# STABILITY DERIVATIVE $L_p$, ROLLING MOMENT DUE TO ROLLING FOR SWEPT AND TAPERED WINGS

## 1. NOTATION AND UNITS

The derivative notation used is that proposed in ARC R&M 3562 (Hopkin, 1970) and described in Item No. 86021. Coefficients and aeronormalised derivatives are evaluated in aerodynamic body axes with origin at the aircraft centre of gravity and with the wing span as the characteristic length. The derivative $L_p$ is often written as $C_{lp}$ in other systems of notation, but attention must be paid to the reference dimensions used. In particular, in forming $C_{lp}$ differentiation of $C_l$ may be carried out with respect to $pb/2V$ not $pb/V$ as implied in the Hopkin sytem. It is also to be noted that a constant datum value of $V$ is employed by Hopkin.

| | | *SI* | *British* |
|---|---|---|---|
| $A$ | aspect ratio, $b^2/S$ | | |
| $(a_{10})_M$ | two-dimensional lift-curve slope of wing section at Mach number $M$ | radian$^{-1}$ | radian$^{-1}$ |
| $b$ | wing span | m | ft |
| $C_l$ | rolling moment coefficient, $\mathscr{L}/½\rho V^2 Sb$ | | |
| $c_r$ | wing root chord | m | ft |
| $c_t$ | wing tip chord | m | ft |
| $\mathscr{L}$ | rolling moment | N m | lbf ft |
| $L_p$ | rolling moment derivative due to rolling, $(\partial\mathscr{L}/\partial p)/½\rho VSb^2$ | | |
| $M$ | Mach number | | |
| $p$ | angular velocity in roll | radian/s | radian/s |
| $S$ | wing area | m$^2$ | ft$^2$ |
| $V$ | forward velocity | m/s | ft/s |
| $\beta$ | Prandtl-Glauert factor, $(1 - M^2)^{½}$ | | |
| $\kappa$ | parameter $\beta(a_{10})_M/2\pi$ | | |
| $\Lambda_{¼}$ | sweepback of quarter-chord line | degree | degree |
| $\Lambda_E$ | equivalent sweepback at Mach number $M$, $\tan^{-1}\left(\frac{\tan\Lambda_{¼}}{\beta}\right)$ | degree | degree |
| $\lambda$ | taper ratio, $c_t/c_r$ | | |
| $\rho$ | density of air | kg/m$^3$ | slug/ft$^3$ |

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## 2. NOTES

Figure 1 gives the variation of $-\beta L_p/\kappa$ with $\Lambda_E$ for a range of values of $\beta A/\kappa$ and for four values of $\lambda$.

The curves in Figure 1 have been obtained by the use of Weissinger's simplified lifting-surface theory as described in Derivation 1 and have been found to give values of $L_p$ which generally agree with experimental results to within $\pm 10$ per cent for a wide range of wing planforms. Weissinger's theory normally applies only to thin wings in inviscid incompressible flow, but by the use of the subsonic linearised-theory similarity law and the parameter $\kappa$ it has been extended to cover the case of wings of finite thickness in viscous compressible flow, provided that there is no boundary-layer separation from the wing.

Limited experimental data from tests with wings at incidence indicate that the curves in Figure 1 may be used for lift coefficients up to 0.5. At high values of the wing lift coefficient the value of $-\beta L_p/\kappa$ generally becomes smaller and may change sign. Experimental data also indicate that the curves may be used at Mach numbers up to the critical.

## 3. DERIVATION


1. De YOUNG, J. — Theoretical anti-symmetric span loading for wings of arbitrary plan form at subsonic speeds. NACA Rep. 1056, 1951.


## 4. EXAMPLE

Find the rolling moment derivative due to rolling at a Mach number of 0.7 for a wing for which $A = 3.5$, $\Lambda_{1/4} = 30°$ and $\lambda = 0.5$. At $M = 0.7$ the value of $(a_{10})_M$ is 7.80 radian$^{-1}$.

From the given information

$$\kappa = \frac{\beta(a_{10})_M}{2\pi} = \frac{0.714 \times 7.80}{2\pi} = 0.886,$$

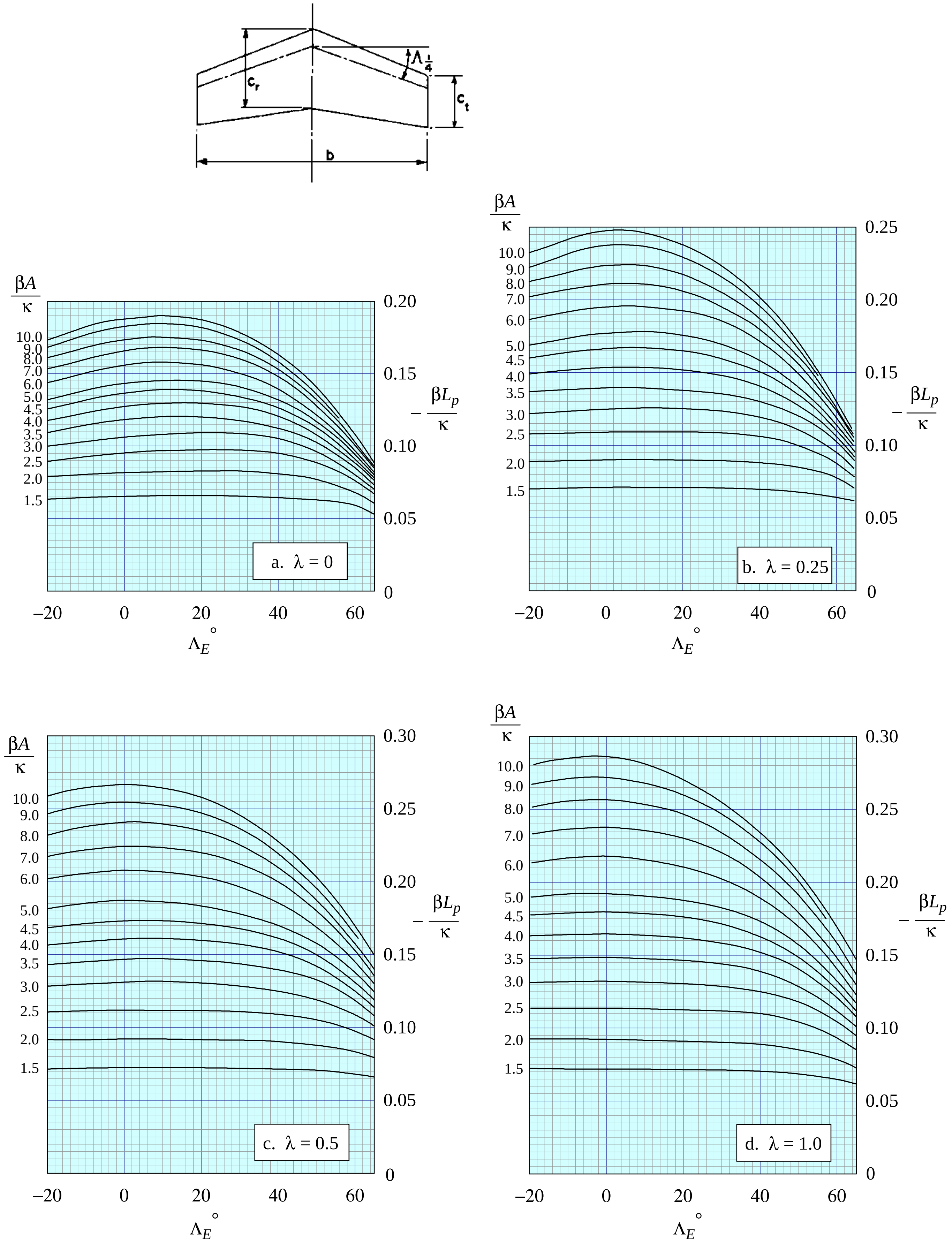
and therefore $$\frac{\beta A}{\kappa} = \frac{0.714 \times 3.5}{0.886} = 2.82.$$

The equivalent sweepback is given by

$$\Lambda_E = \tan^{-1}\left(\frac{\tan 30}{0.714}\right) = 39.0°,$$

and from Figure 1c with $\lambda = 0.5$, $-\beta L_p/\kappa = 0.120$.

Hence $$L_p = \frac{-0.120 \times 0.886}{0.714} = -0.149.$$



$c_r$
$\Lambda_{\frac{1}{4}}$
$c_t$
$b$
$\frac{\beta A}{\kappa}$
10.0
9.0
8.0
7.0
6.0
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
$-\frac{\beta L_p}{\kappa}$
a. $\lambda = 0$
b. $\lambda = 0.25$
c. $\lambda = 0.5$
d. $\lambda = 1.0$
0
0.05
0.10
0.15
0.20
0.25
0.30
–20
0
20
40
60
$\Lambda_E$ °


**FIGURE 1**