AERO-NORMALISED STABILITY DERIVATIVES: EFFECT OF WING ON YAWING MOMENT DUE TO YAWING

1. NOTATION AND UNITS

The derivative notation used is that proposed in ARC R&M 3562 (Hopkin, 1970) and described in Item No. 86021. Coefficients and aeronormalised derivatives are evaluated in aerodynamic body axes with origin at the aircraft centre of gravity and with the wing span as the characteristic length. The derivative N_r is often written as C_{nr} in other systems of notation, but attention must be paid to the reference dimensions used. In particular, in forming C_{nr} differentiation C_n may be carried out with respect to $rb/2V$ not rb/V as implied in the Hopkin system. It is also to be noted that a constant datum value of V is employed by Hopkin.

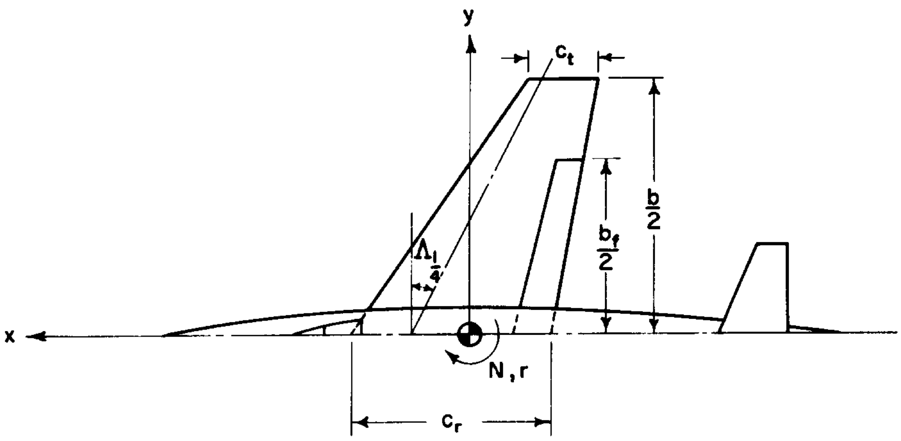
		SI	*British*
A	aspect ratio, b^2/S		
b	wing span	m	ft
b_f	flap span	m	ft
C_{D0}	wing drag coefficient at zero lift		
C_L	wing lift coefficient		
C_n	yawing moment coefficient, $N/½\rho V^2 Sb$		
c_r	root chord	m	ft
c_t	tip chord	m	ft
f	function of b_f/b and λ, see Equation (3.1)		
M	Mach number		
$\mathcal{N}$	yawing moment about the origin	N m	lbf ft
N_r	aero-normalised yawing moment derivative due to yawing, $(\partial\mathcal{N}/\partial r)/½\rho VSb^2$		
r	angular velocity in yaw	rad/s	rad/s
S	gross wing area	m^2	ft^2
V	aircraft velocity relative to air	m/s	ft/s
ΔC_{D0}	increment in C_{D0} due to deflection of trailing-edge flaps		
ΔN_{r0}	increment in N_{r0} due to deflection of trailing-edge flaps		

Issued September 1971

$\Lambda_{1/4}$	sweepback of ¼-chord line	degree	degree
λ	taper ratio, c_t / c_r		
ρ	density of air	kg/m^3	slug/ft^3

Suffixes

v	denotes quantity arising from lift-dependent drag due to trailing vortices
w	denotes wing total quantity
λ	denotes quality relating to wing of taper ratio λ
0	denotes quality arising from wing drag at zero lift.



2. PLAIN WINGS

Figures 1 and 2 present data for the contribution of a straight-tapered swept wing to the aero-normalised yawing moment derivative due to yawing in low-speed flow at lift coefficients for which the flow remains fully attached (see Item No. 66033 for wings with symmetrical sections).

Figures 1a and 1b present data for N_{r0}, the component of N_{rw} derived by consideration of the asymmetric distribution of the profile drag (Derivations 1 and 2). Data are presented to cover useful ranges of aspect ratio and sweepback of the ¼-chord line.

Figures 2a to 2d present data for N_{rv}, the component of N_{rw} derived by consideration of the asymmetric distribution of the lift-dependent drag due to the trailing vortex system (Derivation 2) for taper ratios $\lambda = 0$, 0.25, 0.5 and 1.0 respectively over the same ranges of values for aspect ratio and sweepback as those of Figure 1a. The data given in these Figures strictly apply only to the case when the centre of gravity of the aircraft (assumed to be the centre of rotation) coincides with the wing aerodynamic centre. The effect of there being a finite distance between the centre of gravity and aerodynamic centre is small over the range likely to be experienced in practice. For further information see Derivation 2.

The total contribution of the wing to the aeronormalised yawing moment derivative due to yawing is given by the sum of the two components obtained from Figures 1 and 2, *i.e*.

$$N_{rw} = \frac{N_{r0}}{C_{D0}} C_{D0} + \frac{N_{rv}}{C_L^2} C_L^2 . \tag{2.1}$$

The curves given in the Figures were derived from simple strip considerations using lifting-line theory. Since the wing contribution to N_r is generally small compared with that from the fin and rudder assembly (see Item No. Aero A.07.01.00), the unsophisticated treatment was felt to suffice.

Agreement between values of N_{rw} estimated from this Item and the available wind-tunnel data*, for fully-attached flow conditions (Derivation 1 and References 4, 6 and 7), is in the worst cases within experimental scatter (± 0.01).

Experimental data for the effect of dihedral (Reference 5) show that this parameter has a negligible effect provided that the flow remains fully-attached.

No theoretical or experimental data are available concerning the effects of compressibility on N_{rw}. A considerable part of its effect could be accounted for simply by use of the C_L and C_{D0} values appropriate to the required Mach number and by replacing A and $\Lambda_{1/4}$ in Figure 2 by $A(1-M^2)^{1/2}$ and $\tan^{-1}[(1-M^2)^{-1/2}\tan\Lambda_{1/4}]$, respectively.

3. WINGS WITH FLAPS

There are few data available concerning the effects of flaps on the yawing stability derivative, N_r. From these data the indications are that the effects of leading-edge flaps are negligible (Reference 8). For these cases, therefore, the plain wing data can be assumed to apply to the wing-flap combination, for lift coefficients well below the stall. For wings with trailing-edge flaps Derivation 1 gives an expression for ΔN_{r0}, the increment in N_{r0} due to flap deflection, for unswept wings, and this is here tentatively modified to account for wing sweep to accord with some experimental data given in Reference 8, giving

$$\Delta N_{r0} = \left(\frac{N_{r0}}{C_{D0}}\right) f \sec^2\Lambda_{1/4}\Delta C_{D0} , \tag{3.1}$$

where (N_{r0}/C_{D0}) is for the plain wing, and is obtained from Figure 1. The parameter f, which is a function of b_f/b and λ, is presented in Figure 3. The total component due to the drag at zero lift is given by the sum of N_{r0} (see Section 2) and ΔN_{r0}. The component due to the trailing vortex system of the wing-flap combination may be obtained from Figure 2 using the C_L appropriate to the combination.

Agreement with the experimental data (Derivation 1 and References 3, 4 and 8) using the method of this Item is within ± 0.01 for lift coefficients well below the stall.

The use of this Item for wings fitted with flaps should be considered tentative in view of the small amount of experimental evidence available.

* Where the results are derived from oscillatory measurements it was found possible to ignore the influence of frequency and amplitude.

4. SEPARATED FLOW

As stated in Section 2 the data given in this Item apply strictly only to those cases where fully-attached flow conditions exist. Under separated flow conditions the yawing moment is likely to be highly non-linear and two further parameters, frequency and amplitude of the oscillatory motion, become important. These additional considerations, combined with the fact that relatively few test data for separated flow conditions are available, currently preclude the development of a generalised procedure for this regime. For this reason it was felt helpful to include a bibliography of reports for experimental work covering these conditions (see Section 6).

5. DERIVATION AND REFERENCES

Derivation

The Derivation lists selected sources that have assisted in the preparation of this Item.

1.	CAMPBELL, J.P. MATHEWS, W.O.	Experimental determination of the yawing moment due to yawing contributed by the wing, fuselage, and vertical tail of a midwing airplane model. NACA ARR 3F28 (TIL 457), 1943.
2.	TOLL, T.A. QUEIJO, M.J.	Approximate relations and charts for low-speed stability derivatives of swept wings. NACA tech. Note 1581, 1948.

References

The References are sources of information supplementary to that in this Item.

3.	HARMON, S.M.	Determination of the damping moment in yawing for tapered wings with partial-span flaps. NACA ARR 3H25 (TIL 481), 1944.
4.	COTTER, W.E.	Summary and analysis of data on damping in yaw and pitch for a number of airplane models. NACA tech. Note 1080, 1946.
5.	QUEIJO, M.J. JAQUET, B.M.	Investigation of effects of geometric dihedral on low-speed static stability and yawing characteristics of an untapered 45° sweptback-wing model of aspect ratio 2.61. NACA tech. Note 1668, 1948.
6.	GOODMAN, A. BREWER, J.D.	Investigation at low speeds of the effect of aspect ratio and sweep on static and yawing stability derivatives of untapered wings. NACA tech. Note 1669, 1948.
7.	LETKO, W. COWAN, J.W.	Effect of taper ratio on low-speed static and yawing stability derivatives of 45° sweptback wings with aspect ratio of 2.61. NACA tech. Note 1671, 1948.
8.	LICHTENSTEIN, J.H.	Effect of high-lift devices on the low-speed static lateral and yawing stability characteristics of an untapered 45° sweptback wing. NACA tech. Note 2689. 1948.

6. BIBLIOGRAPHY

The following bibliography consists of references to reports containing experimental data which it is felt would be useful in analyses concerning N_{rw} in the non-linear (separated flow) regime:

1.	GOODMAN, A. FEIGENBAUM, D.	Preliminary investigation at low speeds of swept wings in yawing flow. NACA RM L7I09 (TIL 1617), 1948.
2.	LICHTENSTE1N, J.H.	Effect of high-lift devices on the low-speed static lateral and yawing stability characteristics of an untapered 45° sweptback wing. NACA tech. Note 2689, 1948.
3.	QUEIJO, M.J. JAQUET, B.M.	Investigation of effects of geometric dihedral on low-speed static stability and yawing characteristics of an untapered 45° sweptback-wing model of aspect ratio 2.61. NACA tech. Note 1668, 1948.
4.	GOODMAN, A. BREWER, J.D.	Investigation at low speeds of the effect of aspect ratio and sweep on static and yawing stability derivatives of untapered wings. NACA tech. Note 1669), 1948.
5.	LETKO, W. COWAN, J.W.	Effect of taper ratio on low-speed static and yawing stability derivatives of 45° sweptback wings with aspect ratio of 2.6l. NACA tech. Note 1671, 1948.
6.	BIRD, J.D. JAQUET, B.M.	A study of the use of experimental stability derivatives in the calculation of the lateral disturbed motions of a swept-wing airplane and comparison with flight results. NACA tech. Rep. 1031, 1951.
7.	BIRD, J.D. JAQUET, B.M. COWAN, J.W.	Effect of fuselage and tail surfaces on low-speed yawing characteristics of a swept-wing model as determined in curved flow test section of Langley Stability Tunnel. NACA tech. Note 2483, 1951.
8.	WILLIAMS, J.L.	Measured and estimated lateral static and rotary derivatives of a 1/12-scale model of a high-speed fighter airplane with unswept wings. NACA RM L53K09 (TIL 5187), 1954.
9.	QUEIJO, M.J. *et al*.	Preliminary measurements of the aerodynamic yawing derivatives of a triangular, a swept and an unswept wing performing pure yawing oscillations, with a description of the instrument employed. NACA RM L55LI4 (TIL 5046), 1956.
10.	JOHNSON, J.L. Jr	Low-speed measurements of static stability, damping in yaw and damping in roll of a delta, a swept and an unswept wing for angles of attack from 0° to 90°. NACA RM L56B01 (TIL 5069), 1956.
11.	FISHER, L.R.	Experimental determination of the effects of frequency and amplitude on the lateral stability derivatives for a delta, a swept and an unswept wing oscillating in yaw. NACA tech. Rep. 1357, 1958.
12.	LETKO, W. FLETCHER, H.S.	Effects of frequency and amplitude on the yawing derivatives of triangular, swept and unswept wings and of a triangular wing-fuselage combination with and without a triangular tail performing sinusoidal yawing oscillations. NACA tech. Note 4390, 1958.

7. EXAMPLE

Estimate the aeronormalised yawing moment derivative due to yawing in low-speed flow at $C_L = 0.7$ for a wing with a deflected trailing-edge flap. The geometrical and aerodynamic data for the wing-flap combination are as follows:

$$A = 8, \quad \lambda = 0.25, \quad \Lambda_{1/4} = 40°, \quad b_f/b = 0.6, \quad C_{D0} = 0.0095, \quad \Delta C_{D0} = 0.06.$$

From Figures 1a and 1b for $A = 8$, $\Lambda_{1/4} = 40°$ and $\lambda = 0.25$,

$$\frac{N_{r0}}{C_{D0}} = \left(\frac{N_{r0}}{C_{D0}}\right)_{\lambda = 1} \frac{(N_{r0}/C_{D0})_{\lambda = 0.25}}{(N_{r0}/C_{D0})_{\lambda = 1}} = -0.239 \times 0.70 = -0.167.$$

Thus, $N_{r0} = -0.167 \times 0.0095 = -0.00159$.

From Figure 3, for $b_f/b = 0.6$ and $\lambda = 0.25$, $f = 0.327$.

Hence, from Equation (3.1)

$$\begin{aligned} \Delta N_{r0} &= \left(\frac{N_{r0}}{C_{D0}}\right) f \sec^2 \Lambda_{1/4}\, \Delta C_{D0} \\ &= -0.167 \times 0.327 \times (1.305)^2 \times 0.06 \\ &= -0.00558. \end{aligned}$$

From Figure 2b, for $A = 8$ and $\Lambda_{1/4} = 40°$,

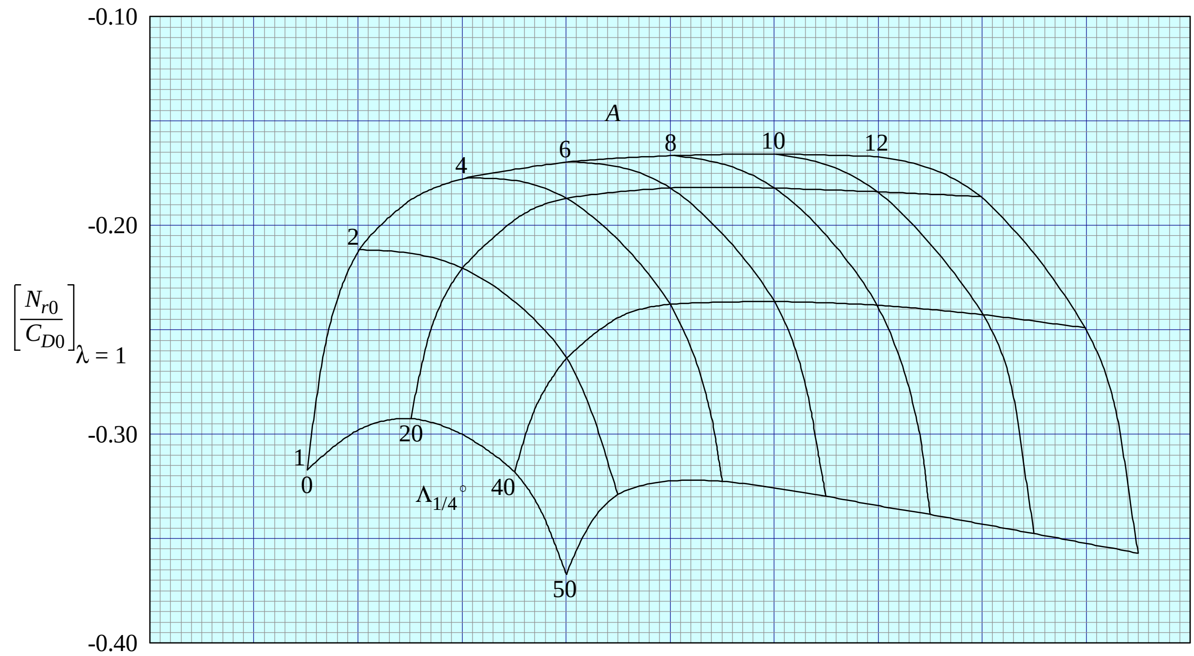
$$\frac{N_{rv}}{C_L^2} = -0.00295.$$

Hence,

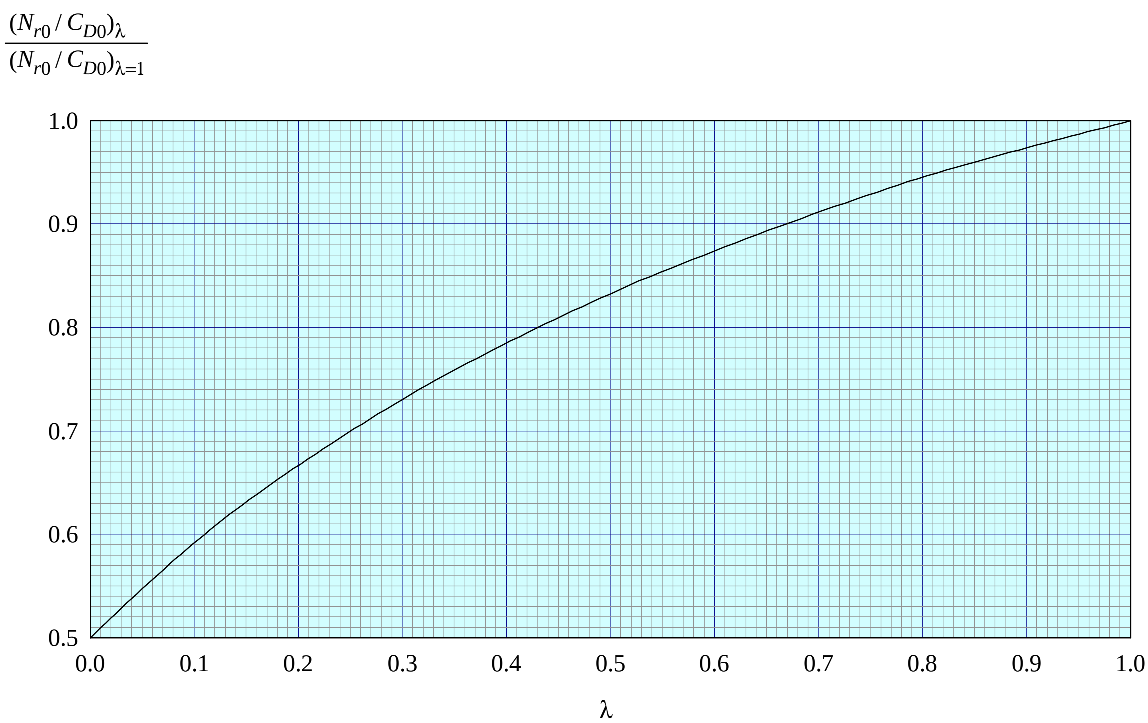
$$\begin{aligned} N_{rv} &= -0.00295 \times 0.7^2 \\ &= -0.00145. \end{aligned}$$

Therefore

$$\begin{aligned} N_{rw} &= N_{r0} + \Delta N_{r0} + N_{rv} \\ &= -0.00159 - 0.00558 - 0.00145 \\ &= -0.0086. \end{aligned}$$

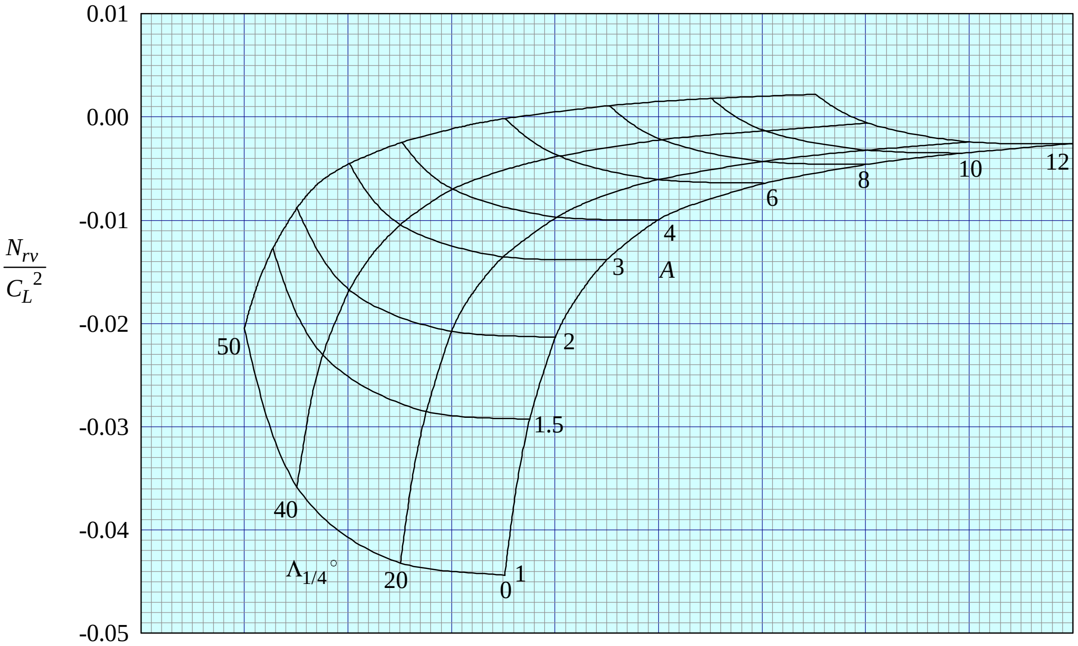


a. λ = 1.0

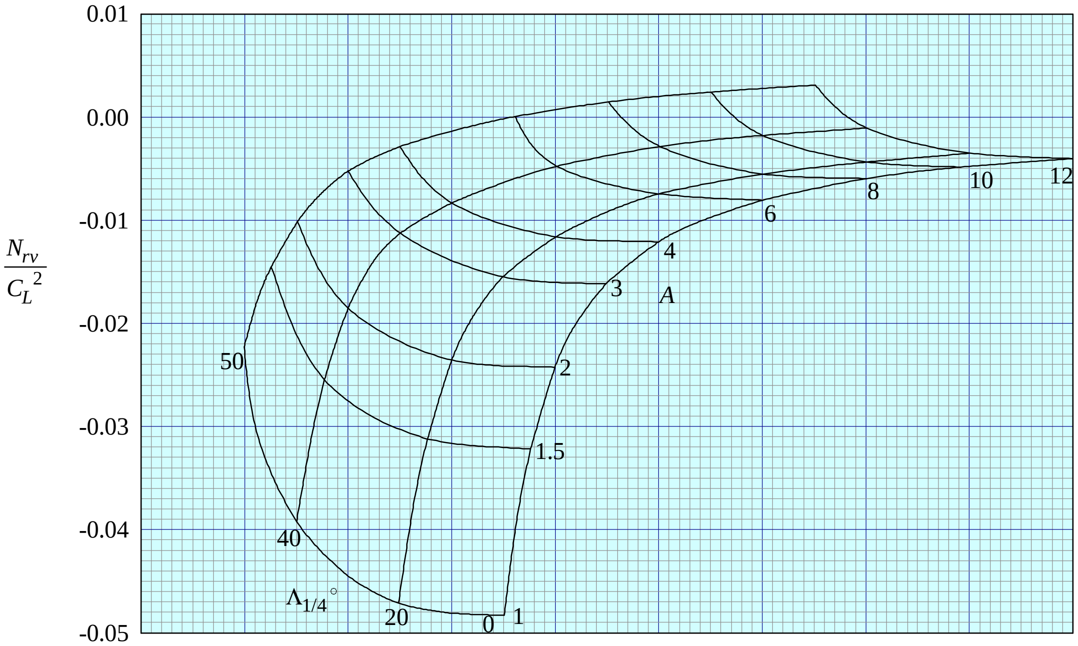


b.

FIGURE 1

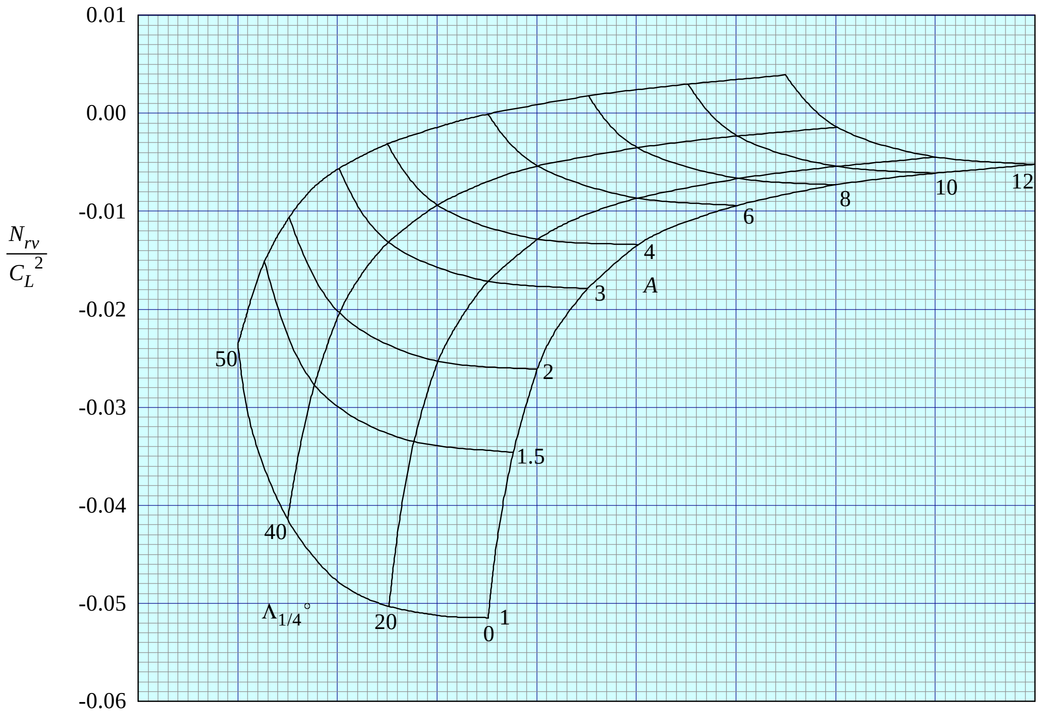


a. λ = 0

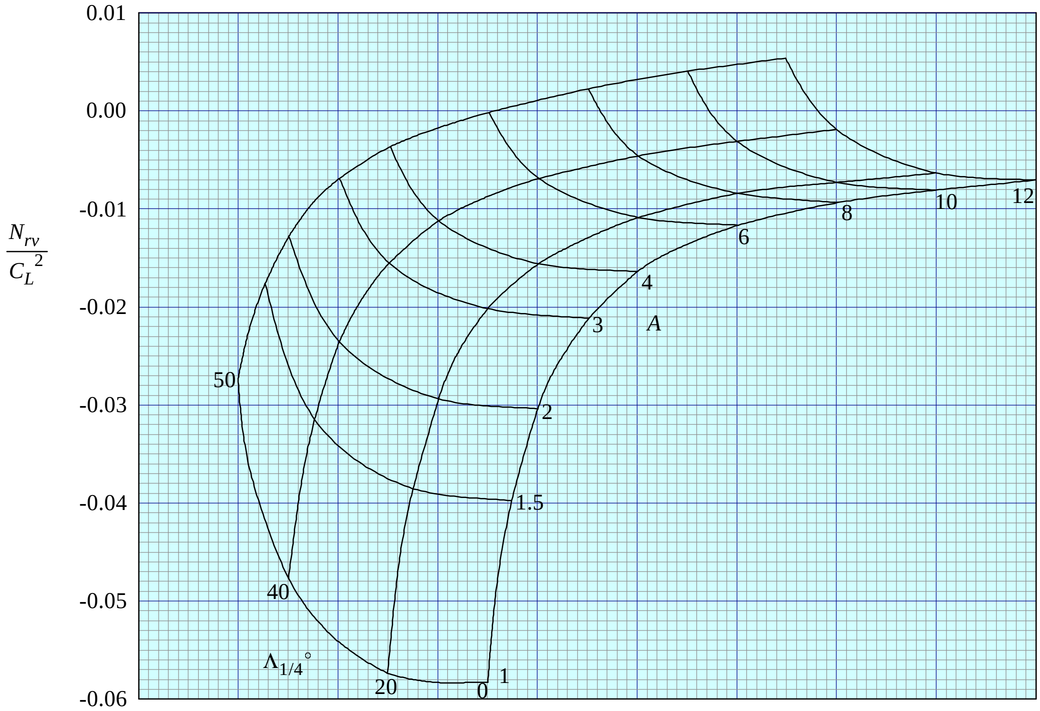


b. λ = 0.25

FIGURE 2

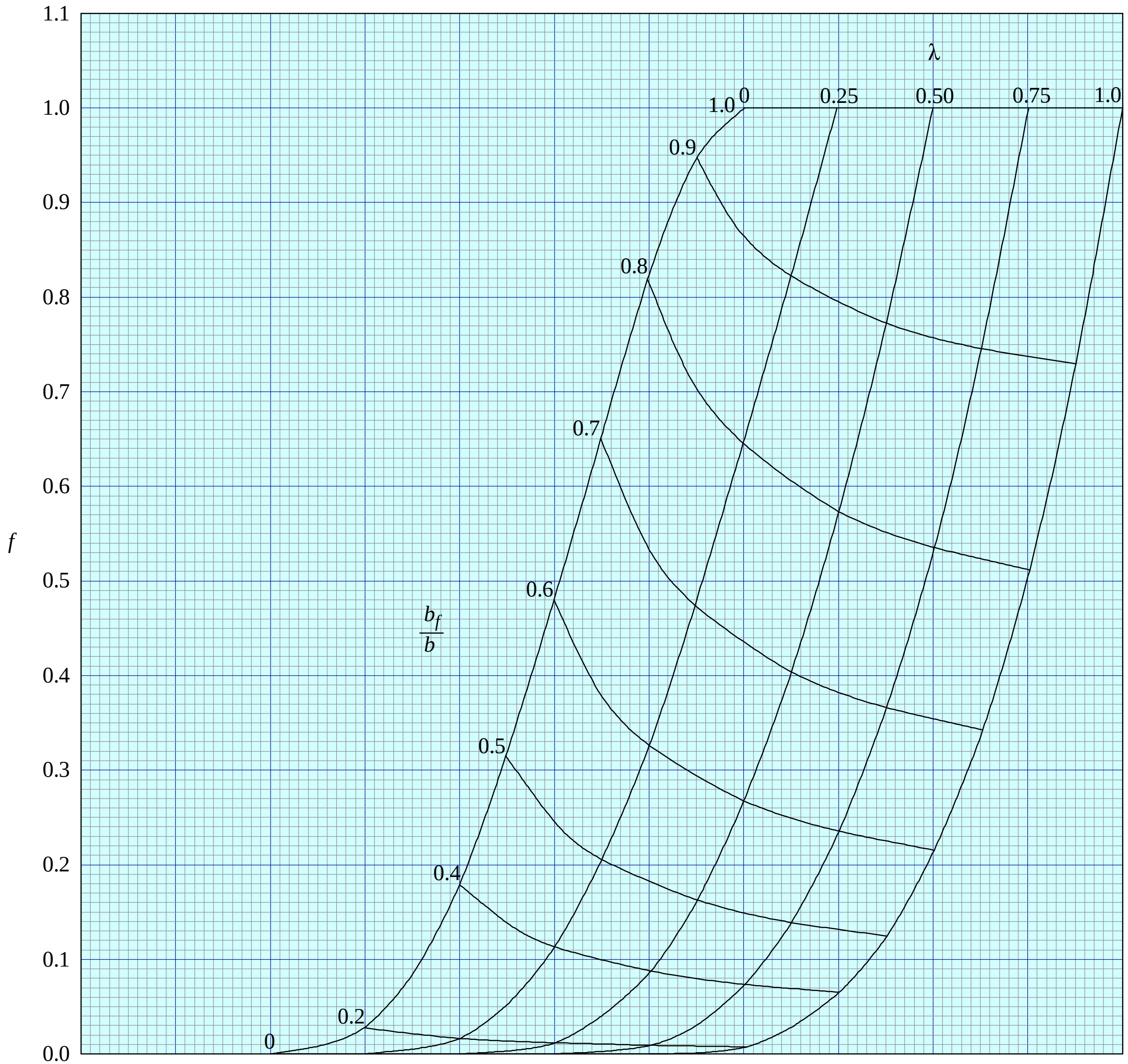


c. λ = 0.5



d. λ = 1.0

FIGURE 2 (Concluded)



1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
f
λ
0
0.25
0.50
0.75
1.0
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.2
0
$\frac{b_f}{b}$

FIGURE 3

THE PREPARATION OF THIS DATA ITEM

The work on this particular Item, which supersedes Item Number Aero A.07.01.02, was monitored and guided by the Aerodynamics Committee which first met in 1942 and now has the following membership:

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