CONTRIBUTION OF WING DIHEDRAL TO SIDEFORCE, YAWING MOMENT AND ROLLING MOMENT DERIVATIVES DUE TO RATE OF ROLL AT SUBSONIC SPEEDS, $(Y_p)_\Gamma$, $(N_p)_\Gamma$ AND $(L_p)_\Gamma$

1. NOTATION AND UNITS (see Sketch 1.1)

The derivative notation used is that proposed in ARC R&M 3562 (Hopkin, 1970) and described in Item No. 86021. Coefficients and aeronormalised derivatives are evaluated in aerodynamic body axes with origin at the aircraft centre of gravity and with the wing span as the characteristic length. The derivatives Y_p, N_p and L_p are often written as C_{Yp}, C_{np} and C_{lp} in other systems of notation, but attention must be paid to the reference dimensions used. In particular, in forming C_{Yp}, C_{np} and C_{lp} differentiation of C_Y, C_n and C_l may be carried out with respect to $pb/2V$ not pb/V as implied in the Hopkin system. It is also to be noted that a constant datum value of V is employed by Hopkin.

		SI	*British*
A	aspect ratio		
b	wing span	m	ft
C_L	lift coefficient, $L/½\rho V^2S$		
C_l	rolling moment coefficient, $\mathscr{L}/½\rho V^2Sb$		
C_n	yawing moment coefficient, $\mathscr{N}/½\rho V^2Sb$		
C_Y	sideforce coefficient, $Y/½\rho V^2S$		
L	lift	N	lbf
$\mathscr{L}$	rolling moment	N m	lbf ft
L_p	rolling moment derivative due to rate of roll, $L_p = (\partial\mathscr{L}/\partial p)/½\rho VSb^2$		
M	Mach number		
$\mathscr{N}$	yawing moment	N m	lbf ft
N_p	yawing moment derivative due to rate of roll, $N_p = (\partial\mathscr{N}/\partial p)/½\rho VSb^2$		
p	rate of roll	rad/s	rad/s
S	wing planform (reference) area	m^2	ft^2
s	wing semi-span	m	ft
V	velocity of aircraft relative to air	m/s	ft/s
x	distance of moment reference centre ahead of aerodynamic centre	m	ft

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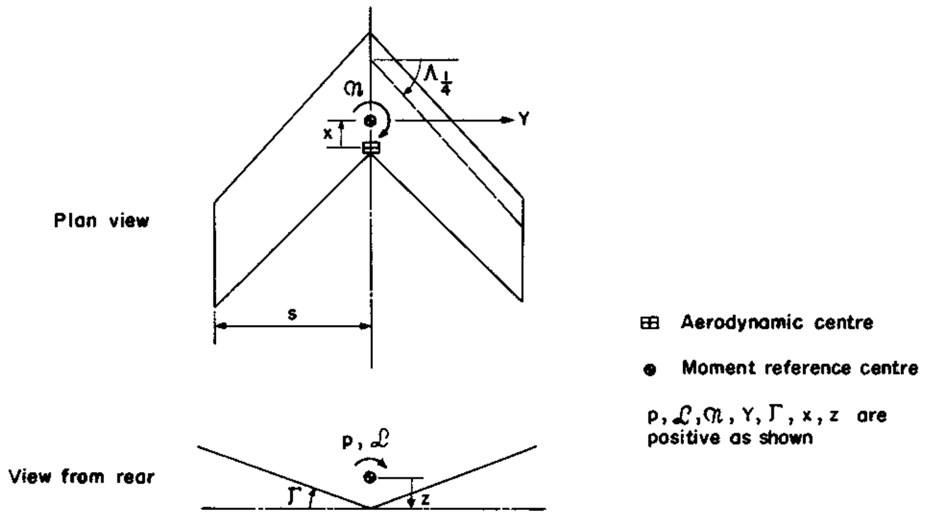
Y	sideforce	N	lbf
Y_p	sideforce derivative due to rate of roll, $Y_p = (\partial Y/\partial p)/\frac{1}{2}\rho VSb$		
z	perpendicular distance of wing centre-line chord below moment reference centre	m	ft
Γ	dihedral angle	degree	degree
ζ	z/s		
$\Lambda_{1/4}$	wing quarter-chord sweep angle	degree	degree
λ	ratio of wing tip chord to wing centre-line chord		
ξ	x/s		
ρ	density of air	kg/m^3	slug/ft^3

Additional symbols

$(\ )_w$	denotes component due to wing planform
$(\ )_\Gamma$	denotes component due to wing dihedral



Sketch 1.1

2. INTRODUCTION

The dihedral contribution to the sideforce derivative, $(Y_p)_\Gamma$, is often a significant proportion of the total value of Y_p for aircraft with wings of moderate dihedral ($\Gamma = 5°$ to 10°). For wings of small sweep at low C_L the contribution can be sufficient to change the sign of the total value. The dihedral contributions to the yawing and rolling moment derivatives, $(N_p)_\Gamma$ and $(L_p)_\Gamma$ are small for realistic aircraft configurations. To evaluate the total values of Y_p, N_p and L_p for a wing the dihedral contributions must be added to the wing planform contributions, which can be predicted by using Item No. 81014 (Derivation 4) for the sideforce and yawing moment and by using Item No. Aero A.06.01.01 (Derivation 3) for the rolling moment.

This Item uses the methods of Derivations 1 and 2 to predict $(Y_p)_\Gamma$, $(N_p)_\Gamma$ and $(L_p)_\Gamma$. In Derivation 2 the strip theory and simple lifting-line considerations of Derivation 1 are used to produce simple expressions for those derivatives for untapered, swept wings. Using the same techniques the full expressions for tapered wings have been developed for this Item.

For realistic dihedral angles it can be assumed that $\sin\Gamma \approx \Gamma/57.3$ and this simplifies the solutions considerably, provided that the moment reference centre is reasonably close to the wing centre-line chord. The variation of the derivatives with λ is small and the selection of a fixed value, $\lambda = 1/3$ say, leads to very simple formulae. This approach is usually completely satisfactory for estimating the small contributions $(N_p)_\Gamma$ and $(L_p)_\Gamma$. A simplified equation can be used for a rapid estimation of $(Y_p)_\Gamma$ but this Item also gives a family of curves that allows easy access to the full result and shows the effect of variations in taper and vertical location of the moment reference centre.

The expressions involved in the calculation of the dihedral contributions are introduced in Section 3. The accuracy and applicability are discussed in Section 4. The Derivation and References are given in Section 5, and a worked example is set out in Section 6.

3. METHOD

The equations for $(Y_p)_\Gamma$, $(N_p)_\Gamma$ and $(L_p)_\Gamma$ for a straight tapered wing are

$$\frac{(Y_p)_\Gamma}{(L_p)_w} = \frac{4\sin\Gamma}{1+3\lambda}\left[1+2\lambda-3\zeta(1+\lambda)\sin\Gamma\right], \tag{3.1}$$

$$\frac{(N_p)_\Gamma}{(L_p)_w} = \frac{-\sin\Gamma}{3(1+3\lambda)(1+\lambda)}\left[6\xi(1+\lambda)\{1+2\lambda-3\zeta(1+\lambda)\sin\Gamma\}+(1+4\lambda+\lambda^2)\tan\Lambda_{1/4}\right] \tag{3.2}$$

and

$$\frac{(L_p)_\Gamma}{(L_p)_w} = \frac{-2\zeta\sin\Gamma}{1+3\lambda}\left[2+4\lambda-3\zeta(1+\lambda)\sin\Gamma\right]. \tag{3.3}$$

It can be seen that each of the dihedral contributions is related to the wing planform rolling moment contribution $(L_p)_w$, which can be obtained from Item No. Aero A.06.01.01 (Derivation 3). In addition to a dependence on the wing geometry through dihedral, Γ, sweep, $\Lambda_{1/4}$, and taper, λ, the expression for $(N_p)_\Gamma$ involves the distance of the moment reference centre ahead of the aerodynamic centre, $\xi(=x/s)$, and the perpendicular distance of the wing centre-line chord below the moment reference centre, $\zeta(=z/s)$. The expressions for $(Y_p)_\Gamma$ and $(L_p)_\Gamma$ involve ζ but not ξ.

Figure 1 shows $(Y_p)_\Gamma/(L_p)_w$ plotted against Γ for particular values of λ and ζ. Values are plotted for positive ζ. For negative values of ζ, $(Y_p)_\Gamma$ may be evaluated by using the identity

$$(Y_p)_\Gamma(\Gamma,\zeta) = -(Y_p)_\Gamma(-\Gamma,-\zeta)\,. \tag{3.4}$$

For typical aircraft configurations, where ζ is small, $\sin\Gamma \approx \Gamma/57.3$, and for $\lambda = 1/3$, Equations (3.1) to (3.3) become

$$\frac{(Y_p)_\Gamma}{(L_p)_w} = \frac{3.333\Gamma}{57.3} = 0.0582\,\Gamma, \tag{3.5}$$

$$\frac{(N_p)_\Gamma}{(L_p)_w} = -(1.667\xi + 0.306\tan\Lambda_{1/4})\frac{\Gamma}{57.3} = -(0.0291\xi + 0.00533\tan\Lambda_{1/4})\Gamma \tag{3.6}$$

and

$$\frac{(L_p)_\Gamma}{(L_p)_w} = \frac{-3.333\zeta\Gamma}{57.3} = -0.0582\zeta\Gamma\,, \tag{3.7}$$

where Γ is in degrees.

An examination of Figure 1 reveals the variations of $(Y_p)_\Gamma$ with λ and ζ that are neglected by using Equation (3.5) rather than Equation (3.1). For cases of practical importance Equations (3.6) and (3.7) will invariably suffice for predicting $(N_p)_\Gamma$ and $(L_p)_\Gamma$.

4. ACCURACY AND APPLICABILITY

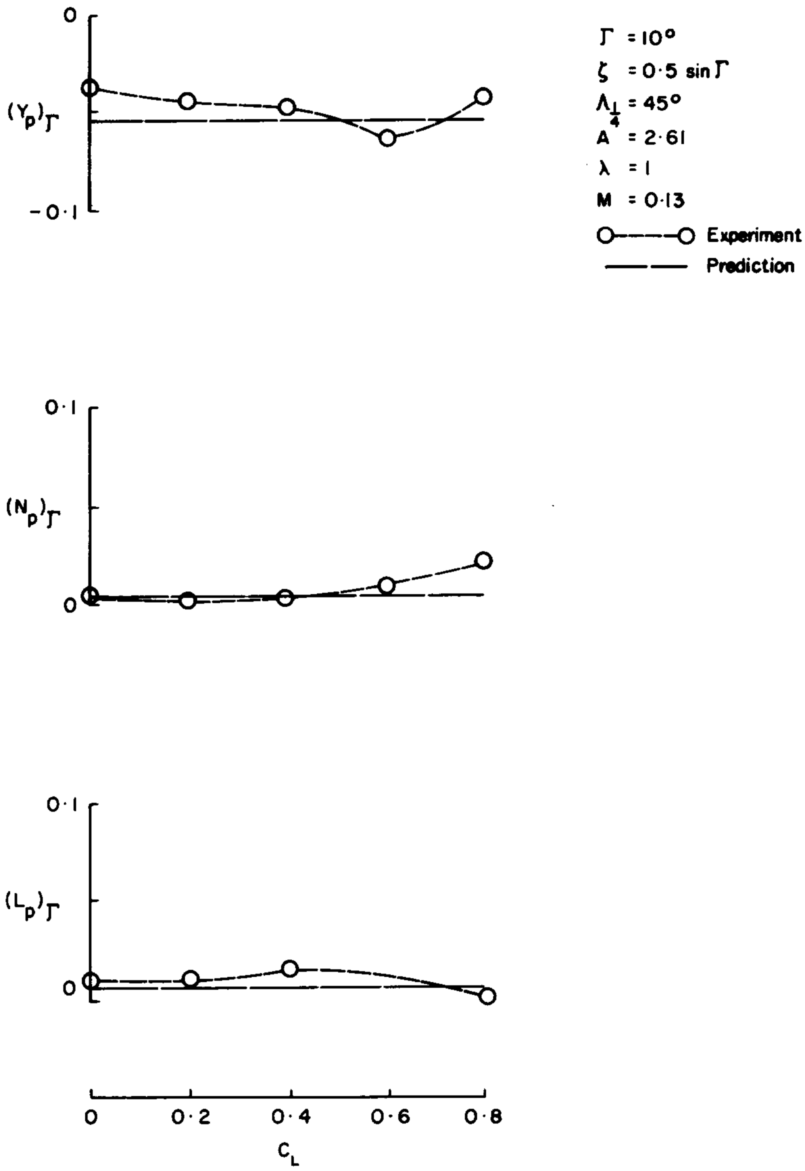
Few data are available from experiments in which a systematic variation of dihedral angle has been conducted during rolling motion tests. Therefore an assessment of the accuracy of the method is difficult. In Derivation 2 an untapered wing of aspect ratio $A = 2.61$ and sweep $\Lambda_{1/4} = 45°$ has been tested systematically, and for those results the method is fairly reliable at low and moderate lift coefficients. Sketch 4.1 compares predicted and experimental values for the wing with $\Gamma = 10°$.

The method assumes attached flow and thus the occurrence of any separation will result in poor agreement between predicted and experimental values. As the method for $(L_p)_w$ in Item No. Aero A.06.01.01 is said to be accurate up to lift coefficients of 0.5 this can probably also be taken as the general limit of reliability for the dihedral contributions.

The effects of compressibility are allowed for by determining $(L_p)_w$ at the Mach number of interest. Since the method for $(L_p)_w$ is limited to subcritical Mach numbers the same restriction applies to the dihedral contributions.

The lifting-line and strip-theory assumptions of the method probably restrict its applicability to wing planforms with $\lambda > 0.25$, $\Lambda_{1/4} < 60°$ and $A > 1.5$, ranges for which these assumptions in predicting $(Y_p)_w$ and $(N_p)_w$ in Item No. 81014 have been found to be acceptable when compared with experimental data.

An examination of roll rate derivatives for a number of aircraft configurations with significant wing dihedral (References 5 to 9) has shown that the method of this Item provides sufficiently accurate estimates of the dihedral contribution for use in the estimation of total values of the lateral stability derivatives. In particular the inclusion of $(Y_p)_\Gamma$ leads to a much improved prediction of the total sideforce derivative.



Sketch 4.1 Comparison of experimental and predicted values. (Derivation 2)

5. DERIVATION AND REFERENCES

5.1 Derivation

The Derivation lists selected sources that have assisted in the preparation of this Item.

1.	TOLL, T.A. QUEIJO, M.J.	Approximate relations and charts for low-speed stability derivatives of swept wings. NACA tech. Note 1581, 1948.
2.	QUEIJO, M.J. JAQUET, B.M.	Calculated effects of geometric dihedral on the low-speed rolling derivatives of swept wings. NACA tech. Note 1732, 1948.
3.	ESDU	Stability derivative L_p, rolling moment due to rolling for swept and tapered wings. Item No. Aero A.06.01.01, ESDU International, 1955.
4.	ESDU	Contribution of wing planform to derivatives of yawing moment and sideforce due to roll rate at subsonic speeds, $(N_p)_w$ and $(Y_p)_w$. Item No. 81014, ESDU International 1981.

5.2 References

The References list selected sources of information supplementary to that given in this Item.

5.	QUEIJO, M.J. GOODMAN, A.	Calculations of the dynamic lateral stability characteristics of the Douglas D-558-II airplane in high-speed flight for various wing loadings and altitudes. NACA RM L50H16a (TIL 3352), 1950.
6.	QUEIJO, M.J. WELLS, G.E.	Wind-tunnel investigation of the low-speed static and rotary stability derivatives of a 0.13 scale model of the Douglas D-558-II airplane in the landing configuration. NACA RM L52G07 (TIL 3502), 1952.
7.	SLEEMAN, W.C. WIGGINS, J.W.	Experimental investigation at high subsonic speeds of the rolling stability derivatives of a complete model with an aspect-ratio-2.52 wing having an unswept 72-percent-chord line and high horizontal tail. NACA RM L54I20 (TIL 6633), 1955.
8.	GRAFTON, S.B. CHAMBERS, J.R. COE, P.L.	Wind-tunnel free-flight investigation of a model of a spin-resistant fighter configuration. NASA tech. Note D-7716, 1974.
9.	TANNER, R.R. MONTGOMERY, T.D.	Stability and control derivative estimates obtained from flight data for the Beech 99 aircraft. NASA tech. Memo 72863, 1979.

6. EXAMPLE

Find the dihedral contributions to the lateral stability derivatives due to rate of roll for a wing for which $A = 3.5$, $\Lambda_{1/4} = 30°$, $\lambda = 0.5$ and $\Gamma = 10°$. The Mach number is 0.7. These are the wing planform and flow conditions used in the example of Item No. A.06.01.01, where the wing planform derivative $(L_p)_w$ is evaluated as – 0.149.

The moment reference centre may be assumed to be a perpendicular distance 0.087*s* above the wing centre-line chord and to be coincident with the wing aerodynamic centre, *i.e.* $\zeta = 0.087$ and $\xi = 0$.

From Figure 1 (or Equation (3.1)) for $\Gamma = 10°$, $\lambda = 0.5$ and $\zeta = 0.087$.

$$\frac{(Y_p)_\Gamma}{(L_p)_w} = 0.537\,.$$

(This compares with a value of 0.582 from the simplified approach of Equation (3.5), with roughly half of the discrepancy due to ignoring the displacement of the moment reference centre from the wing centre-line chord and half due to taking the taper to be 1/3 instead of 0.5.)

Using Equations (3.6) and (3.7),

$$\frac{(N_p)_\Gamma}{(L_p)_w} = -(0.0291\xi + 0.00533\tan\Lambda_{1/4})\Gamma = -(0.0291\times 0 + 0.00533\tan 30°)\times 10 = -0.031$$

and

$$\frac{(L_p)_\Gamma}{(L_p)_w} = -0.0582\zeta\Gamma = -0.0582\times 0.087\times 10 = -0.051\,.$$

(The corresponding values provided by the full Equations (3.2) and (3.3) are –0.029 and –0.048, respectively.)

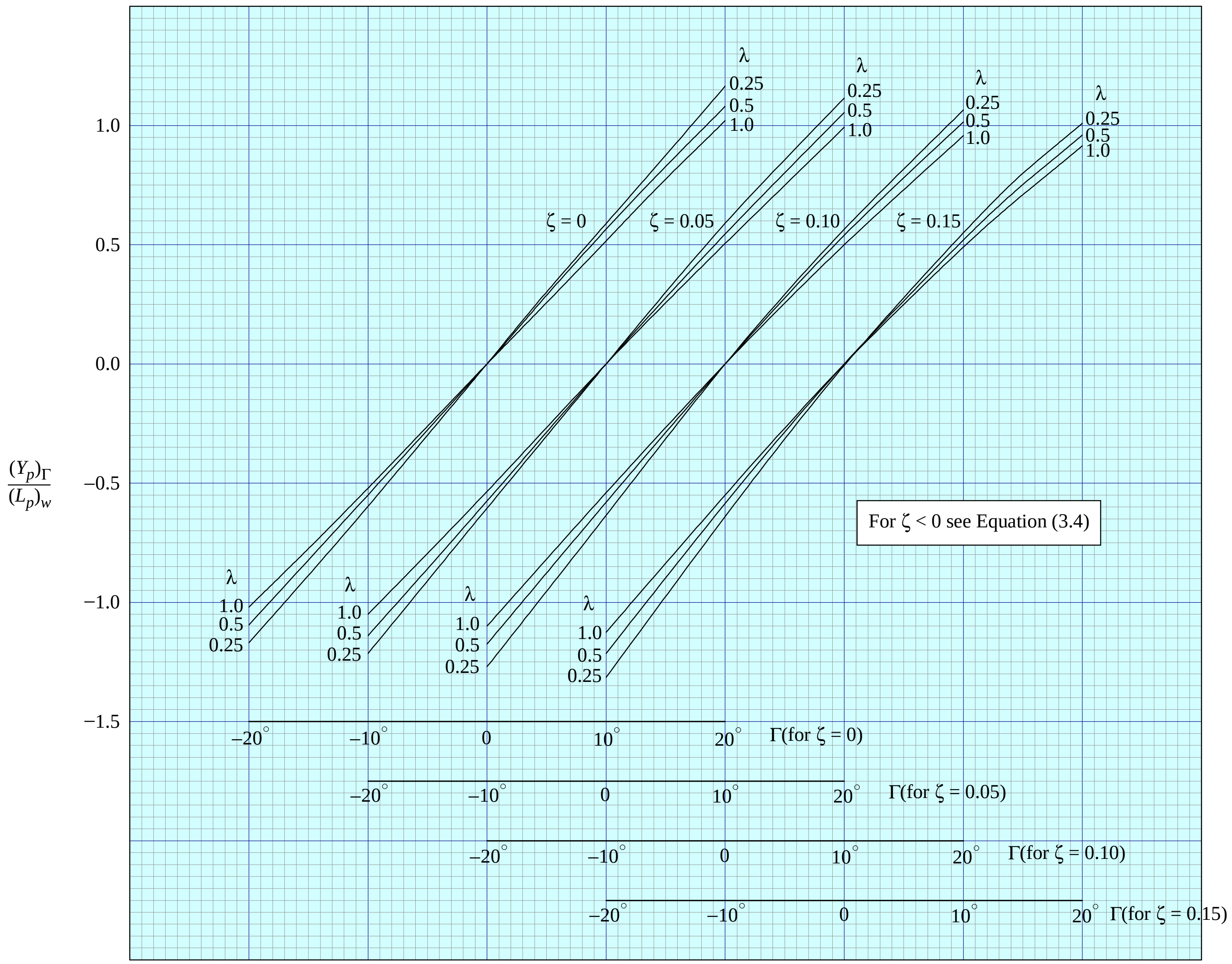
Therefore

$$(Y_p)_\Gamma = 0.537\times(-0.149) = -0.080\,,$$

$$(N_p)_\Gamma = -0.031\times(-0.149) = 0.005$$

and

$$(L_p)_\Gamma = -0.051\times(-0.149) = 0.008\,.$$



$\frac{(Y_p)_\Gamma}{(L_p)_w}$
1.0
0.5
0.0
−0.5
−1.0
−1.5
ζ = 0
ζ = 0.05
ζ = 0.10
ζ = 0.15
λ
0.25
0.5
1.0
For ζ < 0 see Equation (3.4)
Γ(for ζ = 0)
Γ(for ζ = 0.05)
Γ(for ζ = 0.10)
Γ(for ζ = 0.15)
−20°
−10°
0
10°
20°

FIGURE 1

THE PREPARATION OF THIS DATA ITEM

The work on this particular Item was monitored and guided by the Aerodynamics Committee which first met in 1942 and now has the following membership:

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The member of staff who undertook the technical work involved in the initial assessment of the available information and the construction and subsequent development of the Item was

Mr R.W. Gilbey – Senior Engineer.