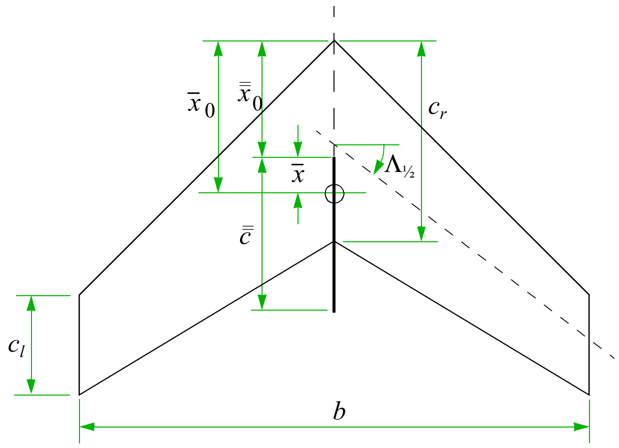
LIFT-CURVE SLOPE AND AERODYNAMIC CENTRE POSITION OF WINGS IN INVISCID SUBSONIC FLOW

1. NOTATIONS AND UNITS

		SI	British
A	aspect ratio, b^2/S		
b	wing span	m	ft
C_L	lift coefficient		
c_r	root chord	m	ft
c_t	tip chord	m	ft
$\bar{\bar{c}}$	aerodynamic mean chord	m	ft
M	free-stream Mach number		
n	fraction of chord		
S	wing area	m^2	ft^2
$\bar{x}$	distance of aerodynamic centre aft of leading edge of aerodynamic mean chord	m	ft
$\bar{x}_0$	distance of aerodynamic centre aft of leading edge of root chord	m	ft
$\bar{\bar{x}}_0$	distance of leading edge of aerodynamic mean chord aft of wing apex	m	ft
α	wing incidence	rad	rad
β	compressibility parameter, $(1 - M^2)^{1/2}$		
Λ_n	sweepback of n-chord line	deg	deg
$\Lambda_{1/2}$	sweepback of mid-chord line	deg	deg
$\Lambda_{1/2M}$	$\tan^{-1}((\tan\Lambda_{1/2})/\beta)$	deg	deg
λ	taper ratio, c_t/c_r		

Issued July 1970
With Amendments A to I, August 1996



Sketch 1.1 Wing notation

2. GENERAL

Figures 1a to 1e and 2a to 2g present data for the lift-curve slope and position of the aerodynamic centre of wings with straight leading and trailing edges and streamwise tips at low incidence in inviscid subsonic flow. See Section 4 for applicability to tailplanes. The data are presented in carpet form. The lift-curve slope parameter, $(1/A)\,(\mathrm{d}C_L/\mathrm{d}\alpha)$, is plotted against βA and $A\tan\Lambda_{1/2}$ for values of taper ratio $\lambda = 0$, 0.125, 0.25, 0.5 and 1.0, while for the purpose of clarity of presentation the aerodynamic centre parameter, $\bar{x}/\bar{\bar{c}}$, is plotted against βA and λ for values of $A\tan\Lambda_{1/2}$ from 0 to 6 in unit steps.

The Item contains three appendices. Appendix A assesses accuracy through comparison of predicted values with experimental data. For information, Appendix B presents the simple Helmbold-Diederich equation for lift-curve slope as this was often used before there was general access to more-soundly based methods such as the lifting-surface theory that has been used as the basis of this Item (see Section 3). Appendix C presents a computer program that reproduces the data of this Item, with a small extension to cover limited forward sweep.

The position of the aerodynamic centre, which is that point on the axis of symmetry of the wing about which the rate of change of pitching moment with incidence is zero, is given in this Item as a fraction of the aerodynamic mean chord aft of the leading edge of the aerodynamic mean chord. The distance of the leading edge of the aerodynamic mean chord aft of the wing apex may, if required, be obtained either from Item No. 76003 (Reference 6) or from the expression

$$\frac{\bar{\bar{x}}_0}{\bar{\bar{c}}} = \frac{(1+2\lambda)(1+\lambda)}{8(1+\lambda+\lambda^2)}\left[A\tan\Lambda_{1/2} + 2\left(\frac{1-\lambda}{1+\lambda}\right)\right]. \tag{2.1}$$

Similarly, the value of the aerodynamic mean chord in terms of the root chord may be found either from Item No. 76003 or from the expression

$$\frac{\bar{\bar{c}}}{c_r} = \frac{2}{3}\left(\frac{1+\lambda+\lambda^2}{1+\lambda}\right). \tag{2.2}$$

The data in this Item are given in terms of the sweepback of the mid-chord line, $\Lambda_{1/2}$, which is related to the sweepback of the n-chord line, Λ_n, via the expression

$$A \tan \Lambda_{1/2} = A \tan \Lambda_n + 2(2n-1)\left(\frac{1-\lambda}{1+\lambda}\right). \tag{2.3}$$

3. BASIS OF DATA

The calculations and data on which the curves of Figures 1a to 1e and 2a to 2g are based over the range $1.5 \le \beta A \le 8$ are detailed in Derivation 1 which also contains full tabulations of the computer solutions together with the spanwise centre of pressure position. The method used was basically the subsonic lifting-surface theory of Reference 4 with the increased rounding of the central kink recommended in Reference 5. For lift-curve slope some additional cases were calculated, using the method of Derivation 2, to extend the applicability up to $\beta A = 12$. Corresponding calculations for aerodynamic centre position were not available and the extensions shown in Figures 2a to 2g were obtained by extrapolation in terms of $1/\beta A$ together with the known asymptote as $\beta A \to \infty$. Slender wing theory (Reference 3) was used to provide solutions for $\beta A = 0$ and the curves in Figures 1a to 1e and 2a to 2g were faired into these data.

4. APPLICABILITY OF DATA

The theory from which the data were derived is linearised with the implication that its application is limited to wings at low incidence and with small thickness, camber and twist. The theory also relates only to inviscid flow. These restrictions are, however, known to be relatively unimportant for most practical purposes in calculating the lift and position of the aerodynamic centre *provided* that the flow remains fully attached over the wing surface, see Appendix A. In particular this condition is not likely to apply, at other than low incidences, to wings with highly swept leading-edges. In addition the theory applies only to those cases for which the flow is subsonic over the whole wing.

Although the Figures were derived from calculations for straight-tapered planforms with streamwise tips, they may be applied to other types of planform using the concept of an "equivalent" wing planform* defined in Addendum A of Item No. 76003. If the ratio of body diameter to wing span is small, the data of Figure 1 may also be used to approximate the lift-curve slope of a wing-body combination (see Appendix A to present Item) but the aerodynamic centre position (Figures 2a to 2g) requires a correction for the effect of a body.

The data for $\beta A < 2$, indicated by broken lines, should be used with caution as there is an element of uncertainty as to how the limiting values for $\beta A = 0$ should be approached. Moreover, on wings with a combination of low aspect ratio and high trailing-edge sweep, a spurious effect of the central rounding may become significant. This is not a serious limitation, however, as most practical planforms lie outside this range. The data for $\beta A > 8$ are shown by the broken lines in Figures 2a to 2g in view of the fact that they were obtained by an extrapolation technique (see Section 3).

The data of Figures 1a to 1e may be applied to tailplanes provided that due allowance is made for airframe interference, see Item No. 89029 (Reference 7). The data of Figures 2a to 2g may be applied directly to tailplanes in the determination of the tail arm for other than very close-coupled configurations (tail arm, $< b/2$, say).

* In essence the equivalent wing planform is straight tapered and has the same overall span and exposed wing planform area (*i.e.* planform area outside the body planform) as the true wing. If the true wing has a streamwise tip chord with no rounded corners then the tip chord of the equivalent wing planform is the same as that of the true wing. If, however, the true wing has a non-streamwise tip chord or a small curved tip then the linear leading and trailing edges just inboard of the tip are extrapolated outboard to the full span to define an equivalent tip chord.

5. DERIVATION AND REFERENCES

The Derivations lists selected sources that have assisted in the preparation of this Item.

Derivation

1.	GARNER, H.C. INCH, S.M.	Subsonic theoretical lift-curve slope, aerodynamic centre and spanwise loading for arbitrary aspect ratio, taper ratio and sweepback. ARC CP 1137, 1970.
2.	LEHRIAN, D.E. GARNER, H.C.	Theoretical calculation of generalised forces and load distribution on wings oscillating at general frequency in a subsonic stream. ARC R & M 3710, 1971.

The References lists sources of information supplementary to that given in this Item.

References

3.	MANGLER, K.W.	Calculation of the pressure distribution over a wing at sonic speeds. ARC R & M 2888, 1951.
4.	GARNER, H.C. FOX, D.A.	Algol 60 programme for Multhopp's low-frequency subsonic lifting-surface theory, ARC R & M 3517, 1966.
5.	ZANDBERGEN, P.J. LABRUJERE, T.E. WOUTERS, J.G.	A new approach to the numerical solution of the equation of subsonic lifting surface theory. NLR Report TR G.49, 1967.
6.	ESDU	Geometrical properties of cranked and straight-tapered wings. ESDU International, Item No. 76003, 1976.
7.	ESDU	Installed tailplane lift-curve slope at subsonic speeds. ESDU International, item No. 89029, 1989.

6. EXAMPLE

Find the lift-curve slope and position of the aerodynamic centre relative to the root chord of a wing at low incidence at a Mach number of 0.6. The wing has the following geometrical properties: $A = 3$, $\Lambda_{1/2} = 60$ degrees, $\lambda = 1.0$.

For this wing, $A \tan\Lambda_{1/2} = 3 \times \tan 60° = 5.2$

$$\text{and } \beta A = (1 - 0.6^2)^{1/2} \times 3 = 0.8 \times 3 = 2.4 .$$

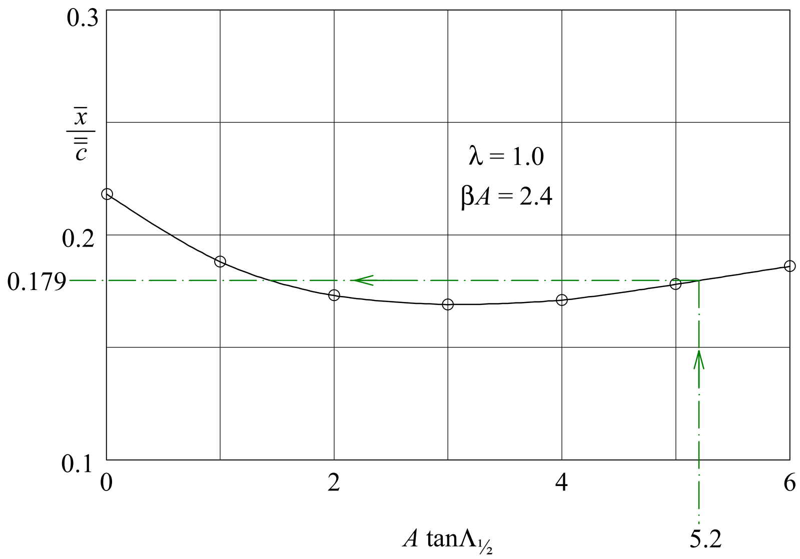
Thus from Figure 1e, for $\lambda = 1.0$, with $A \tan\Lambda_{1/2} = 5.2$ and $\beta A = 2.4$

$$\frac{1}{A}\frac{\mathrm{d}C_L}{\mathrm{d}\alpha} = 0.773 \text{ per rad.}$$

Therefore the lift-curve, $\dfrac{\mathrm{d}C_L}{\mathrm{d}\alpha} = 3 \times 0.773 = 2.32$ per rad.

From Figures 2a to 2g with $\lambda = 1.0$ and $\beta A = 2.4$, the following cross-plot gives for $A \tan\Lambda_{1/2} = 5.2$

$$\frac{\bar{x}}{\bar{\bar{c}}} = 0.179 .$$



From Equations (2.1) and (2.2) with $\lambda = 1.0$ and $A \tan\Lambda_{1/2} = 5.2$

$$\frac{\bar{\bar{x}}_0}{\bar{\bar{c}}} = 1.30 \text{ and } \frac{\bar{\bar{c}}}{c_r} = 1.00 .$$

Hence, the distance of the aerodynamic centre aft of the wing apex, in terms of the root chord, is

$$\frac{\bar{x}_0}{c_r} = \frac{\bar{x} + \bar{\bar{x}}_0}{c_r} = \frac{\bar{\bar{c}}}{c_r}\left(\frac{\bar{x}}{\bar{\bar{c}}} + \frac{\bar{\bar{x}}_0}{\bar{\bar{c}}}\right)$$

$$= 1.00(0.179 + 1.30)$$

$$= 1.48 .$$

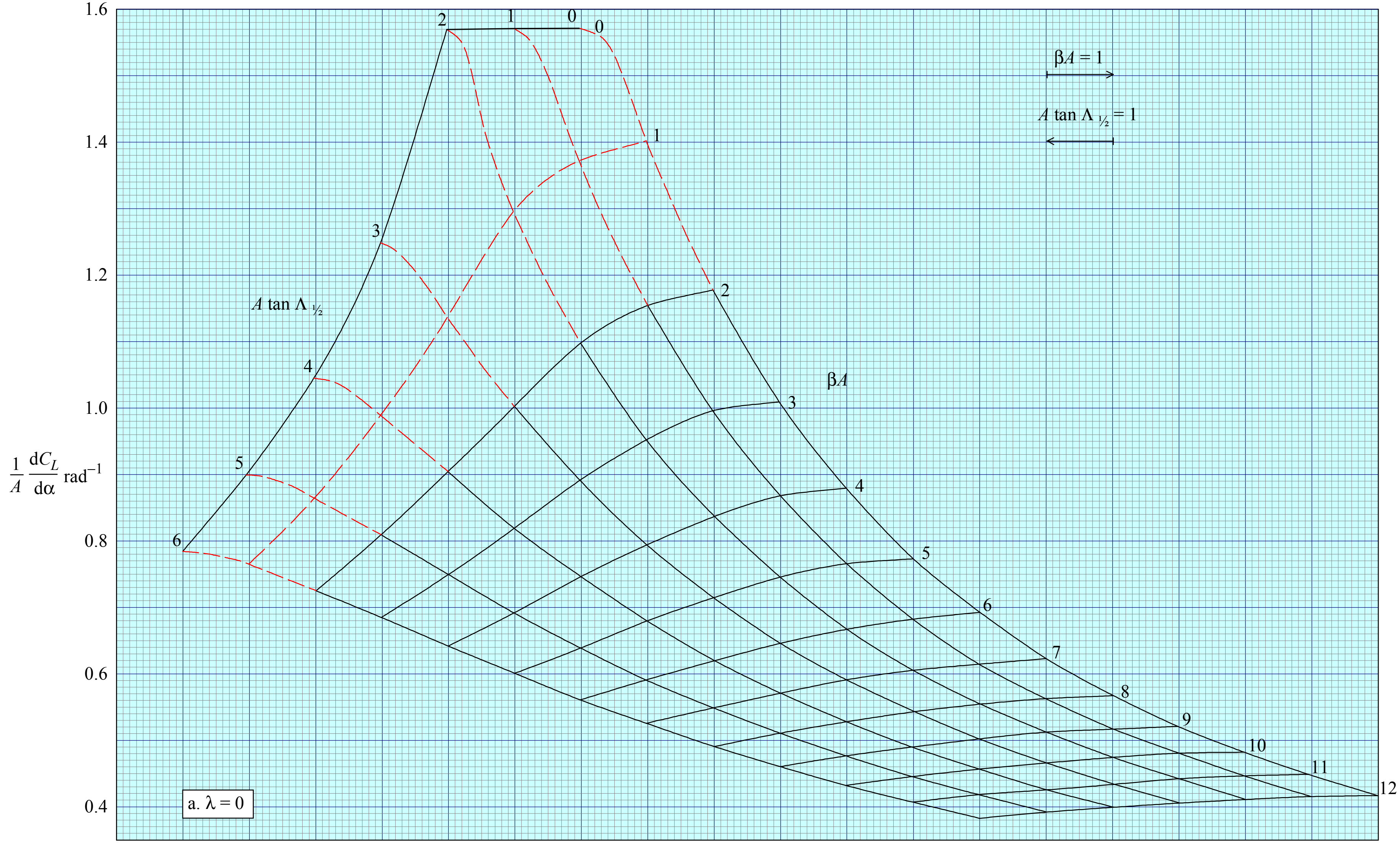


FIGURE 1a LIFT-CURVE SLOPE $\lambda = 0$

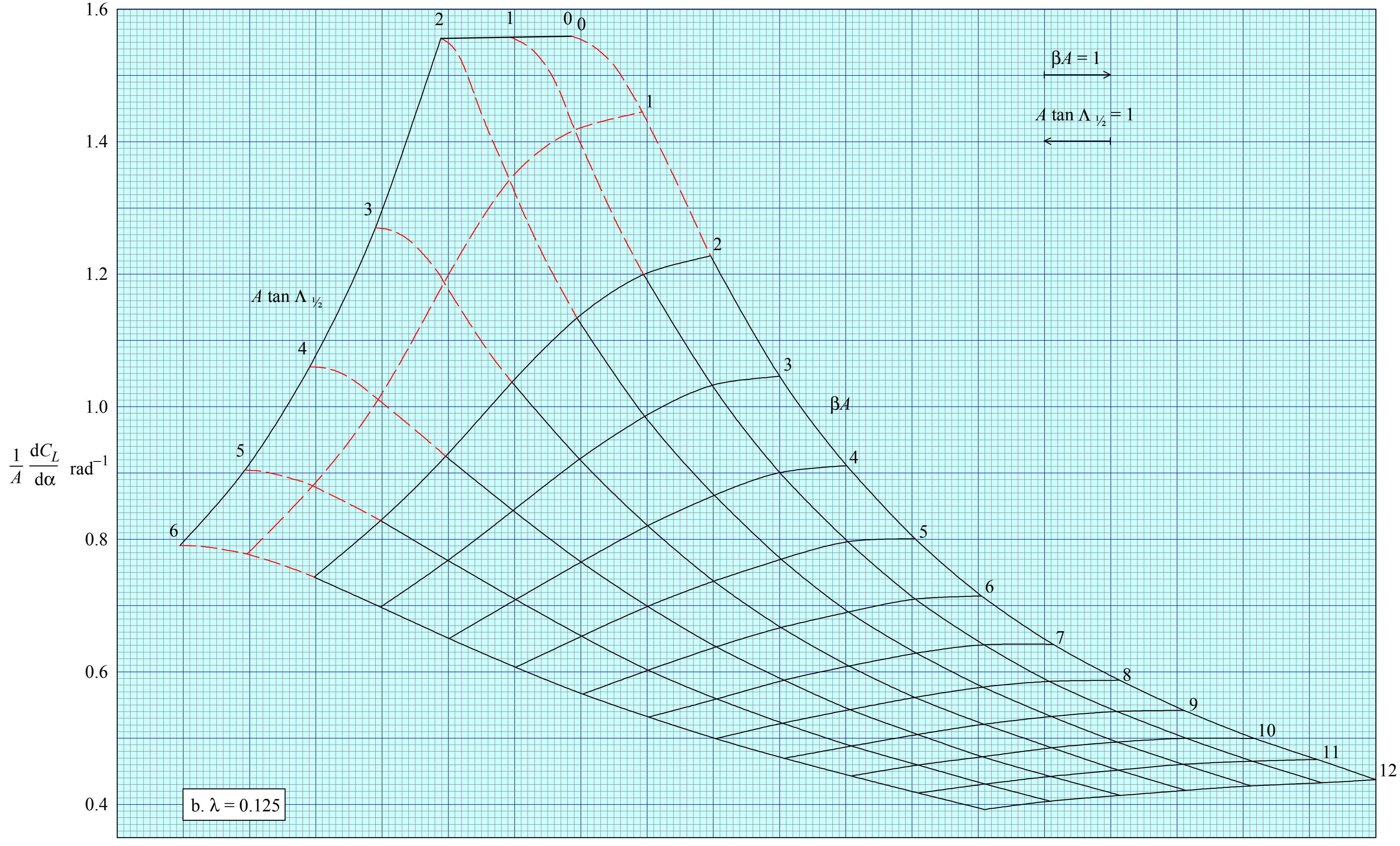


FIGURE 1b LIFT-CURVE SLOPE λ = 0.125

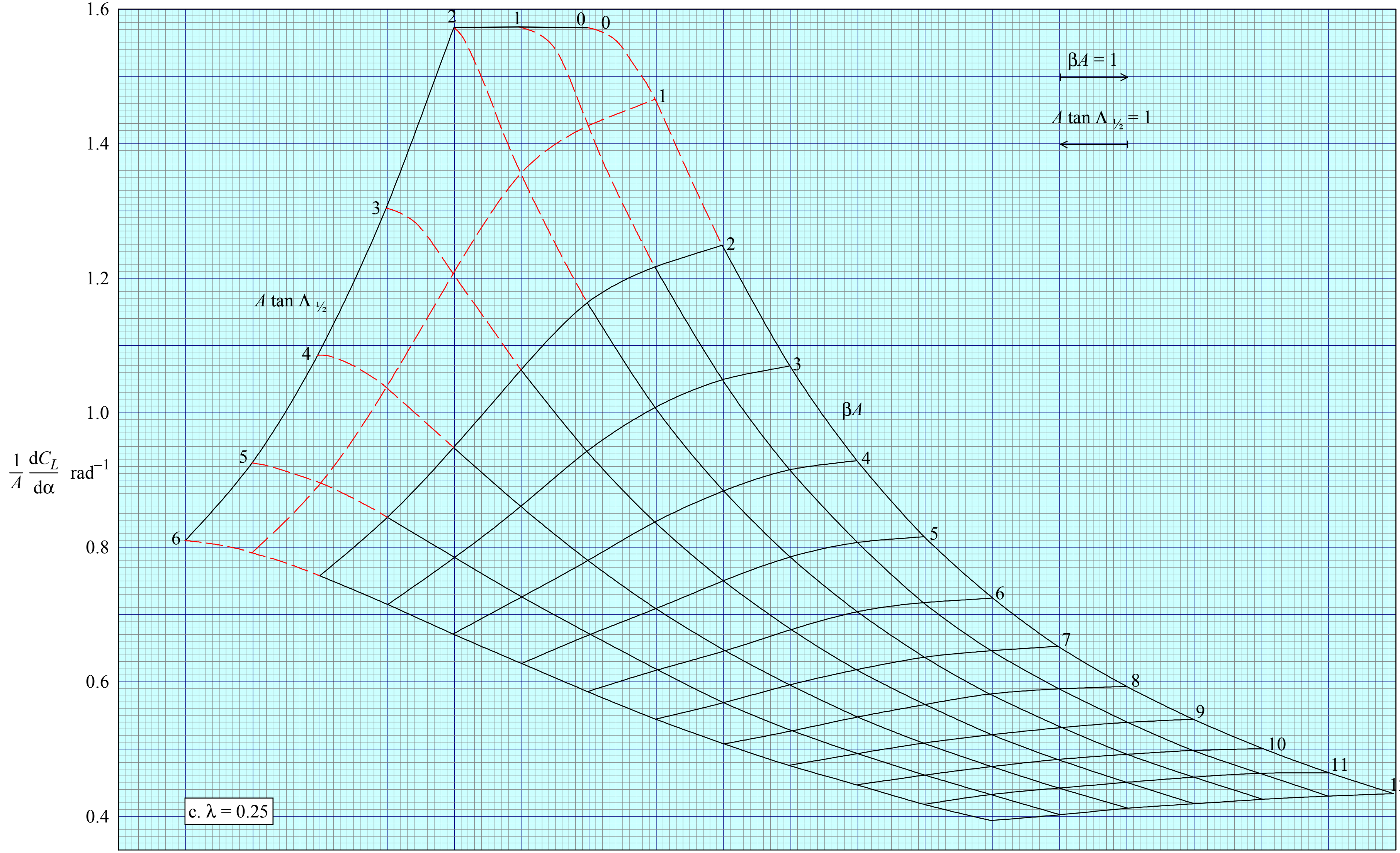


FIGURE 1c LIFT-CURVE SLOPE λ = 0.25

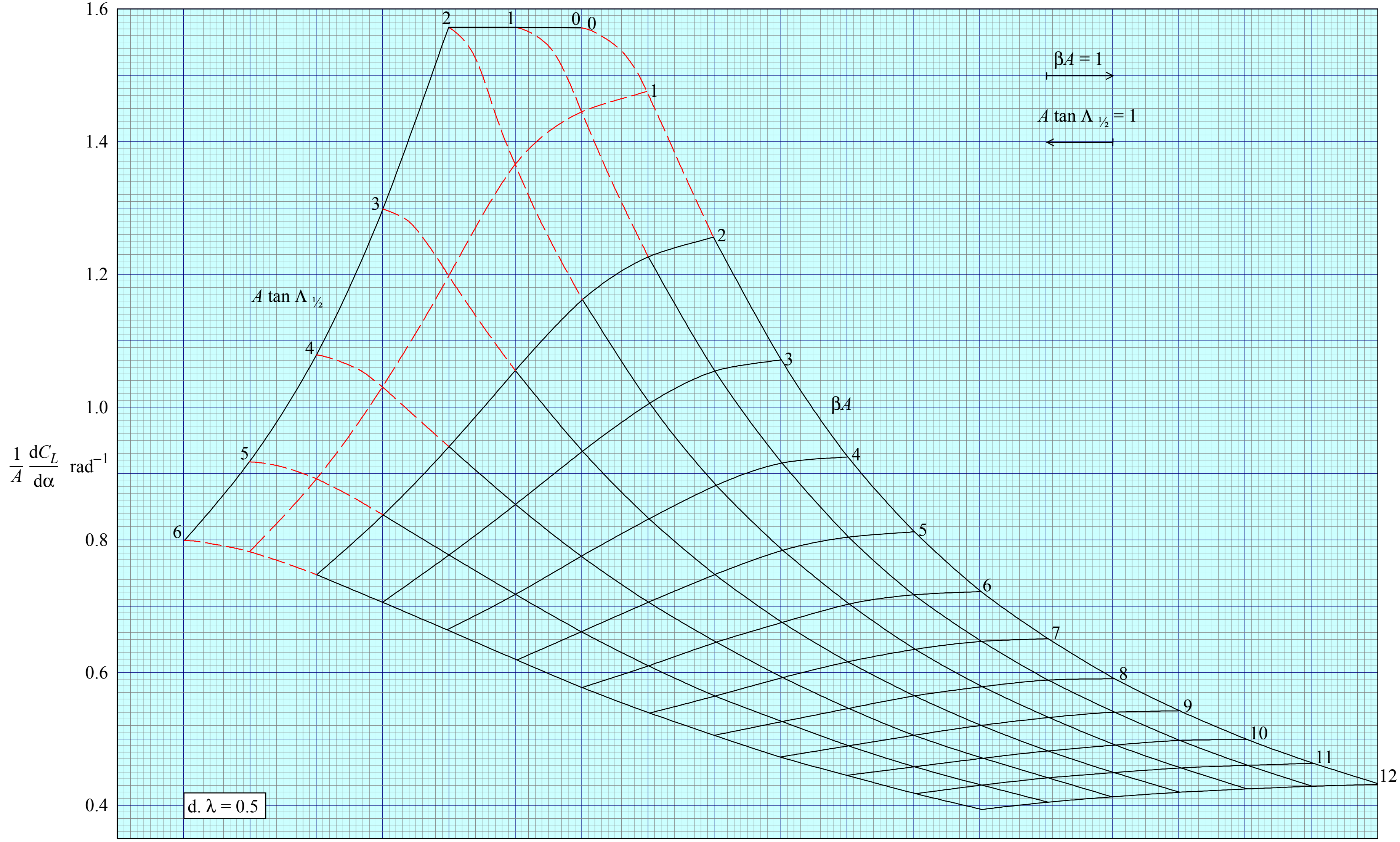


FIGURE 1d LIFT-CURVE SLOPE λ = 0.5

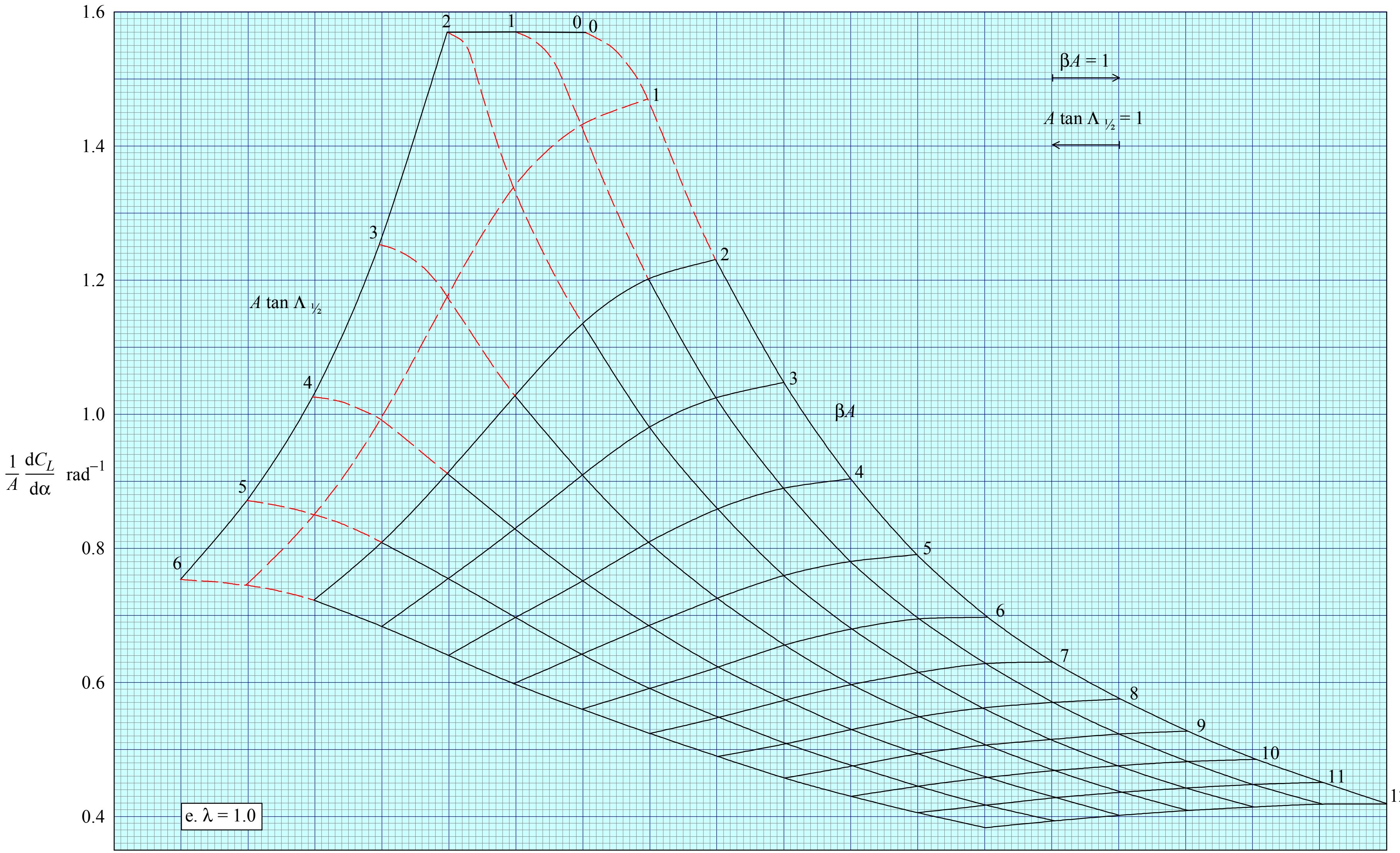


FIGURE 1e LIFT-CURVE SLOPE λ = 1

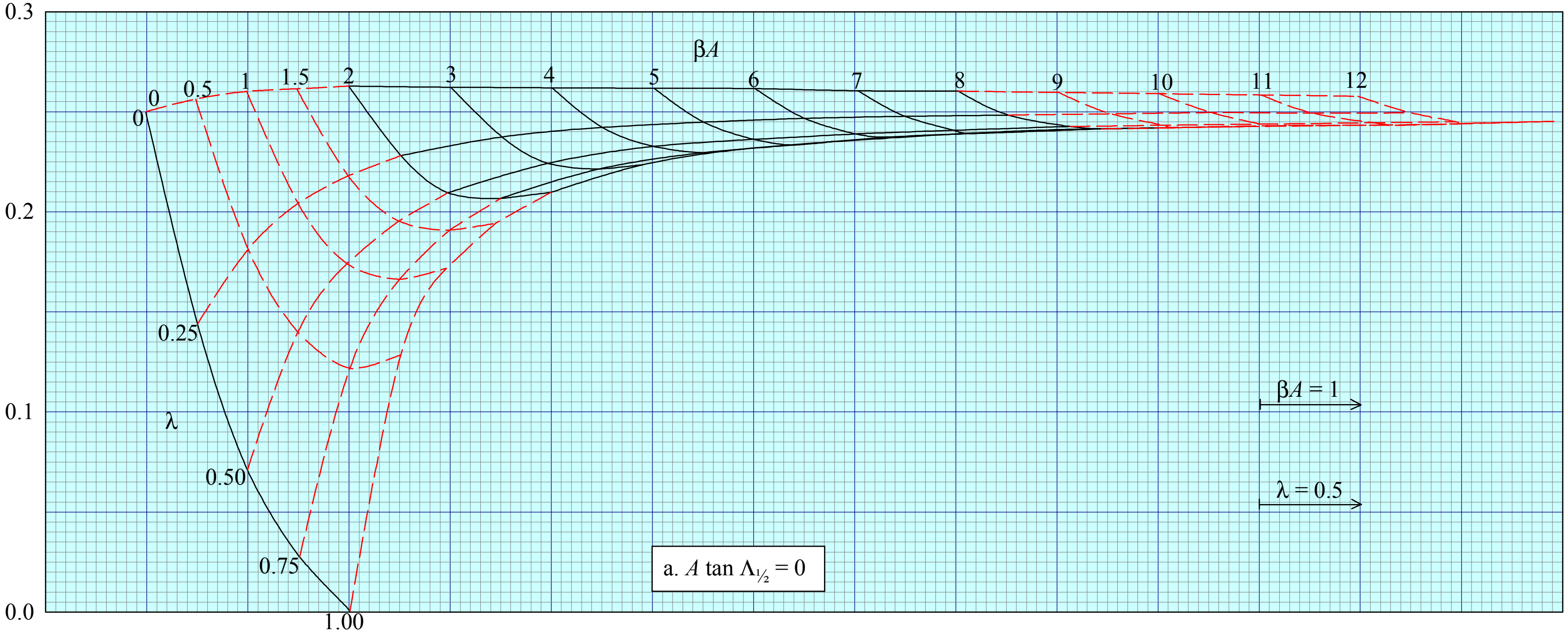


FIGURE 2a AERODYNAMIC CENTRE $A \tan \Lambda_{1/2} = 0$

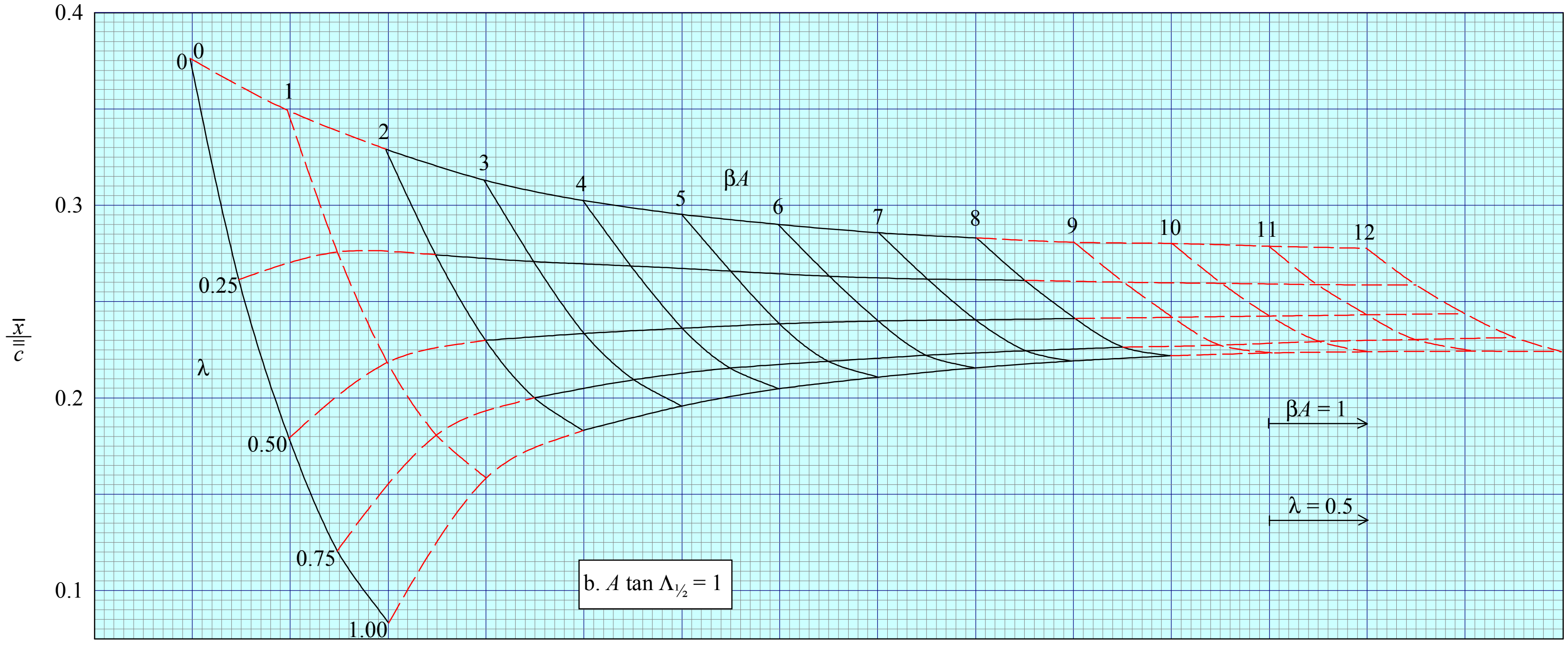


FIGURE 2b AERODYNAMIC CENTRE $A \tan \Lambda_{1/2} = 1$

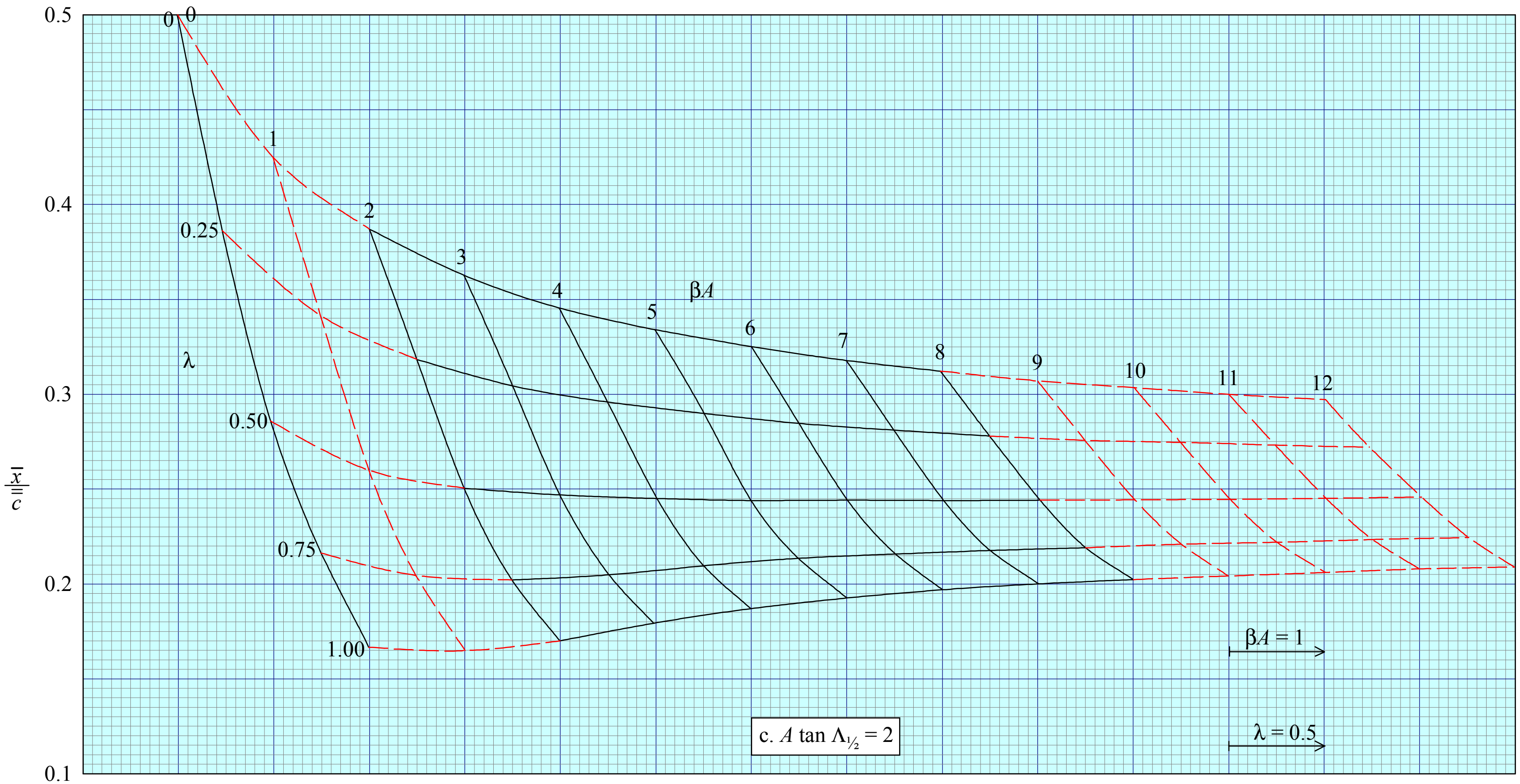


FIGURE 2c AERODYNAMIC CENTRE $A \tan \Lambda_{1/2} = 2$

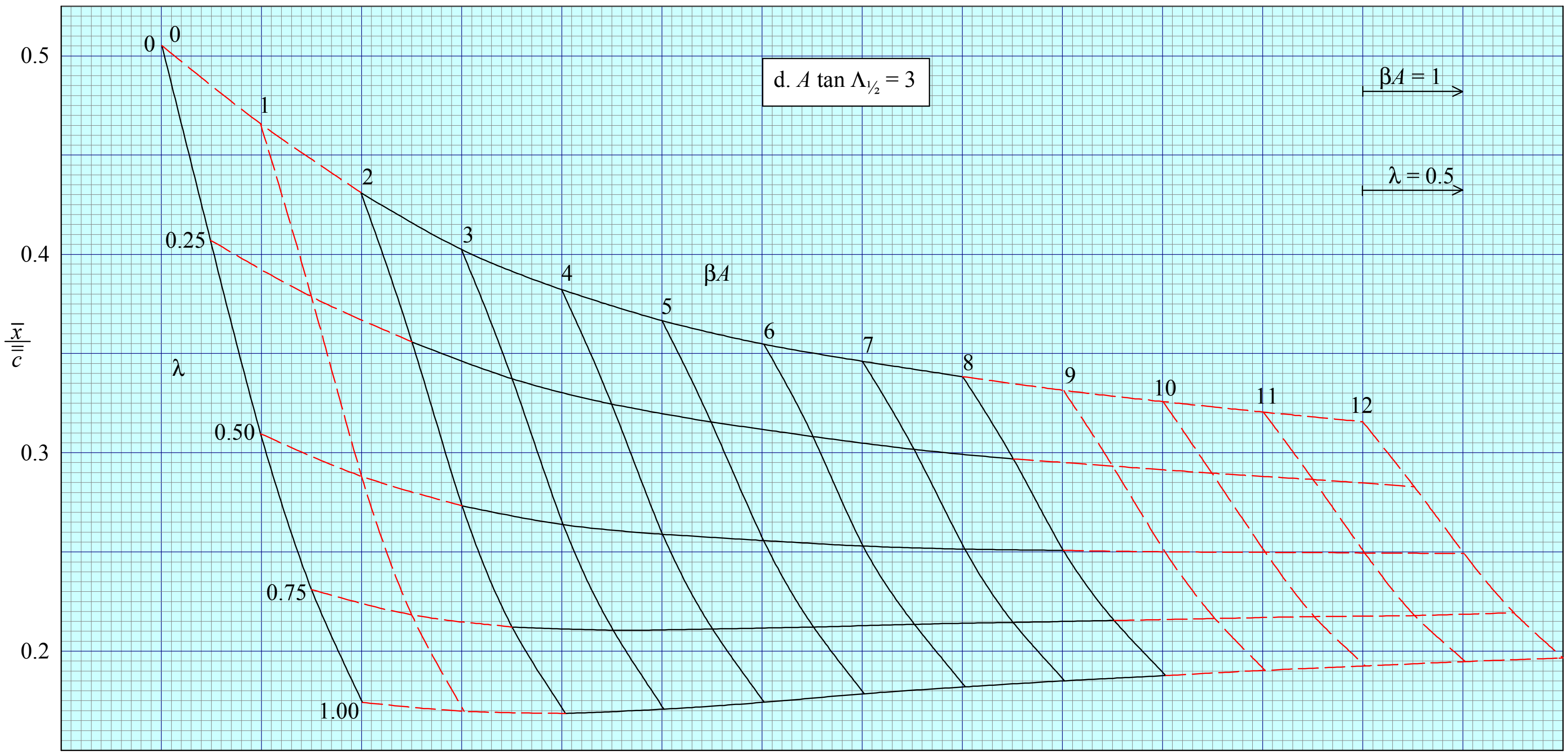


FIGURE 2d AERODYNAMIC CENTRE $A \tan \Lambda_{1/2} = 3$

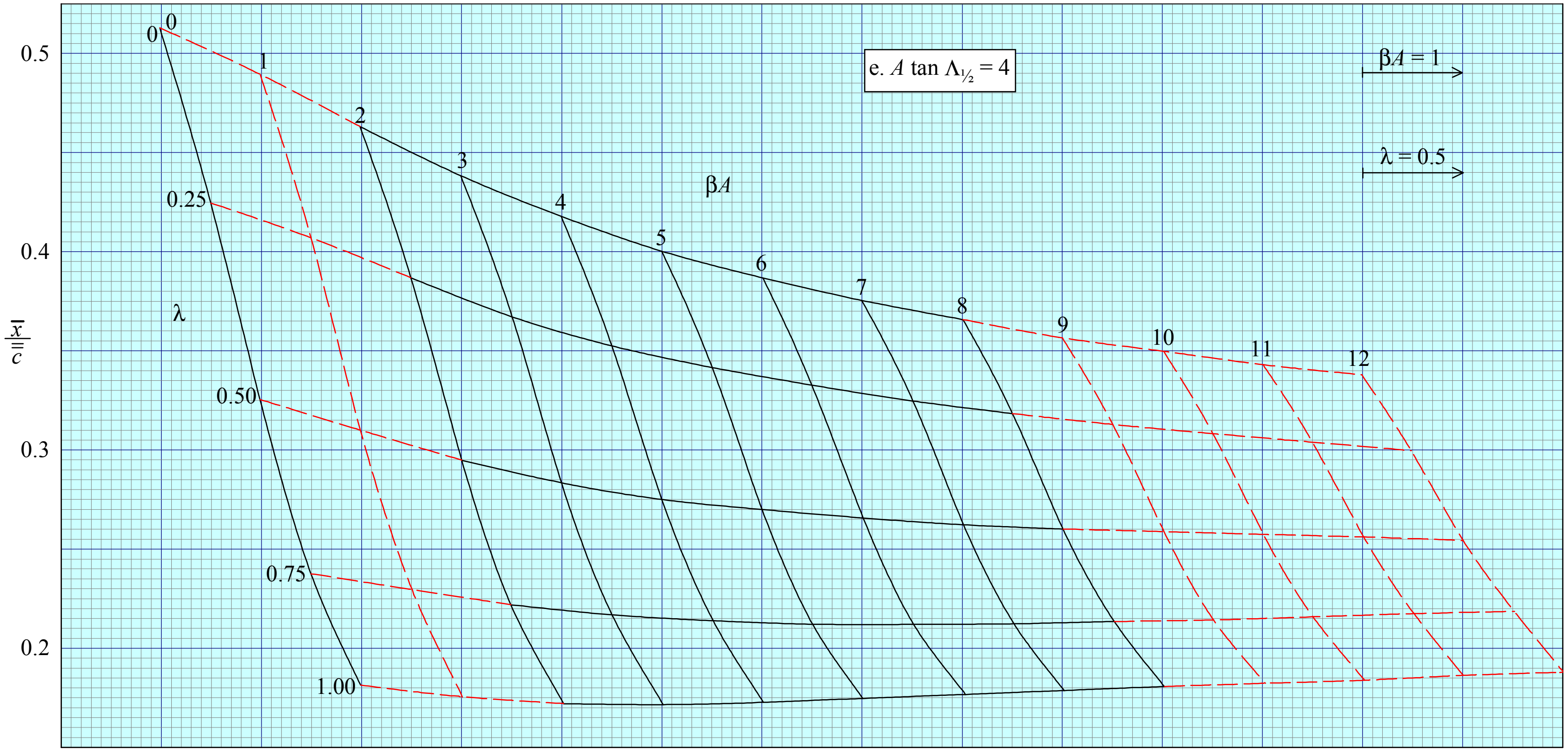


FIGURE 2e AERODYNAMIC CENTRE $A \tan \Lambda_{½} = 4$

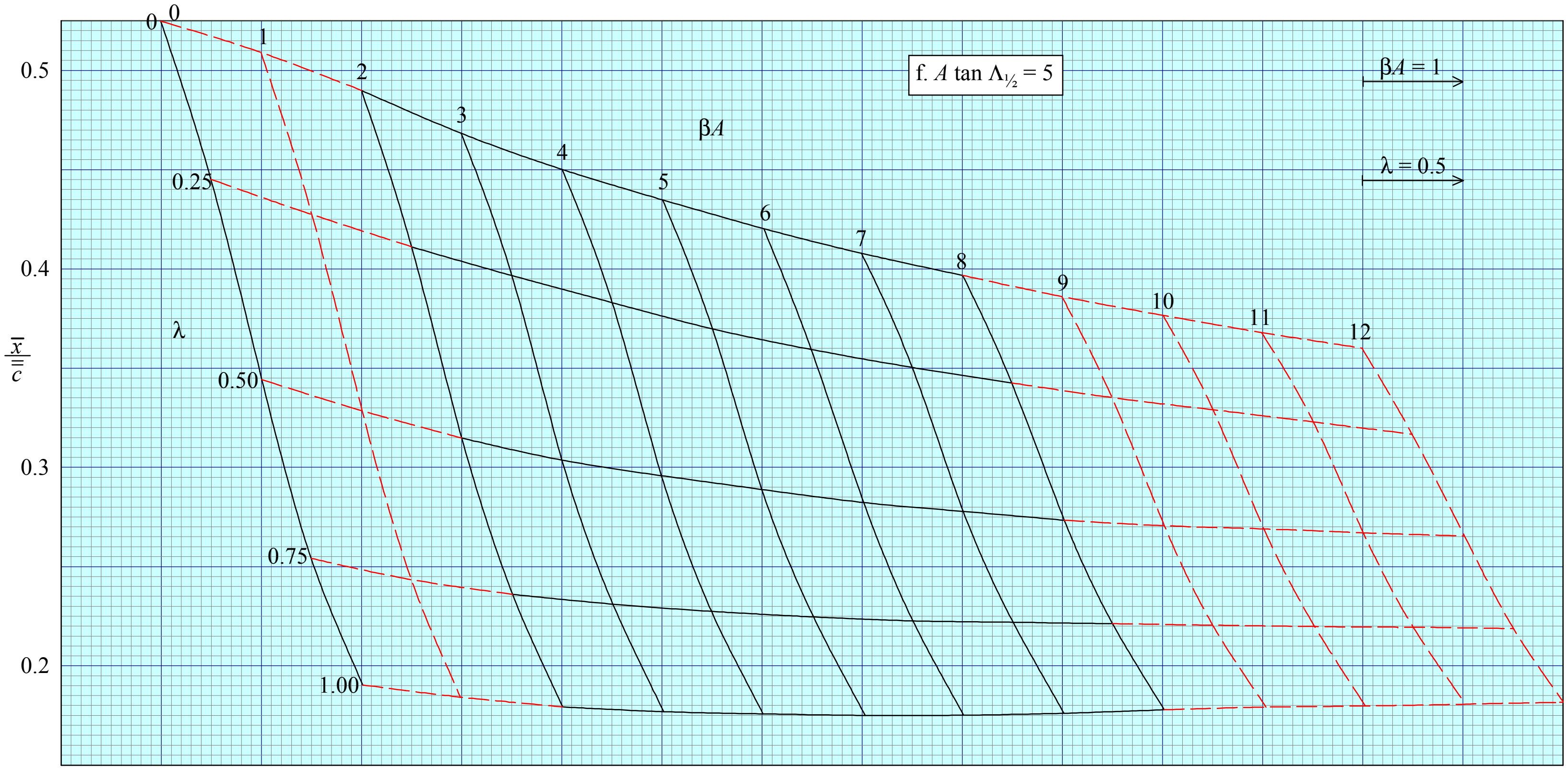


FIGURE 2f AERODYNAMIC CENTRE $A \tan \Lambda_{½}$ = 5

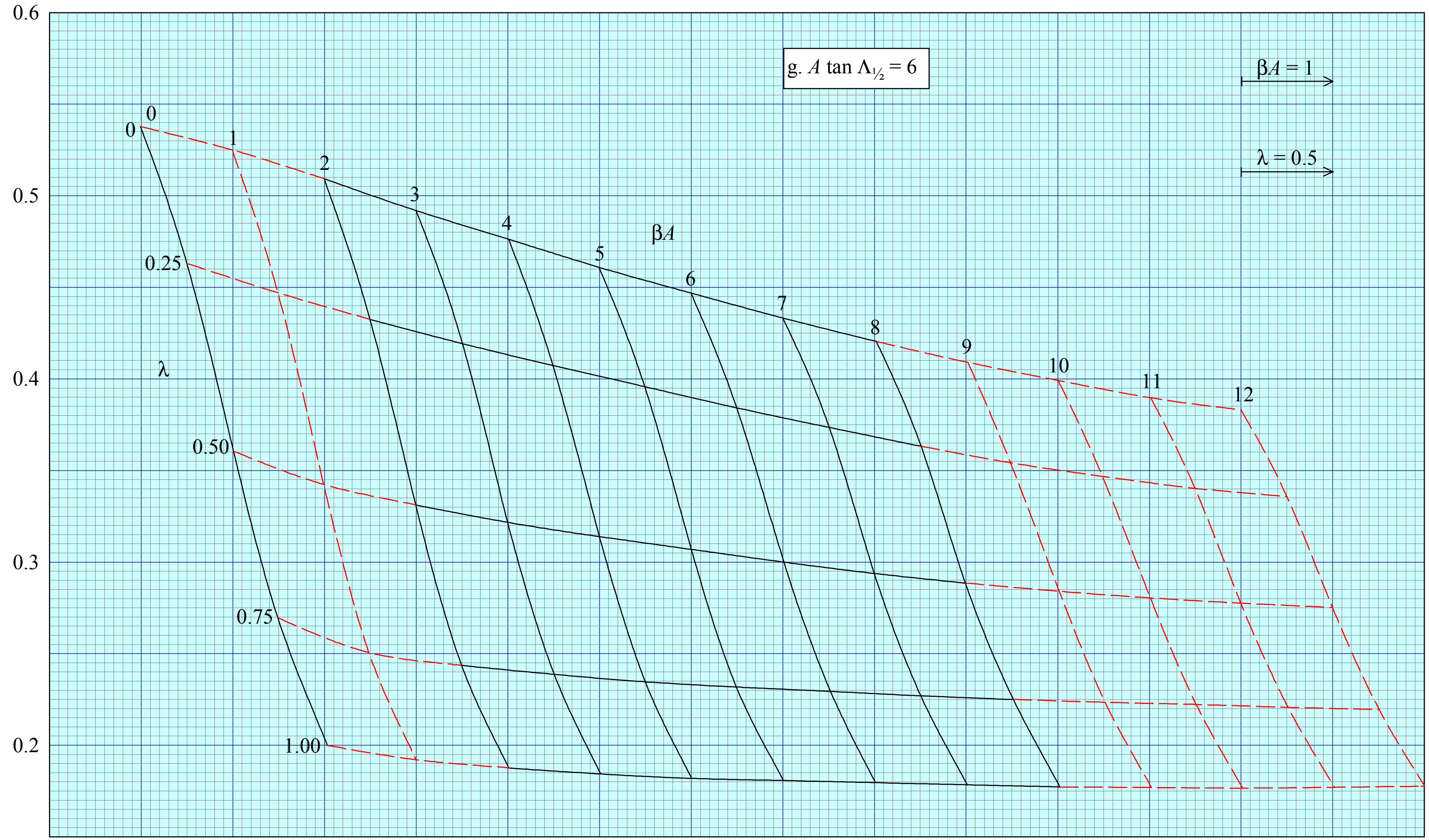


FIGURE 2g AERODYNAMIC CENTRE $A \tan \Lambda_{½} = 6$

APPENDIX A COMPARISONS WITH EXPERIMENTAL DATA

A1. NOTATION AND UNITS

$R_{\bar{\bar{c}}}$	Reynolds number based on aerodynamic mean chord
t/c	mean streamwise thickness/chord ratio

A2. INTRODUCTION

In Section 4 of this Item "Applicability of Data", it is stated that the theory from which the data of Figures 1a to 1e and 2a to 2g were derived is such that its application is limited to wings of small thickness in inviscid flow.

This Appendix assesses the errors introduced by neglecting principally the effects of thickness and viscosity in determining lift-curve slope and aerodynamic centre position. The assessment is limited to wings at low incidence for Mach numbers up to 0.6.

A3. COMPARISONS WITH EXPERIMENTAL DATA

The data were taken from References A1 to A16 These all present lift-curve slope and/or aerodynamic centre position in a form that enables accurate values to be extracted for straight-tapered wings. To increase the size of the data base some tests on wing-body combinations were included, but only for comparisons of lift-curve slope because the presence of a body has a considerable effect on the position of the aerodynamic centre (see Item No. 76015, Reference A17). The geometric parameters of the gross wing were used to estimate the wing-body lift-curve slope*.

In References A2, A8 and A9, the relative effects of having free and fixed transition are considered. In these reports it concluded that there is little or no effect on lift-curve slope or aerodynamic centre position when changing from fixed to free transition. In Reference A15, which had only one data point for both lift-curve slope and aerodynamic centre position, transition was fixed, but in all other cases transition was free.

For each planform and its associated test Mach numbers up to a maximum value of 0.6† the theoretical values of lift-curve slope and aerodynamic centre position were estimated from Figures 1a to 1e and 2a to 2g of this Item and the differences (errors) from the experimental values were calculated. Attempts to correlate these differences in terms of the various geometrical and flow parameters were unsuccessful. It was observed that the wind tunnel corrections which had been applied were of the same order (up to 5 per cent) as the quantities to be correlated, and it is thought that differences in the correction methods used in the various references may contribute to the overall scatter. The results of theory versus experiment are therefore presented here in tabular and simple graphical form.

Table A6.1 for wing-alone configurations shows the arithmetic mean of the modulus of the errors (mean error) in the values of lift-curve slope and aerodynamic centre position estimated from this Item, and Table A6.2 for wing-body configurations gives the mean error in lift-curve slope only.

* For a straight-tapered wing the gross wing planform is constructed by extending to the plane of symmetry the leading and trailing edges of the exposed wing.

† The maximum Mach number considered was 0.6 in order to exclude data for supercritical Mach numbers.

In each table results are given for sub-divided ranges of the thickness/chord ratio, and in each case the overall ranges of various geometric and test parameters are shown, together with the relevant number of test points. It should be noted here that some of the references gave experimental data on only lift-curve slope or aerodynamic centre position, and for each range of *t*/*c* there are different numbers of test points for the two parameters.

Sketch A7.1 shows the estimated values of lift-curve slope plotted against the corresponding experimental values, and Sketch A7.3 shows estimated and experimental values of aerodynamic centre position similarly plotted. On both of these sketches lines of perfect correlation are shown together with lines showing the highest values of the relevant mean error given in Tables A6.1 and A6.2.

A4. CONCLUSIONS

(a) Lift-curve Slope

Use of Figures 1a to 1e of this Item estimates the experimental lift-curve slope within a mean error of 5 per cent. Five points out of a total of 35 were individually more than 10 per cent in error, and, of these, four relate either to high values of $A\tan\Lambda_{1/2}$ (>6) which are outside the range of Figures 1a to 1e of this Item, or to low values of βA (<2) which are in an area of doubtful accuracy (see Section 4).

From Sketches A7.1 and A7.2 it is seen that the Item tends to over-estimate the value of lift-curve slope for wing-alone configurations, and to under-estimate slightly that for wing-body combinations.

Since this Appendix was first issued additional experimental data for the lift-curve slope of 28 wing-body combinations have been compared with values predicted by applying this Item to the gross wing planform. These comparisons were associated with the work on Item Nos 76015 and 77012 and related to wing planforms of medium-to-high aspect ratio $(4 \le \beta A \le 12, A\tan\Lambda_{1/2} \le 4)$ *; the planforms were of straight taper or had a single trailing-edge crank**. *For these cases the errors were generally within* ± 5 *per cent, see Sketch A7.2. It is important to note that the use of the lift-curve slope of the gross wing to provide a rapid estimate of the wing-body lift-curve slope will only be adequate in cases where the ratio of the body diameter to wing span is small,* ≤ 0.15*, say, for* $\lambda \le 0.4$ *and* ≤ 0.09*, say, for* $\lambda = 1$ *(and provided that* $\beta A \ge 3, A\tan\Lambda_{1/2} \le 6$*). Item No. 91007 (Reference A18) gives a general method for the estimation of wing-body lift-curve slope via the use of wing-body interference factors which should be used if the ratio of body diameter to wing span is not small, or if the distribution of lift between wing and body is to be studied.*

(b) Aerodynamic Centre Position

For wing-alone configurations use of Figures 2a to 2g of this Item estimates the experimental aerodynamic centre position within $\pm 0.03\bar{\bar{c}}$.

The quoted errors are limited to the data ranges given in Table A6.1. If estimates are required outside these ranges then caution should be exercised.

* For cranked wings an "equivalent" straight-tapered wing was constructed (see Addendum A of Item No. 76003, Reference 6).

A5. REFERENCES

A1.	REYNOLDS, R.M. SMITH, D.W.	Aerodynamic study of a wing-fuselage combination employing a wing swept back 63° – subsonic Mach and Reynolds number effects on the characteristics of the wing and on the effectiveness of an elevon. NACA RM A8D20 (TIB 1942), October 1948.
A2.	FISCHEL, J. SCHNEITER, L.E.	An investigation at low speed of a 51.3° sweptback semispan wing with a raked tip with 16.7-percent-chord ailerons having three spans and three trailing edge angles. NACA RM L8F29 (TIB 1869), July 1948.
A3.	JOHNSON, H.S. HAGERMAN, J.R.	Wind-tunnel investigation at low speed of an unswept untapered semispan wing of aspect ratio 3.13 equipped with various 25-percent-chord plain flaps. NACA tech. Note 2080, January 1950.
A4.	JOHNSON, H.S. HAGERMAN, J.R.	Wind-tunnel investigation at low speed of a 45° sweptback untapered semispan wing of aspect ratio 1.59 equipped with various 25-percent-chord plain flaps. NACA tech. Note 2169, May 1950.
A5.	CAHILL, J.F. GOTTLIEB, S.M.	Low-speed aerodynamic characteristics of a series of swept wings having NACA 65A006 airfoil sections. NACA RM L50F16 (Revised) (TIB 2555), October 1950.
A6.	SMITH, D.W. HEITMEYER, J.C.	Lift, drag and pitching moment of low-aspect-ratio wings at subsonic and supersonic speeds – plane triangular wing of aspect ratio 2 with NACA 0008-63 sections. NACA RM A50K20 (TIB 2606), February 1951.
A7.	TINLING, B.E. KOLK, W.R.	The effects of Mach number and Reynolds number on the aerodynamic characteristics of several 12-percent-thick wings having 35° of sweepback and various amounts of camber. NACA RM A50K27 (TIB 2629), February 1951.
A8.	KOLBE, C.D. BANDETTINI, A.	Investigation in the Ames 12-foot pressure wind-tunnel of a model horizontal tail of aspect ratio 3 and taper ratio 0.5 having the quarter-chord line swept back 45°. NACA RM A51D02 (TIB 2780), June 1951.
A9.	JOHNSON, B.H. SHIBATA, H.H.	Characteristics throughout the subsonic speed range of a plane wing and of a cambered and twisted wing, both having 45° of sweepback. NACA RM A51D27 (TIB 2812), July 1951.
A10.	HEITMEYER, J.C.	Lift, drag, and pitching moment of low-aspect-ratio wings at subsonic and supersonic speeds – plane triangular wing of aspect ratio 3 with NACA 0003-63 section. NACA RM A51H02 (TIB 2869), September 1951.
A11.	KOLBE, C.D. BOLTZ, F.W.	The forces and pressure distribution at subsonic speeds on a plane wing having 45° of sweepback, an aspect ratio of 3, and a taper ratio of 0.5. NACA RM A51G31 (TIB 2894), October 1951.
A12.	KUHN, R.E. WIGGINS, J.W.	Wind-tunnel investigation of the aerodynamic characteristics in pitch of wing-fuselage combinations at high subsonic speeds. Aspect ratio series. NACA RM L52A29 (TIB 3080), April 1952.

A13. PALMER, W.E. — Effect of reduction in thickness from 6 to 2 percent and removal of the pointed tips on the subsonic static longitudinal stability characteristics of a 60° triangular wing in combination with a fuselage. NACA RM L53 F24 (TIB 3862), August 1953.

A14. JONES, R.; MILES, C.J.W.; PUSEY, P.S. — Experiments in the compressed air tunnel on swept-back wings including two delta wings. ARC R&M 2871, 1954.

A15. DEMELE, F.A.; POWELL, K.H. — The effects of an inverse-taper leading-edge flap on the aerodynamic characteristics in pitch of a wing-body combination having an aspect ratio of 3 and 45° of sweepback at Mach numbers to 0.92. NACA tech. Note 4366, May 1958.

A16. BREBNER, G.G.; WYATT, L.A.; ILOTT, G.P. — Low speed wind-tunnel tests on a series of rectangular wings of varying aspect ratio and aerofoil section. ARC CP 916, 1965.

A17. ESDU — Aerodynamic centre of wing-fuselage combinations. ESDU International, Item No. 76015, 1976.

A18. ESDU — Lift-curve slope of wing-body combinations. ESDU International, Item No. 91007, 1991.

A6. TABLES

TABLE A6.1 Mean error in values of lift-curve slope and aerodynamic centre position obtained from this Item. Wing-alone configurations.

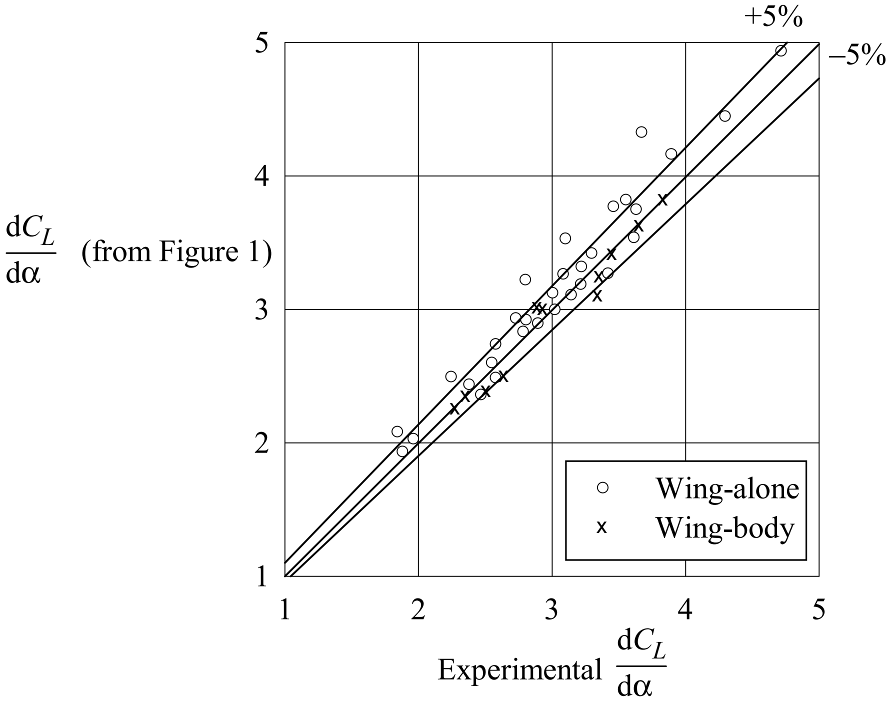
t/c	A	λ	$\Lambda_{1/2}$	M	$R_{\bar{\bar{c}}} \times 10^{-6}$	*Number of points*	*Mean error in lift-curve slope*	*Mean error in aerodynamic centre position*
0.06 to 0.10	1.4 to 6.0	0 to 1.0	0 to 60	0.04 to 0.60	1.6 to 9.3	30	5.0%	–
	2.0 to 6.0	0 to 1.0	0 to 60	0.04 to 0.60	4.0 to 9.3	17	–	$0.013\bar{\bar{c}}$
0.12	3.1 to 10.1	0.48 to 0.71	34 to 45	0.12 to 0.60	2.0 to 2.2	5	3.7%	–
	5.1 to 10.1	0.50 to 0.71	34	0.25 to 0.60	2.0	4	–	$0.028\bar{\bar{c}}$

TABLE A6.2 Mean error in values of lift-curve slope obtained from this Item. Wing-body configurations*

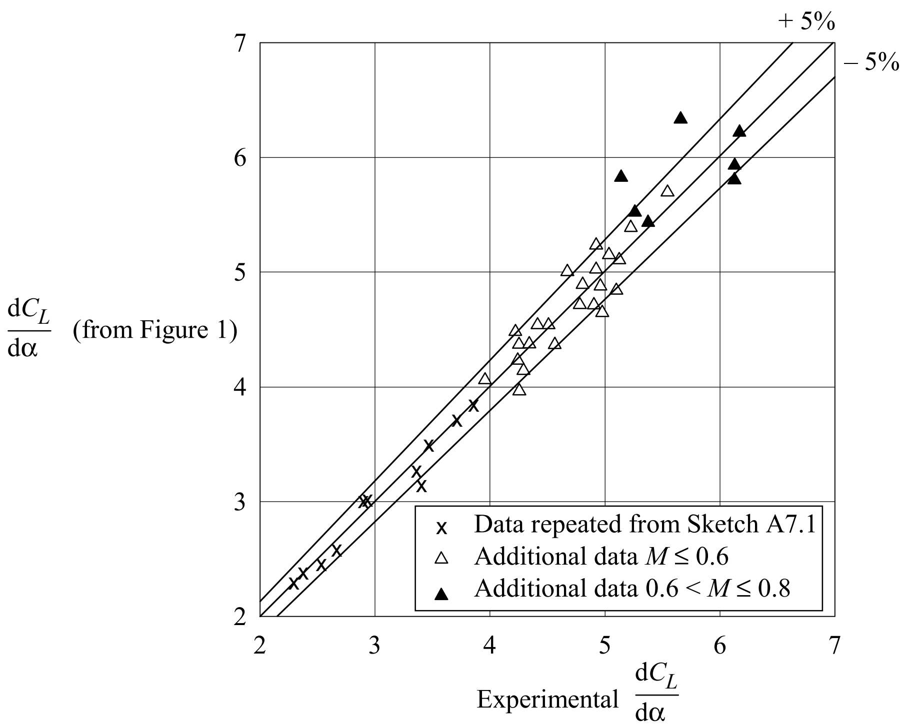
t/c	A	λ	$\Lambda_{1/2}$	M	$R_{\bar{\bar{c}}} \times 10^{-6}$	*Number of points*	*Mean error in lift-curve slope*
0 to 0.05	3.0	0 to 0.50	34 to 36	0.25 to 0.60	4.8 to 5.0	2	4.3%
0.06 to 0.10	2.0 to 6.0	0 to 0.60	39 to 45	0.04 to 0.60	1.6 to 9.5	9	2.0%

* See Section A4.

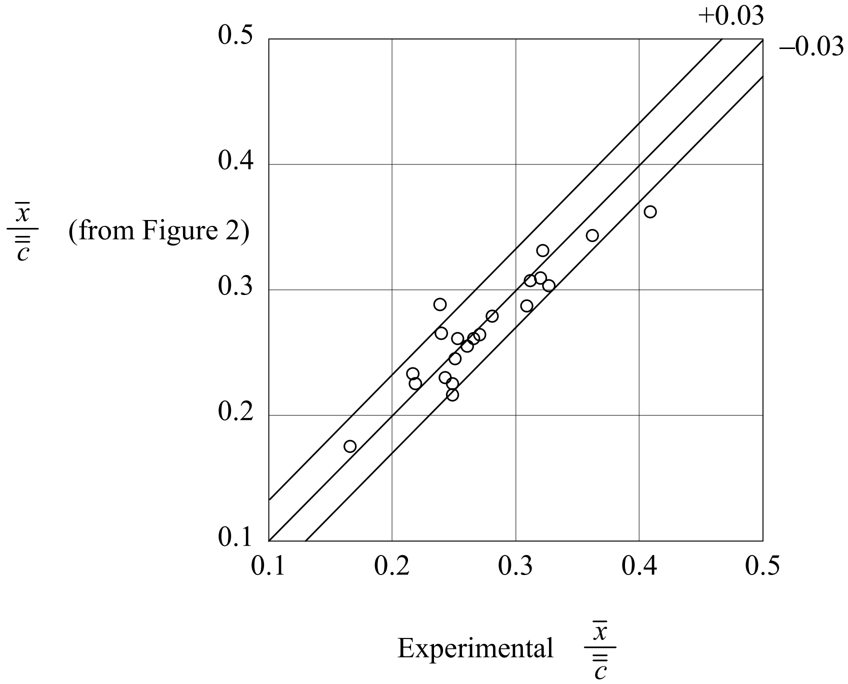
A7. SKETCHES



Sketch A7.1



Sketch A7.2 Wing-body data



Sketch A7.3 Wing-alone data

APPENDIX B ADDITIONAL NOTATION AND UNITS

B1. ADDITIONAL NOTATION AND UNITS

		SI	*British*
$(a_1)_{0M}$	section lift-curve slope in two-dimensional flow at Mach number M	rad^{-1}	rad^{-1}
κ	$(a_1)_{0M}/(2\pi/\beta)$, ratio of section lift-curve slope to theoretical thin-section value.		

B2. INTRODUCTION

Historically, the simple semi-empirical Helmbold-Diederich formula (Reference B1) has been widely used to estimate rapidly the lift-curve slope of straight-tapered wings. More dependable methods such as that provided by this Item are now available, but the formula is presented here for information, without comment as to reliability.

The Helmbold-Diederich formula can be written

$$\left(\frac{\mathrm{d}C_L}{\mathrm{d}\alpha}\right)_{HD} = \frac{2\pi A}{2 + \left[\frac{\beta^2 A^2}{\kappa^2}\left(1 + \frac{\tan^2\Lambda_{½}}{\beta^2}\right) + 4\right]^{½}} . \tag{B2.1}$$

B3. REFERENCE

The Reference is a source of information supplementary to that given in this Appendix.

B1	DIEDERICH, F.W.	A planform parameter for correlating certain aerodynamic characteristics of wings. NACA tech. Note 2335, 1951.

APPENDIX C PROGRAM FOR CALCULATION OF LIFT-CURVE SLOPE AND AERODYNAMIC CENTRE POSITION OF WINGS IN INVISCID SUBSONIC FLOW

Every reasonable effort has been made to ensure that the program performs the intended calculations satisfactorily. However, in common with all providers of software, ESDU International cannot guarantee the suitability or fitness of the program for any particular purpose and no liability for any loss occasioned by any person as a direct or indirect result of use of the program, whether arising from negligence or otherwise, can be accepted. In no event shall ESDU International or any individuals associated with the development of the program be liable for any damage, including loss of profit or consequential loss, arising out of or in connection with the program.

The program, ESDUpac A7011, has been written in “STRICT” Microsoft FORTRAN 77 for use on machines using PC/MS DOS. A disk containing files for the source code, worked examples and an information file is provided in the Aerodynamics Software Volume. Guidance on copying, compilation and running the program is given in the “Introduction to ESDUpacs” in that volume. However, if any difficulty is experienced in using the program please contact ESDU International and we will do all we can to assist in overcoming the problem.

In addition to the source code the program is also given in compiled form as VIEWpac 7011A that may be run in ESDUview, which provides a “user-friendly” interface for operation on an IBM PC compatible system; see “Using ESDUview” in the Aerodynamics Software Volume for further information.

The software is also on CD-Rom.

C1. INTRODUCTION

A computer program, called ESDUpac A7011, has been written by the Computer Products Group of ESDU International to determine the lift-curve slope and aerodynamic centre position of plane straight-tapered wings in inviscid subsonic flow. The numerical results of the lifting-surface theory calculations carried out in the development of Item No. 70011 have been used as the main basis of the program, but as they were restricted to swept-back wings $(A \tan\Lambda_{1/2} \geq 0)$ an extension of the program has been made to incorporate wings with modestly swept-forward half-chord lines $(\Lambda_{1/2M} = \tan^{-1}((\tan\Lambda_{1/2})/\beta) \geq -20°)$, but no forward sweep of the leading edge is permitted $(\Lambda_0 \geq 0)$. The extension in the case of the lift-curve slope is achieved simply by use of the absolute value of $A \tan\Lambda_{1/2}$ to interpolate within the data for Figures 1a to 1e stored in the program. For the calculation of the aerodynamic centre position, data for $A \tan\Lambda_{1/2} < 0$ have been taken from Derivation C2 which presented graphically the results of calculations made by application of the extended lifting-line theory of Derivation C1 to forward-swept wings. In effect, three additional carpets for $\bar{x}/\bar{\bar{c}}$ in terms of βA and λ are contained within the program for $A \tan\Lambda_{1/2} = -1$, –2 and –3. These supplement the seven carpets in Figures 2a to 2g for $A \tan\Lambda_{1/2}$ = 0, 1, . . . , 6. Note that the extra carpets cover a wider range of wing geometries than is consistent with the restrictions on forward sweep imposed via $\Lambda_{1/2M}$ and Λ_0, but this allows better interpolation within the permitted range.

C2. INPUT

Table C2.1 lists the input file parameters for the program, with their name and, where appropriate, their notation in the Item. A single input file allows a run to be made at many values (≤ 50 each) of Mach number M, aspect ratio A, sweep angle Λ_n and taper ratio λ. Note that sweep angle may be defined for any chord line by specification of n in the input.

Table C2.2 translates the output table headings into the notation of the Item.

TABLE C2.1 Input data*

Variable name in program	*Notation in Item*	*Comments*
CHAR1 CHAR2 CHAR3	– – –	The first 3 lines of the input data file are read as text and are written on the output file. Each line may contain 72 characters and must end with a carriage return. A blank line must be entered if no text is available.
NM	–	Number of Mach numbers (≤ 50)
MVALS(1), ..., MVALS(I), ..., MVALS(NM)	M	Values of Mach number ($0 \leq M < 1$)
NA	–	Number of aspect ratios (≤ 50)
AVALS(1), ..., AVALS(I), ..., AVALS(NA)	A	Values of aspect ratios
SN	n	Specifies n^{th} chord line for definition of Λ_n ($0 \leq n \leq 1$)
NLN	–	Number of sweep angles (≤ 50)
LNVALS(1), ..., LNVALS(I), ..., LNVALS(NLN)	Λ_n	Values of sweep angles
NLDA	–	Number of taper ratios (≤ 50)
LVALS(1), ..., LVALS(I), ..., LVALS(NLDA)	λ	Values of taper ratios ($0 \leq \lambda < 1$)
D or R	–	Enter D or R to specify lift-curve slope per degree or per radian

* Formerly Table C.1a, and error messages from the program will use the earlier table number.

Note that for successful interpolation within the program the input data must satisfy the criteria

$$0 \leq \beta A \leq 12$$

$$-2\left(\frac{1-\lambda}{1+\lambda}\right) \leq A \tan\Lambda_{1/2} \leq 6$$

$$\tan^{-1}((A \tan\Lambda_{1/2})/\beta A) \geq -20°$$

which are automatically tested in the program.

TABLE C2.2 Program output headings

Program Heading	M	A	n	Ln	Taper	Beta A	AtanLh	dCL/da	xb/cbb	xb0/cr
Notation in Item	M	A	n	Λ_n	λ	βA	$A \tan\Lambda_{1/2}$	$dC_L/d\alpha$	$\bar{x}/\bar{\bar{c}}$	$\bar{x}_0/c_r$

C3. DERIVATION

The Derivation lists selected additional sources that have assisted in the preparation of this Item,

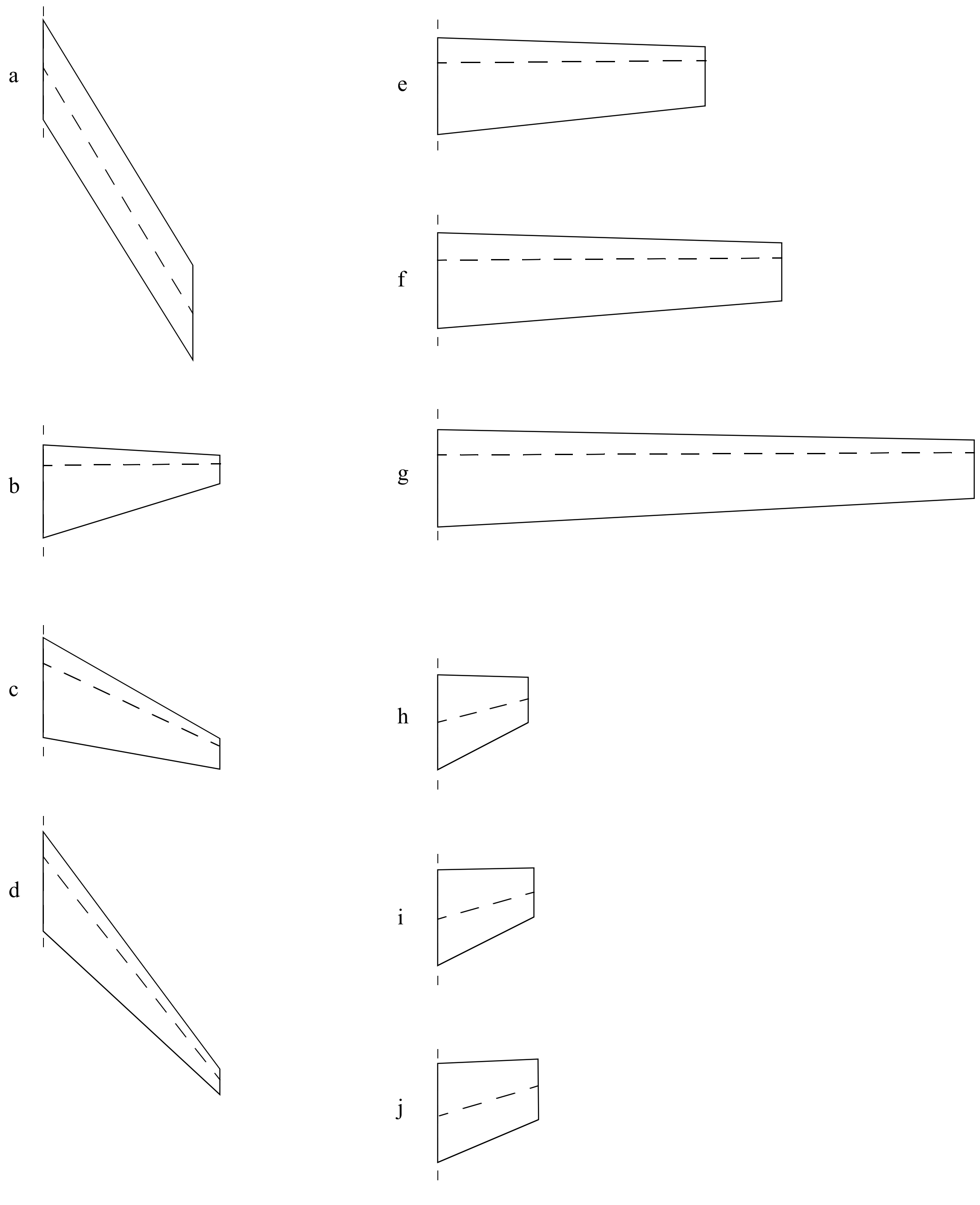
C1.	De YOUNG, J. HARPER, C.W.	Theoretical symmetric span loading at subsonic speeds for wings having arbitrary planform. NACA Rep. 921, 1948.
C2.	STANBROOK, A.	The lift-curve slope and aerodynamic centre position of wings at subsonic and supersonic speeds. RAE tech. Note No. Aero. 2328 (ARC 17615), 1954.

C4. EXAMPLES OF INPUT AND OUTPUT

Five examples of input and output are given. Example C.1 corresponds to the worked example of Section 6. Examples C.2 to C.5 illustrate multiple runs and the triggering of the various warnings on parameter ranges that are contained within the program. Table C4.1 summarises the runs and the Sketch C4.1 shows the wing planforms involved. Table C4.2 gives the values of the parameters calculated within the program that produce the various warning messages.

TABLE C4.1

EXAMPLE	PLANFORM (see Sketch C4.1)	M	A	n	Λ_n (deg)	λ
C.1	a	0.6	3	0.5	60	1.0
C.2	b	0.6	6	0.25	0	0.25
	c	0.6	6	0.25	25	0.25
	d	0.6	6	0.25	50	0.25
C.3	e	0.4	7	0.25	0	0.6
	f	0.4	9	0.25	0	0.6
	g	0.4	14	0.25	0	0.6
C.4	i	0.4	2.67	0.5	–14	0.5
	i	0.6	2.67	0.5	–14	0.5
	i	0.7	2.67	0.5	–14	0.5
	i	0.8	2.67	0.5	–14	0.5
C.5	h	0.7	2.67	0.5	–14	0.4
	i	0.7	2.67	0.5	–14	0.5
	j	0.7	2.67	0.5	–14	0.6
	h	0.8	2.67	0.5	–14	0.4
	i	0.8	2.67	0.5	–14	0.5
	j	0.8	2.67	0.5	–14	0.6



Sketch C4.1 Half-wing geometries of planforms used in examples

TABLE C4.2

EXAMPLE	PLANFORM (see Sketch C4.1)	M	βA	$A \tan \Lambda_{½}$	Λ_0 (deg)	$\Lambda_{½}$ (deg)	$\Lambda_{½M}$ (deg)	*Warnings for*
C.1	a	0.6	2.40	5.20	60.0	60.0	65.2	–
C.2	b	0.6	4.80	–0.60	5.7	–5.7	–7.1	$\Lambda_{½M} < 0$
	c	0.6	4.80	2.20	29.5	20.1	24.6	–
	d	0.6	4.80	6.55	52.3	47.5	53.8	No calculation, $A \tan \Lambda_{½} > 6$
C.3	e	0.4	6.41	–0.25	2.0	–2.0	–2.2	$\Lambda_{½M} < 0$
	f	0.4	8.25	–0.25	1.6	–1.6	–1.7	$\Lambda_{½M} < 0$ and $\beta A > 8$
	g	0.4	12.83	–0.25	1.0	–1.0	–1.1	No calculation, $\beta A > 12$
C.4	i	0.4	2.44	–0.67	0	–14.0	–15.2	$\Lambda_{½M} < 0$
	i	0.6	2.14	–0.67	0	–14.0	–17.4	$\Lambda_{½M} < 0$
	i	0.7	1.90	–0.67	0	–14.0	–19.2	$\Lambda_{½M} < 0$ and $\beta A < 2$
	i	0.8	1.60	–0.67	0	–14.0	–22.6	No calculation, $\Lambda_{½M} < -20°$
C.5	h	0.7	1.91	–0.67	4	–14.0	–19.2	$\Lambda_{½M} < 0$ and $\beta A < 2$
	i	0.7	1.91	–0.67	0	–14.0	–19.2	$\Lambda_{½M} < 0$ and $\beta A < 2$
	j	0.7	1.91	–0.67	–3.6	–14.0	–19.2	No calculation, $\Lambda_0 < 0$
	h	0.8	1.60	–0.67	4	–14.0	–22.6	No calculation, $\Lambda_{½M} < -20°$
	i	0.8	1.60	–0.67	0	–14.0	–22.6	No calculation, $\Lambda_{½M} < -20°$
	j	0.8	1.60	–0.67	–3.6	–14.0	–22.6	No calculation, $\Lambda_{½M} < -20°$ and $\Lambda_0 < 0$

C4.1 Example C.1

Input:

```
EXAMPLE 1 IN APPENDIX C, ( INPUT FILE I01A7011, OUTPUT FILE R01A7011 )
DATA FOR INPUT IN EXAMPLE OF SECTION 6 OF ITEM No. 70011
PLANFORM a
1
0.6
1
3
0.5
1
60
1
1
R
```

Output:

```
****************************************************************************

ESDU International plc

Program                A7011

ESDUpac Number:        A7011
ESDUpac Title:         Lift-curve slope and aerodynamic centre
                       position of wings in inviscid subsonic flow
Data Item Number:      70011
Data Item Title:       Lift-curve slope and aerodynamic centre
                       position of wings in inviscid subsonic flow
ESDUpac Version:       2.1 Issued August 1996, Data Item Amendment I

See Data Item for full input/output specification and interpretation.

****************************************************************************

INPUT DATA
==========

EXAMPLE 1 IN APPENDIX C, ( INPUT FILE I01A7011, OUTPUT FILE R01A7011 )
DATA FOR INPUT IN EXAMPLE OF SECTION 6 OF ITEM No. 70011
PLANFORM a

INPUT DATA ERRORS
-----------------

      No error detected


OUTPUT DATA
===========

----------------------------------------------------------------------------
   M        A      n     Ln     Taper   BetaA    AtanLh    dCL/da   xb/cbb xb0/cr
                        (deg)                              (per rad)
----------------------------------------------------------------------------
 .600     3.000   .500  60.00 1.000     2.40       5.20     2.322    .179   1.478

----------------------------------------------------------------------------

END OF OUTPUT  ------------------------------------------------------------
```

C4.2 Example C.2

Input:

```
EXAMPLE 2 IN APPENDIX C, ( INPUT FILE I02A7011, OUTPUT FILE R02A7011 )
PLANFORMS b , c , d  : QUARTER-CHORD SWEEP DEFINED
dCL/da  per rad
1
0.6
1
6
0.25
3
0,25,50
1
0.25
R
```

Output:

```
****************************************************************************

ESDU International plc

Program              A7011

ESDUpac Number:      A7011
ESDUpac Title:       Lift-curve slope and aerodynamic centre
                     position of wings in inviscid subsonic flow
Data Item Number:    70011
Data Item Title:     Lift-curve slope and aerodynamic centre
                     position of wings in inviscid subsonic flow
ESDUpac Version:     2.1 Issued August 1996, Data Item Amendment I

See Data Item for full input/output specification and interpretation.

****************************************************************************

INPUT DATA
==========

EXAMPLE 2 IN APPENDIX C, ( INPUT FILE I02A7011, OUTPUT FILE R02A7011 )
PLANFORMS b , c , d  : QUARTER-CHORD SWEEP DEFINED
dCL/da  per rad

INPUT DATA ERRORS
-----------------

      No error detected
```

```
OUTPUT DATA
===========

----------------------------------------------------------------------------
   M        A      n     Ln     Taper   BetaA    AtanLh    dCL/da   xb/cbb xb0/cr
                        (deg)                             (per rad)
----------------------------------------------------------------------------
  .600     6.000  .250    .00   .250    4.80       -.60$     4.992   .244   .246

  .600     6.000  .250  25.00   .250    4.80       2.20      4.788   .295   .631

  .600     6.000  .250  50.00   .250    4.80       6.55&&

----------------------------------------------------------------------------

&& WARNING: AtanLh > 6. Out of range of available data.

 $ WARNING: Effective mid-chord line swept forward between 0 and -20
            deg. Data for dCL/da obtained using Atan|Lh|. Data for
            xb/cbb obtained by interpolation with data for forward sweep.

END OF OUTPUT  ------------------------------------------------------------
```

C4.3 Example C.3

Input:

```
EXAMPLE 3 IN APPENDIX C, ( INPUT FILE I03A7011, OUTPUT FILE R03A7011 )
PLANFORMS e , f , g  : QUARTER-CHORD SWEEP DEFINED
dCL/da per degree
1
0.4
3
7,9,14
0.25
1
0
1
0.6
D
```

Output:

```
****************************************************************************

ESDU International plc

Program               A7011

ESDUpac Number:       A7011
ESDUpac Title:        Lift-curve slope and aerodynamic centre
                      position of wings in inviscid subsonic flow
Data Item Number:     70011
Data Item Title:      Lift-curve slope and aerodynamic centre
                      position of wings in inviscid subsonic flow
ESDUpac Version:      2.1 Issued August 1996, Data Item Amendment I

See Data Item for full input/output specification and interpretation.
```

```
***************************************************************************

INPUT DATA
==========

EXAMPLE 3 IN APPENDIX C, ( INPUT FILE I03A7011, OUTPUT FILE R03A7011 )
PLANFORMS e , f , g  : QUARTER-CHORD SWEEP DEFINED
dCL/da per degree

INPUT DATA ERRORS
-----------------

      No error detected



OUTPUT DATA
===========

---------------------------------------------------------------------------
   M        A      n     Ln    Taper   BetaA    AtanLh   dCL/da   xb/cbb xb0/cr
                        (deg)                           (per deg)
---------------------------------------------------------------------------
 .400     7.000  .250    .00   .600    6.42        -.25$    .0843   .241   .243

 .400     9.000  .250    .00   .600    8.25**      -.25$    .0906   .243   .245

 .400    14.000  .250    .00   .600   12.83&

---------------------------------------------------------------------------

** WARNING: BetaA > 8. Data for xb/cbb have been obtained by
            extrapolation, see Sections 3 and 4 of Data Item No. 70011.

 & WARNING: BetaA > 12. Out of range of available data.

 $ WARNING: Effective mid-chord line swept forward between 0 and -20
            deg. Data for dCL/da obtained using Atan|Lh|. Data for
            xb/cbb obtained by interpolation with data for forward sweep.

END OF OUTPUT  ------------------------------------------------------------
```

C4.4 Example C.4

Input:

```
EXAMPLE 4 IN APPENDIX C, ( INPUT FILE I04A7011, OUTPUT FILE R04A7011 )
PLANFORM  i  :  MID-CHORD SWEEP DEFINED
dCL/da per rad
4
0.4,0.6,0.7,0.8
1
2.67
0.5
1
-14
1
0.5
R
```

Output:

```
*****************************************************************************

ESDU International plc

Program               A7011

ESDUpac Number:       A7011
ESDUpac Title:        Lift-curve slope and aerodynamic centre
                      position of wings in inviscid subsonic flow
Data Item Number:     70011
Data Item Title:      Lift-curve slope and aerodynamic centre
                      position of wings in inviscid subsonic flow
ESDUpac Version:      2.1 Issued August 1996, Data Item Amendment I

See Data Item for full input/output specification and interpretation.

*****************************************************************************

INPUT DATA
==========

EXAMPLE 4 IN APPENDIX C, ( INPUT FILE I04A7011, OUTPUT FILE R04A7011 )
PLANFORM  i  :  MID-CHORD SWEEP DEFINED
dCL/da per rad

INPUT DATA ERRORS
-----------------

      No error detected

OUTPUT DATA
===========

-----------------------------------------------------------------------------
   M        A      n     Ln     Taper   BetaA    AtanLh   dCL/da   xb/cbb xb0/cr
                        (deg)                            (per rad)
-----------------------------------------------------------------------------
 .400      2.670  .500 -14.00  .500     2.45       -.67$     3.091   .234   .182

 .600      2.670  .500 -14.00  .500     2.14       -.67$     3.247   .230   .179

 .700x     2.670  .500 -14.00  .500     1.91*      -.67$     3.366   .225   .175

 .800x     2.670  .500 -14.00  .500     1.60*      -.67$$

-----------------------------------------------------------------------------

 * WARNING: 0 <= BetaA <= 2. Use results with caution, see Sections 3 and 4
            of Data Item No. 70011.

 $ WARNING: Effective mid-chord line swept forward between 0 and -20
            deg. Data for dCL/da obtained using Atan|Lh|. Data for
            xb/cbb obtained by interpolation with data for forward sweep.

$$ WARNING: Effective mid-chord line swept forward more than 20 deg:
            Method not applicable.
```

```
 x WARNING: High free-stream Mach number.   The Item neither predicts nor
            caters for cases where the wing flow is supercritical, and if
            this is likely the results should not be relied on.  See also
            discussions in Section 4 and Appendix A of Data Item No.70011.

END OF OUTPUT  ------------------------------------------------------------
```

C4.5 Example C.5

Input:

```
EXAMPLE 5 IN APPENDIX C, ( INPUT FILE I05A7011, OUTPUT FILE R05A7011 )
PLANFORMS h , i , j  : MID-CHORD SWEEP DEFINED
dCL/da per rad
2
0.7,0.8
1
2.67
0.5
1
-14
3
0.4,0.5,0.6
R
```

Output:

```
****************************************************************************

ESDU International plc

Program               A7011

ESDUpac Number:       A7011
ESDUpac Title:        Lift-curve slope and aerodynamic centre
                      position of wings in inviscid subsonic flow
Data Item Number:     70011
Data Item Title:      Lift-curve slope and aerodynamic centre
                      position of wings in inviscid subsonic flow
ESDUpac Version:      2.1 Issued August 1996, Data Item Amendment I

See Data Item for full input/output specification and interpretation.

****************************************************************************

INPUT DATA
==========

EXAMPLE 5 IN APPENDIX C, ( INPUT FILE I05A7011, OUTPUT FILE R05A7011 )
PLANFORMS h , i , j  : MID-CHORD SWEEP DEFINED
dCL/da per rad

INPUT DATA ERRORS
-----------------

      No error detected
```

OUTPUT DATA
===========

M	A	n	Ln (deg)	Taper	BetaA	AtanLh	dCL/da (per rad)	xb/cbb	xb0/cr
.700x	2.670	.500	-14.00	.400	1.91*	-.67$	3.362	.222	.194
.700x	2.670	.500	-14.00	.500	1.91*	-.67$	3.366	.225	.175
.700x	2.670	.500	-14.00	.600	1.91*	-.67#			
.800x	2.670	.500	-14.00	.400	1.60*	-.67$$			
.800x	2.670	.500	-14.00	.500	1.60*	-.67$$			
.800x	2.670	.500	-14.00	.600	1.60*	-.67##			

* WARNING: 0 <= BetaA <= 2. Use results with caution, see Sections 3 and 4 of Data Item No. 70011.

$ WARNING: Effective mid-chord line swept forward between 0 and -20 deg. Data for dCL/da obtained using Atan|Lh|. Data for xb/cbb obtained by interpolation with data for forward sweep.

$$ WARNING: Effective mid-chord line swept forward more than 20 deg: Method not applicable.

\# WARNING: Leading edge swept forward: Method not applicable.

\## WARNING: Leading edge swept forward and effective mid-chord line swept forward more than 20 deg: Method not applicable.

x WARNING: High free-stream Mach number. The Item neither predicts nor caters for cases where the wing flow is supercritical, and if this is likely the results should not be relied on. See also discussions in Section 4 and Appendix A of Data Item No.70011.

END OF OUTPUT --

THE PREPARATION OF THIS DATA ITEM

The work on this particular Item, which supersedes Item Nos Aero W.01.01.01 and W.08.01.01 and in part Item Nos Aero W.S. 01.03.03-06 and W.S. 08.01.02, was monitored and guided by the Aerodynamics Committee which first met in 1942 and now has the following membership:

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Miss J. Willaume	– Aérospatiale, Toulouse, France.

* Corresponding Member

The technical work in the assessment of the available information and the construction and subsequent development of the Data Item was carried out by

Mr P.D. Chappell	– Head of Aircraft Aerodynamics Group
Mr R.W. Gilbey	– Senior Engineer.