EFFECT OF WING ON ROLLING MOMENT DUE TO YAWING

1. NOTATION AND UNITS

The derivative notation used is that proposed in ARC R&M 3562 (Hopkin, 1970) and described in Item No. 86021. Coefficients and aeronormalised derivatives are evaluated in aerodynamic body axes with origin at the aircraft centre of gravity and with the wing span as the characteristic length. The derivative L_r is often written as C_{l_r} in other systems of notation, but attention must be paid to the reference dimensions used. In particular, in forming C_{l_r} differentiation of C_l may be carried out with respect to $rb/2V$ not rb/V as implied in the Hopkin system. It is also to be noted that a constant datum value of V is employed by Hopkin.

		SI	*British*
A	aspect ratio, b^2/S		
a_1	lift-curve slope at zero lift	rad^{-1}	rad^{-1}
$a_{2\infty}$	slope of lift increment curve with flap deflection in two-dimensional flow	rad^{-1}	rad^{-1}
b	wing span	m	ft
$(b_f)_{i/b}$	spanwise distance between inner ends of flaps	m	ft
$(b_f)_{o/b}$	spanwise distance between outer ends of flaps	m	ft
C_L	wing lift coefficient		
C_l	wing rolling moment coefficient, $\mathscr{L}/½\rho V^2Sb$		
c_r	root chord	m	ft
c_t	tip chord	m	ft
$f_1(A)$	function of aspect ratio (see Equation (3.1))		
$f_2(A)$	function of aspect ratio (see Section 3.4)		
$g(\Lambda_{1/4})$	function of sweepback of quarter-chord line		
$\mathscr{L}$	wing rolling moment	N m	lbf ft
L_r	aeronormalised rolling moment derivative for the wing, $(\partial\mathscr{L}/\partial r)/½\rho VSb^2$		
M	Mach number		
r	angular velocity in yaw	rad/s	rad/s

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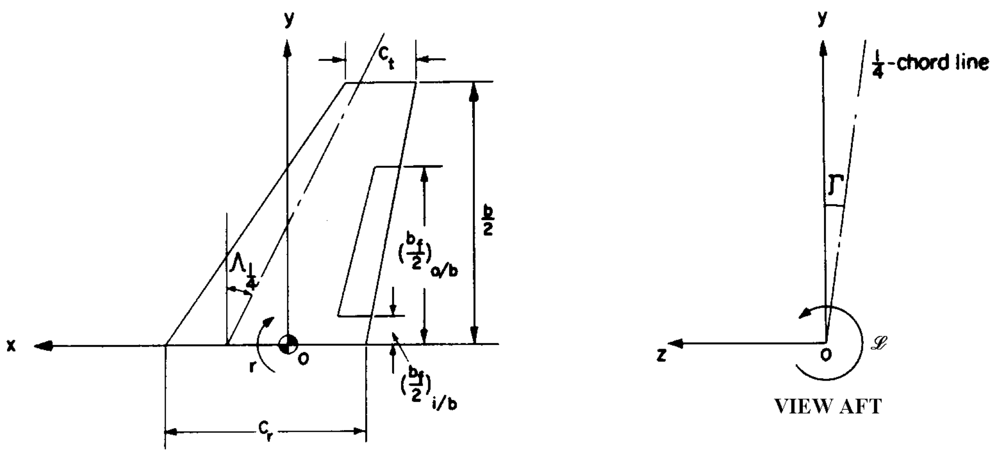
S	wing area	m^2	ft^2
V	velocity of aircraft relative to air	m/s	ft/s
Γ	dihedral angle (see Sketch 1.1)	degree	degree
δ_f	flap deflection	degree	degree
ε	wing twist, *i.e.* angle between no-lift lines of root and tip sections (positive for wash-out) see diagram in Figure 4)		
$\Lambda_{1/4}$	sweepback of quarter-chord line	degree	degree
λ	taper ratio, c_t/c_r		
ρ	density of air	kg/m^3	$slug/ft^3$

Suffixes

f	denotes quantity arising from deflection of trailing-edge flaps
p	denotes planform contribution to L_r
0	denotes quantity in incompressible flow (M = 0)
Γ	denotes quantity arising from dihedral
ε	denotes quantity arising from twist



Sketch 1.1

2. INTRODUCTION

The data given in this Item provide a means of estimating the contribution of a straight-tapered swept wing to the rolling moment derivative due to yawing at lift coefficients for which the flow remains fully attached (see Item No. 66033 for wings with symmetrical sections) and at Mach numbers for which the flow is subsonic over the whole of the wing. Some data are included for slender wings of delta and gothic planform.

The basic planform contribution (p) and the contributions due to dihedral (Γ), twist (ε) and deflected trailing edge flaps (f) can be taken into account, *i.e*.

$$L_r = (L_r)_p + (L_r)_\Gamma + (L_r)_\varepsilon + (L_r)_f. \tag{2.1}$$

3. INCOMPRESSIBLE FLOW (M = 0)

3.1 Planform Contribution, $(L_{r0})_p$

Figure 1a presents data for the planform contribution to L_{r0}, which is proportional to the overall lift coefficient. Ranges of aspect ratio from 1 to 12 and taper ratio from 0 to 1 are covered. The sweepback function $g(\Lambda_{1/4})$, involved in the ordinate parameter of Figure 1a, is given in Figure 1b for angles of sweepback of the quarter-chord line up to 60 degrees.

The curves given in Figure 1a were obtained by the application of lifting-surface theory (Derivation 11) to unswept wings. The sweepback function in Figure 1b is empirical and was obtained using Figure 1a and the wind-tunnel data of Derivations 4, 6, 7 and 12. The data were calculated on the assumption that the centre of gravity of the aircraft (assumed to be the centre of rotation) coincides with the longitudinal position of the wing aerodynamic centre. The effect of the finite distance between the centre of gravity and the aerodynamic centre in practical cases is not however likely to be large.

Agreement between values of $(L_{r0})_p$ estimated from this Item and the available wind-tunnel data for plain wings (Derivations 1, 4, 6, 7 and 12) is generally within ± 10 per cent.

Derivation 14 establishes expressions for the planform contribution to L_{r0} in fully-attached flow relating to slender delta and gothic wings with aspect ratios less than unity. These expressions may be reduced to

$$\frac{(L_{r0})_p}{C_L} = \frac{1}{a_1} f_1(A)\,. \tag{3.1}$$

The function $f_1(A)$ for delta and gothic wings is plotted in Figure 2. Values for a_1, the slope of the lift curve at zero lift, for use in Equation (3.1), may be obtained from Item No.71006.

Comparison of Equation (3.1) with experimental data (Derivation 13) at low incidences for two low aspect ratio (A = 0.53 and 1.07) delta wings obtained under oscillatory conditions are not conclusive (see Derivation 14). Although agreement is within about 20 per cent for the lower aspect ratio model, the experimental data for the higher aspect ratio model are very erratic, being heavily dependent on the oscillation frequency, and the agreement with Equation (3.1) is poor.

3.2 Dihedral Contribution $(L_{r0})_\Gamma$

Figure 3 gives data for the dihedral contribution to L_{r0} in terms of sweepback angle. These data were derived for untapered wings of large aspect ratio (Derivation 5) using lifting-line theory. The method of Derivation 5 indicates that the effect of decreasing the aspect ratio is to decrease the dihedral contribution to L_{r0}, but the effect is negligible for aspect ratios down to about 4 or 5. There is very little experimental evidence with which to compare the theoretical data. Tests on an untapered wing of aspect ratio 2.61 and 45 degrees of quarter-chord sweepback (Derivation 5) indicate that Figure 3 provides values, for this case at least, that are about 20 per cent low. It will be appreciated that this error would have been larger if the effect of finite aspect ratio had been included in Figure 3.

For slender wings, with aspect ratio less than unity, Derivation 14 gives expressions from which the following values may be deduced:–

$$\frac{(L_r)_\Gamma}{\Gamma} = 0.00194 \text{ per degree, for delta wings,} \tag{3.2}$$

and

$$\frac{(L_r)_\Gamma}{\Gamma} = 0.00218 \text{ per degree, for delta wings,} \tag{3.3}$$

3.3 Twist Contribution, $(L_{r0})_\varepsilon$

Figure 4 presents data for the contribution of wing twist to L_{r0} for unswept wings. These data apply to those cases where twist is present on an unswept wing such that the local incidence is reduced linearly with spanwise distance from the root (wash-out) – see diagram in Figure 4.

The data are based on lifting-line calculations in Derivation 3 for aspect ratios up to 8. Above this aspect ratio the data are extrapolated using as guides asymptotic values for two-dimensional flow and values estimated using the method of Derivation 2 for wings with rounded tips.

In the absence of other information concerning the effects of sweep, and since on physical grounds the effect of increasing sweep is to increase the magnitude of the twist effect on L_{r0}, it is suggested that tentative use of the sweep factor given in Figure 1b may be made.

No experimental data for the effect of wing twist have been found.

3.4 Flap Contribution, $(L_{r0})_f$

Figure 5a provides data for the contribution to L_{r0} arising from the deflection of trailing-edge flaps on unswept wings of aspect ratio 12. The data are given in terms of an aspect ratio function, $f_2(A)$, which can be obtained from Figure 5b, and in terms of the parameter $[(a_{2\infty}/2\pi)\delta_f]$, which can be considered as the change in incidence at constant lift coefficient equivalent to the deflection of the flaps in two-dimensional flow. This parameter may be satisfactorily estimated for plain flaps by using the inviscid flow value of $a_{2\infty}$ given by Item No. Aero C.01.01.03 and by using the lift increment $(\equiv a_{2\infty}\delta_f/57.3)$ given by Item No. 74009 for split flaps and Item Nos. Aero F.01.01.08 and 09* for slotted flaps. It is useful to note that flaps not extending into the centre line of the wing may be taken into account by subtracting the value of $(L_{r0})_f$ corresponding to $(b_f/b)_{i/b}$ from that corresponding to $(b_f/b)_{o/b}$.

* To convert lift increments from these Items (for $A = 6$) to two-dimensional flow conditions a factor of 1.4 (suggested by lifting-line theory) should be applied.

The curves given in Figures 5a and 5b were derived from lifting-line theory (Derivation 3) and apply only to unswept wings. To account for sweep effects it is suggested that the factor $g(\Lambda_{1/4})$ given in Figure 1b be used.

Using the method outlined above, agreement with the only available experimental data for the effect of flaps on L_{r0} (Derivation 8) is good qualitatively with respect to trends due to the spanwise extent of the flaps but not so good quantitatively. The tests referred to in Derivation 8 were conducted on an untapered wing of aspect ratio 2.61 with 45 degrees of sweepback, fitted with two sets of split flaps,

(i) $(b_f/b)_{i/b} = 0.1$, $(b_f/b)_{o/b} = 0.5$ and

(ii) $(b_f/b)_{i/b} = 0.1$, $(b_f/b)_{o/b} = 1.0$.

The method of this Item provides estimates which give errors of +6 per cent and about –70 per cent when compared with the experimental data for the flap contributions of configurations (i) and (ii) respectively. The reason for the poor agreement for the nearly full-span flap of configuration (ii) is not obvious although it is worth noting that the experimental value of $(L_{r0})_f$ for this configuration is almost as large as that for the nearly semi-span flap of configuration (i). This is not in accord with reasoning on purely physical grounds which concludes that the flap effect of configuration (ii) should be very much less than that of configuration (i), a hypothesis which is substantiated on theoretical grounds by Figure 5a.

Some test data in Derivation 8 indicate that full-span leading-edge flaps contribute negligibly to L_{r0}. There are no data for part-span leading-edge flaps.

4. EFFECTS OF MACH NUMBER

The effects of Mach number on the planform contribution to L_{r0} for untapered wings at a given lift coefficient are established in Derivation 9 by the application of a Prandtl-Glauert factor which is related to the local Mach number at any given spanwise station. Checks with similar calculations for tapered wings (Derivation 15) show that the compressibility effect is generally greater than that for untapered wings. In the worst cases, for pointed wings, the increases are less than about 5 per cent provided that combinations of low aspect ratio and low sweep are avoided ($A \sec \Lambda_{1/4} > 3$, say). The data are here assumed to apply to the overall value of L_{r0} for both tapered and untapered wings. There are however no experimental data against which to check this hypothesis.

Figures 6a to 6e present the variation of L_r/L_{r0} with aspect ratio and sweep for Mach numbers of 0.4, 0.6, 0.7, 0.8 and 0.9 respectively. Linear interpolation between the given Mach number values should be acceptable in most cases.

5. OTHER EFFECTS

The effect of the fuselage is generally to reduce the value of $(L_r)_p/C_L$ by a negligible amount Derivation 10).

The tailplane contribution is usually much smaller than that arising from the wing and as such is usually neglected.

If the centre of gravity of the aircraft is not at the same height as the aerodynamic centre of the wing, there will be a contribution to L_r arising from the side-force derivative Y_r, but this again is usually negligible.

The contribution from the fin and rudder can be obtained from Item No. 82017.

6. LIMITATIONS

The application of the data in this Item is in general limited to straight-tapered swept wings at lift coefficients for which the flow remains fully attached and at Mach numbers for which the flow is subsonic over the whole of the wing. Some additional data concerning delta and gothic wings are given, and these apply only to aspect ratios less than unity. If test data for the sideslip derivative L_v are available for the required configuration then the method of Derivation 10 provides good estimates for L_r throughout the lift coefficient range when compared with test data obtained under steady yawing conditions.

The data given in this Item relating to slender wings, wing twist (particularly when applied to swept wings), dihedral and trailing-edge flaps should be used with caution in view of the few experimental data with which to check them. The same applies to the application of the data for the effect of Mach number, although this is generally small for Mach numbers below the critical value.

Under separated flow conditions the rolling moment due to yawing is likely to be highly non-linear. When an aircraft is oscillating in yaw two further parameters, frequency and amplitude of the oscillatory motion, become important, particularly so under conditions of separated flow. These additional considerations, combined with the fact that relatively few test data are available, currently preclude the development of a generalised procedure for the separated flow regime. For this reason it was felt useful to include a bibliography of reports for experimental work covering these conditions (see Section 8).

7. DERIVATION

The Derivation lists selected sources that have assisted in the preparation of this Item.

1.	HALLIDAY, A.S. BURGE, C.H.	Experiments on the Whirling Arm. Yawing and rolling moments on the Hornbill and various aerofoils. ARC R & M 1642, 1934.
2.	PEARSON, H.A. JONES, R.T.	Theoretical stability and control characteristics of wings with various amounts of taper and twist. NACA Rep. 635, 1938.
3.	PINSKER, W.J.G.	Die aerodynamischen Beiwerte der freien Seitenbewegung. DVL UM 1144/1 - 2, 1943. (DVL UM 1144/1 is translated in Report No. F-TS-619-RE, Air Materiel Command, Wright Field, Dayton, Ohio).
4.	LETKO, W. JAQUET, B.M.	Effect of airfoil profile of symmetrical sections on the low-speed static stability and yawing derivatives of 45° sweptback wing models of aspect ratio 2.61. NACA RM L8H10 (TIB 2004), 1948.
5.	QUEIJO, M.J. JAQUET, B.M.	Investigation of effects of geometric dihedral on low-speed static stability and yawing characteristics of an untapered 45° sweptback-wing model of aspect ratio 2.61. NACA tech. Note 1668, 1948.
6.	GOODMAN, A. BREWER, J.D.	Investigation at low speeds of the effect of aspect ratio and sweep on static and yawing stability derivatives of untapered wings. NACA tech. Note 1669, 1948.
7.	LETKO, W. COWAN, J.W.	Effect of taper ratio on low-speed static and yawing stability derivatives of 45° sweptback wings with aspect ratio 2.61. NACA tech. Note 1671, 1948.

8.	LICHTENSTEIN, J.H.	Effect of high-lift devices on the low-speed static lateral and yawing stability characteristics of an untapered 45° sweptback wing. NACA tech. Note 2689, 1948.
9.	FISHER, L.R.	Approximate corrections for the effects of compressibility on the subsonic stability derivatives of swept wings. NACA tech. Note 1854, 1949.
10.	CAMPBELL, J.P. GOODMAN, A.	A semiempirical method for estimating the rolling moment due to yawing of airplanes. NACA tech. Note 1984, 1949.
11.	MULTHOPP, H.	Method for calculating the lift distribution of wings (subsonic lifting-surface theory). ARC R & M 2884, 1950.
12.	QUEIJO, M.J. FLETCHER, H.S. *et al*.	Preliminary measurements of the aerodynamic yawing derivatives of a triangular, a swept, and an unswept wing performing pure yawing oscillations, with a description of the instrumentation employed. NACA RM L55L14 (TIL 5046), 1956.
13.	LETKO, W.	Experimental determination at subsonic speeds of the oscillatory and static lateral stability derivatives of a series of delta wings with leading-edge sweep from 30° to 86.5°. NACA RM L57A30 (TIL 5487), 1957.
14.	ROSS, A.J.	The calculation of lateral stability derivatives of slender wings at incidence, including fin effectiveness, and correlation with experiment. ARC R & M 3402, 1961.
15.	QUEIJO, M.J.	Theory for computing span loads and stability derivatives due to sideslip, yawing, and rolling for wings in subsonic compressible flow. NASA tech. Note D-4929, 1968.

8. BIBLIOGRAPHY

The following bibliography consists of references to reports, in addition to those given in Section 7, containing experimental data which it is felt would be useful in analyses concerning L_r in the non-linear (separated flow) regime:

1.	GOODMAN, A. FEIGENBAUM, D.	Preliminary investigation at low speeds of swept wings in yawing flow. NACA RM L7I09 (TIB 1617), 1948.
2.	BIRD, J.D. JAQUET, B.M. COWAN, J.W.	Effect of fuselage and tail surfaces on low-speed yawing characteristics of a swept-wing model as determined in curved-flow test section of Langley Stability Tunnel. NACA tech. Note 2483, 1951.
3.	QUEIJO, M.J. WELLS, E.G.	Wind-tunnel investigation of the low-speed static and rotary stability derivatives of a 0.13-scale model of the Douglas D-558-II airplane in the landing configuration. NACA RM L52G07 (TIB 3502), 1952.
4.	WILLIAMS, J.L.	Measured and estimated lateral static and rotary derivatives of a 1/12-scale model of a high-speed fighter airplane with unswept wings. NACA RM L53K09 (TIL 5187), 1954.

5. LICHTENSTEIN, J.H. WILLIAMS, J.L. — Effect of frequency of sideslipping motion on the lateral stability derivatives of a typical delta-wing airplane. NACA RM L57F07 (TIL 5664), 1957.

6. FISHER, L.R. — Experimental determination of the effects of frequency and amplitude on the lateral stability derivatives for a delta, a swept, and an unswept wing oscillating in yaw. NACA tech. Rep. 1357, 1958.

7. LETKO, W. FLETCHER, H.S. — Effects of frequency and amplitude on the yawing derivatives of triangular, swept, and unswept wings and of a triangular-wing-fuselage combination with and without a triangular tail performing sinusoidal yawing oscillations. NACA tech. Note 4390, 1958.

9. EXAMPLE

9.1 Example 1

Estimate the rolling moment derivative due to yawing in low-speed flow at $C_L = 0.95$ for a wing with deflected trailing-edge flaps. The geometrical and aerodynamic data for the wing and flaps are as follows:

$$A = 8,\quad \lambda = 0.25,\quad \Lambda_{1/4} = 40°,\quad \Gamma = -5°,\quad \varepsilon = 3°,$$

$$(b_f/b)_{i/b} = 0.1,\quad (b_f/b)_{o/b} = 0.6,\quad a_{2\infty} = 3.5 \text{ per rad and } \delta_f = 30°.$$

From Figures 1a and 1b for $A = 8$, $\lambda = 0.25$ and $\Lambda_{1/4} = 40°$,

$$\frac{(L_{r0})_p}{C_L} = \frac{1}{g(\Lambda_{1/4})}\frac{(L_{r0})_p}{C_L} \times g(\Lambda_{1/4})$$

$$= 0.101 \times 1.78$$

$$= 0.180.$$

Thus $\quad (L_{r0})_p = 0.180 \times 0.95 = 0.171.$

From Figure 3 for $\Lambda_{1/4} = 40°$,

$$\frac{(L_{r0})_\Gamma}{\Gamma} = 0.00147$$

Thus $\quad (L_{r0})_\Gamma = 0.00147 \times -5 = -0.00735.$

From Figure 4 for $A = 8$ and $\lambda = 0.25$,

$$\frac{(L_{r0})_\varepsilon}{\varepsilon} = -0.00175.$$

On application of the sweep factor from Figure 1b,

$$(L_{r0})_\varepsilon = -0.00175 \times 3 \times 1.78 = -0.00934.$$

From Figures 5a and 5b for $A = 8$, $\lambda = 0.25$, $(b_f/b)_{o/b} = 0.6$, $(b_f/b)_{i/b} = 0.1$ and $(a_{2\infty}/2\pi)\delta_f = (3.5/6.28)30 = 16.7$ degrees,

$$(L_{r0})_f = 0.86 \times 16.7\,[-0.003\,30 - (-0.000\,98)] = -0.0333$$

for $\Lambda_{1/4} = 0$. On application of the sweep factor from Figure 1b,

$$(L_{r0})_f = 1.78 \times -0.0333 = -0\,.0593.$$

The total derivative is therefore

$$L_{r0} = (L_{r0})_p + (L_{r0})_\Gamma + (L_{r0})_\varepsilon + (L_{r0})_f$$

$$= 0.171 - 0.00735 - 0.00934 - 0.0593.$$

Thus $\qquad L_{r0} = L_r = 0.095.$

9.2 Example 2

Find also the value of the derivative corresponding to the cruise condition for the wing of Section 9.1 with flaps undeflected at $C_L = 0.5$ and $M = 0.8$.

From Section 9.1, $\dfrac{(L_{r0})_p}{C_L} = 0.180.$

Therefore $\qquad (L_{r0})_p = 0.180 \times 0.5 = 0.090.$

Also, $(L_{r0})_\Gamma = -0.00735$, $(L_{r0})_\varepsilon = -0.00935$ and $(L_{r0})_f = 0$.

Thus $\qquad L_{r0} = 0.090 - 0.00735 - 0.00934$

$$= 0.073.$$

From Figure 6d for $A = 8$ and $\Lambda_{1/4} = 40°$,

$$\frac{L_r}{L_{r0}} = 1.223.$$

Therefore $\qquad L_r = 0.073 \times 1.223$

$$= 0.089.$$

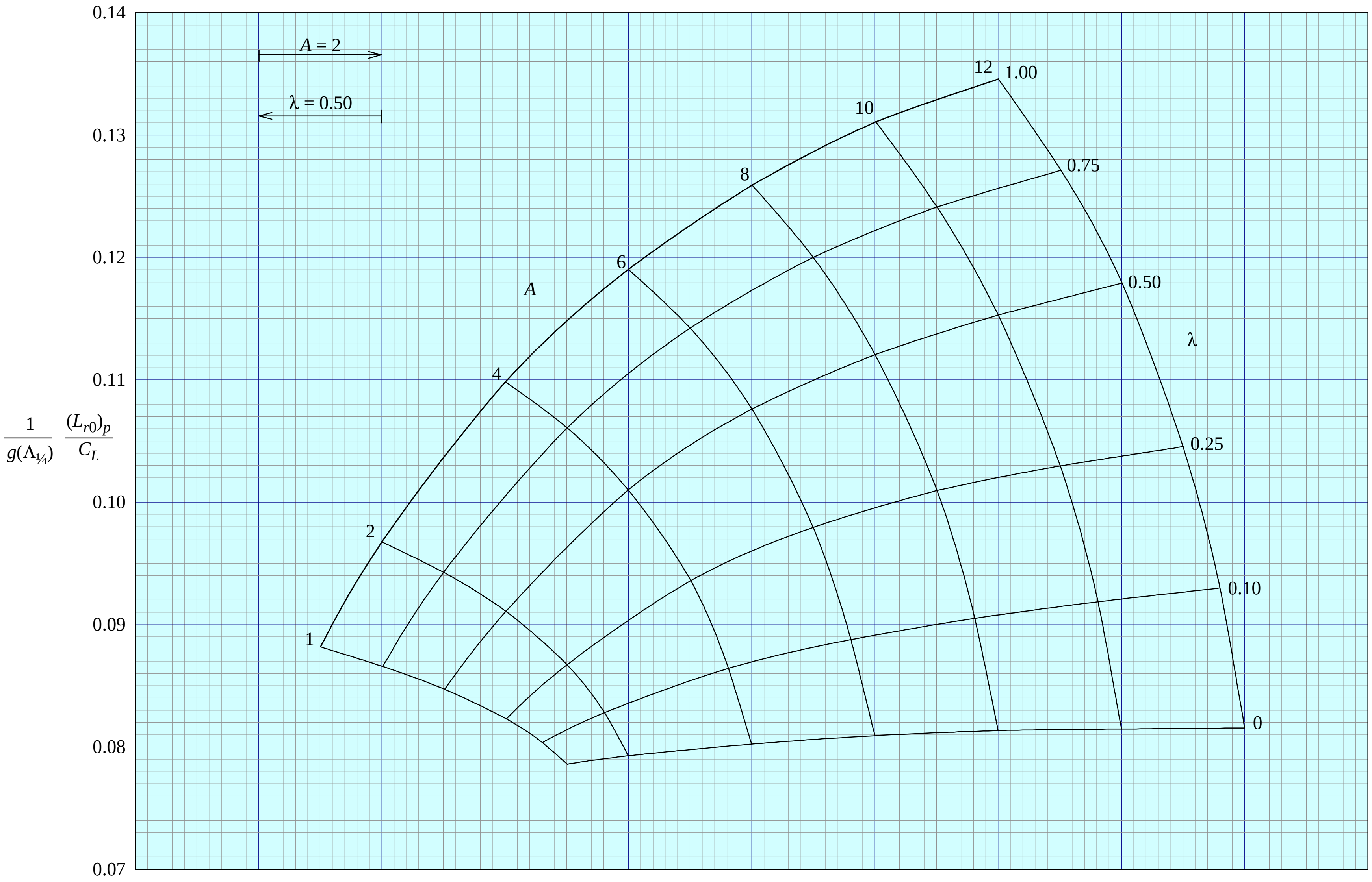
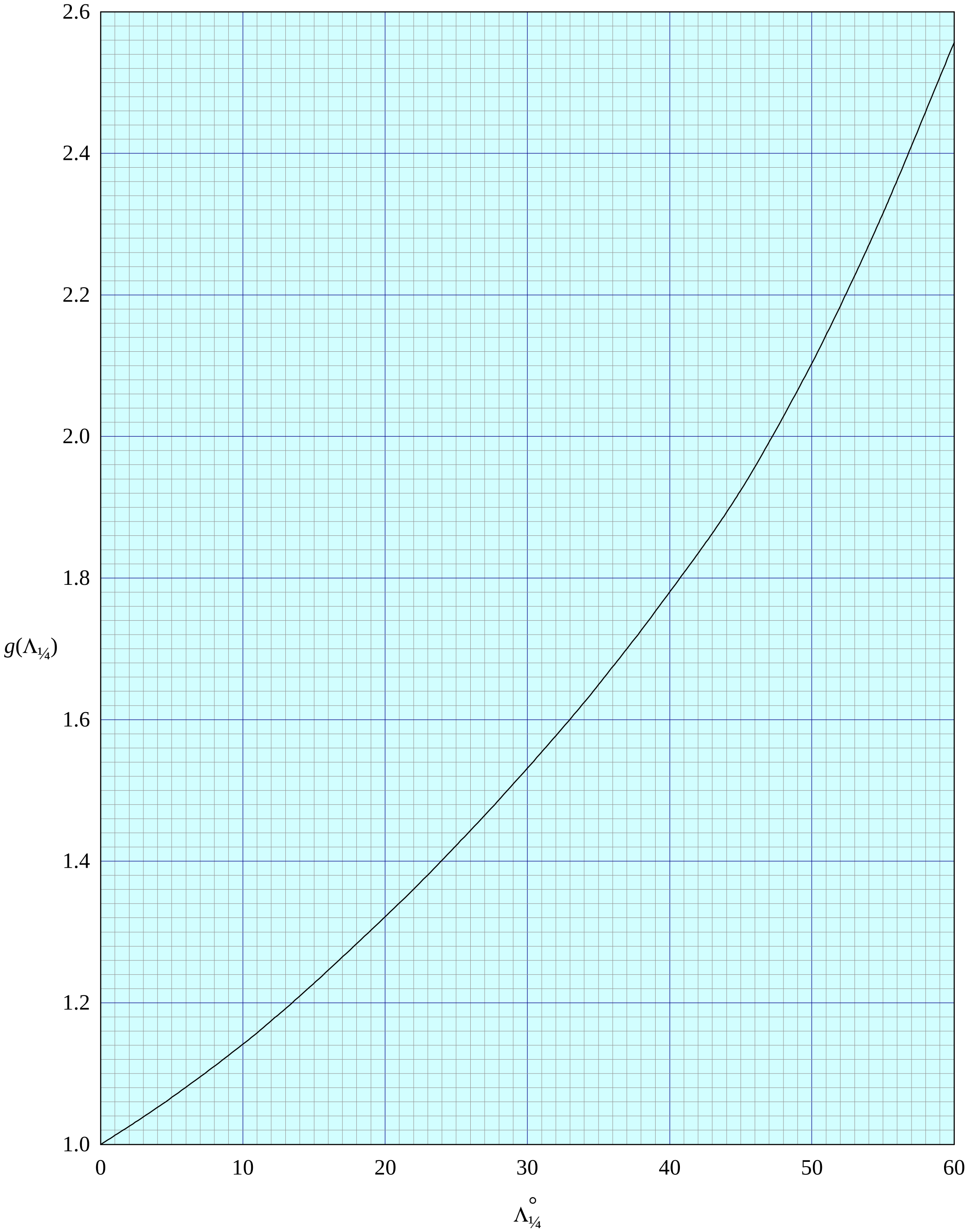

0.14
0.13
0.12
0.11
0.10
0.09
0.08
0.07
$\frac{1}{g(\Lambda_{1/4})} \frac{(L_{r0})_p}{C_L}$
$A = 2$
$\lambda = 0.50$
A
λ
1
2
4
6
8
10
12
1.00
0.75
0.50
0.25
0.10
0

FIGURE 1a



2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
$g(\Lambda_{1/4})$
0
10
20
30
40
50
60
$\Lambda_{1/4}^{\circ}$

FIGURE 1b

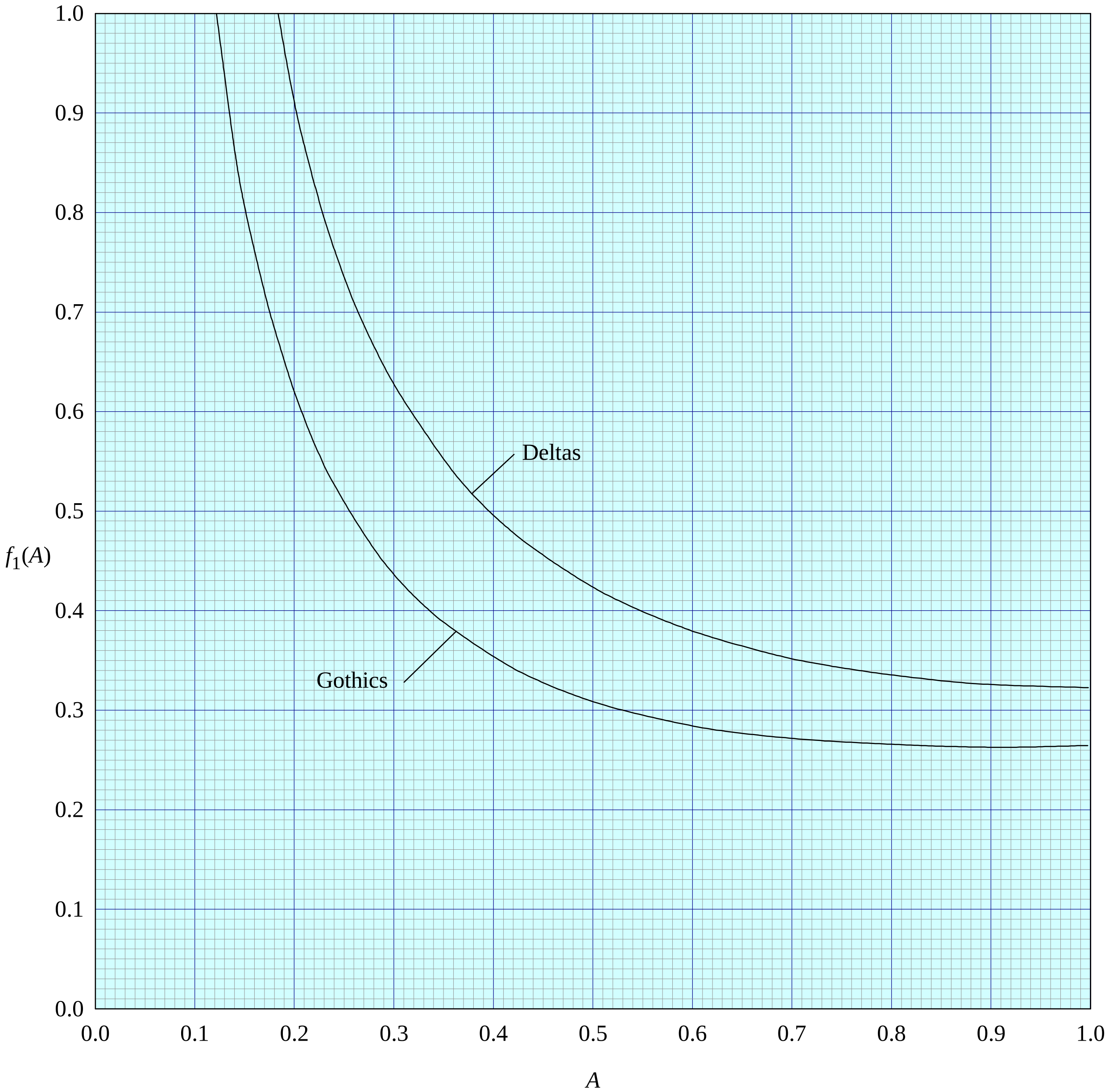

1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
$f_1(A)$
Deltas
Gothics
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
A

FIGURE 2

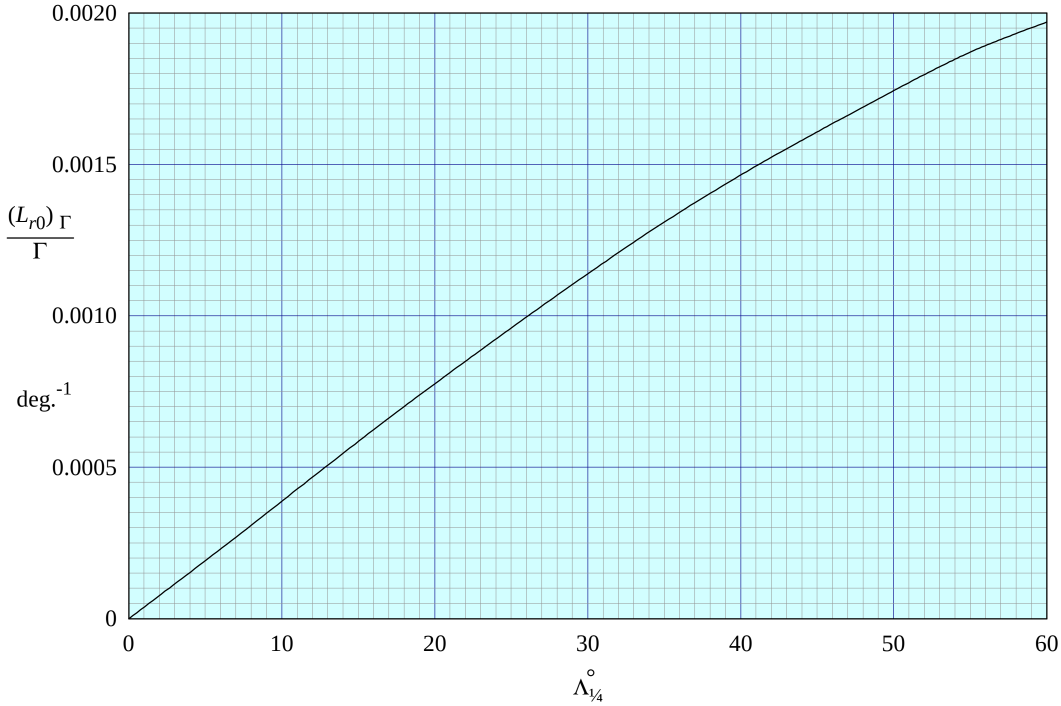


FIGURE 3

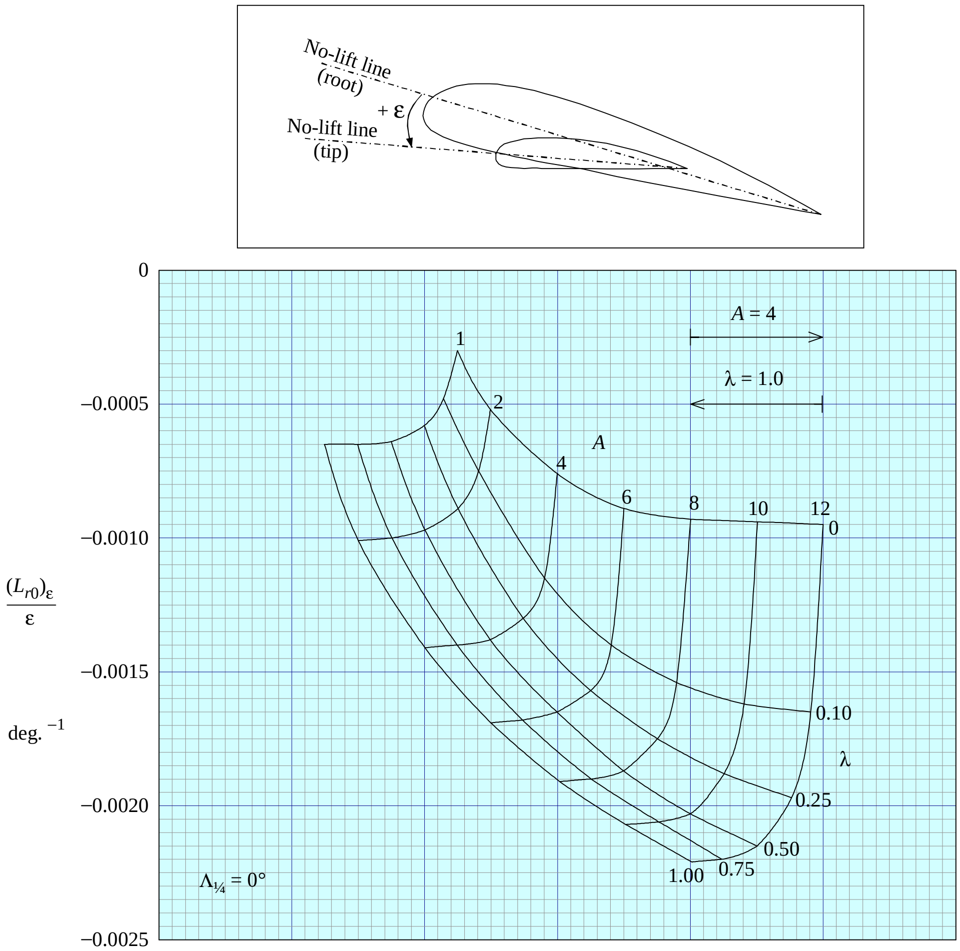


FIGURE 4

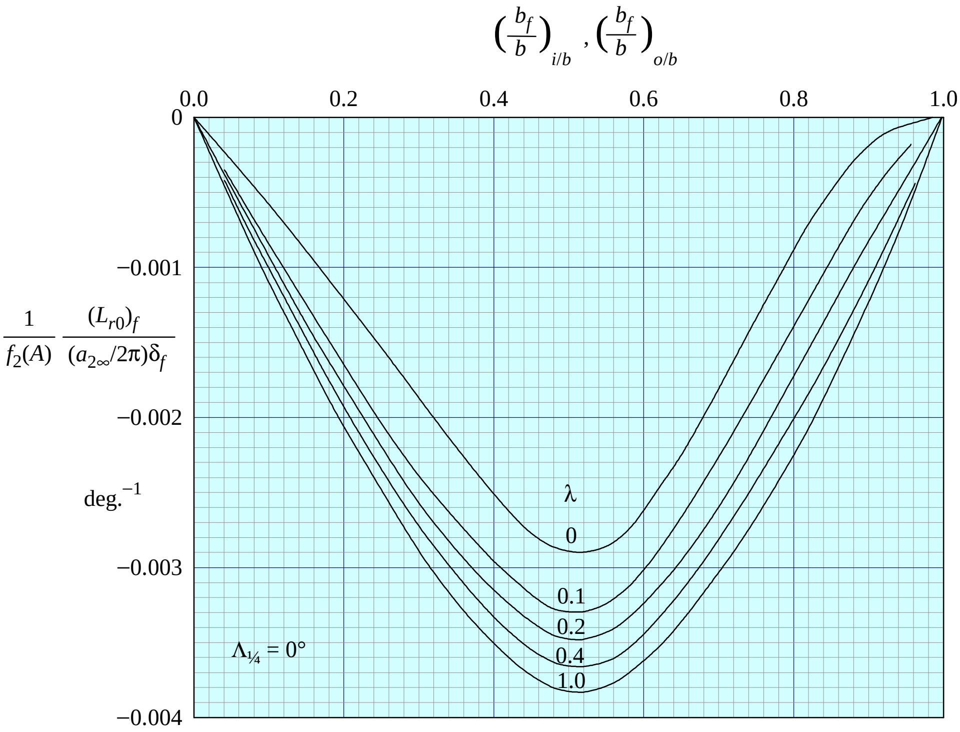


FIGURE 5a

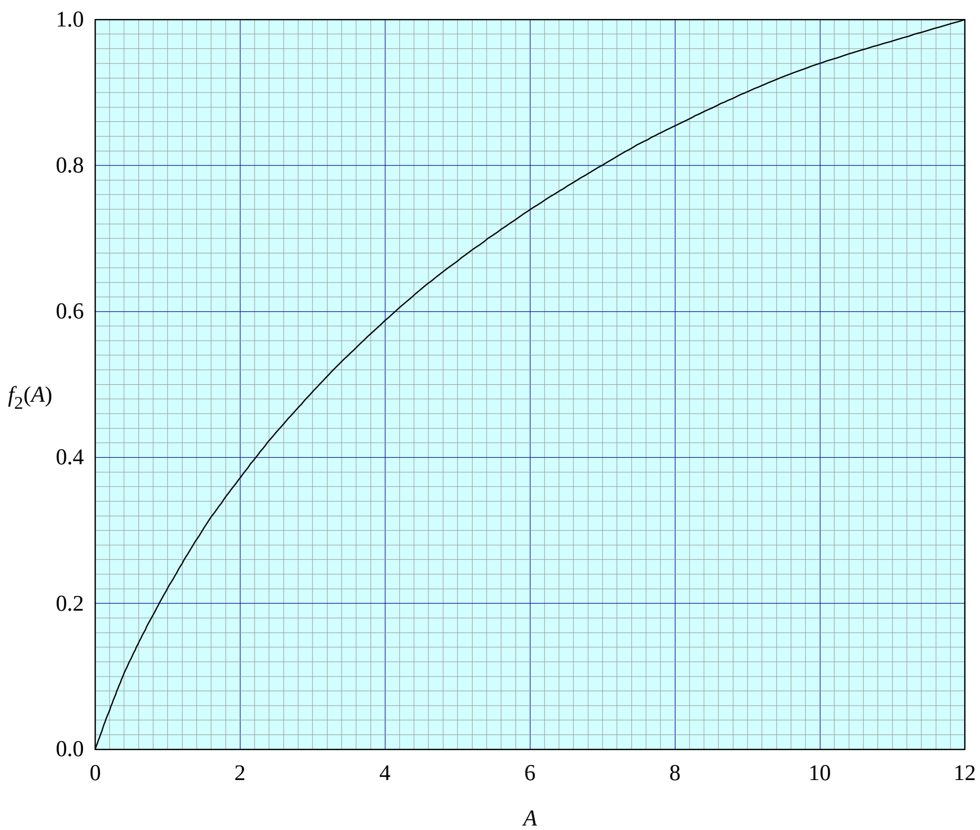
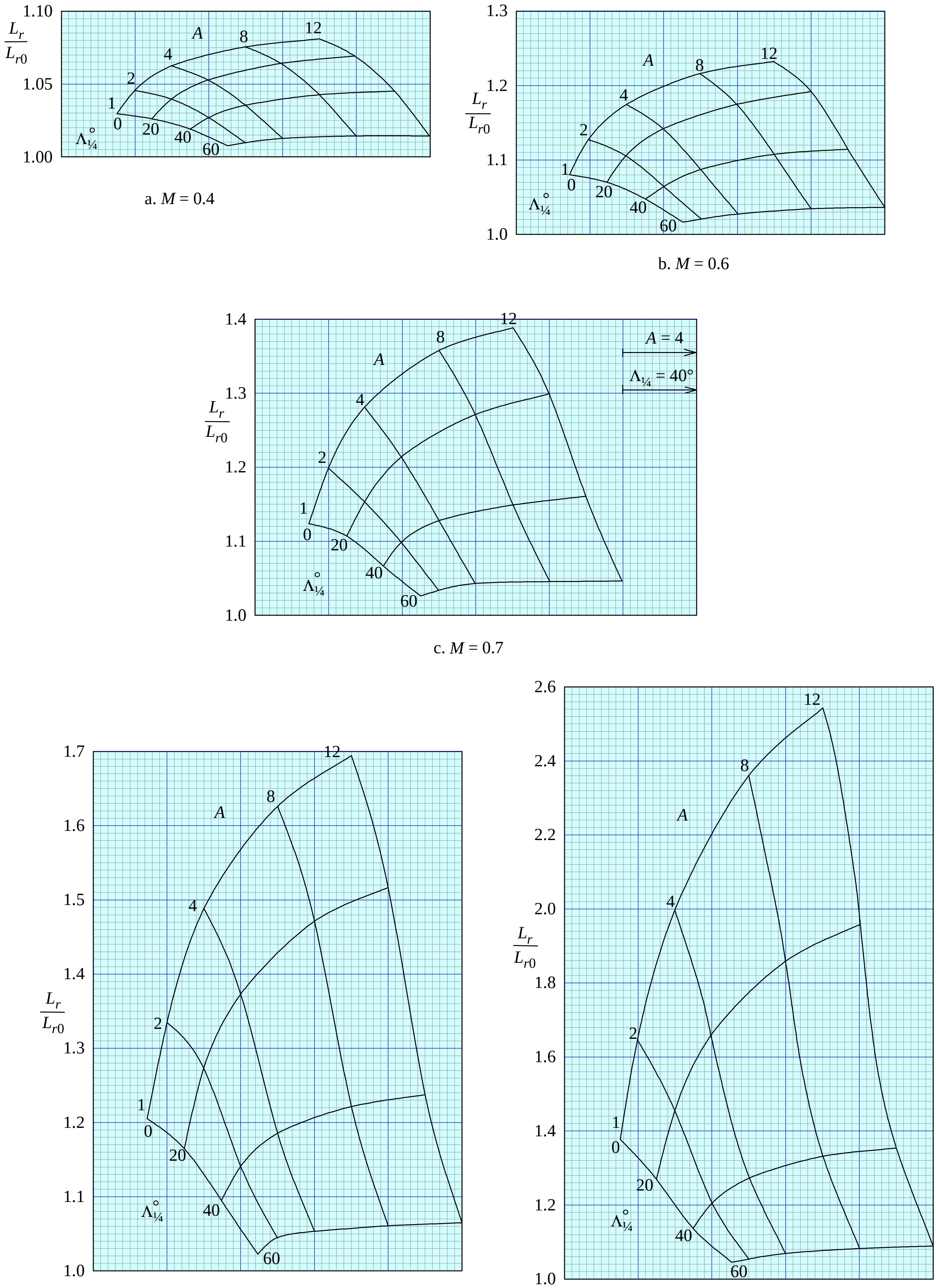


FIGURE 5b



a. $M = 0.4$

b. $M = 0.6$

c. $M = 0.7$

d. $M = 0.8$

e. $M = 0.9$

FIGURE 6

THE PREPARATION OF THIS DATA ITEM

The work on this particular Item, which supersedes Item Numbers Aero A.06.01.02 and A.06.01.08, was monitored and guided by the Aerodynamics Committee which first met in 1942 and now has the following membership:

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