

CONTRIBUTION OF WING PLANFORM TO ROLLING MOMENT DERIVATIVE DUE TO SIDESLIP, $(L_v)_w$, AT SUBSONIC SPEEDS

1. NOTATION AND UNITS

The derivative notation used is that proposed in ARC R&M 3562 (Hopkin, 1970) and described in Item No. 86021. Coefficients and aerodynamically normalised derivatives are evaluated in aerodynamic body axes with origin at the aircraft centre of gravity and with the wing span as the characteristic length. The derivative L_v is often written as $\partial C_l / \partial \beta$ or $C_{l\beta}$ in other systems of notation, but attention must be paid to the reference dimensions used and it is to be noted that a constant datum value of V is employed in the Hopkin system.

| | | <i>SI</i> | <i>British</i> |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------|-----------|----------------|
| A | aspect ratio, b^2/S | | |
| A_* | modified aspect ratio, $A \sec \Lambda_{1/2}$ | | |
| b | wing span | m | ft |
| C_L | wing lift coefficient | | |
| C_l | rolling moment coefficient, $\mathcal{L} / \frac{1}{2} \rho V^2 S b$ | | |
| $f(A_*)$ | function of A_* , see Equation (3.3) | | |
| $f_1(\lambda), f_2(\lambda)$ | functions of λ , see Equations (3.7) and (3.8) | | |
| K_M | factor for Mach number effect on $(L_v)_w$, see Section 3.2 | | |
| \mathcal{L} | rolling moment | N m | lbf ft |
| L_v | aerodynamically normalised rolling moment derivative due to sideslip, $L_v = (\partial \mathcal{L} / \partial v) / \frac{1}{2} \rho V S b$ | | |
| $(L_v)_w$ | wing planform contribution to L_v | | |
| $[(L_v)_w]_0$ | zero sweep contribution to $(L_v)_w$ | | |
| $[(L_v)_w]_{\Lambda_{1/2}}$ | sweep contribution to $(L_v)_w$ | | |
| M | free-stream Mach number | | |
| $R_{\bar{c}}$ | Reynolds number based on wing aerodynamic mean chord | | |
| S | wing reference area | m^2 | ft^2 |
| V | velocity of aircraft relative to air | m/s | ft/s |

| | | | |
|-----------------|------------------------------------------------------------------------|-------------------|----------------------|
| v | sideslip velocity | m/s | ft/s |
| β | sideslip angle, $\sin^{-1}(v/V)$ | radian | radian |
| $\bar{\eta}$ | spanwise location of centre of pressure, as fraction of wing semi-span | | |
| $\Lambda_{1/2}$ | sweepback of wing half-chord line | degree | degree |
| λ | ratio of wing tip chord to centre line chord | | |
| ρ | density of air | kg/m ³ | slug/ft ³ |

2. INTRODUCTION

This Item provides a semi-empirical method for predicting $(L_v)_w$, the contribution of the wing planform to the rolling moment derivative due to sideslip. The method applies for subsonic speeds and angles of attack and sideslip for which the variation of lift coefficient with angle of attack and rolling moment coefficient with sideslip angle are linear, *i.e.* for wholly attached flow. The basis of the method is outlined in Section 3 while Section 4 discusses the accuracy and applicability of the method. Section 6 gives a worked example.

The various other contributions to L_v for the aircraft, including body effects, are dealt with in other Items, detailed in Item No. Aero A.06.01.00 (Reference 45).

3. THE METHOD

3.1 Incompressible Flow

The wing planform contribution to the rolling moment derivative due to sideslip, $(L_v)_w$, may be considered to consist of two components, one, $[(L_v)_w]_0$, being independent of wing sweepback effects and the other, $[(L_v)_w]_{\Lambda_{1/2}}$, being largely dependent on wing sweepback, so that in incompressible flow

$$(L_v)_w = [(L_v)_w]_0 + [(L_v)_w]_{\Lambda_{1/2}}.$$

Each of the two components may be assumed to be linear with wing lift coefficient provided the flow remains fully attached, so that L_v/C_L is the relevant parameter, *i.e.*

$$(L_v)_w = C_L \left\{ \frac{[(L_v)_w]_0}{C_L} + \frac{[(L_v)_w]_{\Lambda_{1/2}}}{C_L} \right\}. \quad (3.1)$$

Figures 1a to 1d present $-(L_v)_w/C_L$ as a function of aspect ratio, A , and sweepback of the half-chord line, $\Lambda_{1/2}$, for wings with $\lambda = 0, 0.25, 0.5$ and 1.0 respectively. The data were obtained as follows.

The wing sweepback contribution to $(L_v)_w$ was estimated using the equation

$$-[(L_v)_w]_{\Lambda_{1/2}} / C_L = \frac{1}{2} \bar{\eta} \tan \Lambda_{1/2} f(A_*) \quad (3.2)$$

where $\bar{\eta}$ is the spanwise location of the centre of pressure, which was obtained from Item No. T.D. Memor. 6403 (Derivation 43) in the Transonic Aerodynamics Sub-series, and

$$f(A_*) = \frac{2 + (4 + A_*^2)^{1/2}}{2 + (4 + A_*^2/4)^{1/2}} \left[1 - \frac{A_*^2/8}{4 + A_*^2/4 + 2(4 + A_*^2/4)^{1/2}} \right] \quad (3.3)$$

where $A_* = A \sec \Lambda_{1/2}$. (3.4)

Equation (3.2) was obtained from Derivation 42 which uses a lifting-line approach with simple sweep considerations applied to the leading and trailing halves of the sideslipping wing. The method was developed in terms of an untapered wing and in order to take some account of the effects of wing taper it is suggested in Derivation 42 that the sweepback of the half-chord line be used rather than the quarter-chord sweepback customarily associated with lifting-line theory, and this artifice has been adopted here.

The relationship between the half-chord and quarter-chord sweepback angles is given by the equation

$$\Lambda_{1/2} = \tan^{-1} \left[\tan \Lambda_{1/4} - \frac{1}{A} \left(\frac{1-\lambda}{1+\lambda} \right) \right] \quad (3.5)$$

Figure 2 presents $\Lambda_{1/2}$ as a function of $\Lambda_{1/4}$ and $\frac{1}{A} \left(\frac{1-\lambda}{1+\lambda} \right)$.

The zero sweep contribution to $(L_v)_w$ is given by the equation

$$-\frac{[(L_v)_w]_0}{C_L} = \frac{f_1(\lambda)}{A} - f_2(\lambda) \quad (3.6)$$

in which $f_1(\lambda)$ and $f_2(\lambda)$ are functions of λ given by

$$f_1(\lambda) = 0.25 + 0.79\lambda - 0.34\lambda^2 \quad (3.7)$$

and $f_2(\lambda) = 0.05 + 0.08\lambda - 0.04\lambda^2$. (3.8)

Equation (3.6) is empirical, being obtained from an analysis of experimental data for $(L_v)_w$ (Derivations 1 to 13, 15 to 41 and 44) in conjunction with Equations (3.1) and (3.2).

The experimental data included in the Derivation of Equation (3.6) include low aspect ratio ($A < 2$) delta wings. The use of the method of this Item for such wings agrees quite well with experimental data and with the slender body equation from Derivation 14, *i.e.*

$$-\frac{(L_v)_w}{C_L} = \frac{2}{3A} \quad (3.9)$$

3.2 Compressible Flow

Theoretical studies of compressibility using a Prandtl-Glauert transformation applicable to sweptback wings (see Derivation 42 or Reference 46, for example) showed an effect of Mach number only on the sweepback component to $(L_v)_w$, which by reference to experimental data for unswept wings is clearly incorrect. This being so, resort is made here to an empirical method which is based on systematic experimental data given in Derivation 42. The data, for ten wings mid-mounted on axisymmetric bodies, were analysed in terms of a factor $K_M (= [(L_v)_w]_M / [(L_v)_w]_{M=0})$ in order to minimise any effects of the body. The data were found to correlate quite well with the component of free-stream Mach number normal to the half-chord line. The results of the analysis are given in Figure 3 which should be considered somewhat tentative in view of the small number of available data. Extrapolations (linear in aspect ratio) beyond the area covered by the experimental data are shown as broken lines and should be used with caution.

4. ACCURACY AND APPLICABILITY

4.1 Accuracy

For wings with aspect ratios from 1 to 6, comparisons of the values of L_v , measured in low-speed wind-tunnel tests on many isolated wings indicate that for 90 per cent of the experimental data Figures 1a to 1d predict $(L_v)_w / C_L$ to within ± 20 percent when $(L_v)_w / C_L \geq 0.25$ and to within ± 0.05 when $(L_v)_w / C_L \leq 0.25$. There are very few suitable experimental data available from tests on isolated wings with aspect ratios between 6 and 12, and assessment of the accuracy of Figures 1a to 1d for this range is limited to comparisons with a small number of wind-tunnel data extracted from tests on wing-body combinations (no fin or tailplane) typical of civil transport aircraft. These data are not ideal for comparison purposes, not only because of the body but also because of the presence of wing twist, wing dihedral and, sometimes, cranks in the wing planform, all of which may affect the planform component slightly. Nevertheless, these data suggest that for wing aspect ratios between 6 and 12 Figures 1a to 1d can be expected to predict $(L_v)_w / C_L$ to within ± 0.03 . Only a limited number of data are available for assessing the accuracy of Figure 3 but these suggest that it predicts the effects of compressibility to within about ± 5 per cent for Mach numbers up to that at which the aerodynamic characteristics start to change rapidly.

4.2 Applicability

The method is applicable to angles of attack and sideslip for which the variation of lift coefficient with angle of attack and rolling moment coefficient with sideslip angle are linear, *i.e.* for fully attached flow. The method also applies for Mach numbers up to that at which the aerodynamic characteristics start to change rapidly.

The method has been developed from data for straight tapered wings. For other wings, with a cranked trailing-edge for example, an equivalent straight tapered wing with the same wing area and tip chord should be constructed as described in Item No. 76003 (Reference 47).

The data presented in Figures 1a to 1d relate only to sweptback wings ($\Lambda_{1/2} > 0$). For aircraft with wings where the leading-edge is unswept or slightly sweptback but, due to the wing taper, the wing half-chord line is swept slightly forward ($-10^\circ < \Lambda_{1/2} < 0$) comparisons with low-speed wind-tunnel data on L_v indicate that a better prediction of the overall value of L_v is obtained for wing-body and wing-body-tail combinations if $(L_v)_w$ is estimated from Figures 1a to 1d at a value of $\Lambda_{1/2} = 0$, rather than by extrapolating the curves to negative values of wing sweep. It is recommended that this procedure is adopted in such cases.

In principle, the method forming the basis of this Item for low speeds is applicable to wings with large amounts of forward sweep in that the wing contribution due to forward sweep may be determined by using

Equation (3.2), provided that appropriate data are used for $\bar{\eta}$ (see Reference 46, for example). However, until the values predicted for swept forward wings can be verified against a substantial number of experimental data they can only be regarded as tentative. There are no compressible flow data for wings with significant forward sweep with which to establish the applicability of Figure 3 to such wings. In lieu of better information it is suggested that Figure 3 be used with caution for swept forward wings.

Table 4.1 shows the ranges of geometric and flow parameters considered in the development of this Item. The experimental data indicate no significant effects of either section shape or Reynolds number over the range considered.

TABLE 4.1 Range of Experimental Data

| <i>Parameter</i> | <i>Range</i> | <i>Parameter</i> | <i>Range</i> |
|------------------|--------------------------|------------------|---------------------------------------|
| A | 1 to 7 | λ | 0 to 1 |
| $\Lambda_{1/2}$ | -7° to 70° | $R_{\bar{c}}$ | 0.5×10^6 to 13×10^6 |

5. DERIVATION AND REFERENCES

5.1 Derivation

The Derivation lists selected sources that have assisted in the preparation of this Item.

1. MÖLLER, V.E. Sechskomponenten- Messungen an Rechteckflügeln mit V-Form und Pfeilform in einem grossen Schiebewinkelbereich. Luftfahrtforschung 18, I 243, 1941.
2. MÖLLER, V.E. Systematische Sechskomponenten- Messungen an Flügel Rumpf – Anordnungen. Jahrbuch 1942 der Deutschen Luftfahrtforschung, I 336.
3. JACOBS, W. Systematische Sechskomponenten Messungen an Pfeilflügeln. Ber.44/21, Aerodynamisches Institut der T.H.Braunschweig, 1944.
4. BREWER, J.D. Tests of the Northrop MX-334 glider airplane in the NACA full scale tunnel. NACA MR L4A13 (TIB1336), 1944.
5. TROUNCER, J.
KETTLE, D. Low speed model tests on two V wings. ARC R&M 2364, 1946.
6. GDALIAHU, M. A summary of the results of some German model tests on wings of small aspect ratio. RAE tech. Note Aero. 1767, 1946.
7. LETKO, W.
GOODMAN, A. Preliminary wind tunnel investigation at low speed of stability and control characteristics of swept back wings. NACA tech. Note 1046, 1946.
8. NEUMARK, S. Lateral characteristics from some German model tests on wings of small aspect ratio. RAE tech. Note Aero. 1917, 1947.
9. TOSTI, L.P. Low speed static stability and damping in roll characteristics of some swept and unswept low aspect ratio wings. NACA tech. Note 1468, 1947.

10. BIRD, J.D.
LICHTENSTEIN, J.H.
JAQUET, B.M. Investigation of the influence of fuselage and tail surfaces on low-speed static stability and rolling characteristics of a swept-wing model. NACA tech. Note 2741, 1947.
11. SALMI, R.J.
CONNER, D.W.
GRAHAM, R.R. Effects of a fuselage on the aerodynamic characteristics of a 42° sweptback wing at Reynolds numbers to 8,000,000. NACA res. Memor. L7E13 (TIB 1185), 1947.
12. McCORMACK, G.M.
STEVENS, V.I. An investigation of the low speed stability and control characteristics of swept-forward and swept-back wings in the Ames 40- by 80-ft wind tunnel. NACA res. Memor. A6K15 (TIB 1362), 1947.
13. TROUNCER, J.
MOSS, G.F. Low-speed wind-tunnel tests on two 45 deg sweptback wings of aspect ratios 4.5 and 3.0 (Models A and B). ARC R&M 2710, 1947.
14. RIBNER, H. The stability derivatives of low aspect ratio triangular wings at subsonic and supersonic speeds. NACA tech. Note. 1423, 1947.
15. PURSER, P.E.
SPEARMAN, M.L. Wind-tunnel tests at low speed of swept and yawed wings having various plan forms. NACA tech. Note 2445, 1947.
16. GOODMAN, A.
BREWER, J.D. Investigation at low speeds of the effect of aspect ratio and sweep on static and yawing stability derivatives of untapered wings. NACA tech. Note 1669, 1948.
17. LETKO, W.
COWAN, J.W. Effect of taper ratio on low-speed static and yawing stability derivatives of 45° sweptback wings with aspect ratio 2.61. NACA tech Note 1671, 1948.
18. McKINNEY, M.O.
DRAKE, H.M. Flight characteristics at low speeds of delta wing models. NACA res. Memor. L7K07 (TIB 1537), 1948.
19. SALMI, R.J. Yaw characteristics of a 52° sweptback wing of NACA 64-112 section with a fuselage and with leading-edge and split flaps at Reynolds numbers from 1.93×10^6 to 6.00×10^6 . NACA res. Memor. L8H12 (TIB 1984), 1948.
20. LETKO, W.
JAQUET, B.M. Effect of airfoil profile of symmetrical sections on the low-speed static stability and yawing derivatives of 45° sweptback wing models of aspect ratio 2.61. NACA res. Memor. LSH10 (TIB 2004), 1948.
21. LANGE/WACKE Test report on three- and six-component measurements on a series of tapered wings of small aspect ratio (partial report: triangular wings). NACA tech. Memor. 1176, 1948.
22. LOCK, R.C.
ROSS, J.G.
MEIKLEM, P. Wind tunnel tests on a 90° apex delta wing of variable aspect ratio (sweptback 36.8°). Part II – Measurement of downwash and effect of high lift devices. ARC CP83, 1948.
23. HOLME, O. Wind tunnel tests of tapered wings with various amounts of dihedral and sweepback. FFA Rep. 17, 1948.
24. CAMPBELL, J.P.
GOODMAN, A. A semi-empirical method for estimating the rolling moment due to yawing of airplanes. NACA tech. Note 1984, 1949.

25. JAQUET, B.M.
BREWER, J.D. Low speed static stability and rolling characteristics of low aspect ratio wings of triangular and modified triangular planforms. NACA res. Memor. L8L29 (TIB 2062), 1949.
26. McCORMACK, G.M.
WALLING, W. Aerodynamic study of a wing fuselage combination employing a wing swept back 63° . Investigation of a large scale model at low speed. NACA res. Memor. A8D02 (TIB 2064), 1949.
27. JAQUET, B.M.
BREWER, J.D. Effects of various outboard and central fins on low speed static stability and rolling characteristics of a triangular wing model. NACA res. Memor. L9E18 (TIB 2137), 1949.
28. RIEBE, J.M.
FIKES, J.E. Preliminary aerodynamic investigation of the effect of camber on a 60° delta wing with round and bevelled leading edges. NACA res. Memor. L9F10 (TIB 2178), 1949.
29. ANDERSON, A.E. An investigation at low speed of a large scale triangular wing of aspect ratio 2. III. Characteristics of wing with body and vertical tail. NACA res. Memor. A9H04 (TIB 2246), 1949.
30. BREWER, J.D.
LICHTENSTEIN, J.H. Effect of horizontal tail on low speed static stability characteristics of a model having 45° swept-back wing and tail surfaces. NACA tech. Note 2010, 1950.
31. NAESETH, R.L.
O'HARE, W.M. The effect of aileron span and spanwise location on the low speed lateral control characteristics of an untapered wing of aspect ratio 2.09 and 45° sweepback. NACA res. Memor. L9L09a (TIB 2355), 1950.
32. RIEBE, J.M.
WATSON, J.H. The effect of end plates on swept-wings at low speed. NACA tech. Note 2229, 1951.
33. GOODMAN, A. Effects of wing position and horizontal-tail position on the static stability characteristics of models with unswept and 45° sweptback surfaces with some reference to mutual interference. NACA tech. Note 2504, 1951.
34. QUEIJO, M.J.
WOLHART, W.D. Experimental investigation of the effect of vertical-tail size and length and of fuselage shape and length on the static lateral stability characteristics of a model with 45° sweptback wing and tail surfaces. NACA Rep. 1049, 1951.
35. NAESETH, R.L.
O'HARE, W.M. Effect of aspect ratio on the low speed lateral control characteristics of unswept untapered low aspect ratio wings. NACA tech. Note 2348, 1951.
36. FISHER, L.R.
MICHAEL, W.H. An investigation of the effect of vertical fin location and area on low speed lateral stability derivatives of a semi-tailless airplane model. NACA res. Memor. L51A10 (TIB 2655), 1951.
37. GOODMAN, A.
THOMAS, D.F. Effects of wing position and fuselage size on the low-speed static and rolling stability characteristics of a delta-wing model. NACA Rep. 1224, 1953.
38. KUHN, R.E.
DRAPER, J.W. Wind tunnel investigation of the effects of geometric dihedral on the aerodynamic characteristics in pitch and sideslip of an unswept- and a 45° sweptback-wing-fuselage combination at high subsonic speeds. NACA res. Memor. L53F09 (TIL 3829), 1953.

39. FOURNIER, P.G. Wind-tunnel investigation of the aerodynamic characteristics in pitch and sideslip at high subsonic speeds of a wing-fuselage combination having a triangular wing of aspect ratio 4. NACA res. Memor. L53G14a (TIL 3887), 1953.
40. CHRISTENSEN, F.B. An experimental investigation of four triangular-wing-body combinations in sideslip at Mach numbers 0.6, 0.9, 1.4 and 1.7. NACA res. Memor. A53L22 (TIL 4119), 1953.
41. LETKO, W. Experimental determination at subsonic speeds of the oscillatory and static lateral stability derivatives of a series of delta wings with leading-edge sweep from 30° to 86.5°. NACA res. Memor. L57A30 (TIL 5487), 1957.
42. POLHAMUS, E.C.
SLEEMAN, W.C. The rolling moment due to sideslip of swept wings at subsonic and transonic speeds. NASA tech. Note D-209, 1960.
43. ESDU Method for the rapid estimation of theoretical spanwise loading due to a change of incidence. Item No. T.D. Memor. 6403. Engineering Sciences Data Unit, London, March 1964. (See also Item No. 83040.)
44. ISOGAI, K.
ICHIKAWA, T. On a lifting surface theory for a wing oscillating in yaw and sideslip with an angle of attack. *AIAA J.* Vol. 11, No.5, pp.599 to 606, May 1973.

5.2 References

The References list selected sources of information supplementary to that given in this Item.

45. ESDU Information on the use of Data Items on the rolling moment derivatives of an aeroplane. Item No. Aero A.06.01.00. Engineering Sciences Data Unit, London, March 1958.
46. QUEIJO, M.J. Theory for computing span loads and stability derivatives due to sideslip, yawing and rolling for wings in subsonic compressible flow. NASA tech. Note D-4929, 1968.
47. ESDU Geometrical properties of cranked and straight tapered wing planforms. Item No. 76003. Engineering Sciences Data Unit, London, January 1976.

6. EXAMPLE

It is required to estimate the planform contribution to L_v for a wing at a lift coefficient of 0.5 and a Mach number of 0.7 with geometrical parameters $A = 6$, $\Lambda_{1/4} = 30$ degrees and $\lambda = 0.25$.

From Figure 2 with $\Lambda_{1/4} = 30$ degrees and $\frac{1}{A} \left(\frac{1-\lambda}{1+\lambda} \right) = \frac{1}{6} \times \left(\frac{1-0.25}{1+0.25} \right) = 0.1$, $\Lambda_{1/2} = 25.5$ degrees.

From Figure 1b with $A = 6$ and $\Lambda_{1/2} = 25.5$ degrees,

$$\frac{(L_v)_w}{C_L} = -0.119.$$

Therefore, for incompressible flow, $(L_v)_w = [(L_v)_w]_{M=0} = -0.119 \times 0.5 = -0.0595$.

From Figure 3 with $M \cos \Lambda_{1/2} = 0.7 \times \cos 25.5^\circ = 0.63$ and $A = 6$, $K_M = 1.19$.

$$\begin{aligned} \text{Therefore } [(L_v)_w]_{M=0.7} &= K_M \times [(L_v)_w]_{M=0} \\ &= -1.19 \times 0.0595 \\ &= -0.071. \end{aligned}$$

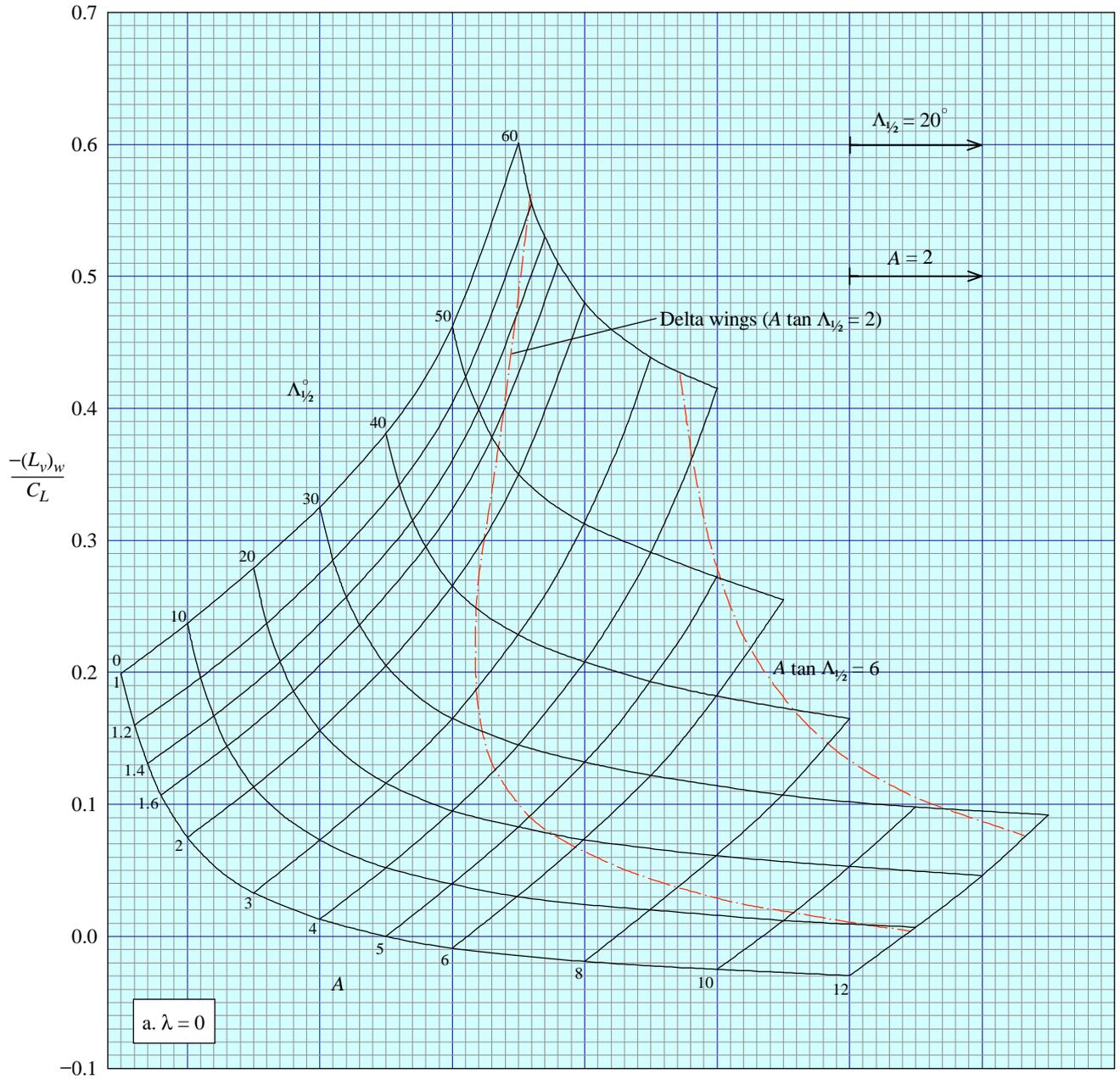


FIGURE 1a WING PLANFORM CONTRIBUTION TO L_p AT LOW SPEEDS

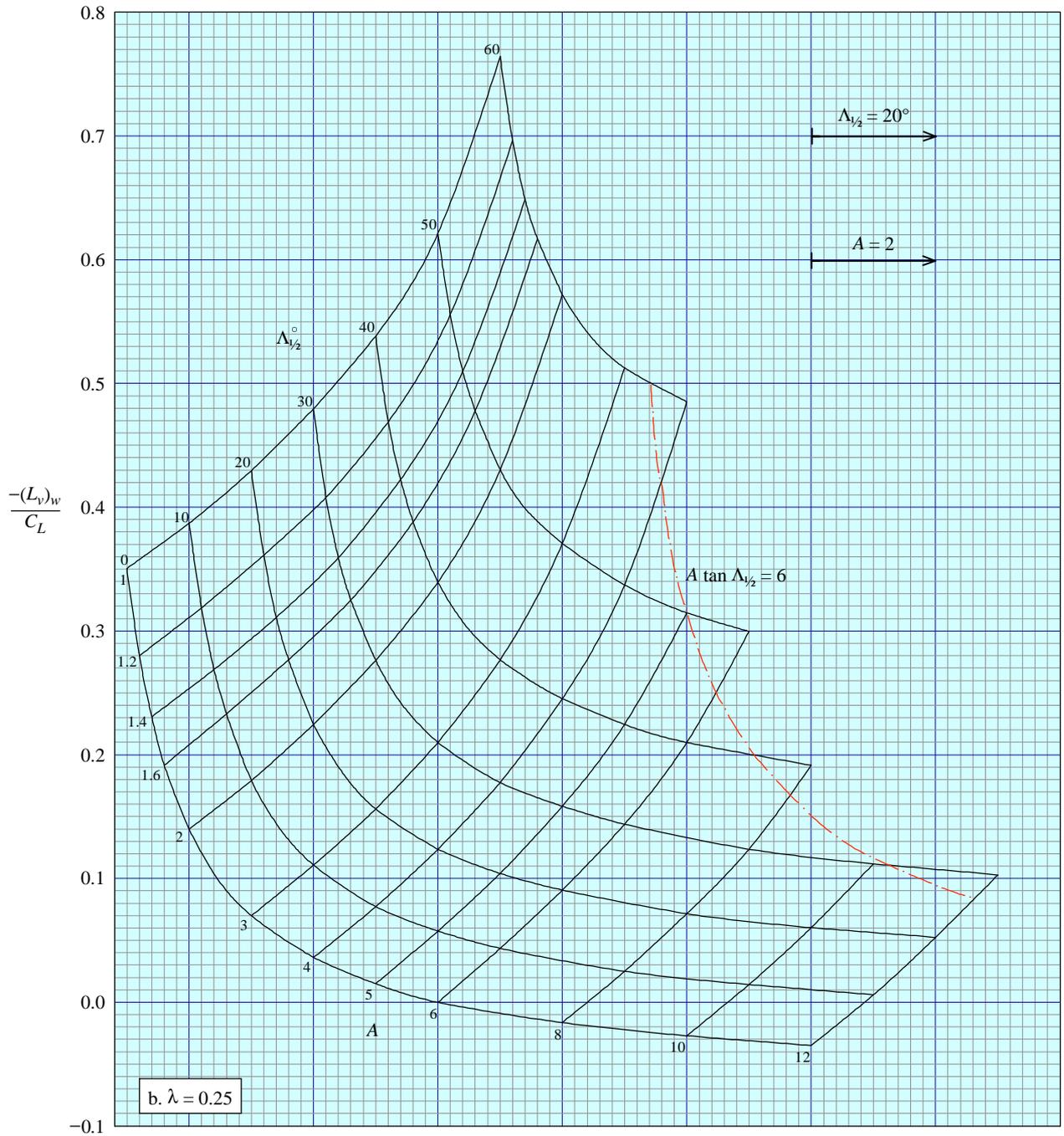


FIGURE 1b

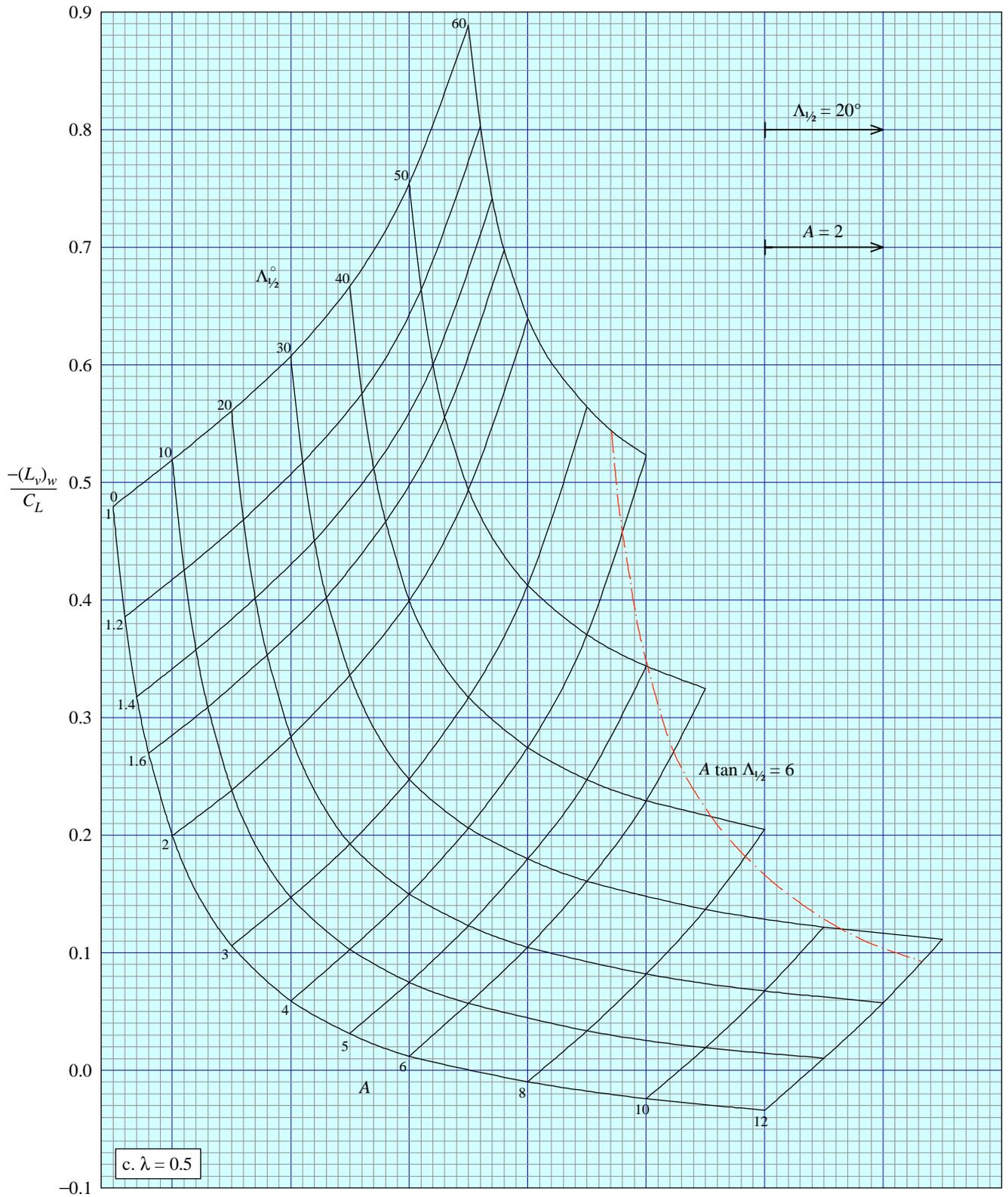


FIGURE 1c

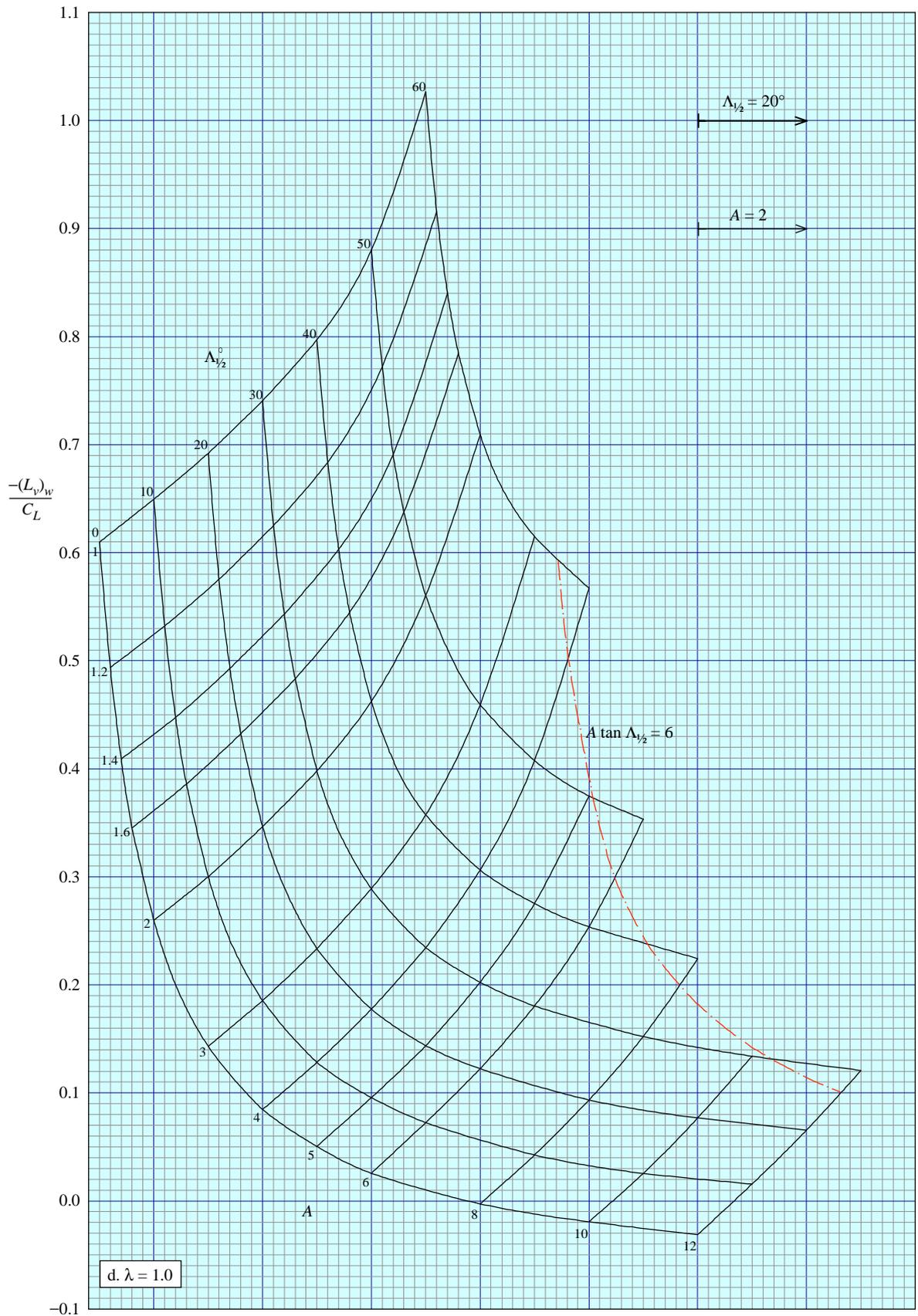


FIGURE 1d

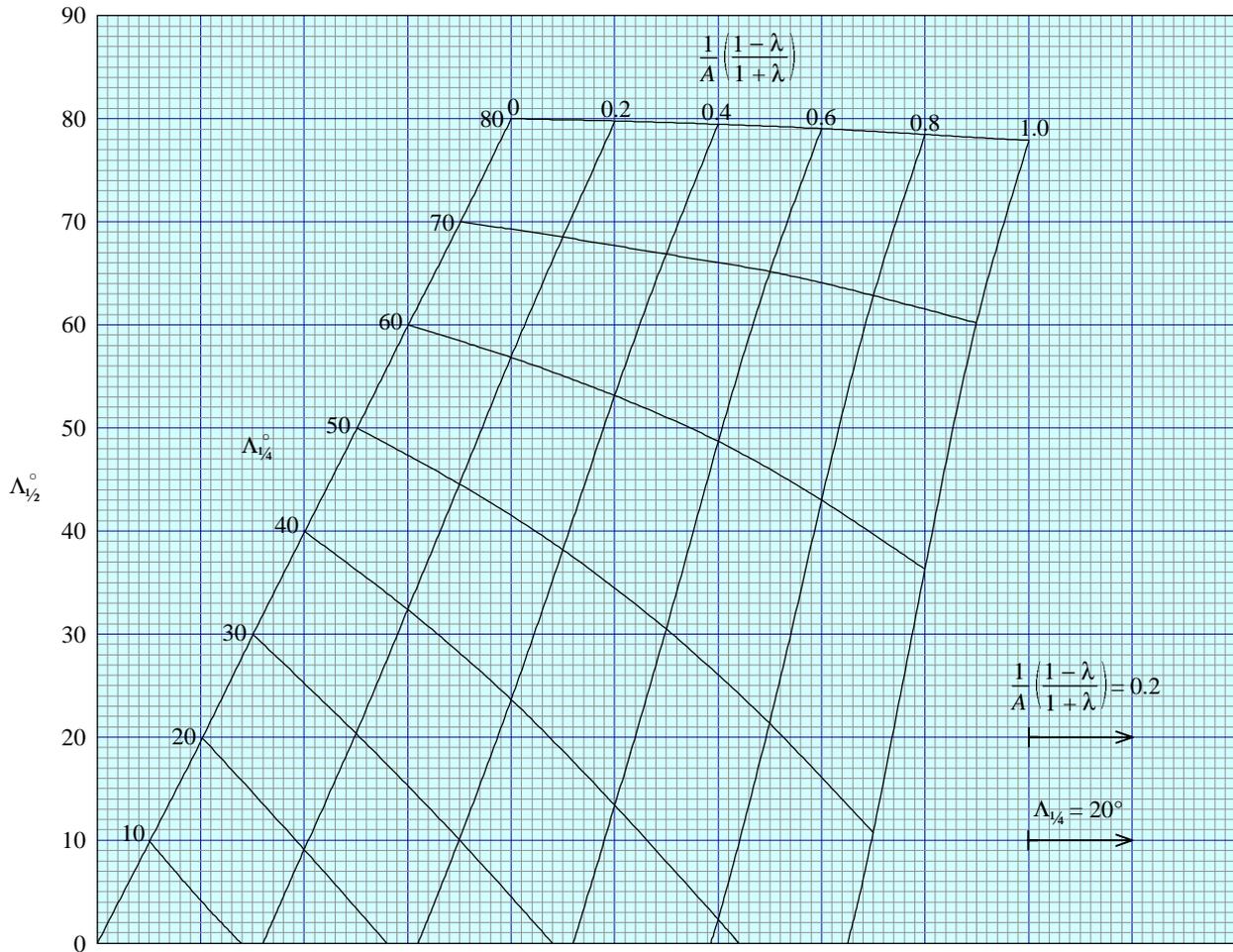


FIGURE 2 SWEEPBACK OF WING HALF-CHORD LINE

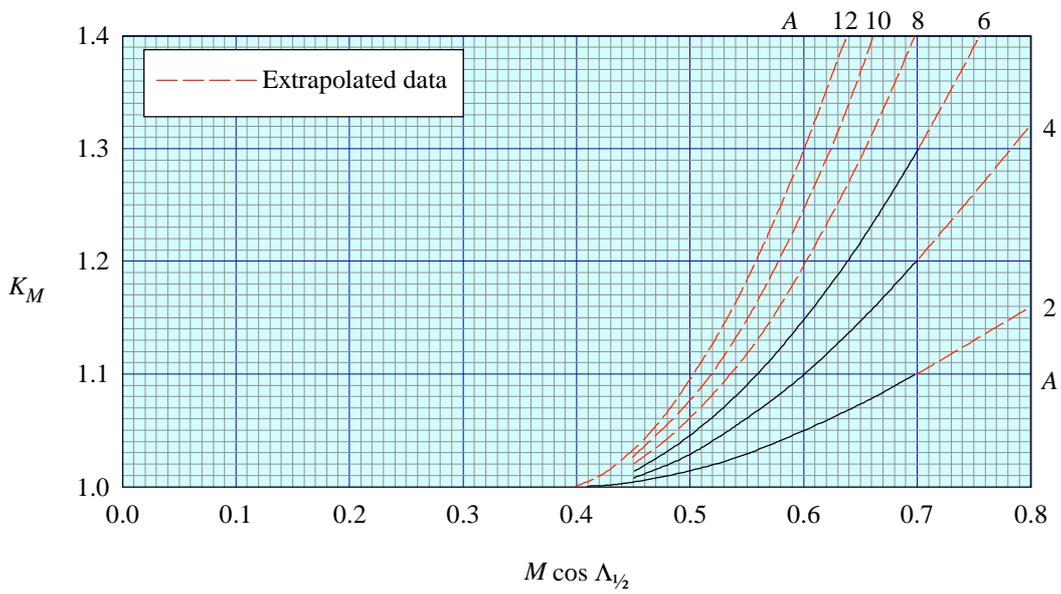


FIGURE 3 MACH NUMBER FACTOR ON $(L_v)_w$

THE PREPARATION OF THIS DATA ITEM

The work on this particular Item, which supersedes Item No Aero A.06.01.04, was monitored and guided by the Aerodynamics Committee which first met in 1942 and now has the following membership:

Chairman

Mr P.K. Jones – British Aerospace, Manchester Division

Vice-Chairman

Mr J. Weir – Salford University

Members

Mr D. Bonenfant – Aérospatiale, Toulouse, France

Mr E.A. Boyd – Cranfield Institute of Technology

Mr K. Burgin – Southampton University

Mr E.C. Carter – Aircraft Research Association

Mr J.R.J. Dovey – British Aerospace, Warton Division

Dr J.W. Flower – Bristol University

Mr H.C. Garner – Royal Aircraft Establishment

Mr A. Hipp – British Aerospace, Stevenage-Bristol Division

Dr B.L. Hunt* – Northrop Corporation, Hawthorne, Calif., USA

Mr J. Kloos* – Saab-Scania, Linköping, Sweden

Mr J.R.C. Pedersen – Independent

Mr I.H. Rettie* – Boeing Aerospace Company, Seattle, Wash., USA

Mr F.W. Stanhope – Rolls-Royce Ltd, Derby

Mr H. Vogel – British Aerospace, Weybridge-Bristol Division.

* Corresponding Member.

The member of staff who undertook the technical work involved in the initial assessment of the available information and the construction and subsequent development of the Item was

Mr P.D. Chappell – Group Head.