LIFT-CURVE SLOPE OF WING-BODY COMBINATIONS

1. NOTATION AND UNITS (see Sketch 1.1)

		SI	*British*
A	net wing aspect ratio, $4(s-r)^2/S$		
C_L	lift coefficient based on net wing area		
C_N	normal force coefficient based on body maximum cross-sectional area		
c_r	wing chord at wing-body junction	m	ft
c_t	wing tip chord	m	ft
K_B	ratio of body-alone lift to lift of net wing, see Equation (3.2)		
$K_{B(W)}$	ratio of change in body lift due to presence of wing to lift of net wing, see Equation (3.4)		
K_C	ratio of wing-body lift to lift of net wing, see Equation (3.1)		
$K_{W(B)}$	ratio of wing lift in presence of body to lift of net wing, see Equation (3.3)		
$k_{B(W)}$	ratio of body lift at zero angle of attack due to presence of deflected wing to lift of net wing, see Equation (3.9)		
$k_{W(B)}$	ratio of lift of deflected wing in presence of body at zero angle of attack to lift of net wing, see Equation (3.8)		
L	lift	N	lbf
l_A	afterbody length, see Sketch 1.1	m	ft
M	free-stream Mach number		
r	body maximum radius	m	ft
S	net wing area $(s-r)(c_r+c_t)$	m^2	ft^2
s	gross wing semi-span, see Sketch 1.1	m	ft
α	angle of attack of body centre-line or wing alone	radian	radian
β	compressibility parameter, $(M^2-1)^{½}$		
δ	all-moving wing deflection angle, see Sketch 1.1	radian	radian

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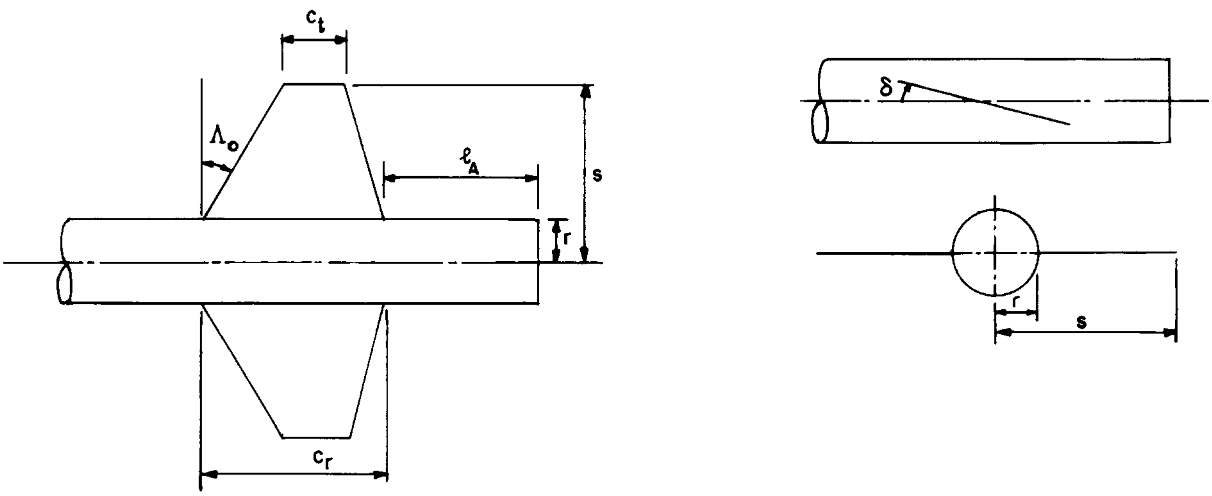
Λ_0	sweepback of wing leading-edge	deg	deg
λ	ratio of wing tip chord to chord at wing-body junction, c_t/c_r		

Subscripts

B	denotes body alone
$B(W)$	denotes change in contribution of body due to presence of wing
C	denotes combination
W	denotes net wing
$W(B)$	denotes contribution of wing in presence of body



Sketch 1.1 Wing-Body Geometry

2. INTRODUCTION

This Item presents a means of estimating the lift-curve slope of configurations comprising an axisymmetric body with a wing mounted at mid-height, see Sketch 1.1, given the body-alone and net wing lift-curve slopes. In addition the Item provides a means of calculating the change in lift coefficient due to small all-moving wing deflections. The method is that of Derivation 1, which is further developed in Derivations 2 and 3.

The method was originally devised for missile-type configurations, where the diameter of the body compared to the span of the wing is large enough for the interference loads to be a large proportion of the total. At subsonic speeds experience has shown that for small r/s a good approximation to the wing-body lift-curve slope can be made rapidly by simply estimating the lift-curve slope for the gross wing planform, see Appendix A to Item No. 70011 (Reference 4).

Within the limits of accuracy of this Item the lift-curve slope may be taken as equal to the normal force-curve slope.

The data are presented graphically but a complete list of the equations is given in Appendix A. The use of the method is illustrated by means of a worked example in Section 6.

A method for the prediction of aerodynamic centre position of wing-body combinations is given in Item No. 92024 (Reference 8). Item No. 91042 (Reference 7) presents a method for estimating the normal force coefficient of bodies in combination with delta, cropped delta and rectangular wings up to high angles of attack at supersonic speeds.

Item No. 95009 (Reference 9) presents a method for estimating the effect of wing height on lift and aerodynamic centre of slender wing-body combinations.

3. METHOD

3.1 Effect of Angle of Attack ($\delta = 0$)

The principal lift components on a wing-body combination are

(i) the lift of the body alone, L_B,

(ii) the lift on the wing in the presence of the body, $L_{W(B)}$

and (iii) the change of lift on the body due to the presence of the wing, $L_{B(W)}$.

The ratio, K_C, of the lift of a wing-body combination to that of the net wing is therefore given by

$$K_C = K_B + K_{W(B)} + K_{B(W)}. \tag{3.1}$$

The ratio, K_B, for the body alone is given by

$$K_B = \frac{\pi r^2 (\partial C_N/\partial\alpha)_B}{S(\partial C_L/\partial\alpha)_W}, \tag{3.2}$$

where the wing-alone lift-curve slope may be found from Item Nos 70011 (Reference 4) or 70012 (Reference 5) for subsonic or supersonic speeds, respectively, and the normal-force-curve slope for forebody-cylinder bodies may be obtained from Item No. 89008 (Reference 6).

The lift-curve slope of the wing in the presence of the body is given by

$$\left(\frac{\partial C_L}{\partial\alpha}\right)_{W(B)} = K_{W(B)}\left(\frac{\partial C_L}{\partial\alpha}\right)_W \tag{3.3}$$

in which the factor $K_{W(B)}$, obtained from slender-body theory (Derivation 1), is plotted in Figure 1 as a function of r/s, the ratio of the body maximum radius to the gross wing semi-span (Equation (A3.1) of Appendix A).

The change in lift-curve slope of the body due to the presence of the wing is given by

$$\left(\frac{\partial C_L}{\partial\alpha}\right)_{B(W)} = K_{B(W)}\left(\frac{\partial C_L}{\partial\alpha}\right)_W. \tag{3.4}$$

At subsonic speeds and at supersonic speeds where the Mach line originating from the wing tip leading-edge cuts the root chord upstream of the trailing edge, *i.e.*

$$\beta A(1+\lambda)\left(\frac{1}{\beta\cot\Lambda_0}+1\right) < 4 \text{ or, equivalently, } \tan\Lambda_0+\beta < \frac{c_r}{s-r},$$

$K_{B(W)}$ is also obtained from slender-body theory (Derivation 1) and is plotted as a function of r/s in Figure 1 (Equation (A4.1) of Appendix A). It is useful to note that the sum of the slender body values for $K_{W(B)}$ and $K_{B(W)}$ is given by the simple equation

$$K_{W(B)} + K_{B(W)} = \left(1+\frac{r}{s}\right)^2. \tag{3.5}$$

In cases where the Mach line originating from the wing tip leading-edge cuts the root chord extended downstream of the trailing edge, *i.e.*

$$\beta A(1+\lambda)\left(\frac{1}{\beta\cot\Lambda_0}+1\right) > 4 \text{ or, equivalently, } \tan\Lambda_0+\beta > \frac{c_r}{s-r},$$

the factor $K_{B(W)}$, obtained from Derivations 1, 2 and 3 is plotted in parametric form in Figure 2 for configurations with zero to full afterbody. A full afterbody is defined as one which extends sufficiently far aft to contain all of the lift carry-over generated by the wing, *i.e.* for $l_A/2r\beta \geq 1$. Interpolation in the sweepback parameter is in terms of $\beta\cot\Lambda_0$ for low values up to 0.5, whereafter the reciprocal parameter $1/(\beta\cot\Lambda_0) \leq 2.0$ is used. Figures 2a to 2e present data for $K_{B(W)}$ for values of $l_A/2r\beta$ of 0, 0.25, 0.5, 0.75 and 1.0. Estimates at intermediate values may be obtained by cross-plotting. Since wing carry-over load does not extend beyond the full afterbody length, the value of $K_{B(W)}$ for $l_A/2r\beta > 1$ equals that at $l_A/2r\beta = 1$ and is thus given by Figure 2e. The full equations leading to Figure 2, taken from Derivation 3, are given in Appendix A.

In summary,

$$\left(\frac{\partial C_L}{\partial\alpha}\right)_C = K_C\left(\frac{\partial C_L}{\partial\alpha}\right)_W = (K_B + K_{W(B)} + K_{B(W)})\left(\frac{\partial C_L}{\partial\alpha}\right)_W \tag{3.6}$$

or, since K_B is used less often than $K_{W(B)}$ and $K_{B(W)}$,

$$\left(\frac{\partial C_L}{\partial\alpha}\right)_C = \frac{\pi r^2}{S}\left(\frac{\partial C_N}{\partial\alpha}\right)_B + (K_{W(B)} + K_{B(W)})\left(\frac{\partial C_L}{\partial\alpha}\right)_W. \tag{3.7}$$

3.2 Effect of All-moving Wing Deflection ($\alpha = 0$)

The method of Derivation 1 also provides an analogous means of calculating the slope of the lift versus wing deflection curve for zero angle of attack. The equations for the lift of the wing in the presence of the body and the lift of the body in the presence of the wing are

$$\left(\frac{\partial C_L}{\partial \delta}\right)_{W(B)} = k_{W(B)}\left(\frac{\partial C_L}{\partial \alpha}\right)_W \tag{3.8}$$

and

$$\left(\frac{\partial C_L}{\partial \delta}\right)_{B(W)} = k_{B(W)}\left(\frac{\partial C_L}{\partial \alpha}\right)_W \text{, respectively.} \tag{3.9}$$

Values of $k_{W(B)}$ and $k_{B(W)}$ may be obtained from Figure 3 which is derived from slender-body theory (Equations (A6.1) and (A7.1), respectively, of Appendix A). However, Derivation 1 advises that for the particular case of rectangular wings at supersonic speeds, where $\beta A > 2$, linear theory provides more accurate estimates for $k_{W(B)}$, which may be found in Figure 4.

3.3 Combined Effect

For combinations of small angles of attack and wing deflection the lift coefficient for a wing-body combination may be obtained from the integration and combination of Equations (3.6) or (3.7), (3.8) and (3.9), which yield

$$(C_L)_C = \left(\frac{\partial C_L}{\partial \alpha}\right)_C \alpha + (k_{W(B)} + k_{B(W)})\left(\frac{\partial C_L}{\partial \alpha}\right)_W \delta . \tag{3.10}$$

4. ACCURACY AND APPLICABILITY

The lift-curve slopes and lift coefficients for wing-body combinations derived from the method of this Item are applicable to low angles of attack and wing deflection (α, $\delta \le 5°$, say), when viscous cross-flow effects are unimportant.

At supersonic speeds the experimental data show no consistent deviation from the theoretical values, but there is a scatter of ±10%. As the Mach number is reduced below unity the agreement with experiment becomes worse, *e.g.* at high subsonic speeds the lift-curve slope may be underestimated by this Item by about 10%; the error band at low subsonic Mach numbers is again ±10%.

Although this Item may strictly be used only to estimate lift-curve slopes of wing-body combinations with wings having unswept or swept-forward trailing-edges, a limited amount of experimental evidence indicates that estimates which are within the general accuracy of the Item may be made for wings with moderately swept-back trailing-edges, although the applicability is unknown for wings having extreme values. This suggests that the method should be applicable to swept-forward leading edges by consideration of the reverse flow theorem, simply by substituting the angle of trailing-edge sweep in place of Λ_0, where required. Experimental data indicate that the method can produce good results for wings with values of forward sweep up to 45 degrees.

5. DERIVATION AND REFERENCES

5.1 Derivation

The Derivation lists selected sources of information that have assisted in the preparation of this Item.

1.	PITTS, W.C. NIELSEN, J.N. KAATTARI, G.E.	Lift and center of pressure of wing-body-tail combinations at subsonic, transonic and supersonic speeds. NACA Rep. 1307, 1957.
2.	VUKELICH, S.R. WILLIAMS, J.E.	Wing-body carryover at supersonic speeds. *AIAA J*., Vol. 19, No. 5, May 1981.
3.	VIRA, N.R. FAN, D.N.	Closed form solutions of supersonic wing-body interference. *AIAA J*., Vol. 20, No. 6, June 1982.

5.2 References

The References list selected sources of information supplementary to that given in this Item.

4.	ESDU	Lift-curve slope and aerodynamic centre position of wings in inviscid subsonic flow. ESDU International, Item No. 70011, 1970. ESDUpac A7011.
5.	ESDU	Lift-curve slope and aerodynamic centre position of wings in inviscid supersonic flow. ESDU International, Item No. 70012, 1970. ESDUpac A7012.
6.	ESDU	Normal-force-curve and pitching-moment-curve slopes of forebody-cylinder combinations at zero angle of attack at Mach numbers up to 5. ESDU International, Item No. 89008, 1989.
7.	ESDU	Normal force of low aspect ratio wing-body combinations up to high angles of attack at supersonic speed. ESDU International, Item No. 91042, 1991.
8.	ESDU	Aerodynamic centre of wing-body combinations. ESDU International, Item No. 92024, 1992.
9.	ESDU	Effect of wing height on lift and aerodynamic centre for a slender wing-body combination. ESDU International, Item No. 95009, 1995.

6. EXAMPLE

A wing-body combination has a wing geometry given by $S = 4.5\ \text{m}^2$, $c_r = 1.81$ m, $\lambda = 0.4$, $\Lambda_0 = 20°$ and $A = 2.8$. The body consists of a cone-cylinder with $r = 0.3$ m; also $r/s = 0.145$. The all-moving wing is mounted so that there is an afterbody with $l_A = 0.6$ m. Find both the lift-curve slope for the combination and the equation for the lift coefficient at angle of attack including wing deflection

(i) at a Mach number of 0.9 given that $(\partial C_L/\partial\alpha)_W = 3.88\ \text{rad}^{-1}$ and $(\partial C_N/\partial\alpha)_B = 2.47\ \text{rad}^{-1}$

and (ii) at a Mach number of 1.9 given that $(\partial C_L/\partial\alpha)_W = 2.35\ \text{rad}^{-1}$ and $(\partial C_N/\partial\alpha)_B = 2.66\ \text{rad}^{-1}$.

(i) *Calculation for* $M = 0.9$

From Equation (3.2),

$$K_B = \frac{\pi \times (0.3)^2 \times 2.47}{4.5 \times 3.88} = 0.040 .$$

From Figure 1, for $r/s = 0.145$,

$$K_{W(B)} = 1.115 \text{ and } K_{B(W)} = 0.196 .$$

Therefore, using Equation (3.1),

$$K_C = 0.040 + 1.115 + 0.196 = 1.351 .$$

(Alternatively, using Equation (3.5),

$$K_{W(B)} + K_{B(W)} = (1 + 0.145)^2 = 1.311 ;$$

therefore, using Equation (3.1),

$$K_C = 0.040 + 1.311 = 1.351.)$$

Hence, using Equation (3.6),

$$\left(\frac{\partial C_L}{\partial\alpha}\right)_C = K_C\left(\frac{\partial C_L}{\partial\alpha}\right)_W = 1.351 \times 3.88 = 5.24\,\text{rad}^{-1}, \text{ based on net wing area.}$$

From Figure 3, for $r/s = 0.145$,

$$k_{W(B)} = 0.953 \text{ and } k_{B(W)} = 0.162 .$$

Hence, from Equation (3.10)

$$\begin{aligned}(C_L)_C &= \left(\frac{\partial C_L}{\partial\alpha}\right)_C \alpha + (k_{W(B)} + k_{B(W)})\left(\frac{\partial C_L}{\partial\alpha}\right)_W \delta . \\ &= 5.24\alpha + (0.953 + 0.162)(3.88)\delta \\ &= 5.24\alpha + 4.33\delta, \text{ where } \alpha \text{ and } \delta \text{ are in radians.}\end{aligned}$$

(ii) *Calculation for M* = 1.9

From Equation (3.2),

$$K_B = \frac{\pi \times (0.3)^2 \times 2.66}{4.5 \times 2.35} = 0.0711.$$

Also $\beta = (M^2 - 1)^{½} = (1.9^2 - 1)^{½} = 1.616.$

Now

$$\beta A(1+\lambda)\left(\frac{1}{\beta \cot \Lambda_0} + 1\right) = 1.616 \times 2.8 \times 1.4\left(\frac{1}{1.616 \times 2.747} + 1\right)$$

$$= 7.76,\ i.e.\ > 4,$$

so that Figure 2 is used for the factor $K_{B(W)}$.

The configuration has an afterbody of length 0.6 m; therefore

$$\frac{l_A}{2r\beta} = \frac{0.6}{2 \times 0.3 \times 1.616} = 0.619,$$

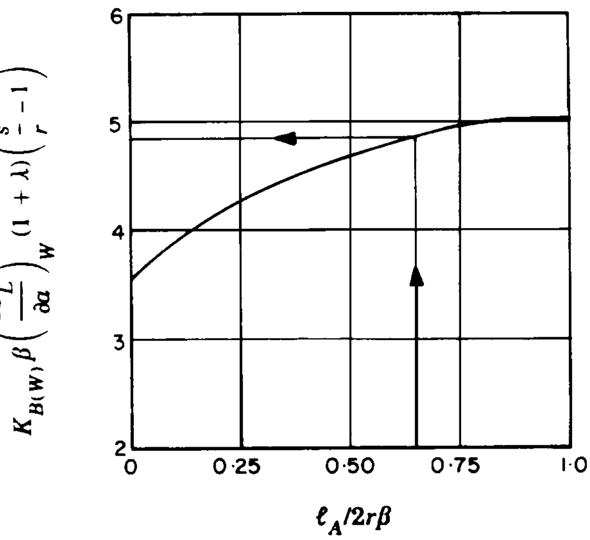
$$\frac{2r\beta}{c_r} = \frac{2 \times 0.3 \times 1.616}{1.81} = 0.536$$

and

$$\frac{1}{\beta \cot \Lambda_0} = \frac{1}{1.616 \times 2.747} = 0.225,$$

so that, from Figures 2a to 2e, the following table is obtained.

$l_A/2r\beta$	$K_{B(W)}\beta(\partial C_L/\partial a)_W(1+\lambda)((s/r)-1)$
0	3.57
0.25	4.23
0.5	4.69
0.75	4.96
1.00	5.05



Sketch 6.1 Cross-plot for $l_A/2r\beta = 0.619$

A cross-plot for $l_A/2r\beta = 0.619$, see Sketch 6.1, gives

$$K_{B(W)}\beta\left(\frac{\partial C_L}{\partial\alpha}\right)_W(1+\lambda)\left(\frac{s}{r}-1\right) = 4.84$$

and so $$K_{B(W)} = \frac{4.84}{1.616\times 2.35\times 1.4\times[(1/0.145)-1]} = 0.1544\,.$$

From Figure 1, with $r/s = 0.145$,

$$K_{W(B)} = 1.115\,.$$

Therefore, using Equation (3.1),

$$\begin{aligned} K_C &= 0.0711 + 1.115 + 0.1544 \\ &= 1.340\,. \end{aligned}$$

From Equation (3.6),

$$\left(\frac{\partial C_L}{\partial\alpha}\right)_C = K_C\left(\frac{\partial C_L}{\partial\alpha}\right)_W = 1.340\times 2.35 = 3.15\,\text{rad}^{-1}$$, based on net wing area.

From Figure 3, for $r/s = 0.145$,

$$k_{W(B)} = 0.953 \text{ and } k_{B(W)} = 0.162\,.$$

Hence, from Equation (3.10),

$$\begin{aligned} (C_L)_C &= \left(\frac{\partial C_L}{\partial\alpha}\right)_C\alpha + (k_{W(B)} + k_{B(W)})\left(\frac{\partial C_L}{\partial\alpha}\right)_W\delta\,. \\ &= 3.15\alpha + (0.953 + 0.162)(2.35)\delta \\ &= (3.15\alpha + 2.62\delta) \end{aligned}$$

, where α and δ are in radians.

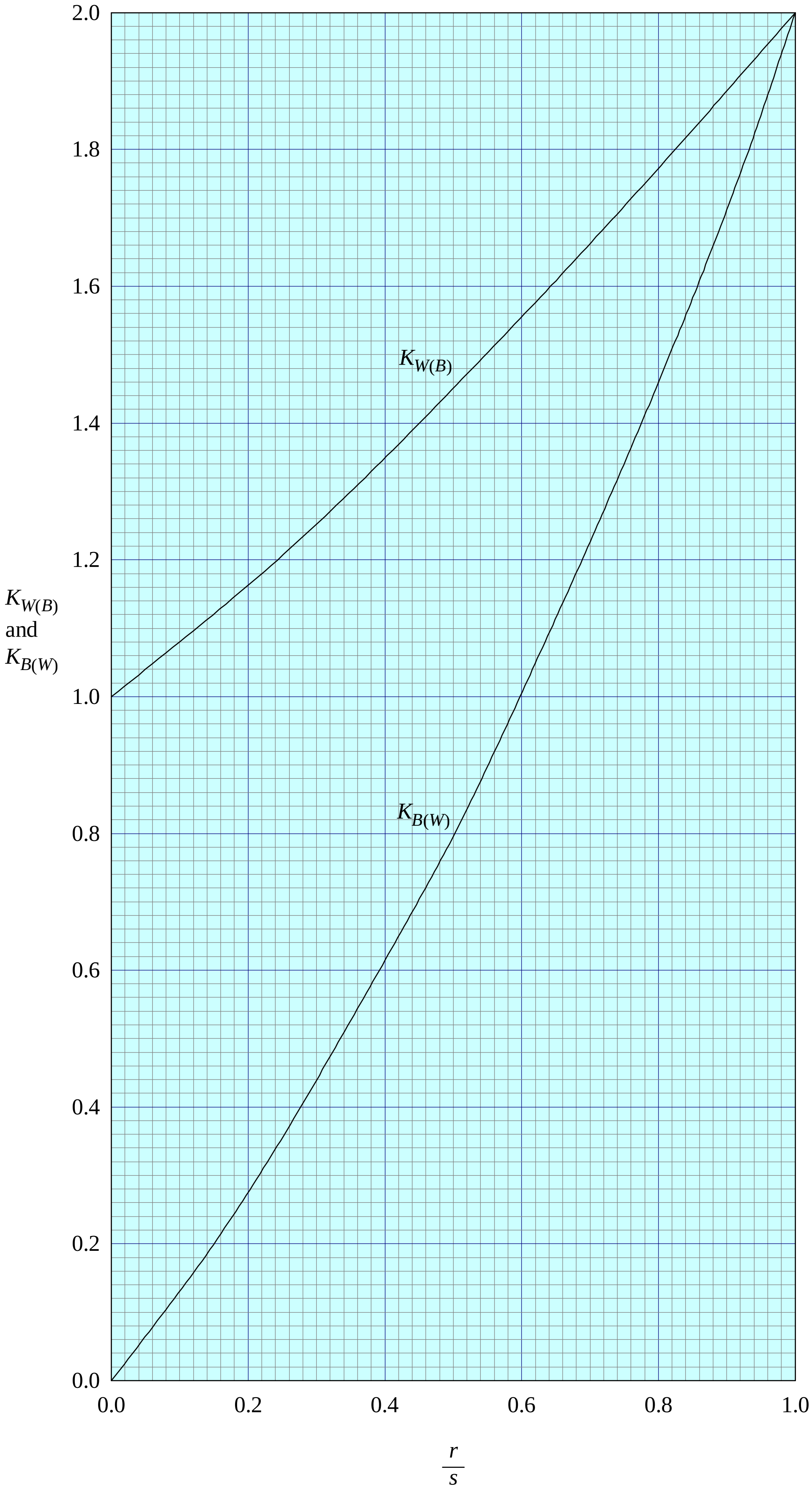


FIGURE 1 $K_{W(B)}$ AND $K_{B(W)}$ FOR $\beta A(1+\lambda)\left(\frac{1}{\beta \cot \Lambda_0}+1\right)<4$

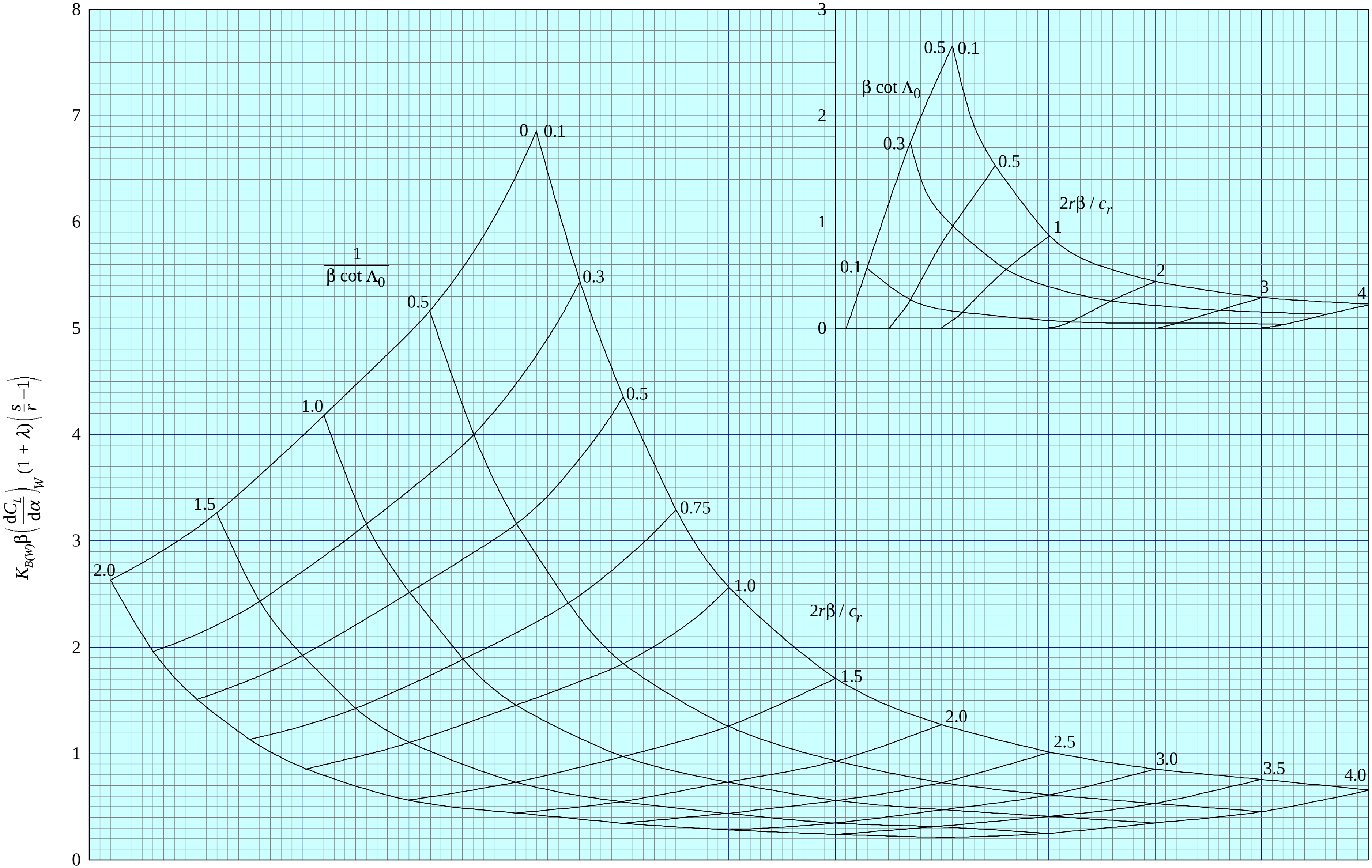


FIGURE 2a $l_A/2r\beta = 0$

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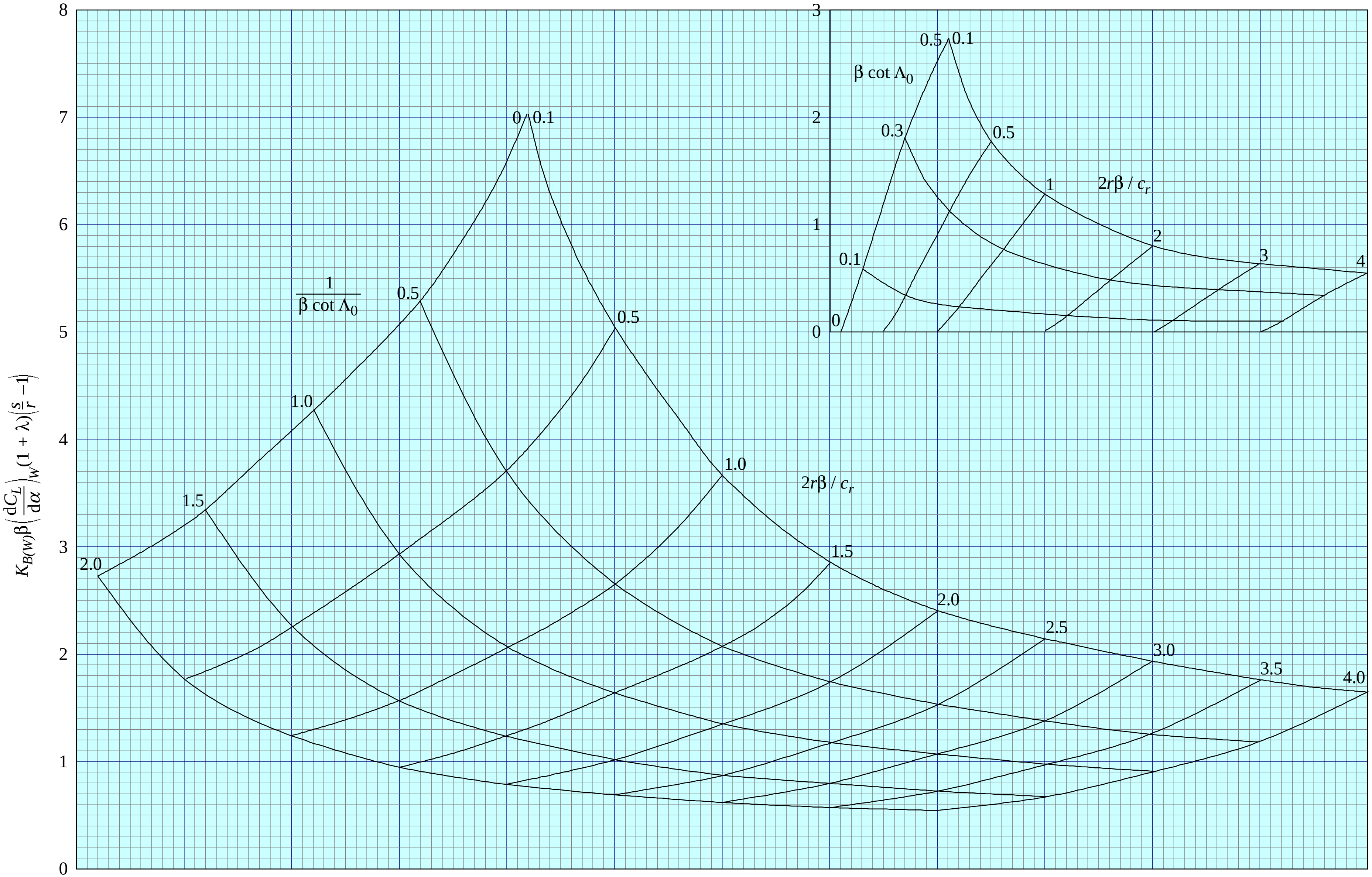

FIGURE 2b $l_A/2r\beta = 0.25$

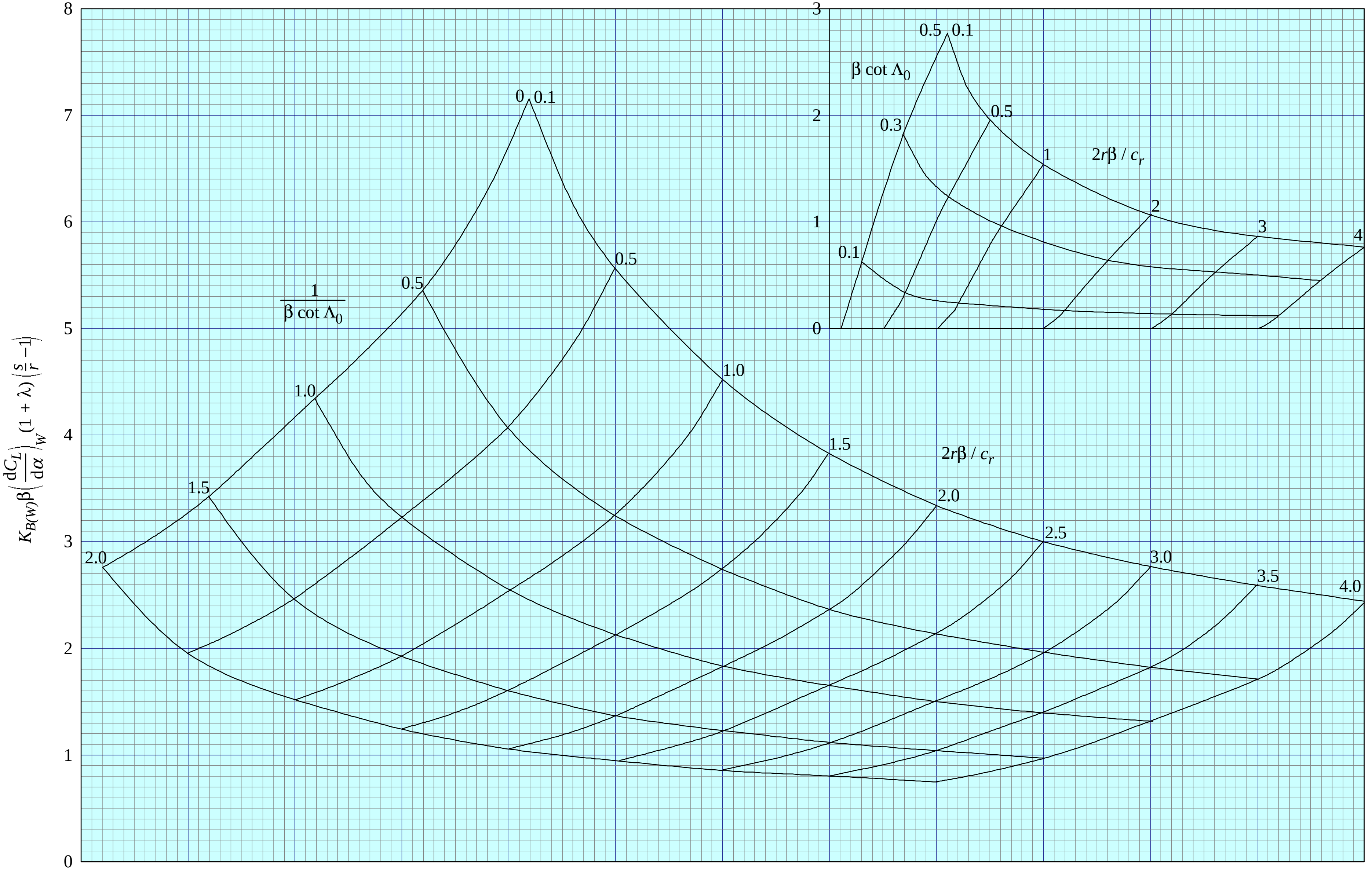

FIGURE 2c $l_A/2r\beta = 0.5$

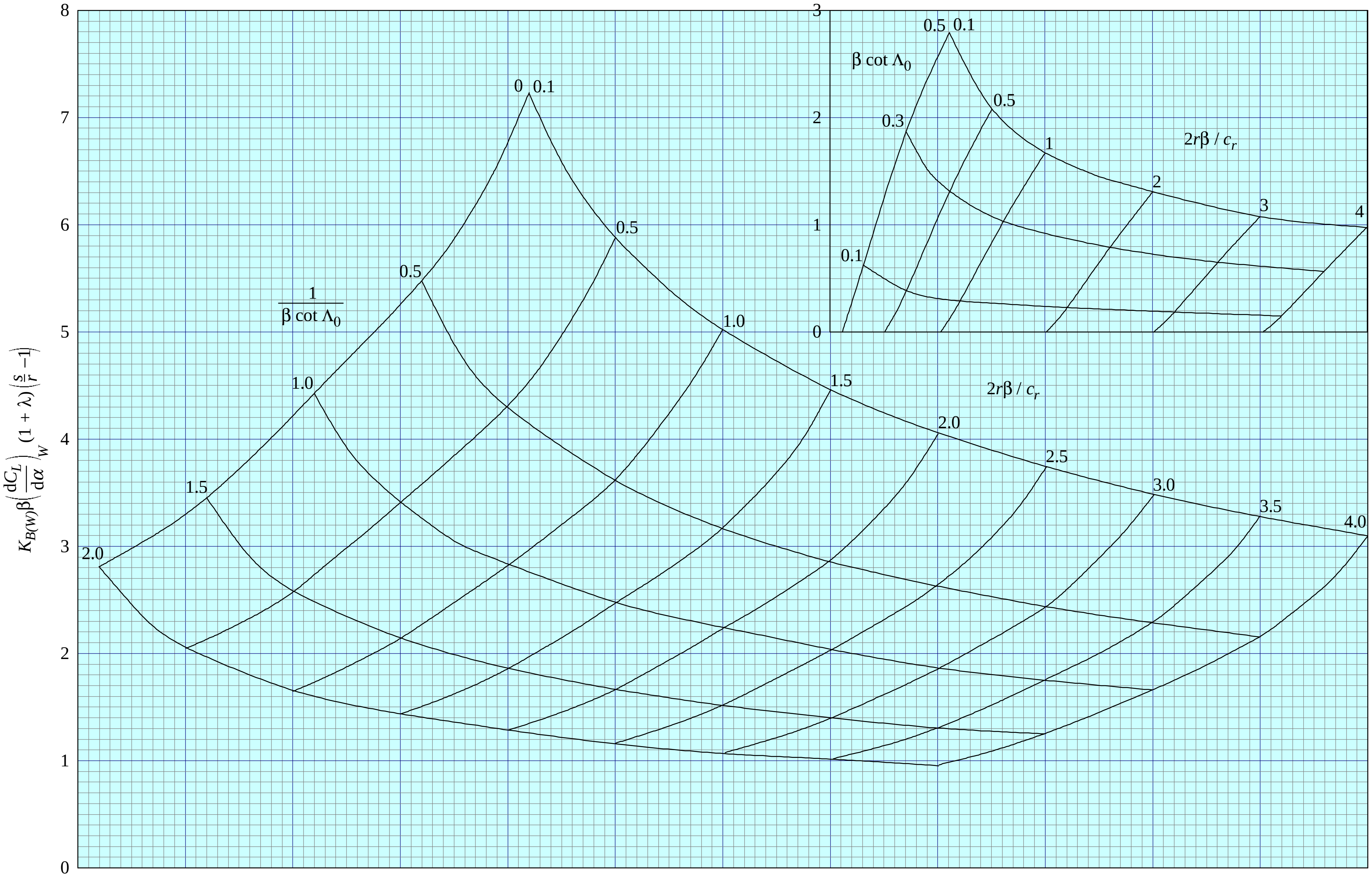

FIGURE 2d $l_A/2r\beta = 0.75$

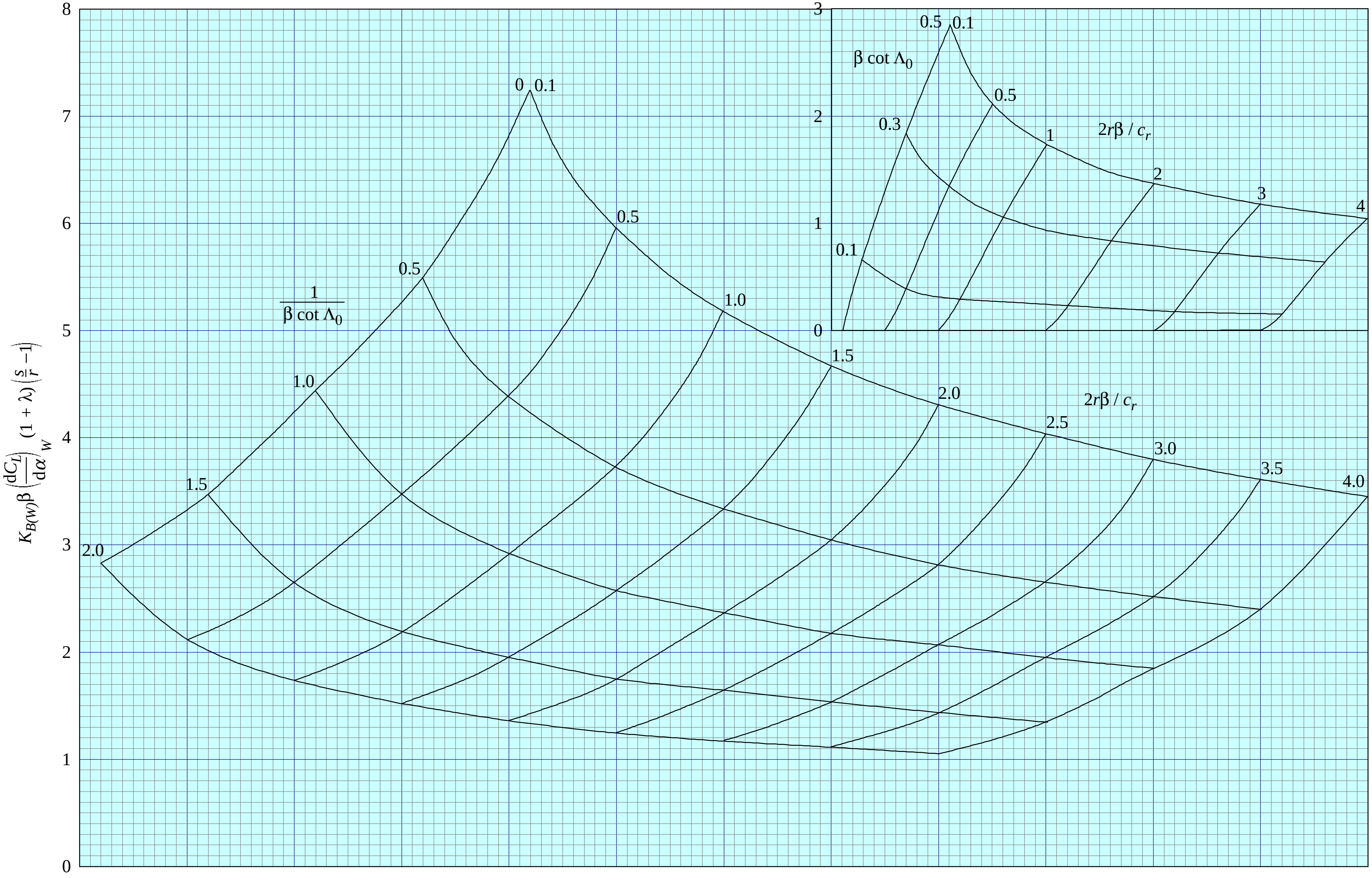

FIGURE 2e $l_A/2r\beta \geq 1$

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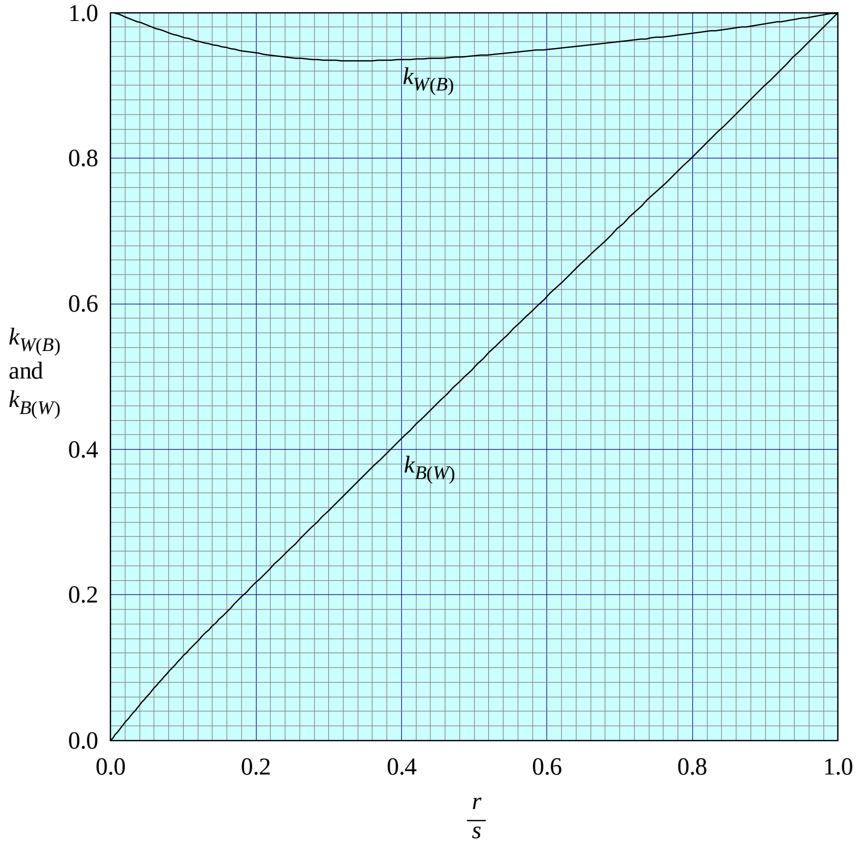


FIGURE 3 $k_{W(B)}$ AND $k_{B(W)}$ FROM SLENDER-BODY THEORY

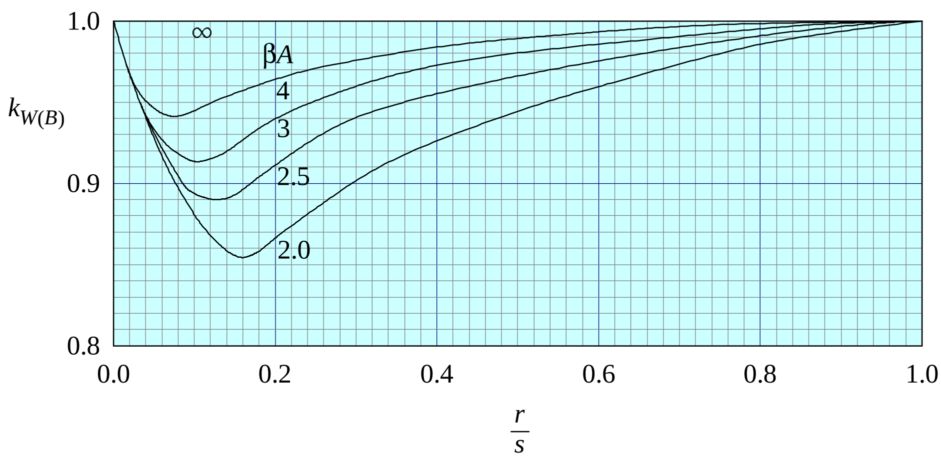


FIGURE 4 $k_{W(B)}$ FOR RECTANGULAR WING-BODY COMBINATIONS AT SUPERSONIC SPEEDS WITH $\beta A \geq 2$

APPENDIX A EQUATIONS FOR $K_{W(B)}$, $K_{B(W)}$, $k_{W(B)}$ AND $k_{B(W)}$

A1. ADDITIONAL NOTATION

The following parameters are introduced in the interest of brevity.

B	dimensionless parameter, $\beta \cot \Lambda_0$
D	dimensionless parameter, $2r\beta/c_r$
$\bar{K}_{B(W)}$	$K_{B(W)}\beta\left(\frac{\partial C_L}{\partial \alpha}\right)_W (1 + \lambda)\left(\frac{s}{r} - 1\right)$
P	dimensionless parameter, $l_A/2r\beta$
R	dimensionless parameter, $(l_A + c_r)/2r\beta (= P + 1/D)$

A2. INTRODUCTION

This appendix presents the equations for $K_{W(B)}$, $K_{B(W)}$, $k_{W(B)}$ and $k_{B(W)}$ from Derivations 1, 2 and 3. The results given in Derivation 1 for $K_{B(W)}$ when $\beta A(1+\lambda)(1/(\beta\cot\Lambda_0)+1) > 4$ are restricted to the special cases of zero afterbody ($P = 0$) and full afterbody ($P \geq 1$). This work is extended in Derivations 2 and 3 and a closed-form solution presented for afterbodies of any length. This avoids the use of linear interpolation between the zero and full afterbody results to obtain an estimate for an intermediate afterbody length, which can underestimate $K_{B(W)}$ by up to 20%.

A3. THE EQUATION FOR $K_{W(B)}$

The equation for $K_{W(B)}$*, from Derivation 1, is

$$K_{W(B)} = \frac{\frac{2}{\pi}\left\{\left(1 + \frac{r^4}{s^4}\right)\left[\frac{1}{2}\tan^{-1}\frac{1}{2}\left(\frac{s}{r} - \frac{r}{s}\right) + \frac{\pi}{4}\right] - \frac{r^2}{s^2}\left[\left(\frac{s}{r} - \frac{r}{s}\right) + 2\tan^{-1}\frac{r}{s}\right]\right\}}{\left(1 - \frac{r}{s}\right)^2} \qquad \text{(A3.1)}$$

A4. THE EQUATION FOR $K_{B(W)}$ WHEN $\beta A(1+\lambda)(1/(\beta\cot\Lambda_0)+1) < 4$

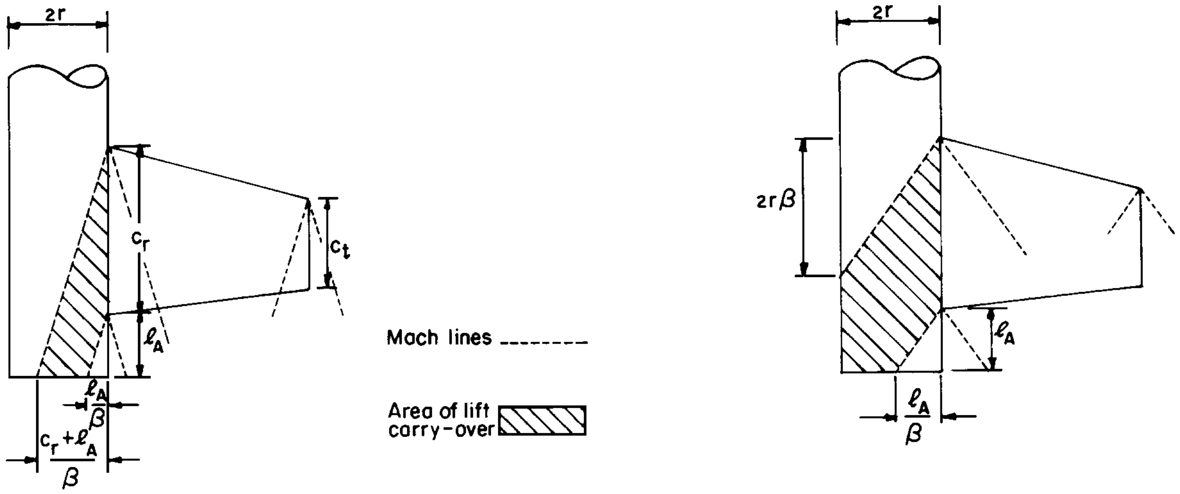
The equation for $K_{B(W)}$*, from Derivation 1, is

$$K_{B(W)} = \frac{\left(1 - \frac{r^2}{s^2}\right)^2 - \frac{2}{\pi}\left\{\left(1 + \frac{r^4}{s^4}\right)\left[\frac{1}{2}\tan^{-1}\frac{1}{2}\left(\frac{s}{r} - \frac{r}{s}\right) + \frac{\pi}{4}\right] - \frac{r^2}{s^2}\left[\left(\frac{s}{r} - \frac{r}{s}\right) + 2\tan^{-1}\frac{r}{s}\right]\right\}}{\left(1 - \frac{r}{s}\right)^2} \qquad \text{(A4.1)}$$

* Note the simple relationship between $K_{W(B)}$ and $K_{B(W)}$, as derived from slender-body theory, given in Equation (3.5).

A5. THE EQUATIONS FOR $K_{B(W)}$ WHEN $\beta A(1+\lambda)(1/(\beta\cot\Lambda_0)+1)>4$

There are separate pairs of equations that deal with wings with subsonic and supersonic leading edges. The first equation of each pair evaluates the contribution to $K_{B(W)}$ for cases where the Mach line generated by the wing root leading-edge cuts the base of the afterbody $(R \le 1)$, and the second equation where the Mach line cuts the side of the afterbody $(R \ge 1)$, see Sketch A5.1.



$R \le 1$ $R \ge 1$

Sketch A5.1

Obviously, where the afterbody is longer than that required to contain all of the lift carry-over $(P>1)$ the excess afterbody length is of no interest, because $K_{B(W)}(P>1) = K_{B(W)}(P=1)$. Therefore, if $P>1$ set $P = 1$ for the purposes of the calculation.

A5.1 Subsonic Leading-edge $(B<1)$

The equation for $\bar{K}_{B(W)}$ when $R \le 1$ is

$$\bar{K}_{B(W)}(R \le 1) = \frac{16B^{1/2}D}{\pi(B+1)}\left\{\frac{B^{3/2}}{D^2(1+B)}\left[\left(\frac{B+(1+B)PD}{B}\right)^{1/2}-2\right]\right.$$

$$\left.-\left(\frac{B}{1+B}\right)\frac{1}{D^{1/2}}(BR+P)^{3/2}+B(1+B)R^2\tan^{-1}\left(\frac{1/D}{BR+P}\right)^{1/2}\right\}. \qquad \text{(A5.1)}$$

The equation for $\bar{K}_{B(W)}$ when $R \ge 1$ is

$$\bar{K}_{B(W)}(R \ge 1) = \bar{K}_{B(W)}(R \le 1) + \frac{16B^{1/2}D}{\pi(B+1)}\left\{(BR+1)[(R-1)(BR+1)]^{1/2}\right.$$

$$\left.-\frac{(B+1)}{B^{1/2}}\tanh^{-1}\left(\frac{BR-B}{BR+1}\right)^{1/2}-B(1+B)R^2\tan^{-1}\left(\frac{R-1}{BR+1}\right)^{1/2}\right\}. \qquad \text{(A5.2)}$$

A5.2 Supersonic Leading-edge $(B > 1)$

The equation for $\bar{K}_{B(W)}$ when $R \le 1$ is

$$\bar{K}_{B(W)}(R \le 1) = \frac{8D}{\pi(B^2-1)^{1/2}}\left\{\left(\frac{-B}{1+B}\right)(BR+P)^2\cos^{-1}\left(\frac{R+BP}{BR+P}\right)\right.$$
$$+\frac{B}{D^2}\frac{(B^2-1)^{1/2}}{(1+B)}\left[(1+2PD)^{1/2}-1\right]-\frac{B^2}{D^2(1+B)}\cos^{-1}\frac{1}{B}$$
$$\left.+BR^2(B^2-1)^{1/2}\cos^{-1}\frac{P}{R}\right\} . \qquad \text{(A5.3)}$$

The equation for $\bar{K}_{B(W)}$ when $R \ge 1$ is

$$\bar{K}_{B(W)}(R \ge 1) = \bar{K}_{B(W)}(R \le 1) + \frac{8D}{\pi(B^2-1)^{1/2}}$$
$$\times\left\{(BR+1)^2\cos^{-1}\left(\frac{R+B}{BR+1}\right)-(B^2-1)^{1/2}\cosh^{-1}R\right.$$
$$\left.+BR^2(B^2-1)^{1/2}\left[\sin^{-1}\left(\frac{1}{R}\right)-\frac{\pi}{2}\right]\right\}. \qquad \text{(A5.4)}$$

For the limiting case of $B \to \infty$, for rectangular wings, Equations (A5.3) and (A5.4) reduce to

$$\bar{K}_{B(W)}(R \le 1)_{limB \to \infty} = \frac{8D}{\pi}\left\{\left(\cos^{-1}\frac{P}{R}\right)(R^2-2RP)+R^2\left(1-\frac{P^2}{R^2}\right)^{1/2}\right.$$
$$\left.+\frac{1}{D^2}\left[(1+2PD)^{1/2}-1\right]-\frac{\pi}{2D^2}\right\}, \qquad \text{(A5.5)}$$

and

$$\bar{K}_{B(W)}(R \ge 1)_{limB \to \infty} = \bar{K}_{B(W)}(R \le 1)_{limB \to \infty}$$
$$+\frac{8D}{\pi}\left\{2R\cos^{-1}\frac{1}{R}-R(R^2-1)^{1/2}-\cosh^{-1}R\right\}, \qquad \text{(A5.6)}$$

respectively. Note that Equations (A5.5) and (A5.6) must be used to define the limiting case if the method is programmed, because the general Equations (A5.3) and (A5.4) are unsuitable for numerical evaluation as $B \to \infty$.

A6. THE EQUATION FOR $k_{W(B)}$

The Equation for $k_{W(B)}$ is

$$k_{W(B)} = \frac{1}{\pi^2}\left\{\frac{\pi^2(s/r+1)^2}{4(s/r)^2} + \frac{\pi[(s/r)^2+1]^2}{(s/r)^2(s/r-1)^2}\sin^{-1}\left[\frac{(s/r)^2-1}{(s/r)^2+1}\right] - \frac{2\pi(s/r+1)}{s/r(s/r-1)}\right.$$
$$+ \frac{[(s/r)^2+1]^2}{(s/r)^2(s/r-1)^2}\left(\sin^{-1}\left[\frac{(s/r)^2-1}{(s/r)^2+1}\right]\right)^2 - \frac{4(s/r+1)}{s/r(s/r-1)}\sin^{-1}\left[\frac{(s/r)^2-1}{(s/r)^2+1}\right]$$
$$\left. + \frac{8}{(s/r-1)^2}\log_e\left[\frac{(s/r)^2+1}{2s/r}\right]\right\} . \qquad \text{(A6.1)}$$

A7. THE EQUATION FOR $k_{B(W)}$

The Equation for $k_{B(W)}$ is

$$k_{B(W)} = K_{W(B)} - k_{W(B)}, \qquad \text{(A7.1)}$$

where $K_{W(B)}$ and $k_{W(B)}$ are obtained from Equations (A3.1) and (A6.1), respectively.

THE PREPARATION OF THIS DATA ITEM

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