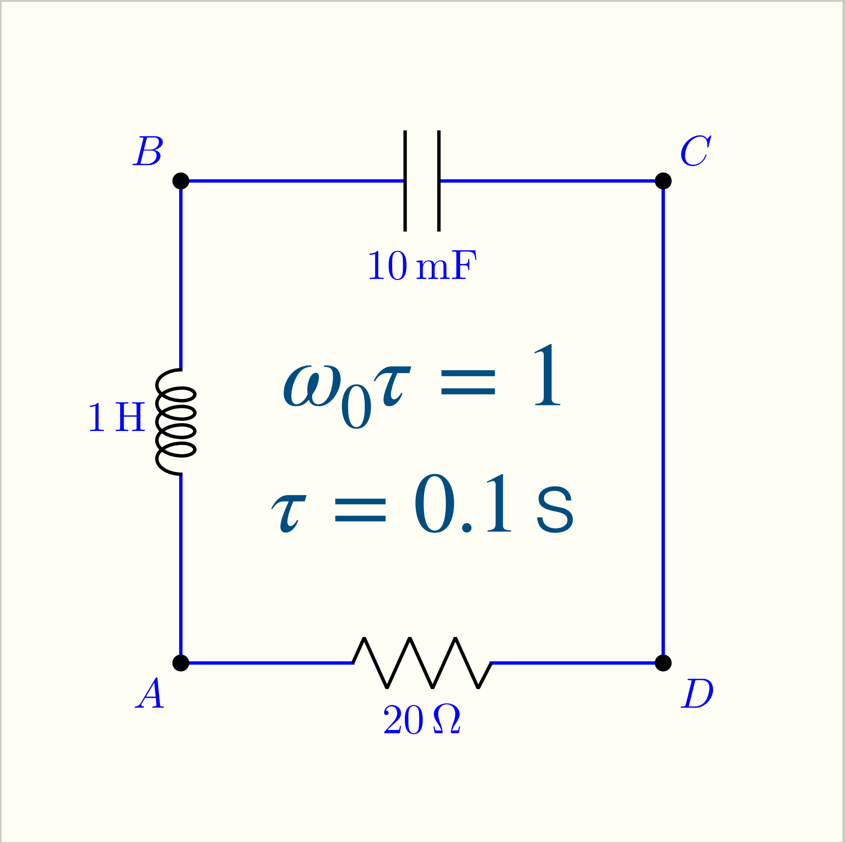

$$Q_x(t) = \exp(-t/\tau)$$

$$Q_y(t) = t\exp(-t/\tau)$$

$$Q(t) = \alpha Q_x + \beta Q_y \Rightarrow \begin{cases} \alpha = 0 \\ \beta = 1 \end{cases}$$

$$\frac{d^2\Delta Q}{dt^2} + \frac{2}{\tau}\frac{d\Delta Q}{dt} + \omega_0^2 \Delta Q = 0$$

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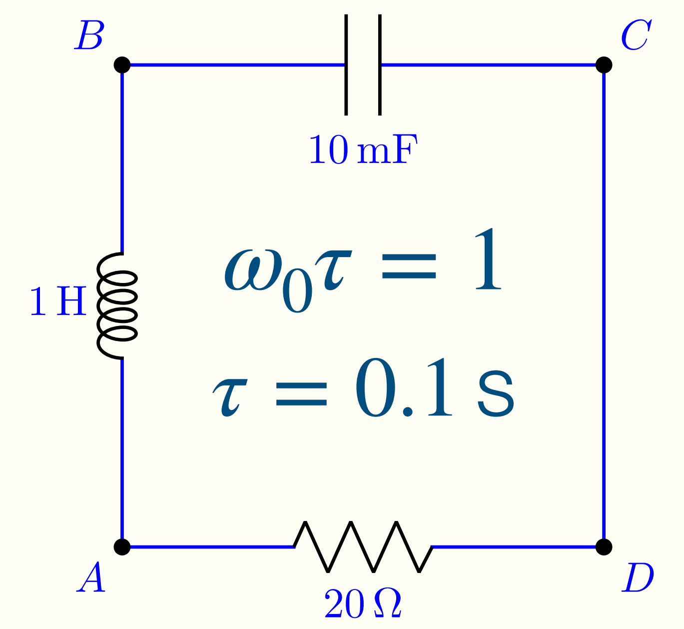


$\omega_0 \tau = 1$

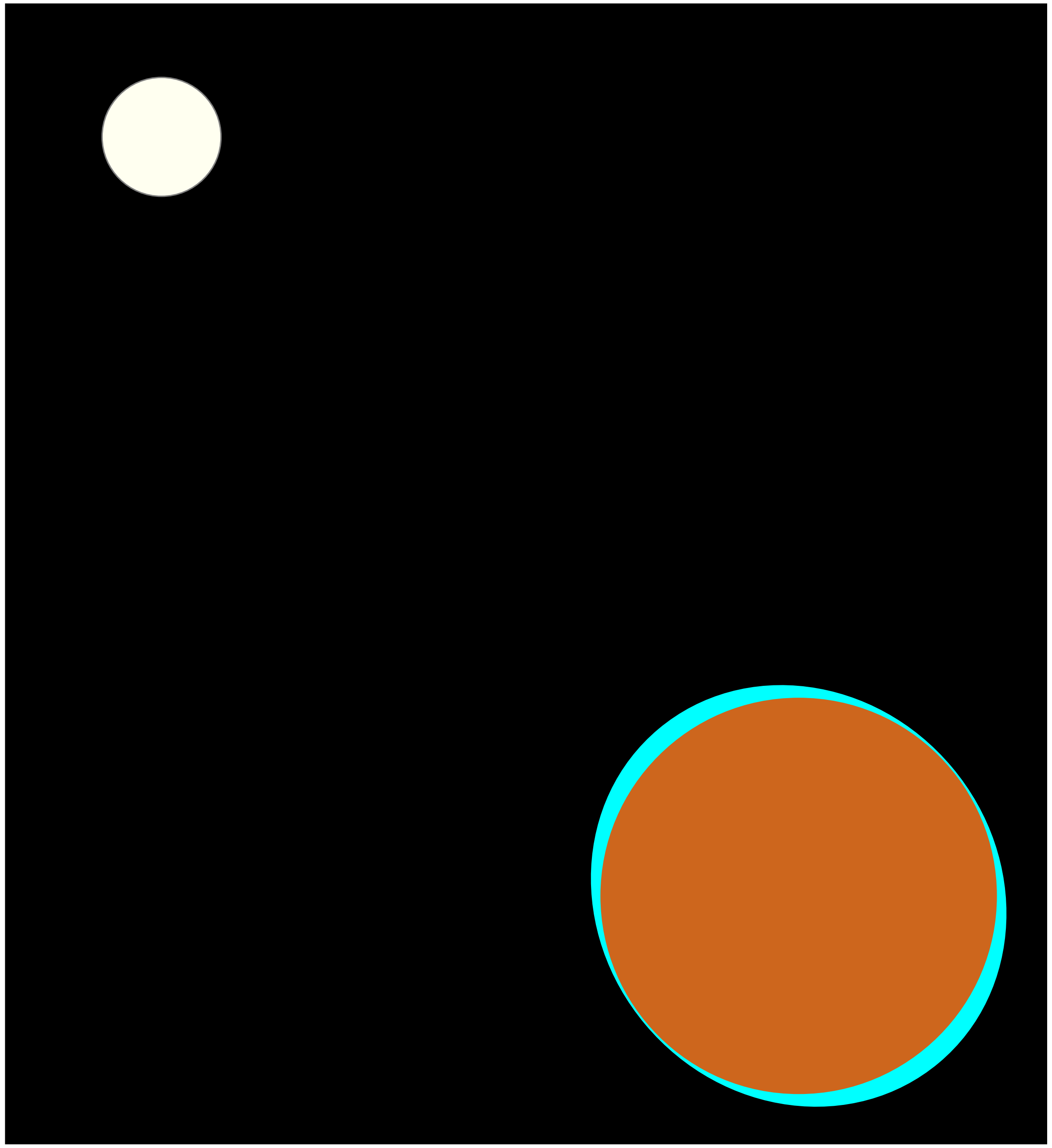
$\tau = 0.1\ \text{s}$

$Q_x(t) = \exp(-t/\tau)$

$Q_y(t) = t\exp(-t/\tau)$

$$Q(t) = \alpha Q_x + \beta Q_y \Rightarrow \begin{cases} \alpha = 0 \\ \beta = 1 \end{cases} \Rightarrow Q(t) = t\exp(-10t)$$

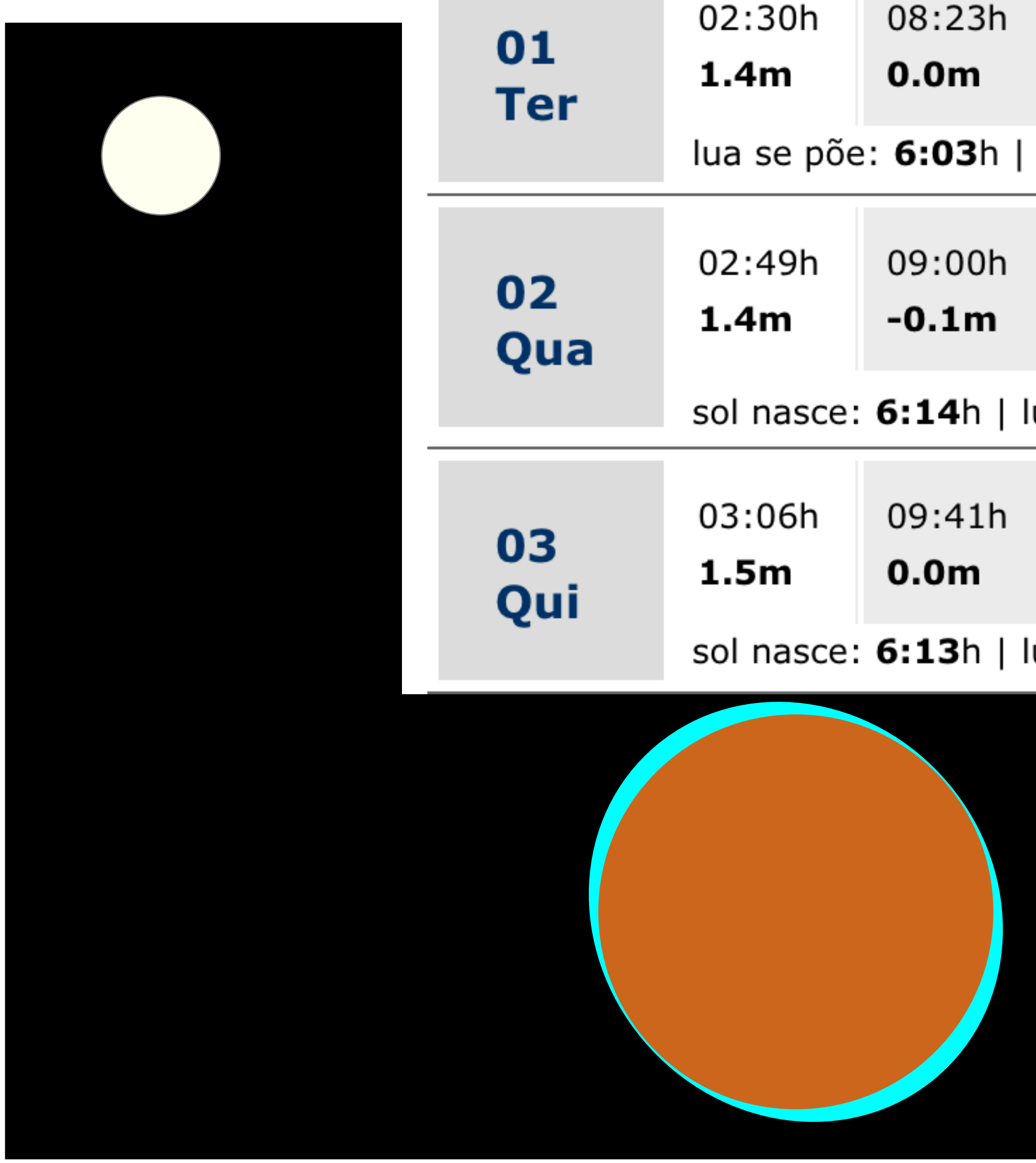
Marés



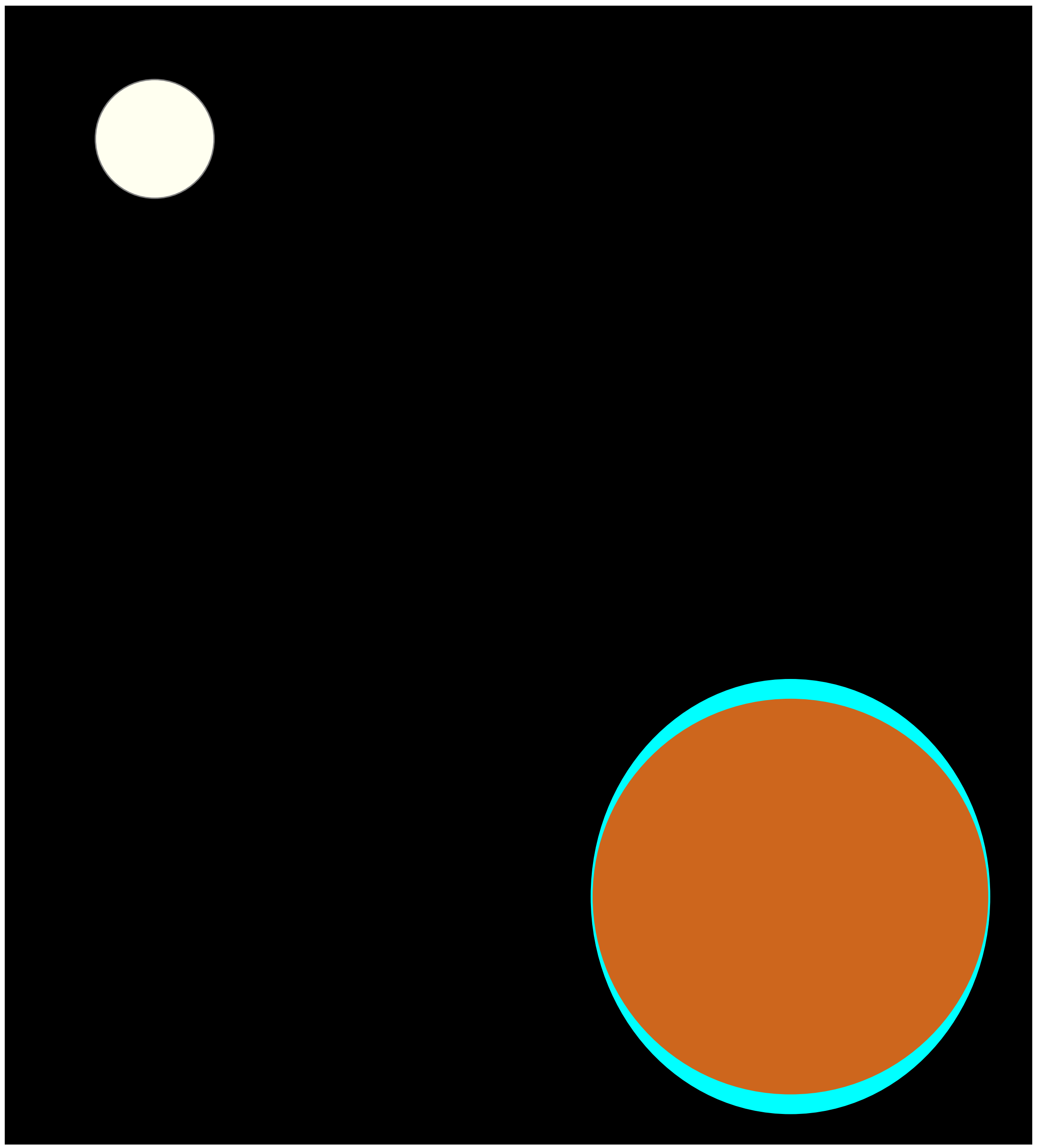
Marés

Maré mais alta: 1.6 m nos dias: 15,16
Maré mais baixa: -0.1 m nos dias: 02,17

01 Ter	02:30h **1.4m**	08:23h **0.0m**	14:39h **1.4m**	20:54h **0.4m**
	lua se põe: **6:03**h | sol nasce: **6:15**h | lua nasce: **17:31**h | sol se põe: **17:54**h			
02 Qua	02:49h **1.4m**	09:00h **-0.1m**	15:00h **1.4m**	21:19h **0.3m**
	sol nasce: **6:14**h | lua se põe: **6:40**h | sol se põe: **17:54**h | lua nasce: **18:24**h			
03 Qui	03:06h **1.5m**	09:41h **0.0m**	15:24h **1.4m**	21:47h **0.3m**
	sol nasce: **6:13**h | lua se põe: **7:13**h | sol se põe: **17:55**h | lua nasce: **19:15**h			

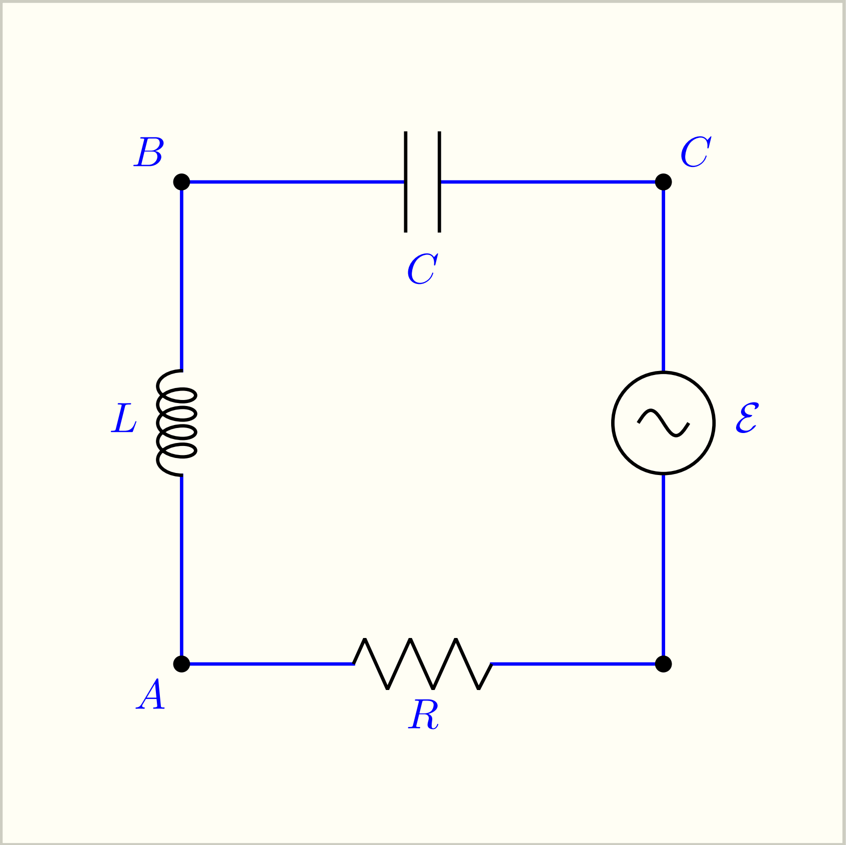


Marés



Solução da equação não-homogênea

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = \mathscr{E}_0 \cos(\omega t)$$



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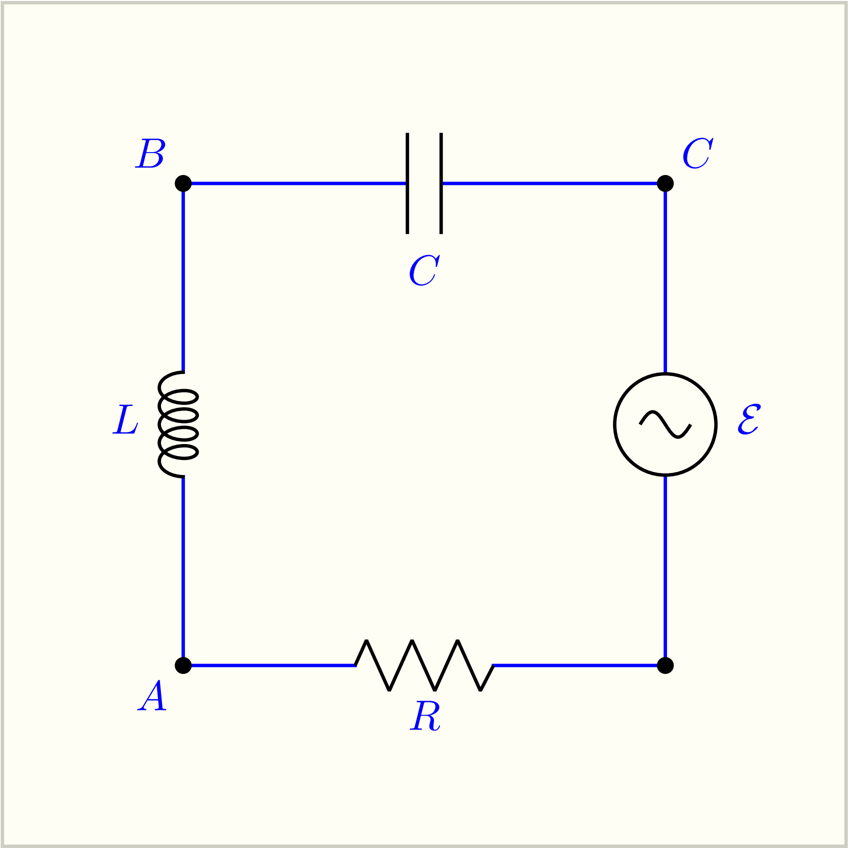
$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = \mathscr{E}_0 \cos(\omega t)$$

$$L\frac{d^2z}{dt^2} + R\frac{dz}{dt} + \frac{z}{C} = \mathscr{E}_0 \exp(i\omega t)$$



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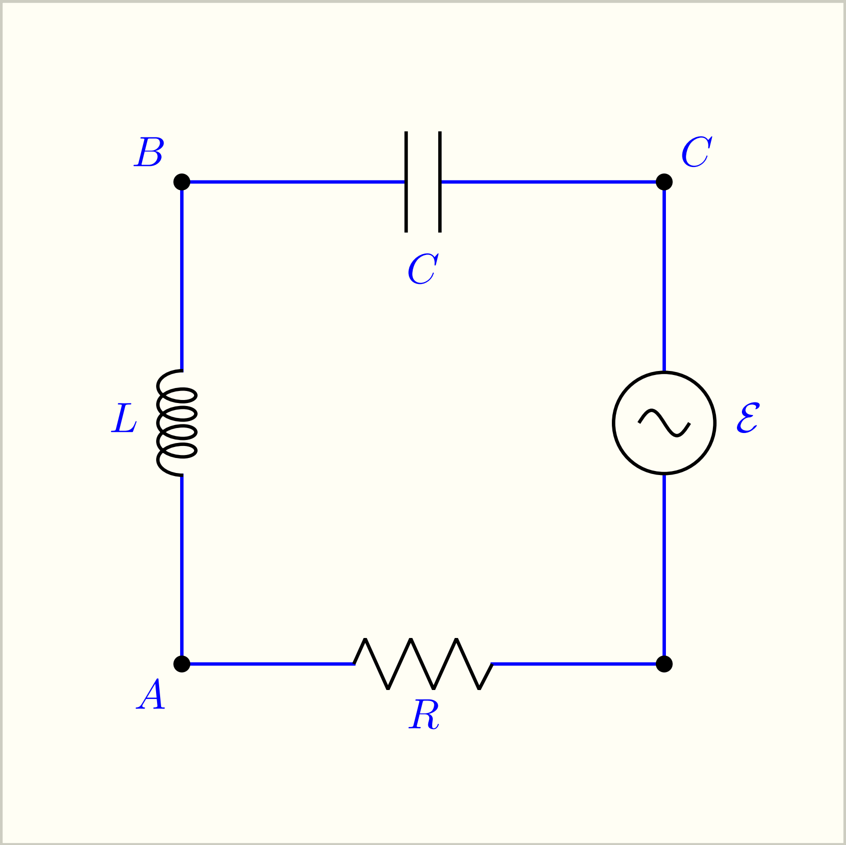
$$Q(t) = \Re z(t)$$



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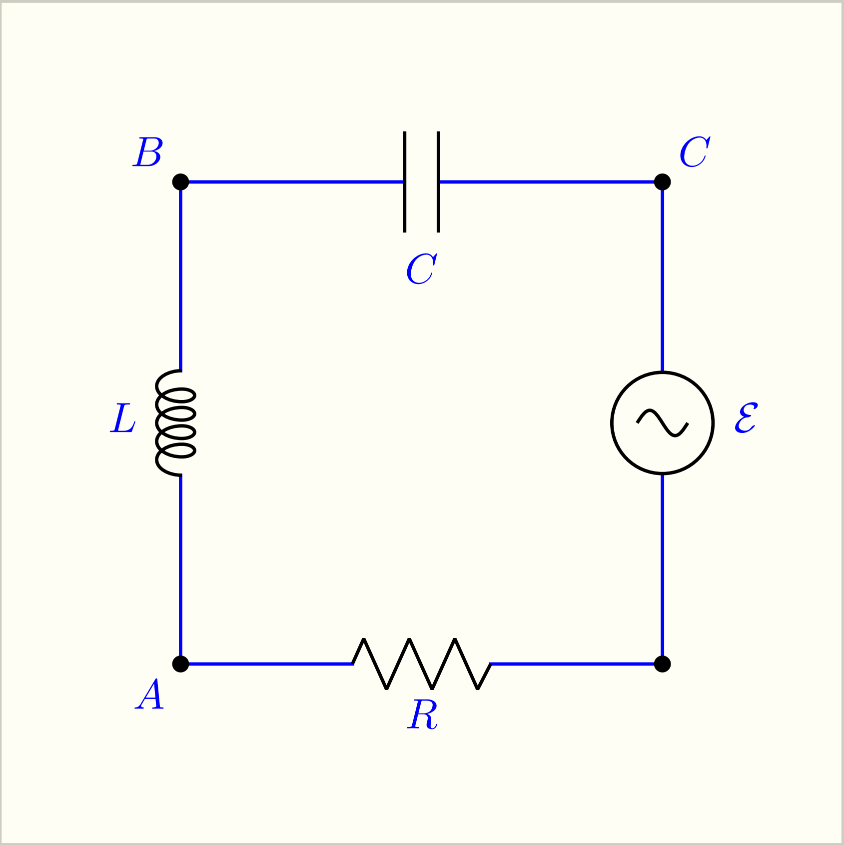
$$L\frac{d^2z}{dt^2} + R\frac{dz}{dt} + \frac{z}{C} = \mathscr{E}_0 \exp(i\omega t)$$

$$\frac{d^2z}{dt^2} + \frac{2}{\tau}\frac{dz}{dt} + \omega_0^2 z = \frac{\mathscr{E}_0}{L} \exp(i\omega t)$$

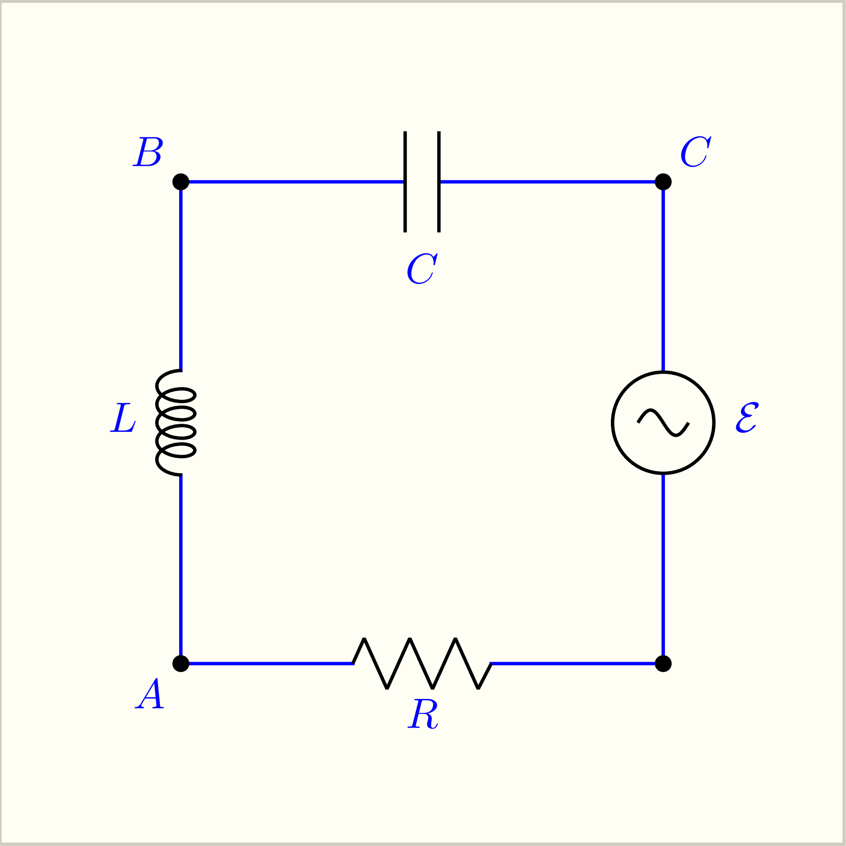


Solução da equação não-homogênea

$$\frac{d^2 z}{dt^2} + \frac{2}{\tau}\frac{dz}{dt} + \omega_0^2 z = \frac{\mathscr{E}_0}{L}\exp(i\omega t)$$

$$Q(t) = \Re z(t)$$

$$z(t) = z_0 \exp(i\omega t)$$



Solução da equação não-homogênea

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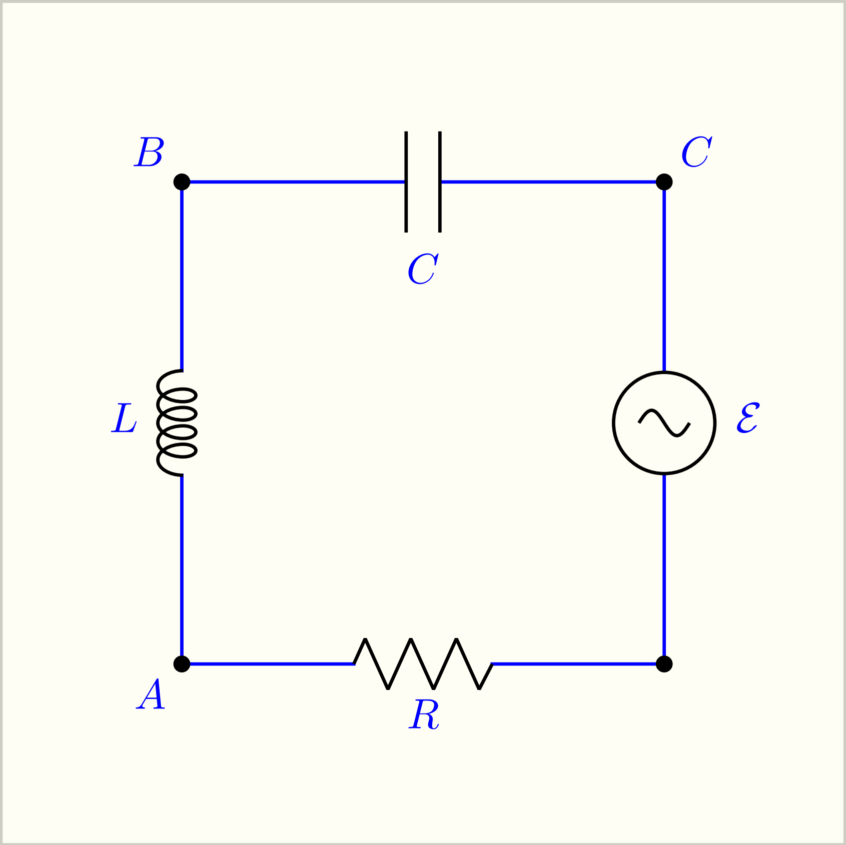
$$Q(t) = \Re z(t)$$

$$z(t) = z_0 \exp(i\omega t)$$

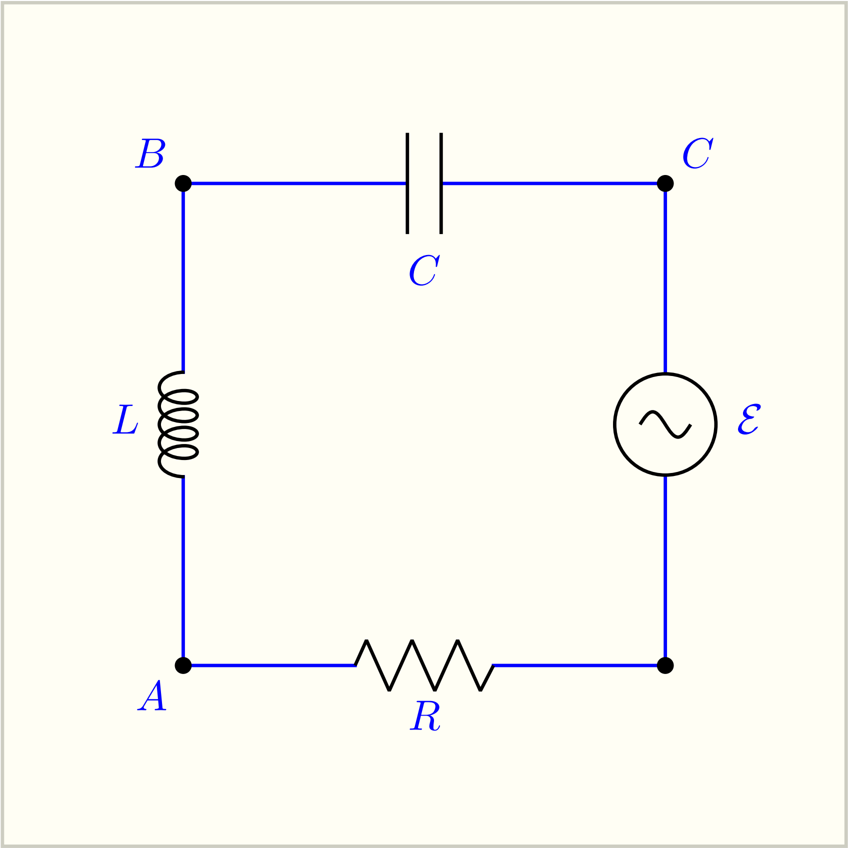
$$-\omega^2 z_0 + i\frac{2\omega}{\tau} z_0 + \omega_0^2 z_0 = \frac{\mathscr{E}_0}{L}$$



Solução da equação não-homogênea

$$\frac{d^2z}{dt^2} + \frac{2}{\tau}\frac{dz}{dt} + \omega_0^2 z = \frac{\mathscr{E}_0}{L}\exp(i\omega t)$$

$$Q(t) = \Re z(t)$$



$$z(t) = z_0 \exp(i\omega t)$$

$$-\omega^2 z_0 + i\frac{2\omega}{\tau} z_0 + \omega_0^2 z_0 = \frac{\mathscr{E}_0}{L}$$

$$z_0 = \frac{\mathscr{E}_0}{L}\frac{1}{-\omega^2 + i\frac{2\omega}{\tau} + \omega_0^2}$$

Solução da equação não-homogênea

$$\frac{d^2z}{dt^2} + \frac{2}{\tau}\frac{dz}{dt} + \omega_0^2 z = \frac{\mathscr{E}_0}{L}\exp(i\omega t)$$

$$Q(t) = \Re z(t)$$

$$z(t) = z_0 \exp(i\omega t)$$

$$-\omega^2 z_0 + i\frac{2\omega}{\tau} z_0 + \omega_0^2 z_0 = \frac{\mathscr{E}_0}{L}$$

$$z_0 = \frac{\mathscr{E}_0}{L}\frac{1}{-\omega^2 + i\frac{2\omega}{\tau} + \omega_0^2}$$

$$z(t) = \frac{\mathscr{E}_0}{L}\frac{\exp(i\omega t)}{\omega_0^2 - \omega^2 + i\frac{2\omega}{\tau}}$$