SLOPE OF LIFT CURVE FOR TWO-DIMENSIONAL FLOW

Item Nos 72024 and 97020 together provide a more accurate value of $(a_1)_0$ and they supersede this Item, which has been temporarily retained within the Sub-series because it is used within the methods of some other Items.

1. NOTATION AND UNITS

		SI	*British*
$(a_1)_0$	slope of lift coefficient curve with incidence for two-dimensional aerofoil in incompressible flow	rad^{-1}	rad^{-1}
$(a_1)_{0T}$	theoretical slope of lift coefficient curve with incidence for two-dimensional aerofoil in incompressible inviscid flow	rad^{-1}	rad^{-1}
c	aerofoil chord	m	ft
t	maximum thickness of aerofoil section	m	ft
x_t	chordwise location of boundary layer transition, measured from aerofoil leading edge	m	ft
y_{90}, y_{99}	thicknesses of aerofoil section at 90, 99 per cent chord	m	ft
τ, τ_a	trailing-edge angles (see sketches on Figures 1 to 3)	deg	deg
R	Reynolds number based on free-stream flow quantities and c		

2. NOTES

Figures 1 and 2 show $(a_1)_0/(a_1)_{0T}$ plotted against $\tan(\tau_a/2)$ and Reynolds number for transition at the aerofoil leading edge and mid-chord positions, respectively. Values of $(a_1)_{0T}$, calculated on the basis of the Kutta-Joukowski hypothesis, are given in Figure 3 as a function of t/c for $\tau = 0$ and $20°$.

Derivation 2 gives the following equations representing Figures 1 to 3:

$$\frac{(a_1)_0}{(a_1)_{0T}} = 1 - \frac{0.1 + (1.05 - 0.5\, x_t/c)\, \tan\left(\tfrac{1}{2}\tau_a\right)}{(\log_{10} R - 5)^{[1 - 2.5\,\tan\left(\frac{1}{2}\tau_a\right)]}} \tag{2.1}$$

for Figures 1 and 2, where x_t/c is the boundary layer transition location,

and
$$(a_1)_{0T} = 2\pi + (4.75 + 0.02\tau)\ t/c \tag{2.2}$$

for Figure 3. There are minor discrepancies between Figures 1 to 3 and values given by these equations, but they are insignificant in practical terms.

The differences between $(a_1)_0$ and $(a_1)_{0T}$ are due essentially to the development of the boundary layer towards the trailing edge and are thus influenced by the flow over the rear of the aerofoil. Therefore, the ratio $(a_1)_0/(a_1)_{0T}$ has been related to the angle, τ_a, where τ_a is defined as the angle between straight lines passing though points at 90 and 99 per cent of the chord on the upper and lower surfaces (see Figures 1 and 2) and is given by the equation $\tan(\tau_a/2) = (y_{90} - y_{99})/0.18c$. The values of $(a_1)_{0T}$ are determined by the Kutta-Joukowski condition of finite velocity at the trailing edge. It may be noted, however, that $(a_1)_{0T}$ is relatively insensitive to the precise value of trailing-edge angle and for most purposes it is sufficiently accurate to replace τ by τ_a.

The data in this Item apply only when there is no separation of flow over the aerofoil, and should therefore be used with caution for aerofoils with maximum thickness exceeding about 0.2*c*, or with trailing-edge angles greater than about $20°$, or at high incidence.

Data for transition positions other than the leading edge or mid-chord may be found either by linear interpolation from the data in Figures 1 and 2 or by means of Equation (2.1).

At Mach numbers below the critical, an estimate of the effect of compressibility on two-dimensional lift-curve slope may be obtained by the method given in Item No. Aero W.01.01.01.

To obtain the slope of the lift coefficient curve for a wing of finite span, the value of $(a_1)_0$ must be corrected for aspect ratio, taper ratio and sweep (Item No. Aero W.01.01.01) and, where applicable, cut-outs (Item No. Aero W.01.01.04), gap (Item No. Aero C.01.01.04) and end-plate effects (Item Nos Aero C.01.01.01 and 92007).

The accuracy of this Item is assessed to be within ± 5 per cent.

3. DERIVATION

1.	BRYANT, L.W. HALLIDAY, A.S. BATSON, A.S.	Two dimensional control characteristics. ARC R&M 2730, 1955.
2.	GARNER, H.C.	Charts for low-speed characteristics of two-dimensional trailing-edge flaps. ARC R&M 3174, 1957.

4. EXAMPLE

Find the slope of the lift coefficient curve for a two-dimensional aerofoil for which $t/c = 0.10$, $y_{90}/c = 0.0202$, $y_{99}/c = 0.0018$ and $\tau = 10°$. Transition occurs at 0.3*c* and the Reynolds number is 3×10^7.

With

$$\log_{10} R = \log_{10}(3 \times 10^7) = 7.477$$

and $$\tan(\tau_a/2) = (0.0202 - 0.0018)/0.18 = 0.102,$$

Figure 1 gives $(a_1)_0/(a_1)_{0T} = 0.896$ for leading-edge transition

and Figure 2 gives $(a_1)_0/(a_1)_{0T} = 0.907$ for mid-chord transition.

Linear interpolation gives

$$\frac{(a_1)_0}{(a_1)_{0T}} = 0.896 + \frac{(0.907 - 0.896)}{0.5} \times 0.3 = 0.903$$

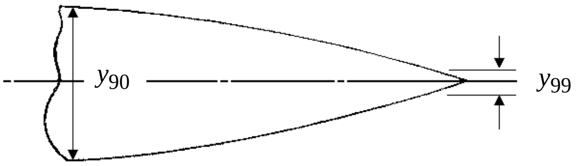
for transition at $0.3c$.

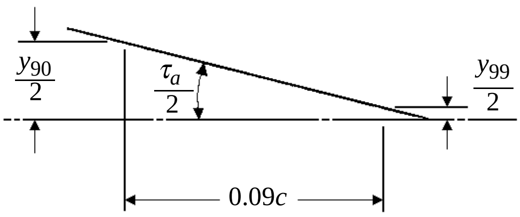
From Figure 3, with $\tau = 10°$ and $t/c = 0.10$,

$$(a_1)_{0T} = 6.775 \text{ per radian.}$$

Therefore, $$(a_1)_0 = 6.775 \times 0.903$$

$$= 6.12 \text{ per radian.}$$





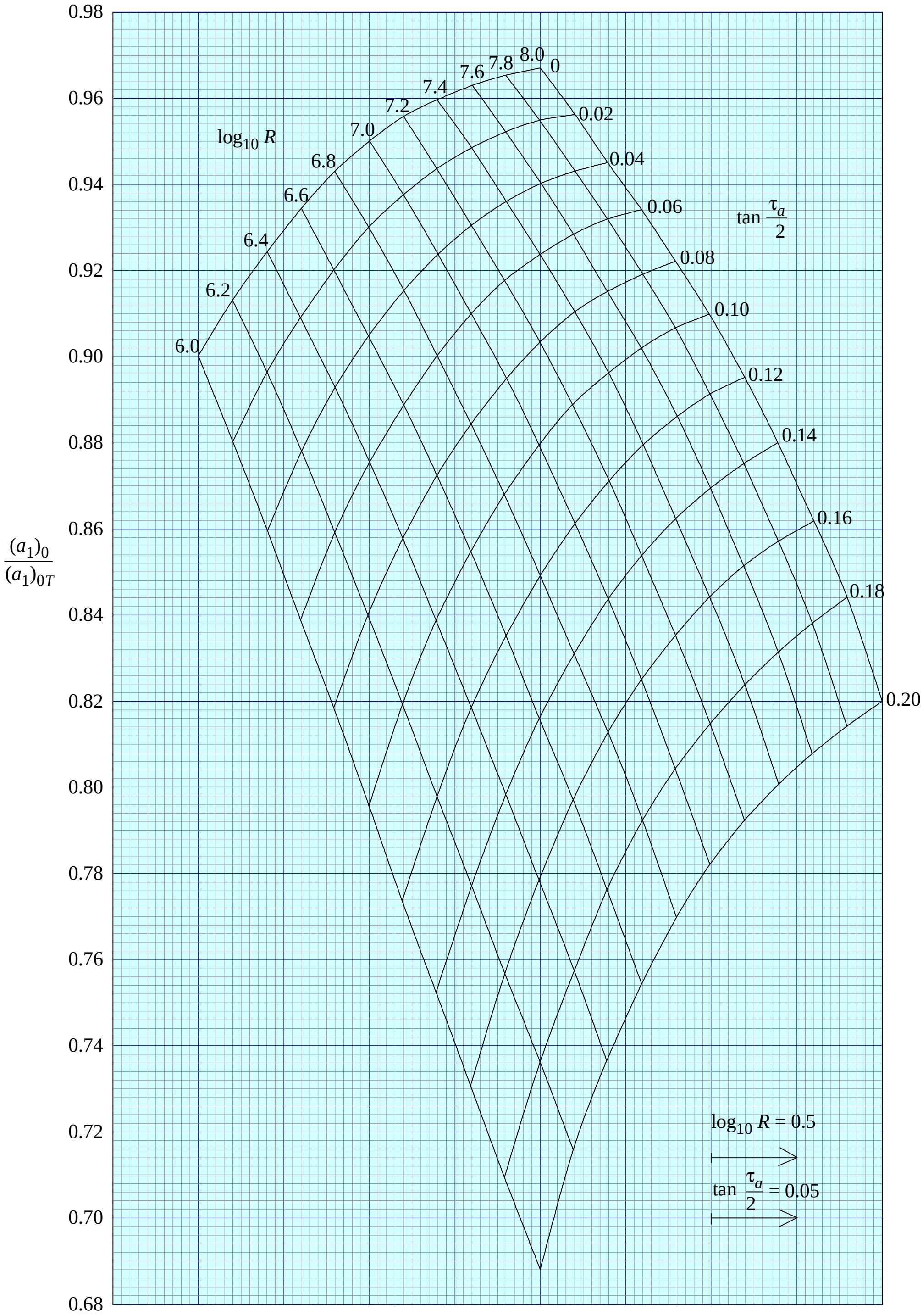
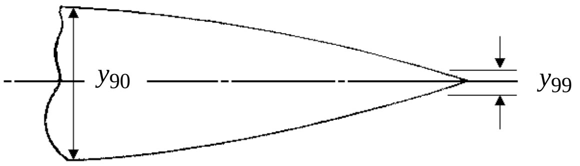
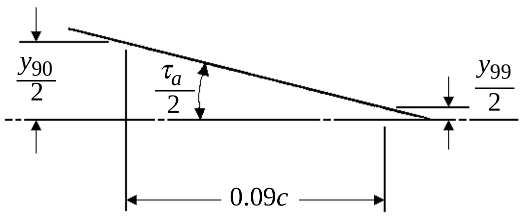


FIGURE 1 $\frac{(a_1)_0}{(a_1)_{0T}}$ FOR LEADING-EDGE TRANSITION





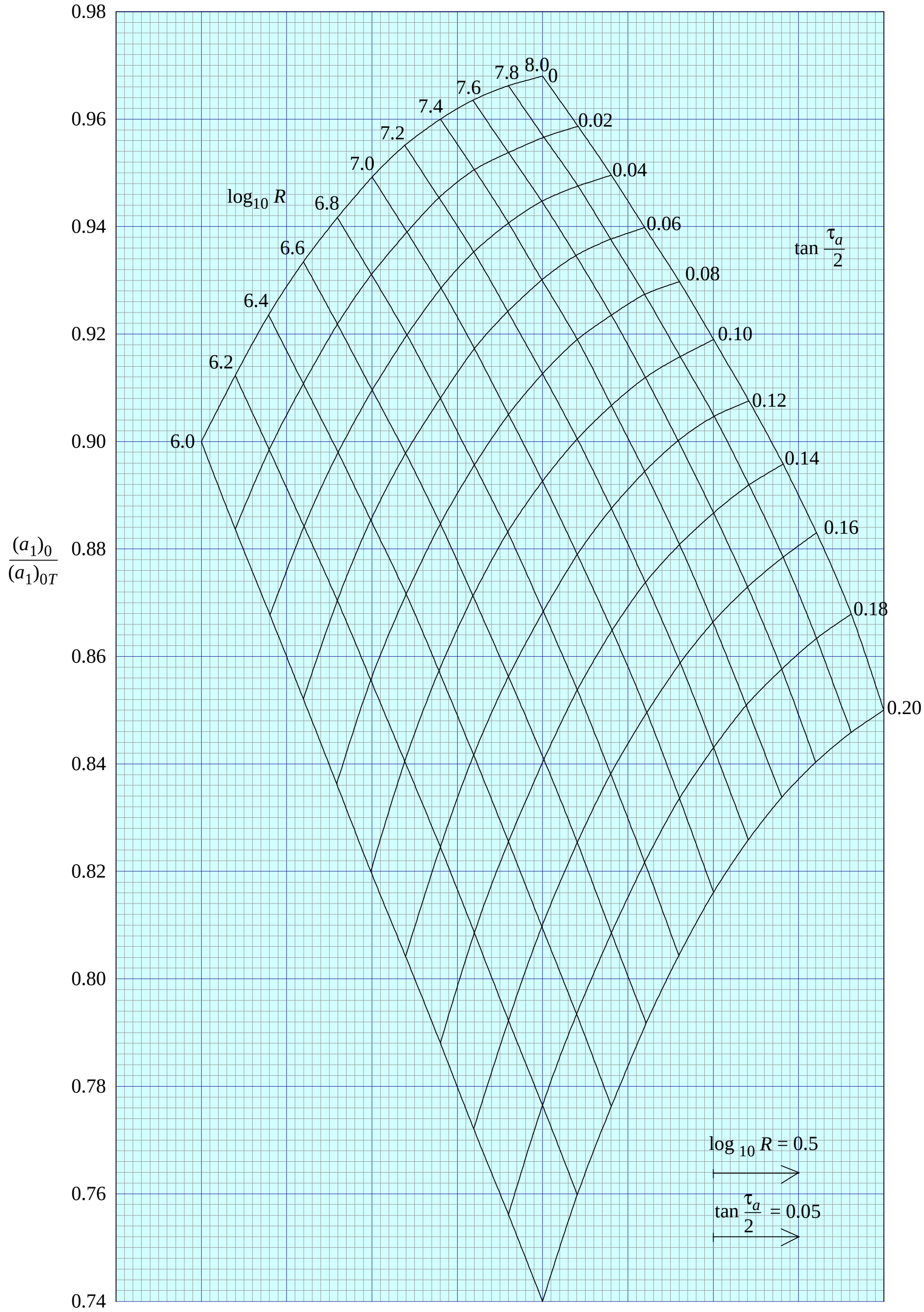


FIGURE 2 $\frac{(a_1)_0}{(a_1)_{0T}}$ FOR MID-CHORD TRANSITION

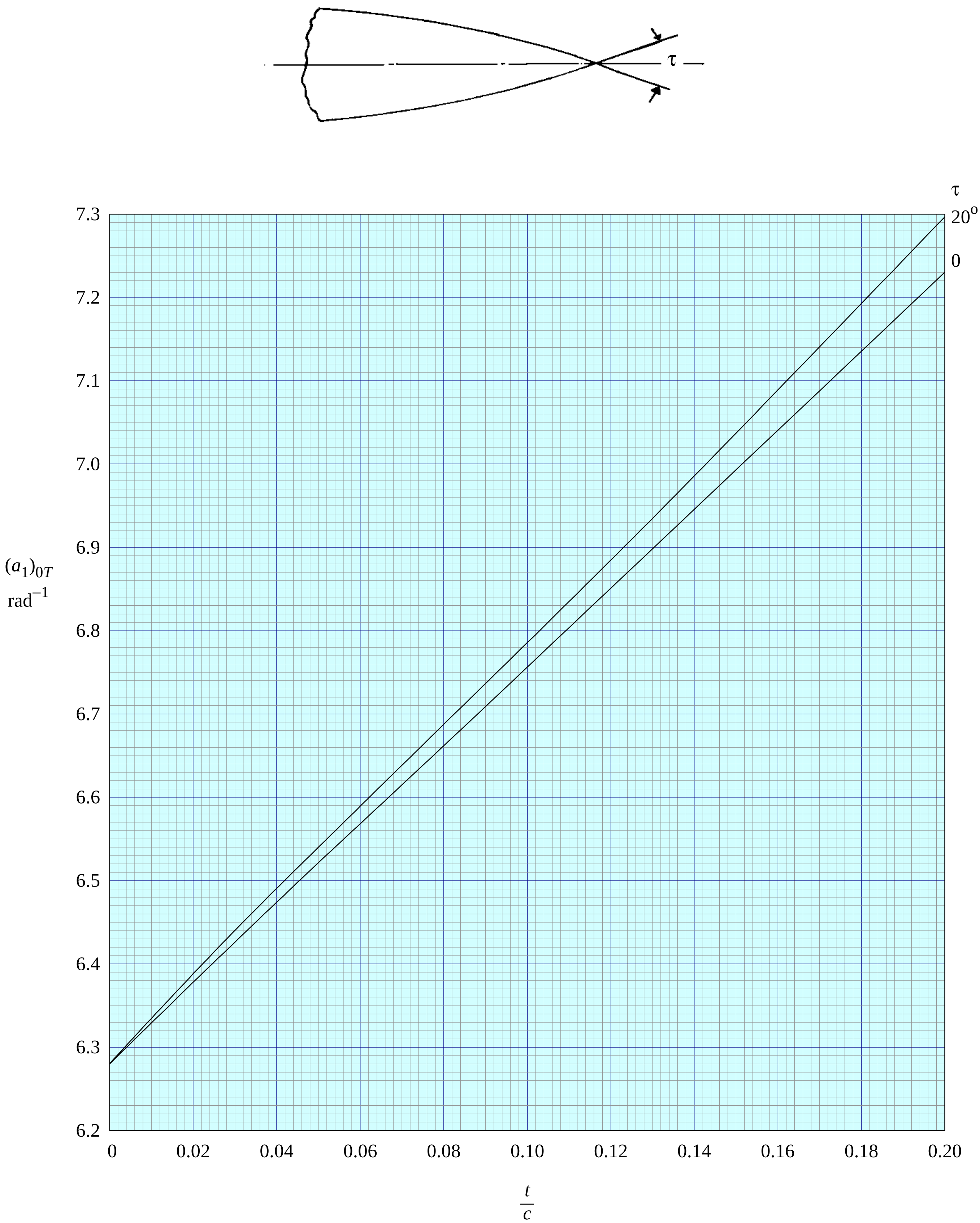


FIGURE 3 THEORETICAL LIFT-CURVE SLOPE