CONTRIBUTION OF FIN TO SIDEFORCE, YAWING MOMENT AND ROLLING MOMENT DERIVATIVES DUE TO RATE OF YAW, $(Y_r)_F$, $(N_r)_F$, $(L_r)_F$

1. NOTATION AND UNITS (see Sketch 1.1)

The derivative notation used is that proposed in ARC R&M 3562 (Hopkin, 1970) and described in Item No. 86021. Coefficients and aeronormalised derivatives are evaluated in aerodynamic body axes with origin at the aircraft centre of gravity and with the wing span as the characteristic length. The derivatives Y_r, N_r and L_r are often written as C_{Yr}, C_{nr} and C_{lr} in other systems of notation, but attention must be paid to the reference dimensions used. In particular, in forming C_{Yr}, C_{nr} and C_{lr} differentiation of C_Y, C_n and C_l may be carried out with respect to $rb/2V$ not rb/V as implied in the Hopkin system. It is also to be noted that a constant datum value of V is employed by Hopkin.

		SI	*British*
b	wing span	m	ft
$(C_{L\alpha})_F$	lift-curve slope of fin, evaluated from Item No. 82010 (Derivation 19)	radian^{-1}	radian^{-1}
C_l	rolling moment coefficient, $\mathscr{L}/\frac{1}{2}\rho V^2 S_w b$		
C_n	yawing moment coefficient $\mathscr{N}/\frac{1}{2}\rho V^2 S_w b$		
C_Y	sideforce coefficient, $Y/\frac{1}{2}\rho V^2 S_w$		
c_{rF}	fin root chord, see also Item No. 82010 (Derivation 19)	m	ft
c_{tF}	fin tip chord, see also Item No. 82010 (Derivation 19)	m	ft
h_{BW}	body height at wing root quarter-chord station	m	ft
h_F	height of fin, measured from fin root chord in direction normal to longitudinal body axis, see also Item No. 82010 (Derivation 19)	m	ft
J_B	fin sideforce correction factor allowing for presence of body, evaluated from Item No. 82010 (Derivation 19)		
J_T	fin sideforce correction factor allowing for presence of tailplane, evaluated from Item No. 82010 (Derivation 19)		
J_W	fin sideforce correction factor allowing for presence of wing, evaluated from Item No. 82010 (Derivation 19) as a function of z_W/h_{BW} when estimating fin contribution to lateral derivatives due to sideslip, set equal to unity when estimating fin contribution to lateral derivatives due to rate of yaw		
$\mathscr{L}$	rolling moment	N m	lbf ft

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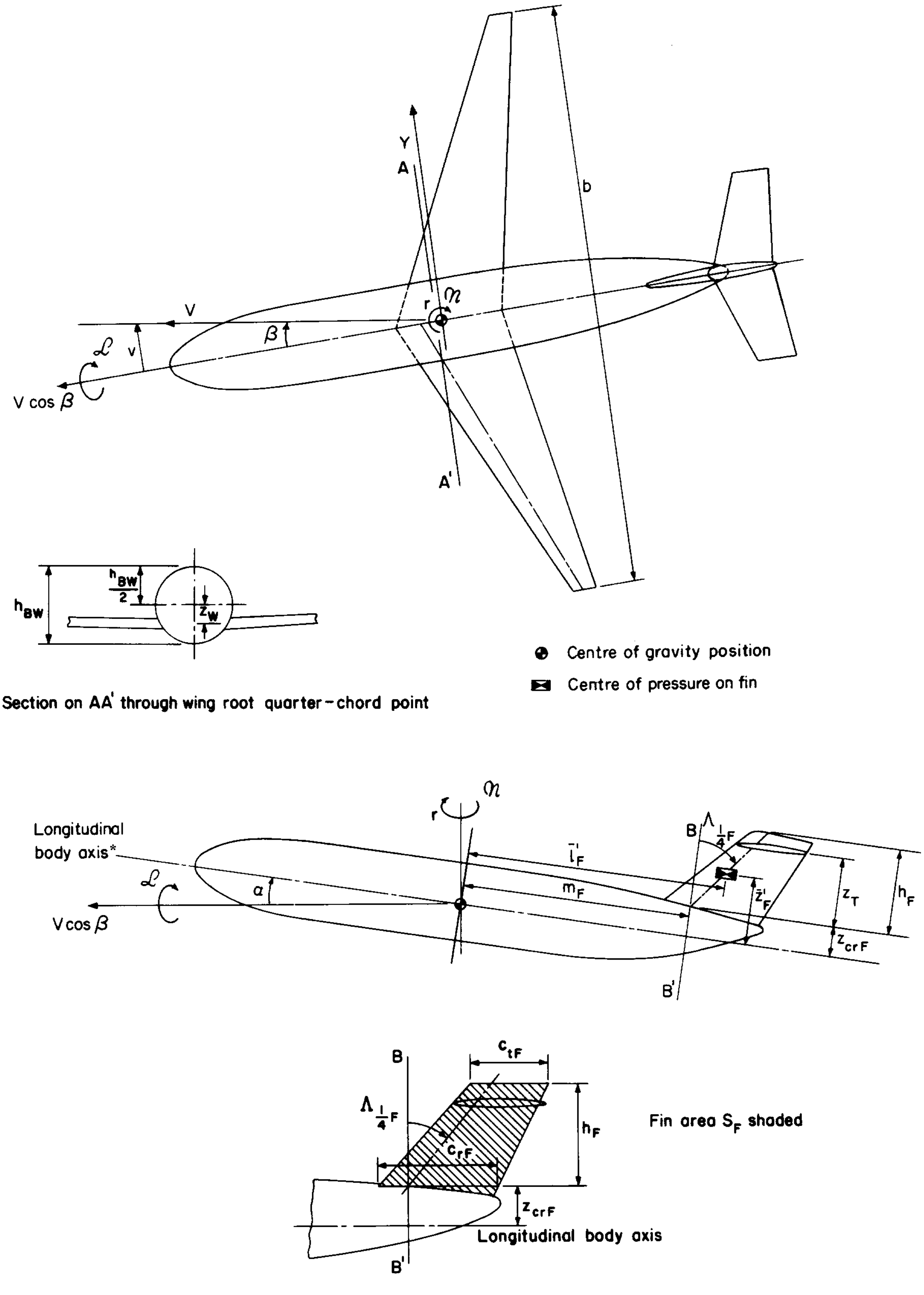
L_r	rolling moment derivative due to rate of yaw $L_r = (\partial\mathscr{L}/\partial r)/½\rho V S_W b^2$		
$(L_r)_F$	fin contribution to L_r in presence of body, wing and tailplane		
L_v	rolling moment derivation due to sideslip, $L_v = (\partial\mathscr{L}/\partial v)/½\rho V S_W b$		
$(L_v)_F$	fin contribution to L_v in presence of body, wing and tailplane		
$\bar{l}'_F$	distance of centre of pressure position of fin sideforce, measured aft from centre of gravity position and parallel to longitudinal body axis; from Item No. 82010 (Derivation 19) $\bar{l}'_F = m_F + 0.7\bar{z}_F \tan\Lambda_{¼F}$	m	ft
m_F	distance of fin root quarter-chord station aft of centre of gravity position, measured parallel to longitudinal body axis, see also Item No. 82010 (Derivation 19)	m	ft
$\mathscr{N}$	yawing moment	N m	lbf ft
N_r	yawing moment derivative due to rate of yaw, $N_r = (\partial\mathscr{N}/\partial r)/½\rho V S_W b^2$		
$(N_r)_F$	fin contribution to N_r in presence of body, wing and tailplane		
N_v	yawing moment derivative due to sideslip, $N_v = (\partial\mathscr{N}/\partial v)/½\rho V S_W b$		
$(N_v)_F$	fin contribution to N_v in presence of body, wing and tailplane		
r	rate of yaw	rad/s	rad/s
S_F	fin area, $h_F(c_{rF} + c_{tF})/2$, see also Item No. 82010 (Derivation 19)	m^2	ft^2
S_W	wing (reference) area	m^2	ft^2
V	velocity of aircraft relative to air	m/s	ft/s
v	slipside velocity	m/s	ft/s
Y	sideforce	N	lbf
Y_r	sideforce derivative due to rate of yaw, $Y_r = (\partial Y/\partial r)/½\rho V S_W b$		
$(Y_r)_F$	fin contribution to Y_r in presence of body, wing and tailplane		

Y_v	sideforce derivative due to sideslip, $Y_v = (\partial Y/\partial v)/\frac{1}{2}\rho V S_W$		
$(Y_v)_F$	fin contribution to Y_v in presence of body, wing and tailplane		
$[(Y_v)_F]_{J_W = 1}$	value of $(Y_v)_F$ with no interference between wing and fin $(J_W = 1)$ evaluated from Item No. 82010 (Derivation 19)		
z_{crF}	height of fin root chord, measured from longitudinal body axis in direction normal to longitudinal body axis, see also Item No. 82010 (Derivation 19)	m	ft
$\bar{z}_F$	height of centre of pressure position of load distribution on fin, measured from fin root chord in direction normal to longitudinal body axis, evaluated from Item No. 82010 (Derivation 19) as function of z_T/h_F	m	ft
$\bar{z}'_F$	distance of centre of pressure position of fin sideforce, measured above and normal to longitudinal body axis; from Item No. 82010 (Derivation 19) $\bar{z}'_F = z_{crF} + 0.85\bar{z}_F$	m	ft
z_T	height of intersection of fin-mounted tailplane with fin, measured from fin root chord in direction normal to longitudinal body axis, see also Item No. 82010 (Derivation 19)	m	ft
z_W	height of wing root quarter-chord point above local body centre-line, positive for low wings	m	ft
α	angle of attack	radian	radian
β	angle of sideslip	radian	radian
$\Lambda_{\frac{1}{4}F}$	fin quarter-chord sweep angle	degree	degree
ρ	density of air	kg/m^3	slug/ft^3



Sketch 1.1 Body, wing and fin geometries

* The longitudinal body axis is a reference axis, fixed in the body in the plane of symmetry and passing through the centre of gravity position. The exact direction of the axis in the plane of symmetry is conventionally determined by considerations of mid-body geometry, the axis being taken parallel to some convenient 'horizontal' datum.

2. INTRODUCTION

This Item shows how the fin contribution to the sideforce, yawing moment and rolling moment derivatives due to rate of yaw, $(Y_r)_F$, $(N_r)_F$ and $(L_r)_F$ can be calculated from the data on the fin contribution to the static stability derivatives due to sideslip that are given in Item No. 82010 (Derivation 19). The method applies at subsonic speeds where the flow over the configuration is wholly subsonic and fully attached. It covers configuration geometries where the fin is essentially a trapezoidal panel mounted on top of the aircraft rear body and in the plane of symmetry (see also comments in Section 3.2).

Section 2.1 gives a résumé of the method in Item No. 82010. Section 3 describes the method for predicting the fin contribution to the yaw rate derivatives, Section 4 discusses its accuracy and applicability, Section 5 gives the Derivation and References and Section 6 gives a worked example.

2.1 Résumé of Method in Item No. 82010

The method that is provided in Item No. 82010 predicts the fin contributions $(Y_v)_F$, $(N_v)_F$ and $(L_v)_F$ by first determining a basic lift-curve slope for the fin, $(C_{L\alpha})_F$. This is then multiplied by the three factors J_B, J_T and J_W to allow for the interference effects of the body, tailplane and wing, respectively. Based on the wing area the sideforce derivative is written

$$(Y_v)_F = -J_B\, J_T\, J_W\, (C_{L\alpha})_F\, S_F / S_W, \tag{2.1}$$

where S_F is the fin area. The yawing moment and rolling moment derivatives are

$$(N_v)_F = -(Y_v)_F(\bar{l}_F' \cos\alpha + \bar{z}_F' \sin\alpha)/b \tag{2.2}$$

and

$$(L_v)_F = (Y_v)_F(\bar{z}_F' \cos\alpha - \bar{l}_F' \sin\alpha)/b\ . \tag{2.3}$$

The lengths $\bar{l}_F'$ and $\bar{z}_F'$ have been introduced for the purposes of this Item in order to simplify the notation. They represent the distances of the centre of pressure position of the fin sideforce relative to the centre of gravity position measured in directions parallel and normal to the longitudinal body axis, respectively.

In the notation of Item No. 82010 they are expressed $\bar{l}_F' = m_F + 0.7\,\bar{z}_F \tan\Lambda_{1/4F}$ and $\bar{z}_F' = z_{crF} + 0.85\,\bar{z}_F$, where m_F, z_{crF}, $\bar{z}_F$ and $\Lambda_{1/4F}$ are determined by the geometry and position of the aircraft tail assembly. The user is referred to Item No. 82010 for a full description of the definition of the geometry, lift-curve slope and centre of pressure position of the fin.

3. METHOD

The method adopted in this Item for calculating the fin contributions to Y_r, N_r and L_r is to assume that in yawing motion a local sideslip velocity arises at the fin that is equal to the product of the yaw rate and the moment arm of the sideforce acting on the fin. If the fin lift-curve slope, the wing-body-fin interference effects and the centre of pressure position of the fin sideforce are the same in yawing motion as they are in steady sideslip, then for an angle of attack α the sideslip velocity due to yawing is $-(\bar{l}_F' \cos\alpha + \bar{z}_F' \sin\alpha)r$, and the fin contribution to Y_r is equal to the product of the steady sideslip derivative $(Y_v)_F$ given by Equation (2.1) and the factor $-(\bar{l}_F' \cos\alpha + \bar{z}_F' \sin\alpha)/b$. The yawing moment and sideforce derivatives are then obtained from the equations $(N_r)_F = -(Y_r)_F(\bar{l}_F' \cos\alpha + \bar{z}_F' \sin\alpha)/b$ and $(L_r)_F = (Y_r)_F(\bar{z}_F' \cos\alpha - \bar{l}_F' \sin\alpha)/b$.

Under this approach the predicted effects of wing-body-fin geometry are essentially the same for both the yaw rate and the steady sideslip derivatives. However, Derivations 9 and 11 report on three systematic series of wind tunnel tests in which wing height on the body was varied for an otherwise unchanging configuration. Examination of the results of those tests shows that although the effect of wing height is large for the derivatives due to sideslip it is very small for the derivatives due to rate of yaw. In sideslip the method in Item No. 82010 models the effect of wing-fin interference through the empirically determined factor J_W, which depends only on wing height on the body, varying with the parameter z_W/h_{BW}. The value of J_W is unity for mid-wing configurations, increasing for low wing configurations and decreasing for high wing configurations. The data in Derivations 9 and 11 suggest that J_W should always be taken as unity in the calculation of the yaw rate derivatives, implying that the wing-fin interference is negligible.

Thus, introducing the derivative

$$[(Y_v)_F]_{J_W=1} = -J_B J_T (C_{L\alpha})_F S_F/S_W, \tag{3.1}$$

and assuming that with the exception of the wing-fin interference effect the behaviour of the fin in steady sideslip and yawing motion is similar, then

$$(Y_r)_F = -[(Y_v)_F]_{J_W=1}(\bar{l}_F' \cos\alpha + \bar{z}_F' \sin\alpha)/b, \tag{3.2}$$

$$(N_r)_F = -(Y_r)_F(\bar{l}_F' \cos\alpha + \bar{z}_F' \sin\alpha)/b \tag{3.3}$$

and

$$(L_r)_F = (Y_r)_F(\bar{z}_F' \cos\alpha - \bar{l}_F' \sin\alpha)/b. \tag{3.4}$$

Values of J_B, J_T, $(C_{L\alpha})_F$, S_F, $\bar{l}_F'$ and $\bar{z}_F'$ are all determined as described in Item No. 82010.

As discussed in Section 4.1 comparisons with experimental data show that Equations (3.2), (3.3) and (3.4) provide good estimates of the yaw rate derivatives for a wide variety of configurations, the overall level of agreement being consistent with that achieved in predicting the corresponding steady sideslip derivatives, which suggests that the assumptions made in deriving the equations are sound.

4. ACCURACY AND APPLICABILITY

4.1 Accuracy

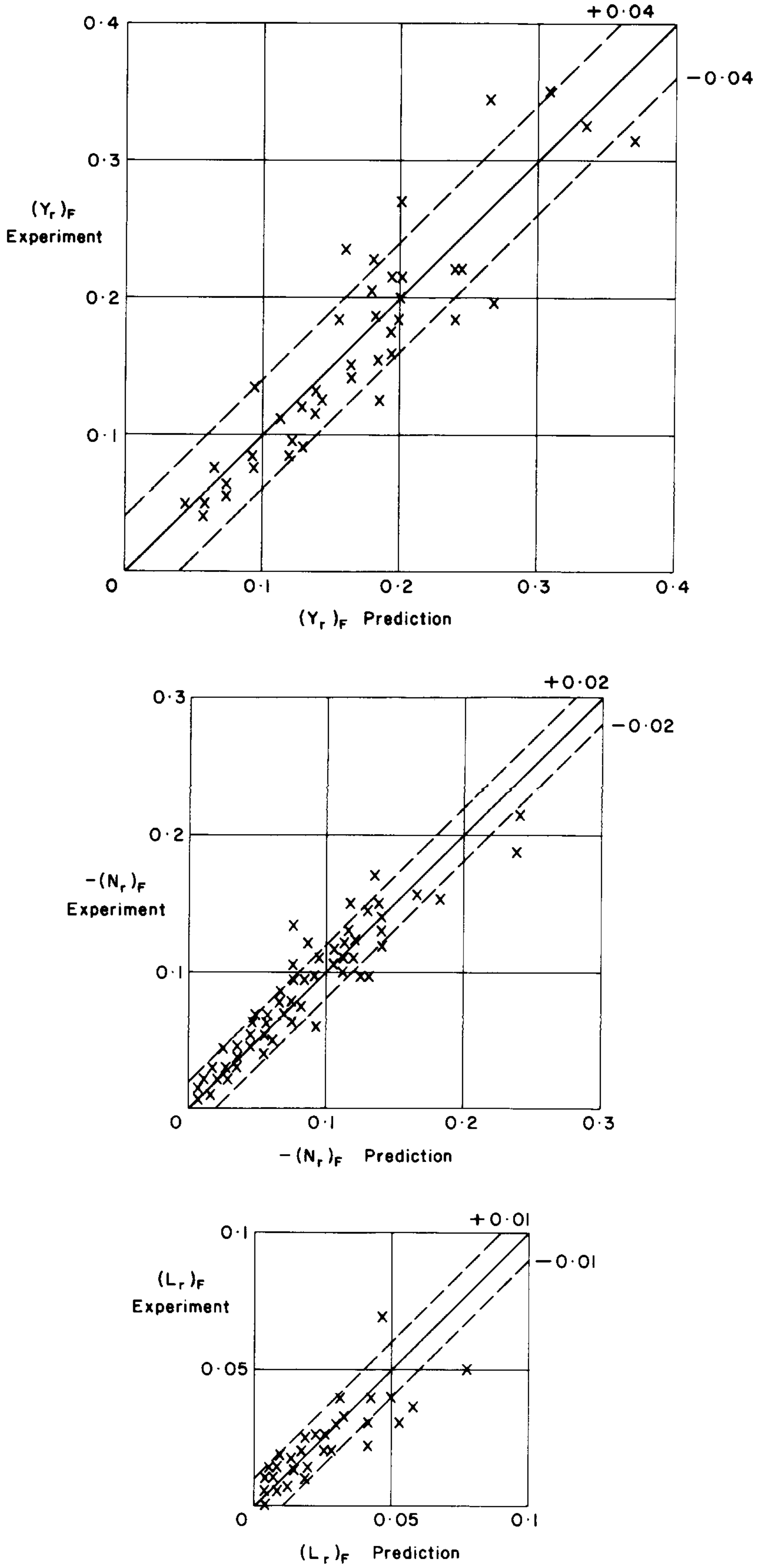
The fin contributions to the yaw rate derivatives predicted by Equations (3.2) to (3.4) have been compared with the experimental data reported in Derivations 1 to 18. Sketch 4.1 demonstrates the overall level of accuracy achieved. In general $(Y_r)_F$ is predicted to within ± 0.04, $(N_r)_F$ to within ± 0.02 and $(L_r)_F$ to within ± 0.01, although the errors are slightly greater in some cases. As the value of $(\bar{l}'_F \cos\alpha + \bar{z}'_F \sin\alpha)/b$ is typically about 0.5 these error bands are to be expected, being half the numerical value of the error bands quoted in Item No. 82010 for the corresponding steady sideslip derivatives.

4.2 Applicability

The limitations on the method in this Item are the same as those on the method in Item No. 82010 since that is used as a basis. The method is thus restricted to subsonic speeds where the flow over the configuration is fully attached and wholly subsonic and to configuration geometries where the fin is essentially a trapezoidal panel mounted on top of the aircraft rear-body and in the plane of symmetry, with the tailplane mounted on the body or on the fin*. The user should refer to the applicability section of Item No. 82010 for further information on the range of fin geometries covered.

The method applies at angles of sideslip and values of yaw rate where the sideforce, yawing moment and rolling moment coefficients vary linearly with those parameters.

* For aircraft geometries where the rear body is very narrow and merges into the shape of the fin Item No. Aero C.01.01.01 (Reference 20) gives steady sideslip derivatives for the fin. Similarly, Item No. Aero C.01.01.02 (Reference 21) gives steady sideslip derivatives for the case of twin fins mounted at the extremities of a tailplane. Both these sets of data may be used to estimate corresponding yaw rate derivatives in the same way as described in this Item, by calculating the local sideslip velocity induced at the fin centre of pressure position by the aircraft yaw rate, and ignoring any wing-fin interference effects.



Sketch 4.1 Comparison of experimental and predicted values

5. DERIVATION AND REFERENCES

5.1 Derivation

The Derivation lists selected sources that have assisted in the preparation of this Item.

1.	COTTER, W.E.	Summary and analysis of data on damping in yaw and pitch for a number of airplane models. NACA tech. Note 1080, 1946.
2.	BIRD, J.D. JAQUET, B.M. COWAN, J.W.	Effect of fuselage and tail surfaces on low-speed yawing characteristics of a swept-wing model as determined in curved-flow test section of Langley stability tunnel. NACA tech. Note 2483, 1948.
3.	GOODMAN, A.	Effect of various outboard and central fins on low-speed yawing stability derivatives of a 60° delta-wing model. NACA RM L50E12a (TIL 2411), 1950.
4.	QUEIJO, M.J. GOODMAN, A.	Calculations of the dynamic lateral stability characteristics of the Douglas D-588-II airplane in high-speed flight for various wing loadings and altitudes. NACA RM L50H16a (TIL 3352), 1950.
5.	FISHER, L.R. MICHAEL, W.H.	An investigation of the effect of vertical-fin location and area on low-speed lateral stability derivatives of a semitailless airplane model. NACA RM L51A10 (TIL 2655), 1951.
6.	LETKO, W.	Effect of vertical-tail area and length on the yawing stability characteristics of a model having a 45° sweptback wing. NACA tech. Note 2358, 1951.
7.	QUEIJO, M.J. WELLS, E.G.	Wind-tunnel investigation of the low-speed static and rotary stability derivatives of a 0.13-scale model of the Douglas D-558-II airplane in the landing configuration. NACA RM L52G07 (TIL 3502), 1952.
8.	BIRD, J.D. FISHER, L.R. HUBBARD, S.M.	Some effects of frequency on the contribution of a vertical tail to the free aerodynamic damping of a model oscillating in yaw. NACA Rep. 1130, 1953.
9.	FISHER, L.R. FLETCHER, H.S.	Effect of lag of sidewash on the vertical-tail contribution to oscillatory damping in yaw of airplane models. NACA tech. Note 3356, 1954.
10.	WILLIAMS, J.L.	Measured and estimated lateral static and rotary derivatives of a 1/12-scale model of a high-speed fighter airplane with unswept wings. NACA RM L53K09 (TIL 5187), 1954.
11.	JAQUET, B.M. FLETCHER, H.S.	Experimental steady-state yawing derivatives of a 60° delta-wing model as affected by changes in vertical position of the wing and in ratio of fuselage diameter to wing span. NACA tech. Note 3843, 1956.
12.	BUELL, D.A. REED, V.D. LOPEZ, A.E.	The static and dynamic-rotary stability derivatives at subsonic speeds of an airplane model with an unswept wing and a high horizontal tail. NACA RM A56I04 (TIL 6655), 1956.

13.	LETKO, W. FLETCHER, H.S.	Effects of frequency and amplitude on the yawing derivatives of triangular, swept and unswept wings and of a triangular-wing-fuselage combination with and without a triangular tail performing sinusoidal yawing oscillations. NACA tech. Note 4390 1958.
14.	HEWES, D.E.	Low-subsonic measurements of the static and oscillatory lateral stability derivatives of a sweptback-wing airplane configuration at angles of attack from –10° to 90°. NASA Memo. 5-20-59L (TIL 6506), 1959.
15.	GRAFTON, S.B. LIBBEY, C.E.	Dynamic stability derivatives of a twin-jet fighter model for angles of attack from –10° to 110°. NASA tech. Note D-6091, 1971.
16.	COE, P.L. NEWSOM, W.A.	Wind-tunnel investigation to determine the low-speed yawing stability derivatives of a twin-jet fighter model at high angles of attack. NASA tech. Note D-7721, 1974.
17.	O'LEARY, C.O.	Wind-tunnel measurements of lateral aerodynamic derivatives using a new oscillatory rig, with results and comparisons for the GNAT aircraft. ARC R & M 3847, 1977.
18.	RAE	Unpublished wind-tunnel data.
19.	ESDU	Contribution of fin to sideforce, yawing moment and rolling moment derivatives due to sideslip, $(Y_v)_F$, $(N_v)_F$, $(L_v)_F$, in the presence of body, wing and tailplane. Item No. 82010, Engineering Sciences Data Unit, London, April 1982.

5.2 References

The References list selected sources of information supplementary to that given in this Item.

20.	ESDU	Lift-curve slope for single fin and rudder. (i) Body shape merging into fin. Item No. Aero C.01.01.01, Engineering Sciences Data Unit, London, January 1955.
21.	ESDU	Lift-curve slope for twin fins and rudders. Item No. Aero C.01.01.02, Engineering Sciences Data Unit, London, March 1955. (Superseded by Item No. 92007.)

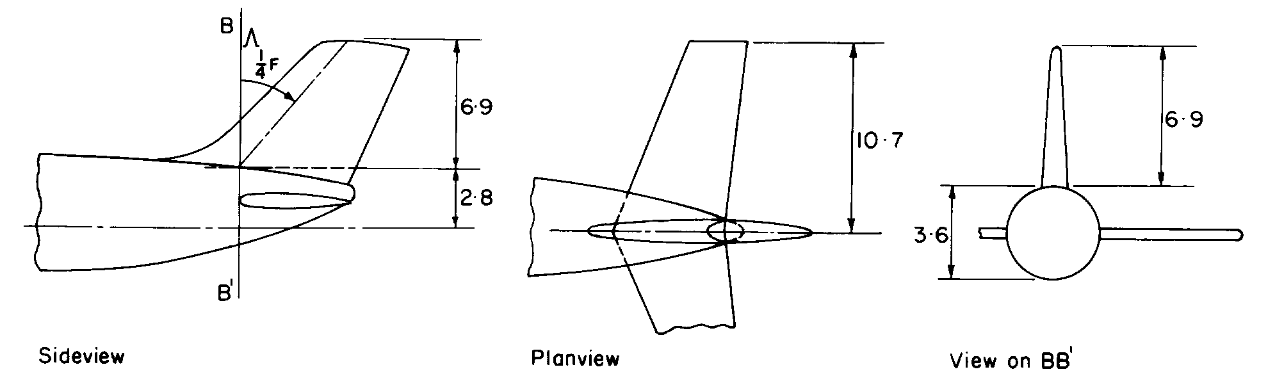
6. EXAMPLE

Calculate the derivatives $(Y_r)_F$, $(N_r)_F$ and $(L_r)_F$ for the three aircraft tail assemblies shown in the Example of Item No. 82010, which are reproduced here as Sketch 6.1. The same Mach number, 0.8, angle of attack, $\alpha = 2°$, and reference dimensions, $S_W = 320\ \text{m}^2$ and $b = 45.0$ m, may be assumed.

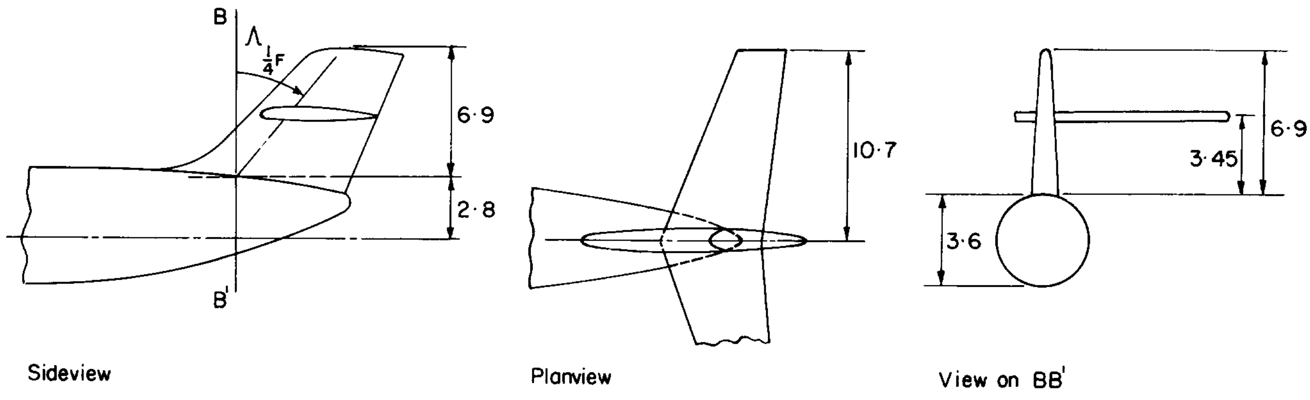
Table 6.1 summarises the results of the calculations for the fin contributions to the yaw rate derivatives, making use of the data given in the Example of Item No. 82010 and Equations (3.1) to (3.4), as shown in the Comments column.

TABLE 6.1

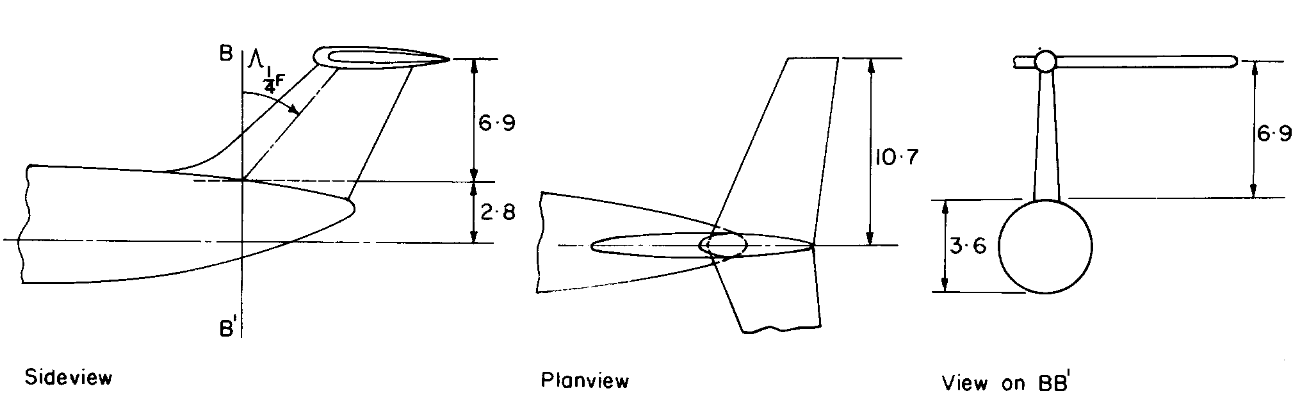
Quantity	*Tailplane Mounted on Body*	*Tailplane at Mid-Fin*	*Tailplane at Top of Fin*	*Comments*
$(C_{L\alpha})_F(\text{radian}^{-1})$	3.01	3.01	3.01	from Item No. 82010
S_F/S_W	42.1/320	42.1/320	42.1/320	from Item No. 82010
J_B	1.13	1.13	1.13	from Item No. 82010
J_T	1.12	0.98	1.30	from Item No. 82010
$[(Y_v)_F]_{J_W=1} = -J_B J_T (C_{L\alpha})_F S_F/S_W$	–0.501	–0.439	–0.582	Equation (3.1)
$\bar{l}_F' = m_F + 0.7\bar{z}_F \tan\Lambda_{1/4F}$	18.32 m	18.62 m	19.13 m	from Item No. 82010
$\bar{z}_F' = z_{crF} + 0.85\bar{z}_F$	5.15 m	5.57 m	6.32 m	from Item No. 82010
α	2°	2°	2°	Given
b	45.0 m	45.0 m	45.0 m	Given
$(Y_r)_F = -[(Y_v)_F]_{J_W=1}(\bar{l}_F' \cos\alpha + \bar{z}_F' \sin\alpha)/b$	0.206	0.183	0.250	Equation (3.2)
$(N_r)_F = -(Y_r)_F(\bar{l}_F' \cos\alpha + \bar{z}_F' \sin\alpha)/b$	–0.085	–0.076	–0.107	Equation (3.3)
$(L_r)_F = (Y_r)_F(\bar{z}_F' \cos\alpha - \bar{l}_F' \sin\alpha)/b$	0.021	0.020	0.031	Equation (3.4)



a. Tailplane mounted on body



b. Tailplane mounted at mid-fin



c. Tailplane mounted at top of fin

Sketch 6.1

THE PREPRARATION OF THIS DATA ITEM

The work on this particular Item, which supersedes, in part, Item No. 70006, was monitored and guided by the Aerodynamics Committee which first met in 1942 and now has the following membership:

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Mr H. Vogel	– British Aerospace, Weybridge-Bristol Division.

* Corresponding Member

The member of staff who undertook the technical work involved in the initial assessment of the available information and the construction and subsequent development of the Item was

Mr R.W. Gilbey – Senior Engineer.