METHOD FOR THE RAPID ESTIMATION OF SPANWISE LOADING OF WINGS WITH CAMBER AND TWIST IN SUBSONIC ATTACHED FLOW

1. NOTATION AND UNITS

		SI	*British*
a_1	lift-curve slope	degree^{-1}	degree^{-1}
A	aspect ratio $(= 2s/\bar{c})$		
A^*	effective aspect ratio, for wings with curved leading edges, see Equation (4.4) or (4.5)		
B_i	coefficients used in estimation of zero-lift angle for cambered wing section, see Equation (3.1)	degree	degree
c	local chord	m	ft
$\bar{c}$	geometric mean chord, $\int_0^1 c \, d\eta$	m	ft
c_r	root chord	m	ft
c_r^*	effective root chord for wings with curved or cranked trailing edges, see Equation	m	ft
C_{LL}	local lift coefficient		
C_{LLT}	local lift coefficient due to wing twist		
$\bar{C}_{LT}$	lift coefficient due to wing twist		
$(\bar{C}_{LT})_{int}$	value of $\bar{C}_{LT}$ obtained by integration of local loading, see Equation (5.8)		
$\bar{C}_{LTA}$	lift coefficient due to unit twist of type A	degree^{-1}	degree^{-1}
$\bar{C}_{LTB}$	lift coefficient due to unit twist of type B	degree^{-1}	degree^{-1}
$\bar{C}_L$	overall lift coefficient		
$F(\eta, \bar{\eta})$, $G(\eta)$, $H(\eta)$	spanwise load functions, see Equation (4.3)		
K_A	effective twist parameter for twist of type A, see Equation (5.2)		

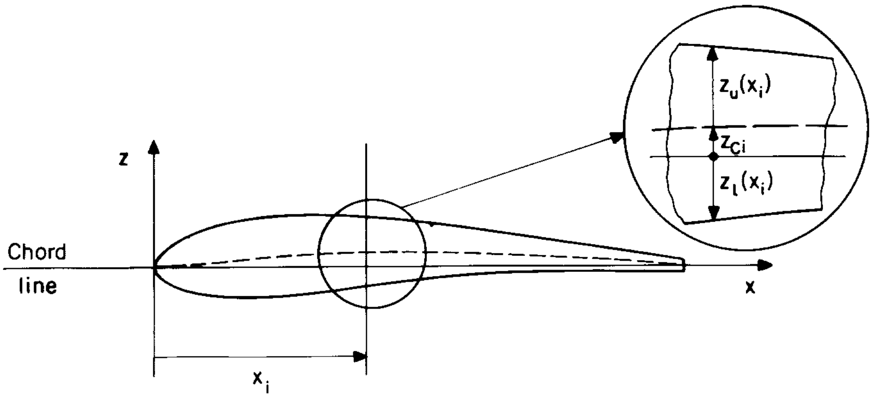
Issued November 1983
With Amendments A to C, July 1995

K_B	effective twist parameter for twist of type B, see Equation (5.2)		
M	Mach number		
s	wing semi-span	m	ft
x_i	chordwise co-ordinate of the i'th station measured from leading edge, see Sketch 1.1	m	ft
$x_T(\eta)$	distance of point on trailing edge measured in free-stream direction from root leading edge	m	ft
z_{Ci}	camber ordinate at x_i, see Equation (3.2)	m	ft
$z_l(x_i)$	lower-surface ordinate at x_i, measured from chord line, see Sketch 1.1	m	ft
$z_u(x_i)$	upper-surface ordinate at x_i, measured from chord line, see Sketch 1.1	m	ft
α	wing incidence measured from reference plane	degree	degree
α'	wing incidence measured from reference plane	radian	radian
α_C	incremental local twist angle for cambered wing section $(= -\alpha_0)$	degree	degree
α_E	effective local twist angle, see Section 5.2	degree	degree
α_G	local value of geometric twist	degree	degree
α_T	local value of total twist $(= \alpha_G + \alpha_C)$	degree	degree
α_{T0}	datum value of α_T, see Sketches 5.3 and 5.6	degree	degree
α_0	zero-lift angle of incidence for wing section, see Equation (3.1)	degree	degree
β	compressibility parameter, $(1 - M^2)^{½}$		
δ	wing twist angle relative to datum value, see Section 5	degree	degree
δ_A	root twist angle relative to datum value, α_{T0}, see Sketch 5.6	degree	degree
δ_B	tip twist angle relative to datum value, α_{T0}, see Sketch 5.6	degree	degree
η	spanwise distance from root as a fraction of semi-span		
$\bar{\eta}$	spanwise centre of pressure position, $\int_0^1 \sigma\eta\,d\eta$		

$\left.\begin{matrix}\eta_A\\ \eta_B\\ \eta_K\end{matrix}\right\}$	spanwise positions of intersection of twist segments, see Sketch 5.7		
η^*	value of $\bar{\eta}$ for wing with curved leading edge		
$\bar{\eta}_\Delta$	value of $\bar{\eta}$ for planform having required chord distribution and span, but with unswept trailing edge		
κ	taper parameter, $\int_0^1 \frac{c}{\bar{c}} \eta \, d\eta$		
λ	taper ratio, tip chord / c_r		
Λ_0	leading-edge sweep angle at $\eta = 0$	degree	degree
$\Lambda_{1/4}$	quarter-chord sweep angle at $\eta = 0$	degree	degree
$\Lambda_{1/2}$	mid-chord sweep angle at $\eta = 0$	degree	degree
Λ_1	trailing-edge sweep angle at $\eta = 0$	degree	degree
Λ_1^*	effective trailing-edge sweep angle for wing having cranked trailing edge, see Equation (4.2)	degree	degree
σ	spanwise loading due to incidence, $\dfrac{C_{LL}c}{\bar{C}_L\bar{c}}$		

Subscripts

$A1$	denotes twist of type A, with kink at η_K
$A2$	denotes twist of type A, with kink at η_A
$B1$	denotes twist of type B, with kink at η_K
$B2$	denote twist of type B with kink at η_B



Sketch 1.1

The chord line is defined as the straight line connecting the leading and trailing-edge points. For a section with a finite base thickness, the trailing-edge point is taken as the mid-thickness point. The leading-edge point is defined as that unique point at which a circle centred at the trailing-edge point is tangential to the section.

2. INTRODUCTION AND SCOPE OF ITEM

2.1 Introduction

A method is presented for the rapid estimation of the theoretical spanwise loading of wings with camber and twist in wholly subsonic flow*. It utilises the basic method of Derivation 2 which gives spanwise loading due to incidence for untwisted and uncambered wings and extends the procedure to take account of camber and twist. The effect of camber is treated by considering it as imparting an equivalent incremental twist to the wing. A simple artifice is used to represent the twist distribution by a combination of straight line distributions, from which values of effective local twist are derived for use in the method.

A step-by-step description of the calculation procedure to be followed for estimating the spanwise loading is given in Section 5.3.

2.2 Scope of Item

The method is applicable to a wide range of wing planforms in unseparated flow at subcritical Mach numbers. The simplifications adopted in accounting for camber and twist can involve some minor reservations on the choice of planform (see Section 6) and there are minor restrictions on the choice of leading-edge and trailing-edge shapes, see Section 4.

3. BASIS OF METHOD

3.1 Uncambered and Untwisted Wings

A rapid method for estimating the theoretical spanwise loading due to incidence was developed in Derivation 2, where a full account can be found. In subsonic compressible flow the method embodies a modified planform in which all spanwise dimensions are reduced by the factor β.

* A computer program, ESDUpac A9510, described in Item No. 95010 (Reference 7), is available that calculates the spanwise loading directly from steady lifting-surface theory using the Multhopp-Richardson solution.

Derivation 2 is applicable to wings for which $\beta A \le 8$, and the charts contained in it are reproduced in this Item. An additional chart has been generated using data obtained from Derivations 3 and 4 to extend the range of applicability to $\beta A = 12$. Details of the procedure are given in Section 4.

3.2 Cambered and Twisted Wings

The extension of Derivation 2 to cater for wings with camber and twist is based on factoring the spanwise leading due to constant unit incidence by an effective local twist angle, α_E. The angle α_E is related to the local total twist α_T that incorporates the effects of both camber and geometric twist. Sections 3.2.1 and 3.2.2 describe the calculation of α_T. The calculation of α_E is described in Section 5.

3.2.1 Representation of camber as a local twist

To cater for wings with camber, the spanwise distribution of the local zero-lift angle, α_0, must first be obtained. Values of α_0 are estimated by the method of Derivation 1. An approximate expression for the zero-lift angle of a cambered wing section is given by

$$\alpha_0 = -2 \sum_{i=1}^{i=14} B_i \frac{z_{Ci}}{c} \tag{3.1}$$

where B_i is obtained from Table 3.1 and z_{Ci} is the camber ordinate given by

$$z_{Ci} = \frac{z_u(x_i) + z_l(x_i)}{2} \tag{3.2}$$

as shown in Sketch 1.1.

The approximate value of zero-lift angle given by Equation (3.1) is then interpreted as an incremental local twist angle, $\alpha_C = -\alpha_0$.

TABLE 3.1

i	1	2	3	4	5	6	7
$(x/c)_i$	0	0.025	0.05	0.1	0.2	0.3	0.4
B_i	1.45	2.11	1.56	2.41	2.94	2.88	3.13
i	8	9	10	11	12	13	14
$(x/c)_i$	0.5	0.6	0.7	0.8	0.9	0.95	1.00
B_i	3.67	4.69	6.72	11.75	21.72	99.85	–164.88

3.2.2 Local total twist due to camber and geometric twist

The local total twist angle, α_T, is obtained by adding the geometric local twist, α_G, and the incremental local twist, α_C, resulting from wing camber. Thus

$$\alpha_T = \alpha_C + \alpha_G; \tag{3.3}$$

for cambered, untwisted wings, $\alpha_G = 0$ and $\alpha_T = \alpha_C$; for uncambered, twisted wings, $\alpha_C = 0$ and $\alpha_T = \alpha_G$.

4. ESTIMATION OF THEORETICAL SPANWISE LOADING DUE TO INCIDENCE

4.1 Scope of Method

The method used is that of Derivation 2*, extended to $\beta A = 12$. Neither $\bar{C}_L$ nor α is required as input. Charts used in the rapid estimation of the theoretical spanwise loading due to incidence are presented as Figures 1 to 8.

The spanwise centre of pressure position, $\bar{\eta}$, is derived from Figures 1 to 5 in terms of aspect ratio, mid-chord sweep, Mach number and a taper parameter. This taper parameter caters for arbitrary leading edges, but excludes wings with inverse taper $(\lambda > 1)$. The method has an upper limit of $\beta A = 12$, with mid-chord sweep angles such that $0 \le A \tan\Lambda_{1/2} \le 6$. The lower limit of βA is 1.5, except that for wings with straight leading and trailing edges and $A \tan\Lambda_{1/2} \le 2$ it is known that $\bar{\eta}$ converges rapidly to the value corresponding to elliptic loading $(\bar{\eta} = 0.4244)$ as $\beta A \to 0$. This may sometimes be used as an extra point on the plot required in step (iii) of Section 4.2.

The method should not be applied to planforms having cranked leading or trailing edges with discontinuities greater than about 40°.

4.2 Wings with Straight or Cranked Leading and Trailing Edges

The procedure of Derivation 2 is as follows.

(i) Evaluate the taper parameter

$$\kappa = \int_0^1 \frac{c}{\bar{c}} \eta \, d\eta \tag{4.1}$$

$$= \frac{1 + 2\lambda}{3(1 + \lambda)} \quad \text{for a straight tapered wing.}$$

(ii) Knowing κ and $A \tan\Lambda_{1/2}$, obtain $\bar{\eta}$ from Figures 1 to 5 for $\beta A = 1.5, 3, 5, 8$ and 12, respectively.

(iii) Plot $\bar{\eta}$ against βA to obtain $\bar{\eta}$ for the required value of M.

(iv) Evaluate $$\tan\Lambda_1^* = \int_0^1 \frac{x_T(\eta)}{s}(12\eta - 6)\, d\eta \tag{4.2}$$

$$= \tan\Lambda_0 - \frac{12}{A}(1 - 2\kappa) \quad \text{for a straight leading edge.}$$

Note that $\Lambda_1^* = \Lambda_1$ for wings with straight trailing edges.

* A computer program listing for the method of Derivation 2 is contained in an Addendum to that Derivation.

(v) Obtain $F(\eta, \bar{\eta})$, $G(\eta)$ and $H(\eta)$ from Figures 6, 7 and 8, for selected values of η corresponding to step (i) in Section 5.2.

(vi) Evaluate the local spanwise loading due to incidence from the equation

$$\sigma = \frac{C_{LL}c}{\bar{C}_L\bar{c}} = F(\eta, \bar{\eta}) + (A\tan\Lambda_1)G(\eta) + (\beta A - 4)(1 + 3.5\beta^{-1}\tan\Lambda_1^*)H(\eta) \quad (4.3)$$

where the last term is omitted if $\beta A \le 4$.

4.3 Swept Wings with Curved Leading Edges

Curved leading edges are defined here as having continuous slope, apart from the root, and streamwise wing tips of zero chord. The procedures used depend on the form of the trailing edge and are only applicable when the sweep angles Λ_0 and Λ_1 are approximately equal.

(a) Complete step (i) of Section 4.2.

(b) Evaluate the empirical parameter A^*.

For straight trailing edges

$$A^* = -1.2A + 8.8\left(\frac{2c_r}{s} + \tan\Lambda_1 - \tan\Lambda_0\right)^{-1}. \quad (4.4)$$

For curved or cranked trailing edges

$$A^* = -1.2A + 8.8\left(\frac{2c_r^*}{s} + \tan\Lambda_1^* - \tan\Lambda_0\right)^{-1} \quad (4.5)$$

where $$c_r^* = \int_0^1 x_T(\eta)(4 - 6\eta)d\eta \quad (4.6)$$

and $\tan\Lambda_1^*$ is obtained from Equation(4.2).

(c) Step (ii) of Section 4.2 now requires the evaluation of

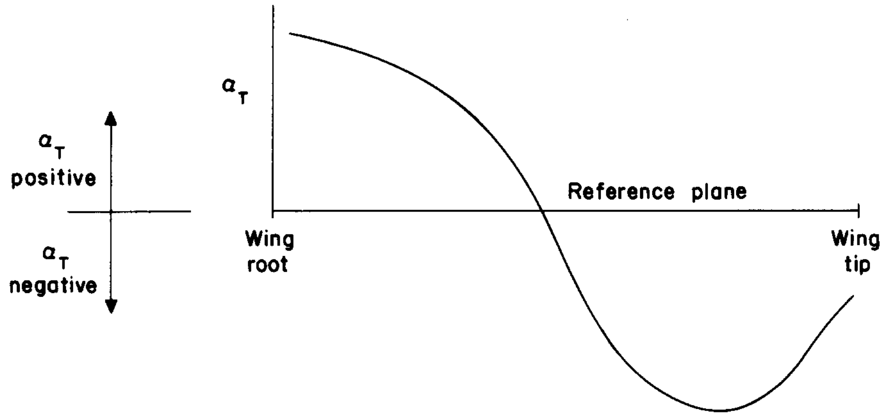
$$\bar{\eta}^* = (\bar{\eta} - \bar{\eta}_\Delta)\frac{A^*}{A} + \bar{\eta}_\Delta, \quad (4.7)$$

where $\bar{\eta}_\Delta$ is obtained from Figures 1 to 5 with the true βA and κ from Equation (4.1) but with $A\tan\Lambda_{1/2} = 6(1-2\kappa)$.

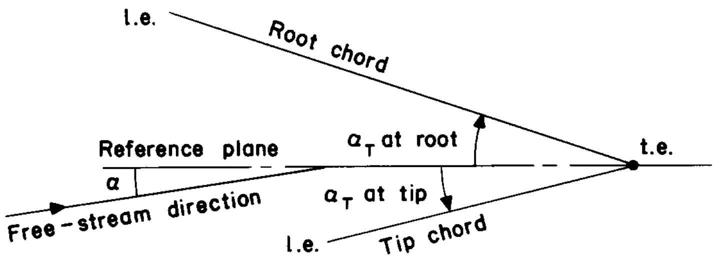
(d) Steps (iii) to (vi) of Section 4.2 follow with $\bar{\eta}$ replaced by $\bar{\eta}^*$.

5. ESTIMATION OF THEORETICAL SPANWISE LOADING FOR CAMBERED AND TWISTED WINGS

The wing incidence, α, and the spanwise variation of α_T must be defined relative to a common reference plane (see Sketches 5.1 and 5.2).



Sketch 5.1 View in the longitudinal direction

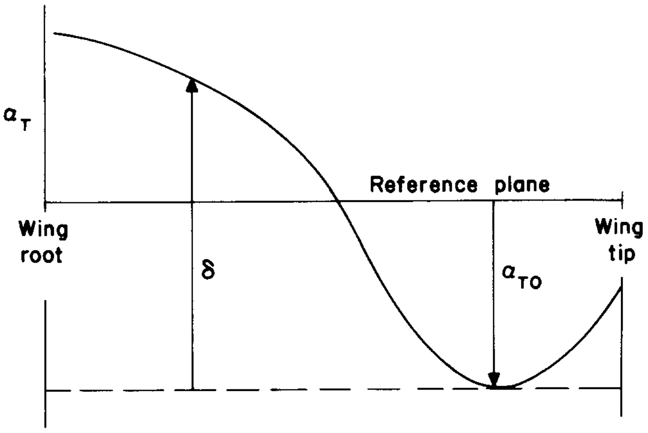


Sketch 5.2 *View in the lateral direction

It is implicit that when the reference plane lies in the stream direction the wing is at zero incidence $(\alpha = 0)$.

For the purpose of this Item α_T is defined in terms of a datum value, α_{T0}, and the twist, δ, relative to this datum value (see Sketch 5.3).

* For purposes of clarity in Sketch 5.2, the root and tip chords are shown as having a common trailing edge (t.e.) location lying in the reference plane. In practice this will not usually be the case.

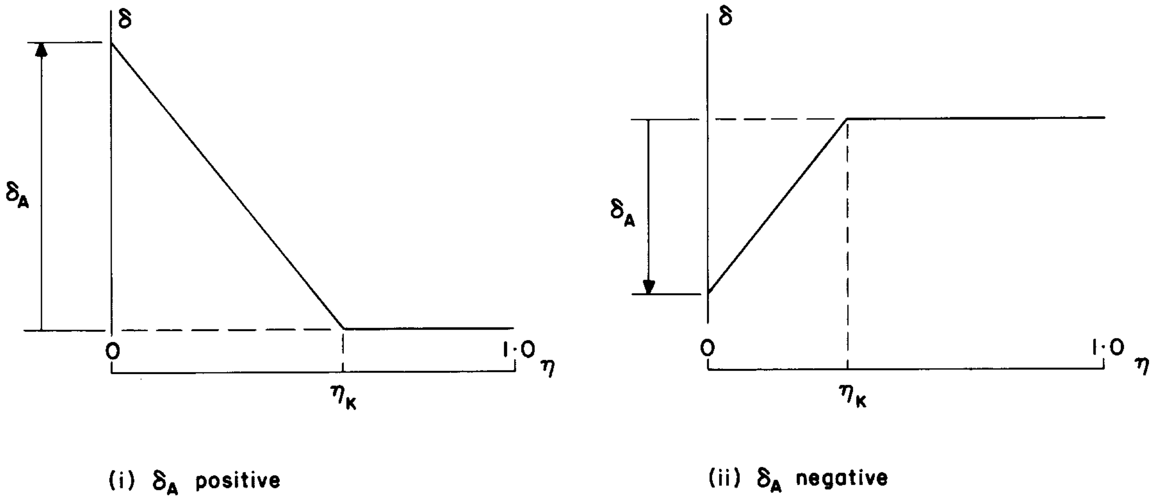


Sketch 5.3 Definition of α_T in terms of α_{T0} and δ (Note that in this sketch α_{T0} is negative)

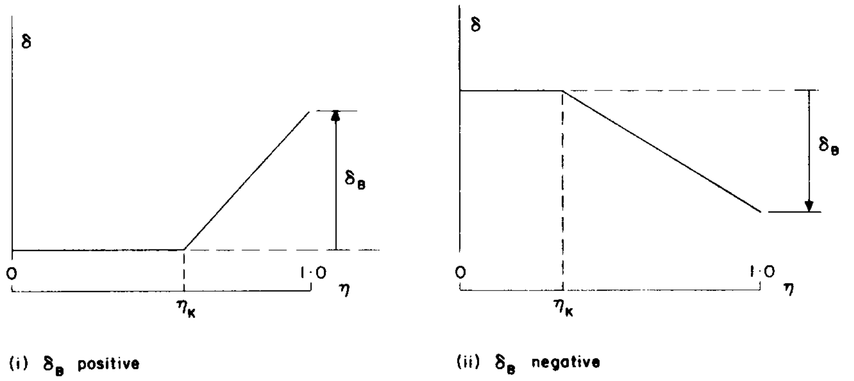
The lift distribution at $\alpha = 0$ then consists of contributions due to α_{T0} and δ. The contribution due to α_{T0} is simply an incidence loading and may be determined as such. Thus it remains to determine the spanwise loading due to the spanwise twist distribution, δ. The procedure adopted here is described in Sections 5.1 to 5.3.

5.1 Representation of Twist Distribution by Linear Segments

As stated in Section 3.2, the method requires a distribution of effective twist, α_E, to be derived from the distribution of twist, α_T. To do this, α_T must first be represented by a series of contiguous linear segments. Two basic types of linear twist distribution are used, as defined in Sketches 5.4 and 5.5 in which δ_A and δ_B respectively are the values of δ at the wing root and wing tip.



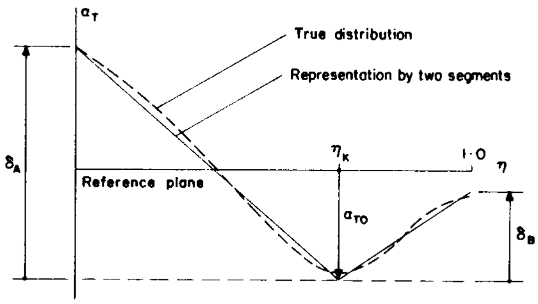
Sketch 5.4 Twist of Type A



Sketch 5.5 Twist of Type B

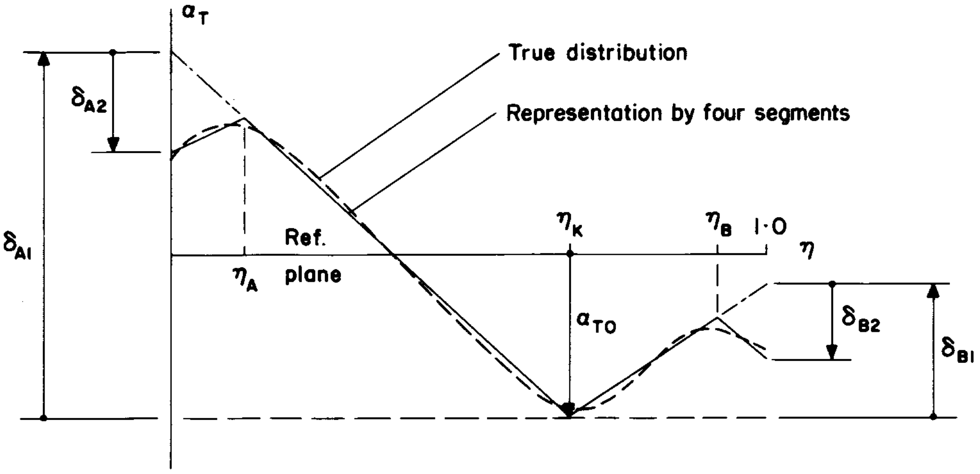
In principle a given twist distribution can be represented by a number of linear segments, but it is likely that most distributions encountered in practice can be adequately represented by up to four linear segments and only such cases are considered here. A two-segment representation, for example, could be used for the twist distribution illustrated in Sketch 5.6. The extension of the method for an arbitrary number of segments is considered in Appendix A.

In Sketch 5.6, α_{T0} is the value of α_T at $\eta = \eta_K$ derived from the two-segment representation, and is negative as drawn. For this illustration δ_A and δ_B are positive. The parameters δ_A, δ_B, η_K and α_{T0} then define the twist distribution across the span (see Example 1 in Section 8.1).



Sketch 5.6

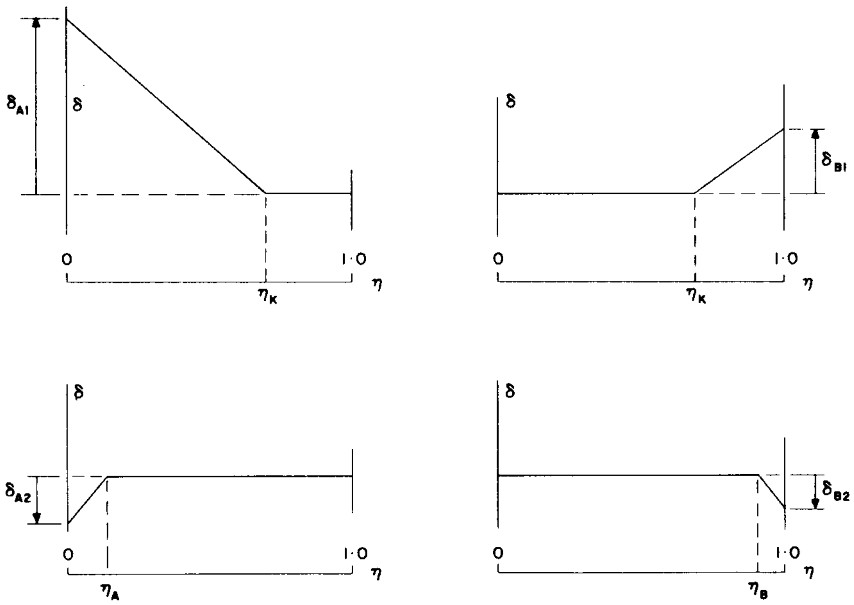
Many practical twist distributions have marked changes in twist near the wing root and tip. For such cases four linear segments can often be used to represent the twist, as shown in Sketch 5.7.



Sketch 5.7

In Sketch 5.7, δ_{A1} and δ_{B1} are positive as drawn, whilst δ_{A1}, δ_{B2} and α_{T0} (the common value of α_T where the segments join at η_K) are negative as drawn. The parameters δ_{A1} and δ_{B1} are obtained by extrapolating the appropriate segments to the root $(\eta = 0)$ and tip $(\eta = 1)$ respectively. The parameters δ_{A1}, δ_{A2}, δ_{B1}, δ_{B2}, η_A, η_B, η_K and α_{T0} then define the twist distribution across the span (see Example 3 in Section 8.3).

It is immediately clear that the full line in Sketch 5.7 comprises two twist distributions of type A and two of type B as shown in Sketch 5.8.



Sketch 5.8

5.2 Evaluation of Effective Local Twist

To determine the incremental spanwise loading, $C_{LLT}(c/\bar{c})$, due to α_T, an effective local twist, α_E, is defined such that

$$C_{LLT}\frac{c}{\bar{c}} = \sigma a_1 \alpha_E , \tag{5.1}$$

where the spanwise load distribution due to incidence, σ, is determined from Section 4 and the lift-curve slope, a_1, is determined from Reference 5. In setting up a procedure for the rapid determination of α_E, the method of Derivation 4 has been used to calculate values of $C_{LLT}(c/\bar{c})$ for a wide range of wing planforms having twist distributions of types A and B. The results have been collapsed in terms of effective twist parameters K_A and K_B by substituting into Equation (5.1)

$$\alpha_E = \alpha_T + \delta_A K_A + \delta_B K_B . \tag{5.2}$$

To a very good approximation (see Section 6), the parameters K_A and K_B depend only on βA and η_K and they are given numerically in Tables 9.1 to 9.5. Their use for two- and four-segment linear twist, distributions is described in step (vi) of Section 5.3. Their use for an arbitrary number of segments is outlined in Appendix A.

5.3 Procedure for Estimating Spanwise Loading for Wings with Camber and Twist

(i) Select from those in Tables 9.1 to 9.5 a number of spanwise locations, η, at which the spanwise loading is to be determined.

(ii) Obtain the theoretical spanwise loading due to incidence, σ, for the required planform and Mach number, as detailed in Section 4.

(iii) Use Reference 5 to derive $(d\bar{C}_L/d\alpha')/A$ which is a function of βA, λ and $A\tan\Lambda_{1/2}$, and hence $d\bar{C}_L/d\alpha'$. For wings with cranked leading or trailing edges, or with curved leading edges, Reference 6 will be required to determine an equivalent planform for use in conjunction with Reference 5.

(iv) Calculate the equivalent local twist, α_C, due to camber from the method in Section 3.2.1 which must be applied for each of the spanwise locations chosen in step (i). Derive values of total twist angle, α_T, by the addition of α_C to the local geometric twist, α_G.

(v) Represent α_T by linear segments as illustrated in Section 5.1.

The remaining steps must be evaluated for each spanwise location chosen in step (i).

(vi) Evaluate the effective local twist angle as follows.

(a) For two linear segments, as shown in Sketch 5.6*,

$$\alpha_E = \alpha_T + \delta_A K_A + \delta_B K_B , \tag{5.3}$$

where K_A and K_B are obtained from Tables 9.1 to 9.5 as functions of βA and η_K. In using Tables 9.1 to 9.5 linear interpolation is satisfactory for values of βA or η_K intermediate to those given, except for low values of η_K.

(b) For four linear segments, as shown in Sketch 5.7*,

* The values of α_T used in Equations (5.3) and (5.4) must be those from the linear representation. The use of the true values in calculating α_E, while having certain attractions, would be inconsistent with the values of $\bar{C}_{LT}$ developed in step (vii).

$$\alpha_E = \alpha_T + \delta_{A1} K_{A1} + \delta_{A2} K_{A2} + \delta_{B1} K_{B1} + \delta_{B2} K_{B2} \tag{5.4}$$

where, using linear interpolation if satisfactory, K_{A1} and K_{A2} are values of K_A obtained from Tables 9.1 to 9.5 as functions of βA and, respectively, η_K and η_A; similarly K_{B1} and K_{B2} are values of K_B obtained from Tables 9.1 to 9.5 as functions of βA and η_K and βA and η_B respectively.

(vii) Derive the incremental lift coefficient due to twist.

(a) For two linear segments

$$\bar{C}_{LT} = \delta_A \bar{C}_{LTA} + \delta_B \bar{C}_{LTB} + \alpha_{T0} a_1 \tag{5.5}$$

where $\bar{C}_{LTA}$ and $\bar{C}_{LTB}$ are obtained from Tables 9.1 to 9.5 as functions of βA and η_K using linear interpolation where necessary. It may be noted that $\bar{C}_{LTA}$ and $\bar{C}_{LTB}$ are the total lift coefficients per degree for twists of type A and type B respectively in the special case where $\alpha_{T0} = 0$ (see Sketch 5.6).

(b) For four linear segments

$$\bar{C}_{LT} = \delta_{A1} \bar{C}_{LTA1} + \delta_{A2} \bar{C}_{LTA2} + \delta_{B1} \bar{C}_{LTB1} + \delta_{B2} \bar{C}_{LTB2} + \alpha_{T0} a_1 \tag{5.6}$$

where $\bar{C}_{LTA1}$, $\bar{C}_{LTA2}$, $\bar{C}_{LTB1}$ and $\bar{C}_{LTB2}$ are also obtained from Tables 9.1 to 9.5 in a manner similar to that used to obtain K_{A1}, K_{A2}, K_{B1} and K_{B2} in step (vi).

In both Equations (5.5) and (5.6), $a_1 = (\mathrm{d}\bar{C}_L/\mathrm{d}\alpha')\pi/180$ is the lift-curve slope per degree, $\mathrm{d}\bar{C}_L/\mathrm{d}\alpha'$ having been determined in step (iii).

(viii) Calculate the spanwise loading due to twist as follows

$$C_{LLT}\frac{c}{\bar{c}} = \sigma a_1 \alpha_E . \tag{5.7}$$

where σ is given by Equation (4.3) and α_E is given by Equations (5.3) and (5.4).

(ix) Calculate the spanwise loading for a wing with twist.

(a) For a given incidence

$$C_{LL}\frac{c}{\bar{c}} = \sigma a_1(\alpha + \alpha_E) = \sigma a_1 \alpha + C_{LLT}\frac{c}{\bar{c}} . \tag{5.8}$$

The total lift coefficient is given by

$$\bar{C}_L = \int_0^1 C_{LL} \frac{c}{\bar{c}} \mathrm{d}\eta$$

which, within the accuracy of the method given here, is identically equal to

$$\bar{C}_L = a_1 \alpha + \bar{C}_{LT} . \tag{5.9}$$

The assumption in the method of this Item is that $\bar{C}_{LT}$ in Equation (5.9) is also given to a satisfactory accuracy by Equation (5.5) or (5.6). In practice it is prudent to check the adequacy of the assumption by comparing the value of $\bar{C}_{LT}$ given by Equation (5.5) or (5.6) with that, $(\bar{C}_{LT})_{int}$, given by an integration of the spanwise loading due to twist, *i.e.*

$$(\bar{C}_{LT})_{int} = \int_0^1 C_{LLT} c / \bar{c} \, \mathrm{d}\eta . \tag{5.10}$$

If the two values are significantly different, by more than the likely error in the integration procedure, 0.0001, say, then the value of $(\bar{C}_{LT})_{int}$ should be used in Equation (5.9) instead of $\bar{C}_{LT}$, for consistency with the spanwise load distribution. In such cases, however, the results should be treated with some caution since the differences between the values of $\Lambda_{1/4}$ and λ for the wing and the datum values of 25° and 0.4 could be significant, see Section 6.

(b) For a given lift coefficient the incidence is given by

$$\alpha = (\bar{C}_L - \bar{C}_{LT}) / a_1 \text{ degrees.} \tag{5.11}$$

Therefore, from Equation (5.8)

$$C_{LL} \frac{c}{\bar{c}} = \sigma(\bar{C}_L - \bar{C}_{LT}) + C_{LLT} \frac{c}{\bar{c}} \tag{5.12}$$

where $\bar{C}_{LT}$ is given by Equation (5.5) or (5.6) and $C_{LLT} c / \bar{c}$ is given by Equation (5.7).

As with (a), for a given incidence, it is prudent to check the compatibility of the value of $\bar{C}_{LT}$ with the value of $(\bar{C}_{LT})_{int}$ obtained via Equation (5.10). As before, if the two values are significantly different, by more than the likely error in the integration procedure, 0.0001, say, then the value of $(\bar{C}_{LT})_{int}$ should by substituted for $\bar{C}_{LT}$ in Equation (5.12). This will at least ensure that the ensuing total spanwise load distribution will be compatible with the requested value of $\bar{C}_L$. As with (a), the results should be applied with caution for such cases.

6. ACCURACY

The accuracy of that part of the method providing the spanwise loading due to incidence (Section 4) is illustrated in Section 5 of Derivation 2 in which results using the rapid method are compared with results from lifting-surface theory and with experimental data.

The accuracy of that part of the method providing the spanwise loading due to camber and twist (Section 5) is comparable with that for the spanwise loading due to incidence. The effective twist parameters (K_A and K_B) in Tables 9.1 to 9.5 have been derived for βA values ranging from 1.5 to 12, but they correspond to fixed values $\lambda = 0.4$ and $\Lambda_{1/4} = 25°$. However, calculations of K_A and K_B for other wing planforms have shown that sweep and taper have an effect on these parameters of secondary importance to that of βA. No significant deterioration in the accuracy of the method should occur for the range of sweep and taper parameters covered in this Item. However, some caution is advised if the sweep parameter, $A \tan \Lambda_{1/2}$, exceeds about 4 in combination with taper ratios much different from 0.4.

The adequacy of the method of this Item in estimating the spanwise loading due to camber and twist for any given case will obviously depend on the adequacy of the linear representation chosen to approximate the total twist distribution (α_T). For a wide range of practical twist distributions the two or four segment representations in Section 5.1 will be satisfactory. For those twist distributions which are not so satisfactorily represented the method of Appendix A should be used.

7. DERIVATION AND REFERENCES

7.1 Derivation

The Derivation lists selected sources that have assisted in the preparation of this Item.

1.	PANKHURST, R.C.	A method for the rapid evaluation of Glauert's expression for the angle of zero lift and the moment at zero lift. ARC R & M 1914, 1944.
2.	ESDU	Method for the rapid estimation of theoretical spanwise loading due to a change of incidence. T.D. Memo 6403, ESDU International Ltd, London, March 1964. Computer program Addendum, August 1983.
3.	GARNER, H.C.	Unpublished work at RAE (Farnborough), 1977.
4.	CAIRNS, I.C.D. CAME, N.J.	FORTRAN program of the Multhopp-Richardson method for estimating spanwise loadings. Unpublished. British Aerospace plc, Weybridge-Bristol Division, January 1980.

7.2 References

The References list selected sources of information supplementary to that given in this Item.

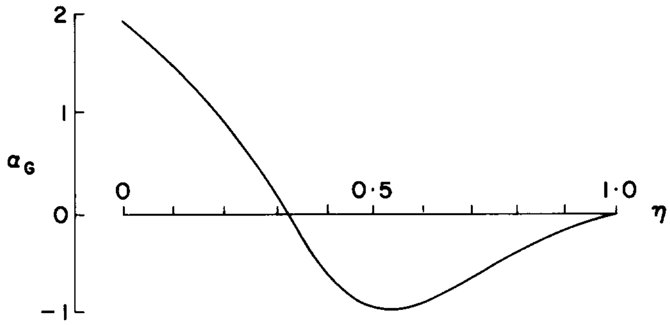
5.	ESDU	Lift-curve slope and aerodynamic centre position of wings in inviscid subsonic flow. Item No. 70011, Engineering Sciences Data Unit, London, 1970.
6.	ESDU	Geometrical properties of cranked and straight tapered wing planforms. Item No. 76003, Engineering Sciences Data Unit, London, 1976.

7. ESDU Computer program for estimation of spanwise loading of wings with camber and twist in subsonic attached flow. ESDU International, Item No. 95010, 1995. ESDUpac A9510.

8. EXAMPLES

8.1 Example 1

For $M = 0$ and $\alpha = 2°$ and $10°$, it is required to obtain the spanwise loading of, and the lift coefficient on, a straight tapered wing with $A = 8$, $\lambda = 0.4$ and $\Lambda_{1/4} = 25°$, without camber and with the twist distribution defined in Sketch 8.1.



Sketch 8.1

The steps in the calculation follow the procedure of Section 5.3.

(i) Select a number of spanwise locations for which spanwise loadings are to be produced.

Use $\eta = 0$ to 0.9 in steps of 0.1, $\eta = 0.95$ and 0.98.

(ii) Obtain the theoretical spanwise loading due to incidence, as detailed in Section 4.

Evaluate $$\kappa = \frac{1 + 2\lambda}{3(1 + \lambda)} \text{ for } \lambda = 0.4 .$$

$$\kappa = \frac{1 + 0.8}{3 \times 1.4} = 0.429 .$$

Derive $A \tan\Lambda_{1/2}$ and $A \tan\Lambda_1$.

From the planform geometry,

$$A \tan\Lambda_{1/2} = A \tan\Lambda_{1/4} - \left(\frac{1-\lambda}{1+\lambda}\right) = 3.7305 - 0.4286 = 3.302$$

and $$A \tan\Lambda_1 = A \tan\Lambda_{1/4} - 3\left(\frac{1-\lambda}{1+\lambda}\right) = 3.7305 - 1.2857 = 2.445 .$$

Derive $\bar{\eta}$.

Since $M = 0$, $\beta A = 8$ and, from Figure 4, $\bar{\eta} = 0.4374$.

Obtain the functions $F(\eta, \bar{\eta})$, $G(\eta)$, and $H(\eta)$ from Figures 6, 7 and 8.

η	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	0.98
$F(\eta, \bar{\eta})$	1.208	1.205	1.194	1.175	1.145	1.102	1.041	0.954	0.825	0.618	0.450	0.290
$G(\eta)$	–0.0148	–0.0060	0.0006	0.0048	0.0067	0.0065	0.0043	0.0009	–0.0031	–0.0060	–0.0059	–0.0044
$H(\eta)$	0.0030	0.0026	0.0013	–0.0004	–0.0022	–0.0034	–0.0036	–0.0023	0.0006	0.0040	0.0047	0.0038

Calculate the spanwise loading due to incidence.

$$\sigma = F(\eta, \bar{\eta}) + (A \tan \Lambda_1) G(\eta) + (\beta A - 4)(1 + 3.5\beta^{-1} \tan \Lambda_1^*) H(\eta) .$$

For this Example, this reduces to

$$\sigma = F(\eta, \bar{\eta}) + 2.445 G(\eta) + 8.279 H(\eta)$$

because $\Lambda_1^* = \Lambda_1$ for a straight tapered wing.

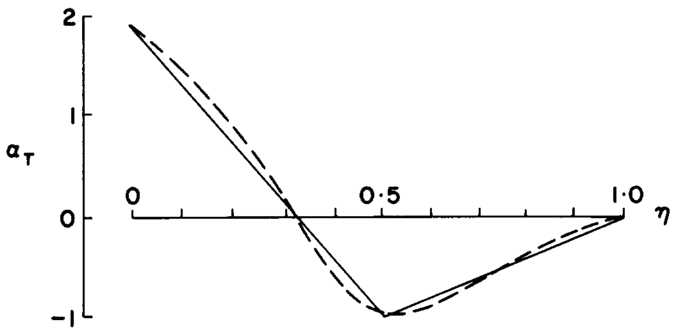
η	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	0.98
σ	1.197	1.212	1.206	1.183	1.143	1.090	1.022	0.937	0.822	0.636	0.474	0.311

(iii) From Reference 5, derive $(\mathrm{d}\bar{C}_L/\mathrm{d}\alpha')/A$.

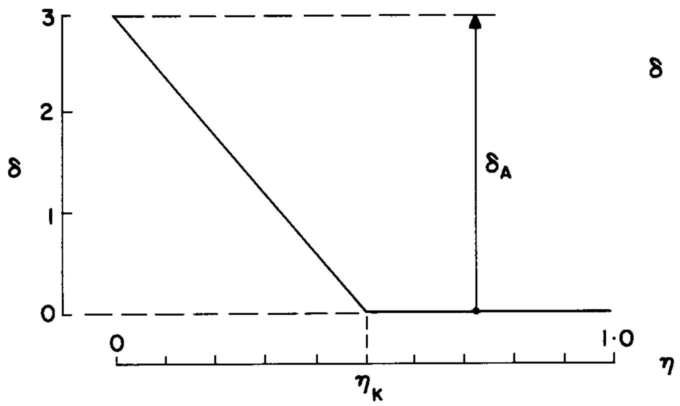
$$\frac{1}{A}\frac{\mathrm{d}\bar{C}_L}{\mathrm{d}\alpha'} = 0.563 \text{, giving } \frac{\mathrm{d}\bar{C}_L}{\mathrm{d}\alpha'} = 4.50 \text{ per radian.}$$

(iv) The wing is uncambered; thus $\alpha_C = 0$ and $\alpha_T = \alpha_G$.

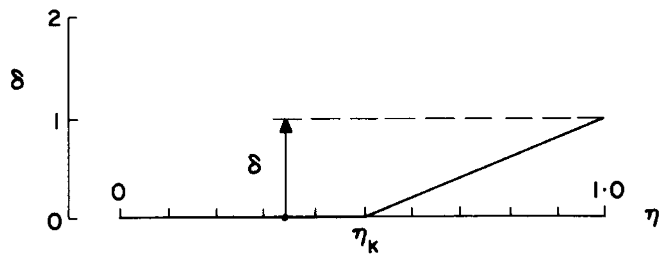
(v) Replace the twist distribution, α_T, by two linear segments as shown below.



This divides into twist distributions of types *A* and *B* as shown below, with $\alpha_{T0} = 1°$.



Type *A*: $\delta_A = 3°$, $\eta_K = 0.5$.



Type *B*: $\delta_B = 1°$, $\eta_K = 0.5$.

(vi) For each spanwise station, evaluate the effective local twist angle for a two linear segment case using Equation (5.3).

$$\alpha_E = \alpha_T + \delta_A K_A + \delta_B K_B ,$$

where $$\alpha_T = \delta_A\left(1 - \frac{\eta}{\eta_K}\right) + \alpha_{T0} \text{ for } 0 \le \eta \le \eta_K$$

and $$\alpha_T = \delta_B\left(\frac{\eta - \eta_K}{1 - \eta_K}\right) + \alpha_{T0} \text{ for } \eta_K \le \eta \le 1 .$$

Obtain K_A and K_B at the required spanwise positions from Table 9.4, for $\beta A = 8$, using $\eta_K = 0.5$. The calculations for α_E are summarised below.

η	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	0.98
α_T	2.0	1.4	0.8	0.2	–0.4	–1.0	–0.8	–0.6	–0.4	–0.2	–0.1	–0.04
K_A	–0.34	–0.18	–0.08	0.02	0.11	0.21	0.15	0.12	0.09	0.08	0.07	0.07
$\delta_A K_A$	–1.02	–0.54	–0.24	0.06	0.33	0.63	0.45	0.36	0.27	0.24	0.21	0.21
K_B	0.02	0.02	0.02	0.02	0.05	0.09	–0.05	–0.13	–0.20	–0.27	–0.29	–0.31
$\delta_B K_B$	0.02	0.02	0.02	0.02	0.05	0.09	–0.05	–0.13	–0.20	–0.27	–0.29	–0.31
α_E	1.00	0.88	0.58	0.28	–0.02	–0.28	–0.40	–0.37	–0.33	–0.23	–0.18	–0.14

(vii) Derive the incremental lift coefficient due to twist for two linear segments, using Equation (5.5) and Table 9.4 obtain $\bar{C}_{LTA}$ and $\bar{C}_{LTB}$.

$$\bar{C}_{LT} = \delta_A \bar{C}_{LTA} + \delta_B \bar{C}_{LTB} + \alpha_{T0} a_1$$

where $a_1 = \dfrac{d\bar{C}_L}{d\alpha'} \dfrac{\pi}{180} = 4.50 \times \dfrac{\pi}{180} = 0.0785$ per degree.

Thus $\bar{C}_{LT} = (3.0 \times 0.0254) + (1.0 \times 0.0112) - (1.0 \times 0.0785) = 0.0089$.

(viii) Calculate the spanwise loading due to twist using Equation (5.7).

$$C_{LLT}\frac{c}{\bar{c}} = \sigma a_1 \alpha_E .$$

η	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	0.98
$C_{LLT}\frac{c}{\bar{c}}$	0.094	0.084	0.055	0.026	–0.002	–0.024	–0.032	–0.027	–0.021	–0.011	–0.007	–0.003

It will be found that the value of $(\bar{C}_{LT})_{int}$ obtained by integrating the spanwise loading due to twist, see Equation (5.10), is in satisfactory agreement with the value of $(\bar{C}_{LT})$ from step (vii), as would be expected for a wing with the datum values of $\Lambda_{1/4}$ and λ, see Section 6.

(ix) Calculate the spanwise loading for a wing with twist using Equation (5.8), for $\alpha = 2°$ and $\alpha = 10°$.

$$C_{LLT}\frac{c}{\bar{c}} = \sigma a_1 \alpha + C_{LLT}\frac{c}{\bar{c}} .$$

The total lift coefficient is given by Equation (5.9)

$$\bar{C}_L = a_1 \alpha + \bar{C}_{LT} .$$

$\alpha°$	$\bar{C}_L$	η	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	0.98
2	0.166	$\sigma a_1 \alpha$	0.188	0.190	0.189	0.186	0.179	0.171	0.160	0.147	0.129	0.100	0.074	0.49
		$C_{LLT}\frac{c}{\bar{c}}$	0.094	0.084	0.055	0.026	–0.002	–0.024	–0.032	–0.027	–0.021	–0.011	–0.007	–0.003
		$C_{LL}\frac{c}{\bar{c}}$	0.282	0.274	0.244	0.212	0.177	0.147	0.128	0.120	0.108	0.089	0.067	0.046
10	0.794	$\sigma a_1 \alpha$	0.940	0.951	0.947	0.929	0.897	0.856	0.802	0.736	0.645	0.499	0.372	0.244
		$C_{LLT}\frac{c}{\bar{c}}$	0.094	0.084	0.055	0.026	–0.002	–0.024	–0.032	–0.027	–0.021	–0.011	–0.007	–0.003
		$C_{LL}\frac{c}{\bar{c}}$	1.034	1.035	1.002	0.955	0.895	0.832	0.770	0.709	0.624	0.488	0.365	0.241

8.2 Example 2

It is required to calculate the spanwise loading of the wing defined in Section 8.1, but at a given value of lift coefficient, $\bar{C}_L = 0.3$. Find also the corresponding incidence.

Steps (i) to (viii) are as for Example 1.

(ix) The incidence is calculated using Equation (5.11), and the values of $\bar{C}_{LT}$ and a_1 from step (vii) of Example 1.

$$\begin{aligned}\alpha &= (\bar{C}_L - \bar{C}_{LT})/a_1\\ &= (0.3 - 0.0089)0.0785\\ &= 3.71 \text{ degrees}\end{aligned}$$

and the spanwise loading for a wing with twist at a given value of $\bar{C}_L = 0.3$ then follows from Equation (5.12).

$$C_{LL}\frac{c}{\bar{c}} = \sigma(\bar{C}_L - \bar{C}_{LT}) + C_{LLT}\frac{c}{\bar{c}}$$

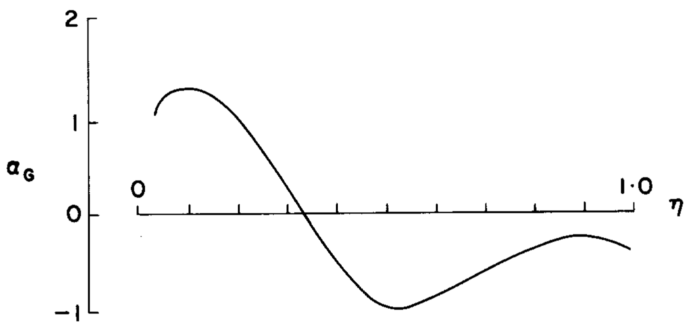
where σ and $C_{LLT}\frac{c}{\bar{c}}$ are obtained from steps (ii) and (viii) of Example 1.

η	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	0.98
σ	1.197	1.212	1.206	1.183	1.143	1.090	1.022	0.937	0.822	0.636	0.474	0.311
$\sigma(\bar{C}_L - \bar{C}_{LT})$	0.348	0.353	0.351	0.344	0.332	0.317	0.298	0.273	0.239	0.186	0.138	0.091
$C_{LLT}\frac{c}{\bar{c}}$	0.094	0.084	0.055	0.026	–0.002	–0.024	–0.032	–0.027	–0.021	–0.011	–0.007	–0.003
$C_{LLT}\frac{c}{\bar{c}}$	0.442	0.437	0.406	0.370	0.331	0.293	0.266	0.246	0.218	0.174	0.131	0.088

Since the wing planform and twist distribution are the same as in Example 1, the spanwise loading due to twist is compatible with the value of $\bar{C}_{LT}$, see step (viii) of Example 1. The overall spanwise loading is therefore also compatible with the required value of $\bar{C}_L = 0.3$.

8.3 Example 3

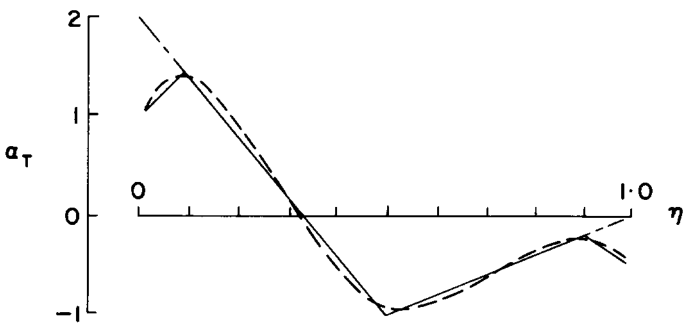
It is required to obtain the spanwise loading for the wing planform used in Example 1 but with the geometric twist distribution defined in Sketch 8.2, and at $\bar{C}_L = 0.3$ as in Example 2.



Sketch 8.2

Steps (i) to (iv) are the same as for Example 1.

(v) Replace the twist distribution, α_T, by four linear segments as shown below.

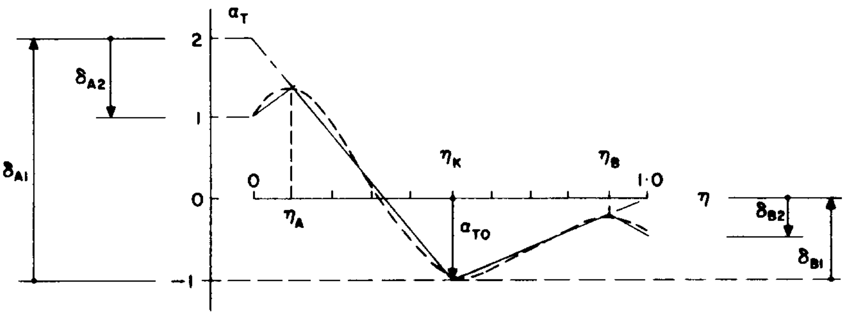


This divides into two twist distributions of type A and two of type B as in Sketch 5.8 with

$\alpha_{T0} = -1°$, such that

$$\delta_{A1} = 3.0°,\ \delta_{A2} = -1.0°,\ \eta_A = 0.1,\ \ \eta_K = 0.5,$$
$$\delta_{B1} = 1.0°,\ \delta_{B2} = -0.5°,\ \text{and}\ \eta_B = 0.9\,.$$

It will be noted that the twist distributions denoted by $A1$ and $B1$ and the value of α_{T0} are the same as those for Example 1.



(vi) For each spanwise station, evaluate the effective local twist angle using Equation (5.4).

$$\alpha_E = \alpha_T + \delta_{A1}K_{A1} + \delta_{A2}K_{A2} + \delta_{B1}K_{B1} + \delta_{B2}K_{B2}$$

where $\alpha_T = \delta_{A1}\left(1 - \dfrac{\eta}{\eta_K}\right) + \delta_{A2}\left(1 - \dfrac{\eta}{\eta_A}\right) + \alpha_{T0}$ for $0 \le \eta \le \eta_A$,

$$\alpha_T = \delta_{A1}\left(1 - \frac{\eta}{\eta_K}\right) + \alpha_{T0}\ \text{ for } \eta_A \le \eta \le \eta_K,$$

$$\alpha_T = \delta_{B1}\left(\frac{\eta - \eta_K}{1 - \eta_K}\right) + \alpha_{T0}\ \text{ for } \eta_K \le \eta \le \eta_B$$

and $\alpha_T = \delta_{B1}\left(\dfrac{\eta - \eta_K}{1 - \eta_K}\right) + \delta_{B2}\left(\dfrac{\eta - \eta_B}{1 - \eta_B}\right) + \alpha_{T0}$ for $\eta_B \le \eta \le 1$.

The values of K_{A1} , K_{A2} ; K_{B1} , K_{B2} are obtained from Table 9.4 at the required spanwise positions and values of η_K, η_A ; η_K, η_B respectively.

η	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	0.98
α_T	1.0	1.40	0.80	0.20	–0.40	–1.00	–0.80	–0.60	–0.40	–0.20	–0.35	–0.44
K_{A1}	–0.34	–0.18	–0.08	0.02	0.11	0.21	0.15	0.12	0.09	0.08	0.07	0.07
$\delta_{A1}K_{A1}$	–1.02	–0.54	–0.24	0.06	0.33	0.63	0.45	0.36	0.27	0.24	0.21	0.21
K_{A2}	–0.73	0.16	0.09	0.06	0.04	0.03	0.03	0.02	0.02	0.01	0.01	0.01
$\delta_{A2}K_{A2}$	0.73	–0.16	–0.09	–0.06	–0.04	–0.03	–0.03	–0.02	–0.02	–0.01	–0.01	–0.01
K_{B1}	0.02	0.02	0.02	0.02	0.05	0.09	–0.05	–0.13	–0.20	–0.27	–0.29	–0.31
$\delta_{B1}K_{B1}$	0.02	0.02	0.02	0.02	0.05	0.09	–0.05	–0.13	–0.20	–0.27	–0.29	–0.31
K_{B2}	0	0	0	0	0	0	0	0.01	0.02	0.08	–0.29	–0.49
$\delta_{B2}K_{B2}$	0	0	0	0	0	0	0	–0.01	–0.01	–0.04	0.15	0.25
α_E	0.73	0.72	0.49	0.22	–0.06	–0.31	–0.43	–0.40	–0.36	–0.28	–0.29	–0.30

(vii) Derive the incremental lift coefficient due to twist using Equation (5.6) and Table 9.4.

$$\bar{C}_{LT} = \delta_{A1}\bar{C}_{LTA1} + \delta_{A2}\bar{C}_{LTA2} + \delta_{B1}\bar{C}_{LTB1} + \delta_{B2}\bar{C}_{LTB2} + \alpha_{T0}a_1$$

where $a_1 = 0.0785$ per degree, as obtained in step (vii) of Example 1.

$$\bar{C}_{LT} = (3.0\times0.0254) + (-1.0\times0.0054) + (1.0\times0.0112) + (-0.5\times0.0013) + (-1.0\times0.0785)$$

$$= 0.0029 .$$

(viii) Calculate the spanwise loading due to twist using Equation (5.7).

$$C_{LLT}\frac{c}{\bar{c}} = \sigma a_1 \alpha_E .$$

η	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	0.98
$C_{LLT}\frac{c}{\bar{c}}$	0.069	0.069	0.046	0.020	–0.005	–0.027	–0.034	–0.029	–0.023	–0.014	–0.011	–0.007

As with Example 1, it will be found that the value of $(\bar{C}_{LT})_{int}$ obtained by integrating the spanwise loading due to twist, see Equation (5.10), is in satisfactory agreement with the value of $\bar{C}_{LT}$ from step (vii).

(ix) Calculate the spanwise loading for a wing with twist at $\bar{C}_L = 0.3$ using Equation (5.12).

$$C_{LL}\frac{c}{\bar{c}} = \sigma(\bar{C}_L - \bar{C}_{LT}) + C_{LLT}\frac{c}{\bar{c}}\,.$$

η	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	0.98
σ	1.197	1.212	1.206	1.183	1.143	1.090	1.022	0.937	0.822	0.636	0.474	0.311
$\sigma(\bar{C}_L - \bar{C}_{LT})$	0.356	0.360	0.358	0.351	0.340	0.324	0.304	0.278	0.244	0.189	0.141	0.092
$C_{LLT}\frac{c}{\bar{c}}$	0.069	0.069	0.046	0.020	–0.005	–0.027	–0.034	–0.029	–0.023	–0.014	–0.011	–0.007
$C_{LLT}\frac{c}{\bar{c}}$	0.425	0.429	0.404	0.371	0.355	0.297	0.270	0.249	0.221	0.175	0.130	0.085

9. TABLES

TABLE 9.1 Values of K_A and K_B for $\beta A = 1.5$

a. Values of K_A

η_K \ η	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	0.98	$\bar{C}_{LTA}$
0.1	–0.85	0.11	0.08	0.06	0.05	0.05	0.04	0.04	0.03	0.03	0.03	0.03	0.0023
0.2	–0.74	–0.28	0.16	0.13	0.11	0.09	0.08	0.07	0.07	0.06	0.06	0.06	0.0045
0.3	–0.65	–0.35	–0.07	0.20	0.17	0.14	0.13	0.11	0.10	0.09	0.09	0.09	0.0068
0.4	–0.58	–0.35	–0.15	0.04	0.23	0.20	0.17	0.15	0.14	0.12	0.12	0.11	0.0089
0.5	–0.52	–0.33	–0.18	–0.03	0.11	0.26	0.22	0.20	0.17	0.16	0.15	0.15	0.0111
0.6	–0.46	–0.31	–0.18	–0.06	0.05	0.16	0.28	0.24	0.21	0.19	0.19	0.18	0.0131
0.7	–0.41	–0.28	–0.17	–0.07	0.02	0.11	0.20	0.29	0.26	0.23	0.22	0.22	0.0151
0.8	–0.37	–0.26	–0.16	–0.07	0.01	0.09	0.16	0.23	0.31	0.27	0.26	0.25	0.0170
0.9	–0.33	–0.23	–0.15	–0.07	0	0.07	0.14	0.20	0.26	0.32	0.30	0.29	0.0188
1.0	–0.30	–0.21	–0.13	–0.06	0	0.06	0.12	0.18	0.23	0.28	0.31	0.33	0.0203

b. Values of K_B

η_K \ η	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	0.98	$\bar{C}_{LTB}$
0.1	0.24	0.24	0.16	0.08	0.01	–0.06	–0.12	–0.18	–0.24	–0.31	–0.33	–0.35	0.0128
0.2	0.19	0.19	0.21	0.11	0.03	–0.05	–0.12	–0.19	–0.26	–0.33	–0.36	–0.38	0.0108
0.3	0.15	0.15	0.16	0.18	0.07	–0.02	–0.11	–0.20	–0.28	–0.36	–0.38	–0.41	0.0090
0.4	0.11	0.11	0.12	0.13	0.15	0.03	–0.08	–0.19	–0.28	–0.38	–0.42	–0.45	0.0072
0.5	0.08	0.08	0.09	0.10	0.11	0.14	–0.02	–0.15	–0.28	–0.40	–0.46	–0.49	0.0055
0.6	0.06	0.06	0.06	0.06	0.07	0.09	0.12	–0.08	–0.25	–0.41	–0.49	–0.54	0.0040
0.7	0.04	0.04	0.04	0.04	0.05	0.05	0.07	0.10	–0.16	–0.40	–0.52	–0.58	0.0026
0.8	0.02	0.02	0.02	0.02	0.02	0.03	0.03	0.05	0.08	–0.32	–0.51	–0.62	0.0014
0.9	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.06	–0.38	–0.62	0.0005

TABLE 9.2 Values of K_A and K_B for $\beta A = 3$

a. Values of K_A

η_K \ η	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	0.98	$\bar{C}_{LTA}$
0.1	–0.82	0.12	0.08	0.06	0.05	0.04	0.04	0.03	0.03	0.03	0.02	0.02	0.0036
0.2	–0.70	–0.25	0.17	0.13	0.10	0.09	0.07	0.06	0.06	0.05	0.05	0.05	0.0071
0.3	–0.60	–0.31	–0.05	0.21	0.16	0.13	0.11	0.10	0.08	0.08	0.07	0.07	0.0106
0.4	–0.53	–0.31	–0.12	0.05	0.23	0.18	0.15	0.13	0.11	0.10	0.10	0.09	0.0140
0.5	–0.47	–0.29	–0.15	–0.02	0.11	0.24	0.20	0.17	0.15	0.13	0.12	0.12	0.0173
0.6	–0.41	–0.27	–0.15	–0.04	0.05	0.15	0.25	0.21	0.18	0.16	0.15	0.15	0.0204
0.7	–0.37	–0.24	–0.14	–0.05	0.02	0.10	0.17	0.26	0.22	0.19	0.18	0.17	0.0234
0.8	–0.33	–0.22	–0.13	–0.06	0.01	0.07	0.14	0.20	0.27	0.23	0.22	0.21	0.0263
0.9	–0.30	–0.20	–0.13	–0.05	0.01	0.06	0.11	0.17	0.22	0.28	0.26	0.25	0.0289
1.0	–0.27	–0.18	–0.11	–0.05	0	0.05	0.10	0.15	0.19	0.24	0.27	0.28	0.0313

b. Values of K_B

η_K \ η	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	0.98	$\bar{C}_{LTB}$
0.1	0.19	0.20	0.12	0.05	–0.02	–0.07	–0.13	–0.18	–0.23	–0.27	–0.29	–0.30	0.0192
0.2	0.15	0.15	0.17	0.09	0.01	–0.06	–0.13	–0.19	–0.24	–0.29	–0.32	–0.33	0.0162
0.3	0.11	0.12	0.13	0.15	0.05	–0.04	–0.11	–0.19	–0.25	–0.32	–0.34	–0.36	0.0134
0.4	0.08	0.08	0.09	0.11	0.13	0.02	–0.09	–0.18	–0.26	–0.34	–0.37	–0.39	0.0107
0.5	0.06	0.06	0.06	0.07	0.09	0.12	–0.02	–0.14	–0.26	–0.36	–0.40	–0.43	0.0082
0.6	0.04	0.04	0.04	0.05	0.06	0.07	0.11	–0.07	–0.23	–0.37	–0.43	–0.47	0.0059
0.7	0.02	0.02	0.03	0.03	0.03	0.04	0.06	0.10	–0.15	–0.36	–0.47	–0.52	0.0039
0.8	0.01	0.01	0.01	0.01	0.02	0.02	0.03	0.04	0.08	–0.30	–0.47	–0.56	0.0021
0.9	0	0	0	0	0.01	0.01	0.01	0.01	0.02	0.07	–0.35	–0.58	0.0008

TABLE 9.3 Values of K_A and K_B for $\beta A = 5$

a. Values of K_A

η_K \ η	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	0.98	$\bar{C}_{LTA}$
0.1	–0.78	0.14	0.09	0.07	0.05	0.04	0.03	0.03	0.02	0.02	0.02	0.02	0.0046
0.2	–0.65	–0.21	0.18	0.13	0.09	0.08	0.06	0.05	0.05	0.04	0.04	0.04	0.0091
0.3	–0.54	–0.26	–0.03	0.21	0.15	0.12	0.10	0.08	0.07	0.06	0.06	0.06	0.0134
0.4	–0.47	–0.26	–0.10	0.06	0.22	0.17	0.13	0.11	0.09	0.08	0.08	0.08	0.0176
0.5	–0.41	–0.24	–0.12	–0.01	0.11	0.23	0.17	0.14	0.12	0.10	0.10	0.10	0.0217
0.6	–0.36	–0.22	–0.12	–0.03	0.05	0.13	0.22	0.18	0.15	0.12	0.12	0.11	0.0256
0.7	–0.32	–0.20	–0.12	–0.04	0.02	0.09	0.15	0.22	0.18	0.15	0.14	0.14	0.0293
0.8	–0.29	–0.18	–0.11	–0.05	0.01	0.06	0.11	0.17	0.23	0.19	0.17	0.17	0.0327
0.9	–0.26	–0.17	–0.11	–0.05	0	0.05	0.09	0.14	0.18	0.23	0.21	0.20	0.0359
1.0	–0.24	–0.15	–0.10	–0.04	0	0.04	0.08	0.12	0.16	0.20	0.22	0.24	0.0388

b. Values of K_B

η_K \ η	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	0.98	$\bar{C}_{LTB}$
0.1	0.14	0.16	0.09	0.02	–0.04	–0.09	–0.13	–0.17	–0.21	–0.24	–0.25	–0.26	0.0232
0.2	0.11	0.11	0.13	0.05	–0.02	–0.08	–0.13	–0.18	–0.22	–0.26	–0.27	–0.27	0.0195
0.3	0.08	0.08	0.09	0.12	0.02	–0.05	–0.12	–0.18	–0.23	–0.28	–0.30	–0.30	0.0161
0.4	0.05	0.06	0.06	0.08	0.11	0	–0.09	–0.17	–0.24	–0.30	–0.32	–0.33	0.0129
0.5	0.04	0.04	0.04	0.05	0.07	0.10	–0.03	–0.14	–0.24	–0.32	–0.35	–0.37	0.0099
0.6	0.02	0.02	0.03	0.03	0.04	0.05	0.09	–0.07	–0.21	–0.33	–0.38	–0.41	0.0071
0.7	0.01	0.01	0.02	0.02	0.02	0.03	0.05	0.09	–0.14	–0.32	–0.41	–0.45	0.0047
0.8	0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.04	0.09	–0.27	–0.42	–0.51	0.0026
0.9	0	0	0	0	0	0	0.01	0.01	0.02	0.07	–0.32	–0.54	0.0011

TABLE 9.4 Values of K_A and K_B for $\beta A = 8$

a. Values of K_A

η_K \ η	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	0.98	$\bar{C}_{LTA}$
0.1	–0.73	0.16	0.09	0.06	0.04	0.03	0.03	0.02	0.02	0.01	0.01	0.01	0.0054
0.2	–0.57	–0.16	0.20	0.13	0.09	0.07	0.06	0.04	0.04	0.03	0.03	0.03	0.0107
0.3	–0.47	–0.20	0	0.21	0.14	0.11	0.08	0.07	0.05	0.05	0.04	0.04	0.0158
0.4	–0.39	–0.20	–0.06	0.08	0.21	0.15	0.11	0.09	0.07	0.06	0.06	0.06	0.0207
0.5	–0.34	–0.18	–0.08	0.02	0.11	0.21	0.15	0.12	0.09	0.08	0.07	0.07	0.0254
0.6	–0.29	–0.17	–0.08	0	0.06	0.13	0.20	0.15	0.11	0.09	0.09	0.09	0.0298
0.7	–0.26	–0.13	–0.08	–0.01	0.04	0.09	0.13	0.19	0.14	0.12	0.11	0.10	0.0339
0.8	–0.23	–0.14	–0.07	–0.02	0.03	0.07	0.10	0.14	0.19	0.14	0.13	0.13	0.0379
0.9	–0.21	–0.13	–0.07	–0.02	0.02	0.05	0.08	0.12	0.15	0.19	0.17	0.16	0.0415
1.0	–0.19	–0.12	–0.07	–0.02	0.01	0.04	0.07	0.10	0.13	0.16	0.18	0.19	0.0448

b. Values of K_B

η_K \ η	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	0.98	$\bar{C}_{LTB}$
0.1	0.11	0.12	0.05	0	–0.05	–0.08	–0.12	–0.15	–0.18	–0.20	–0.20	–0.20	0.0263
0.2	0.07	0.08	0.10	0.03	–0.03	–0.08	–0.12	–0.16	–0.19	–0.21	–0.22	–0.22	0.0221
0.3	0.05	0.05	0.06	0.09	0	–0.05	–0.11	–0.15	–0.20	–0.23	–0.24	–0.24	0.0182
0.4	0.03	0.03	0.04	0.05	0.08	–0.01	–0.09	–0.15	–0.20	–0.25	–0.26	–0.27	0.0146
0.5	0.02	0.02	0.02	0.02	0.05	0.09	–0.05	–0.13	–0.20	–0.27	–0.29	–0.31	0.0112
0.6	0.01	0.01	0.02	0.02	0.03	0.04	0.08	–0.07	–0.18	–0.28	–0.31	–0.34	0.0081
0.7	0.01	0.01	0.01	0.01	0.01	0.02	0.04	0.08	–0.12	–0.28	–0.35	–0.39	0.0054
0.8	0	0	0	0.01	0.01	0.01	0.02	0.03	0.08	–0.24	–0.36	–0.44	0.0030
0.9	0	0	0	0	0	0	0	0.01	0.02	0.08	–0.29	–0.49	0.0013

TABLE 9.5 Values of K_A and K_B for $\beta A = 12$

a. Values of K_A

η_K \ η	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	0.98	$\bar{C}_{LTA}$
0.1	–0.67	0.18	0.08	0.06	0.04	0.03	0.02	0.02	0.01	0.01	0.01	0.01	0.0061
0.2	–0.50	–0.12	0.20	0.12	0.08	0.06	0.05	0.04	0.03	0.02	0.02	0.02	0.0119
0.3	–0.40	–0.15	0.02	0.20	0.13	0.09	0.07	0.05	0.04	0.03	0.03	0.03	0.0174
0.4	–0.33	–0.15	–0.03	0.08	0.20	0.13	0.09	0.07	0.05	0.04	0.04	0.04	0.0226
0.5	–0.29	–0.14	–0.05	0.04	0.11	0.19	0.13	0.09	0.07	0.06	0.06	0.05	0.0276
0.6	–0.25	–0.13	–0.05	0.01	0.06	0.12	0.18	0.12	0.09	0.07	0.06	0.06	0.0323
0.7	–0.22	–0.12	–0.05	0	0.04	0.08	0.12	0.17	0.11	0.09	0.08	0.08	0.0368
0.8	–0.20	–0.11	–0.05	–0.01	0.03	0.07	0.10	0.12	0.16	0.11	0.10	0.09	0.0409
0.9	–0.18	–0.11	–0.06	–0.01	0.03	0.05	0.08	0.10	0.12	0.15	0.13	0.12	0.0448
1.0	–0.17	–0.10	–0.05	–0.01	0.02	0.05	0.07	0.09	0.10	0.12	0.14	0.16	0.0483

b. Values of K_B

η_K \ η	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	0.98	$\bar{C}_{LTB}$
0.1	0.07	0.09	0.03	–0.01	–0.05	–0.08	–0.10	–0.12	–0.15	–0.15	–0.14	–0.14	0.0283
0.2	0.05	0.05	0.07	0.01	–0.04	–0.07	–0.10	–0.13	–0.15	–0.17	–0.16	–0.15	0.0238
0.3	0.03	0.03	0.04	0.07	–0.01	–0.06	–0.10	–0.13	–0.16	–0.18	–0.18	–0.17	0.0197
0.4	0.02	0.02	0.03	0.04	0.07	–0.02	–0.08	–0.12	–0.17	–0.20	–0.19	–0.19	0.0158
0.5	0.02	0.02	0.02	0.02	0.04	0.07	–0.04	–0.11	–0.17	–0.21	–0.22	–0.23	0.0122
0.6	0.01	0.01	0.01	0.01	0.02	0.03	0.07	–0.06	–0.16	–0.23	–0.25	–0.26	0.0089
0.7	0	0.01	0.01	0.01	0.01	0.01	0.03	0.07	–0.11	–0.23	–0.28	–0.30	0.0059
0.8	0	0	0	0	0	0.01	0.01	0.02	0.08	–0.20	–0.30	–0.36	0.0033
0.9	0	0	0	0	0	0	0	0.01	0.02	0.09	–0.25	–0.42	0.0014

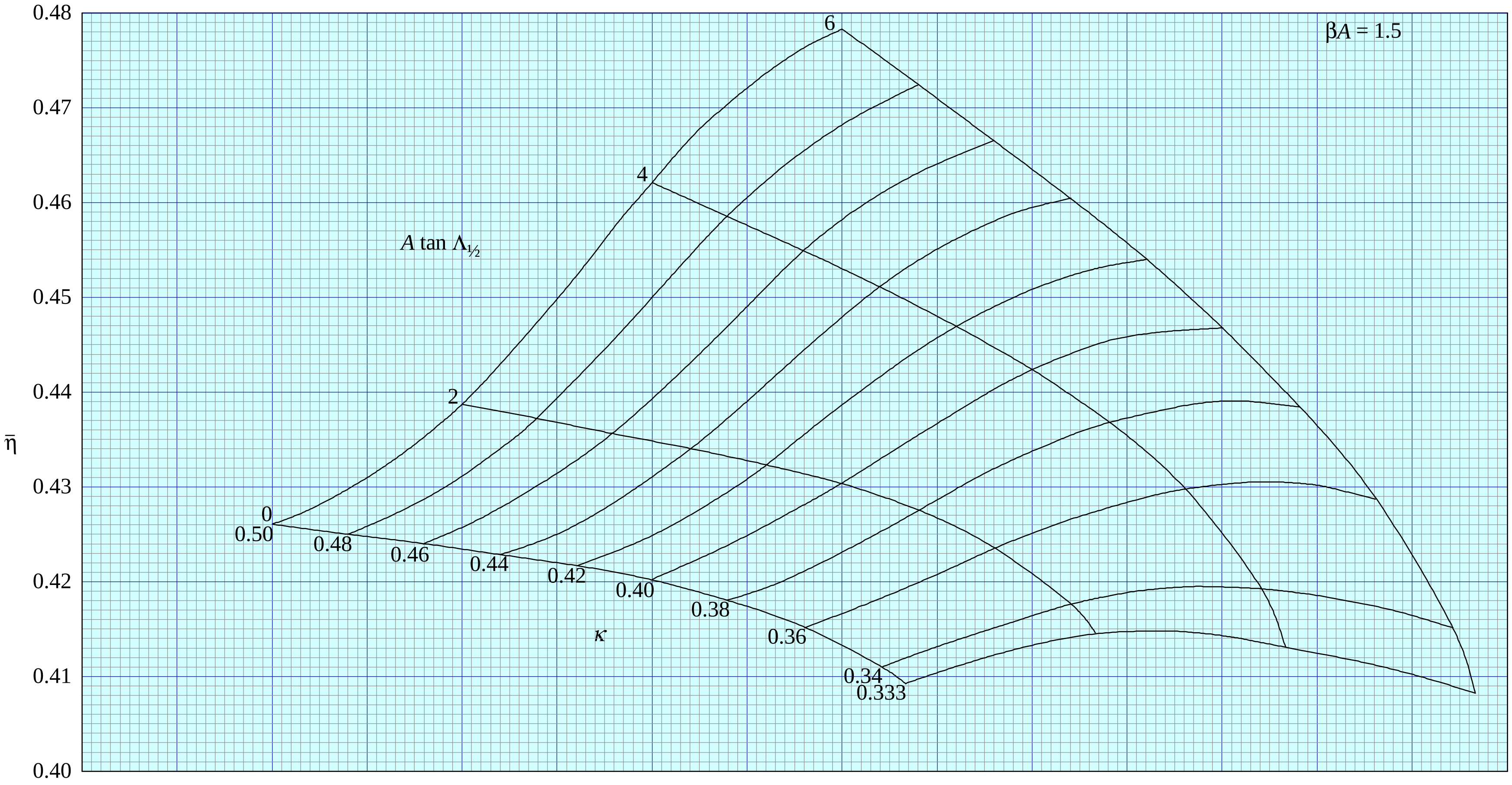


FIGURE 1 THEORETICAL SPANWISE CENTRE OF PRESSURE

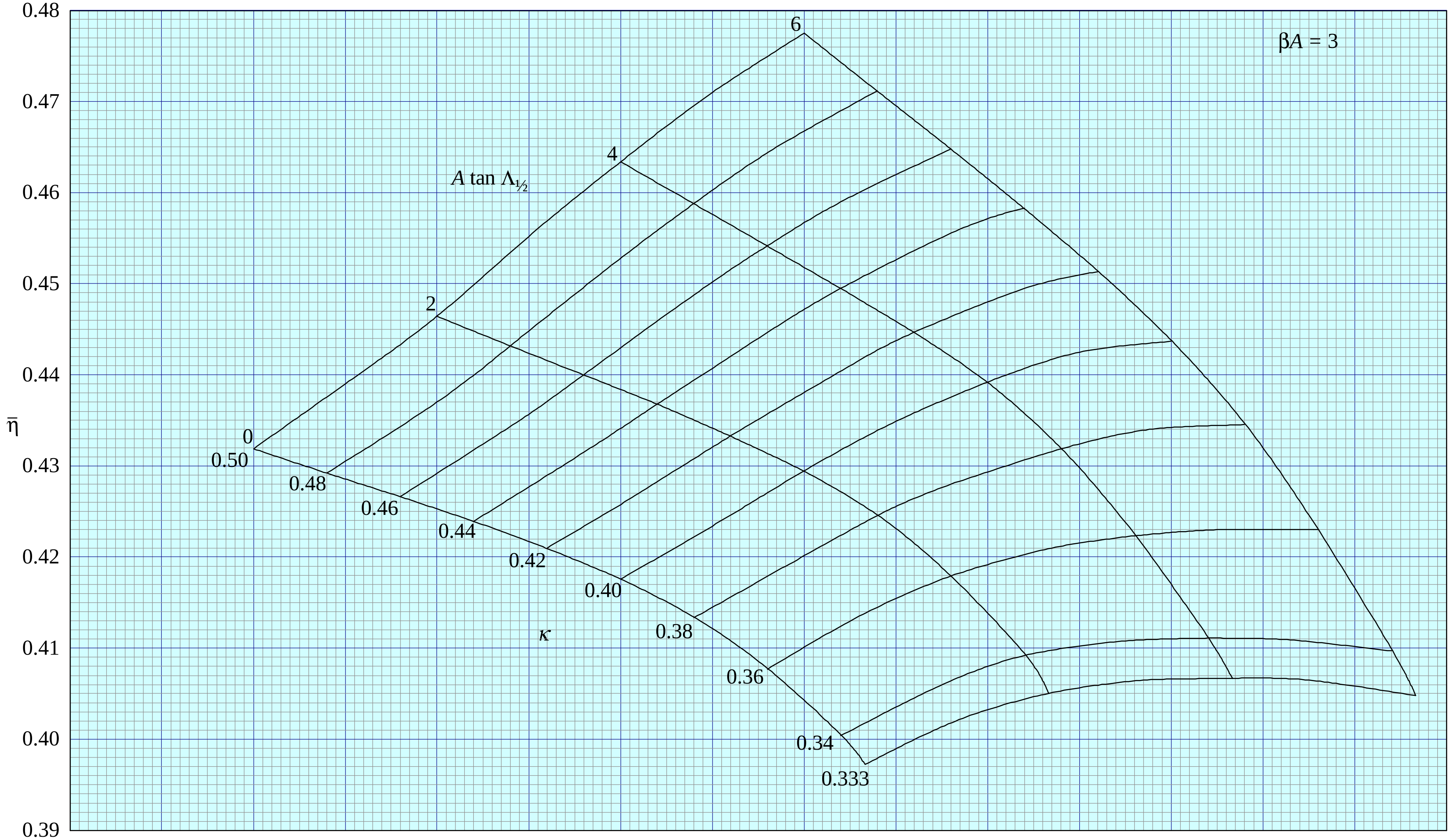


FIGURE 2 THEORETICAL SPANWISE CENTRE OF PRESSURE

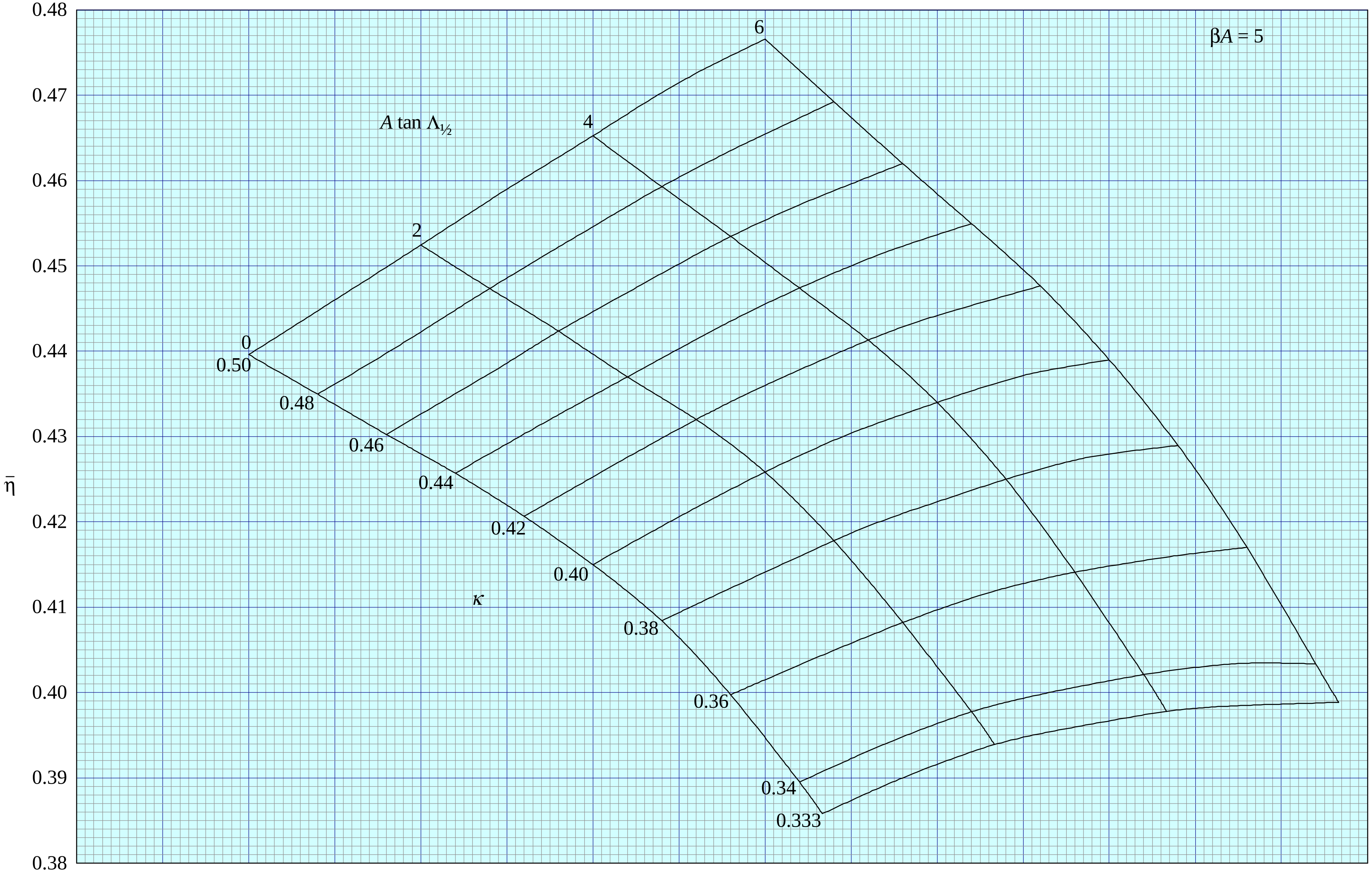


FIGURE 3 THEORETICAL SPANWISE CENTRE OF PRESSURE

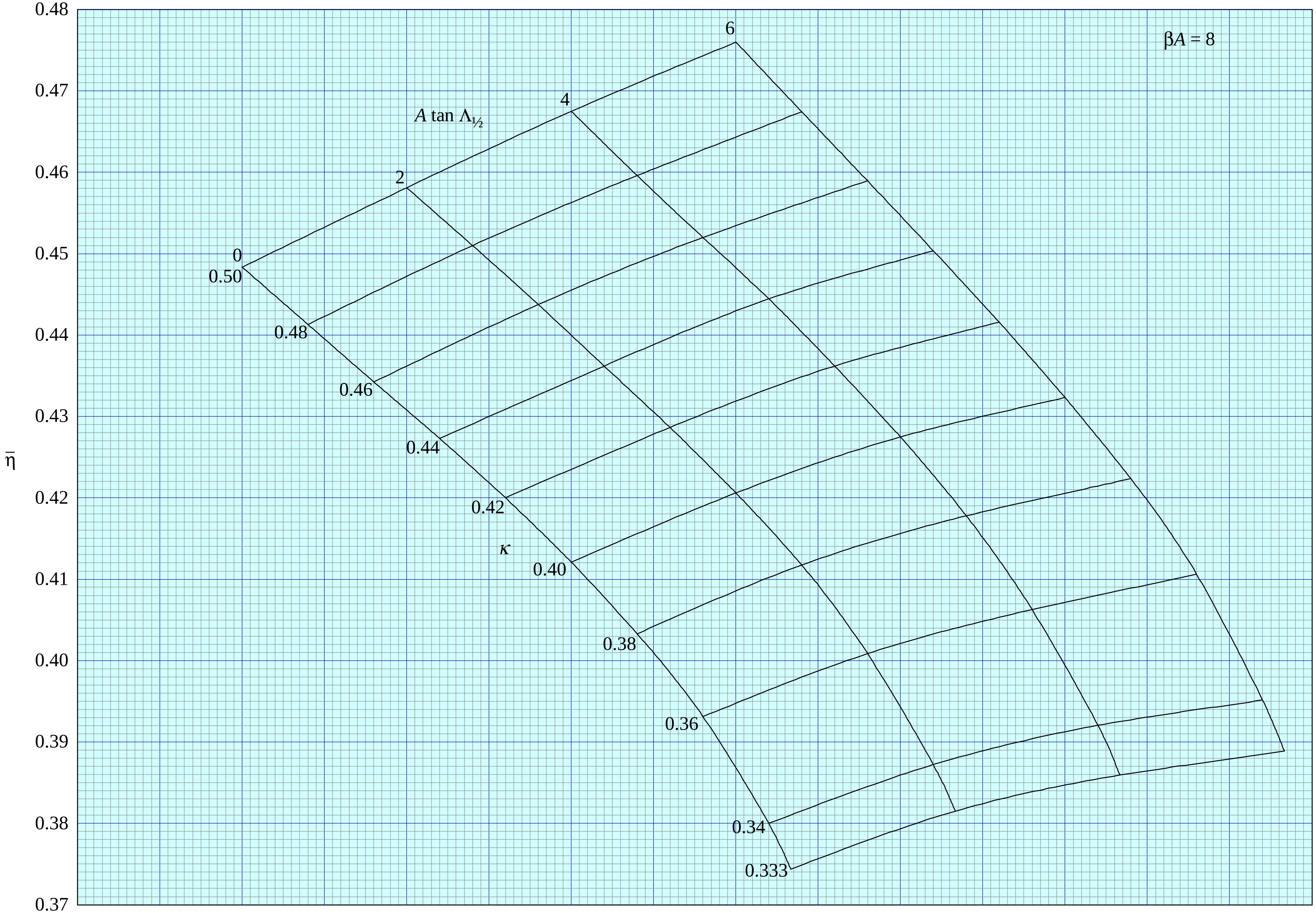


FIGURE 4 THEORETICAL SPANWISE CENTRE OF PRESSURE

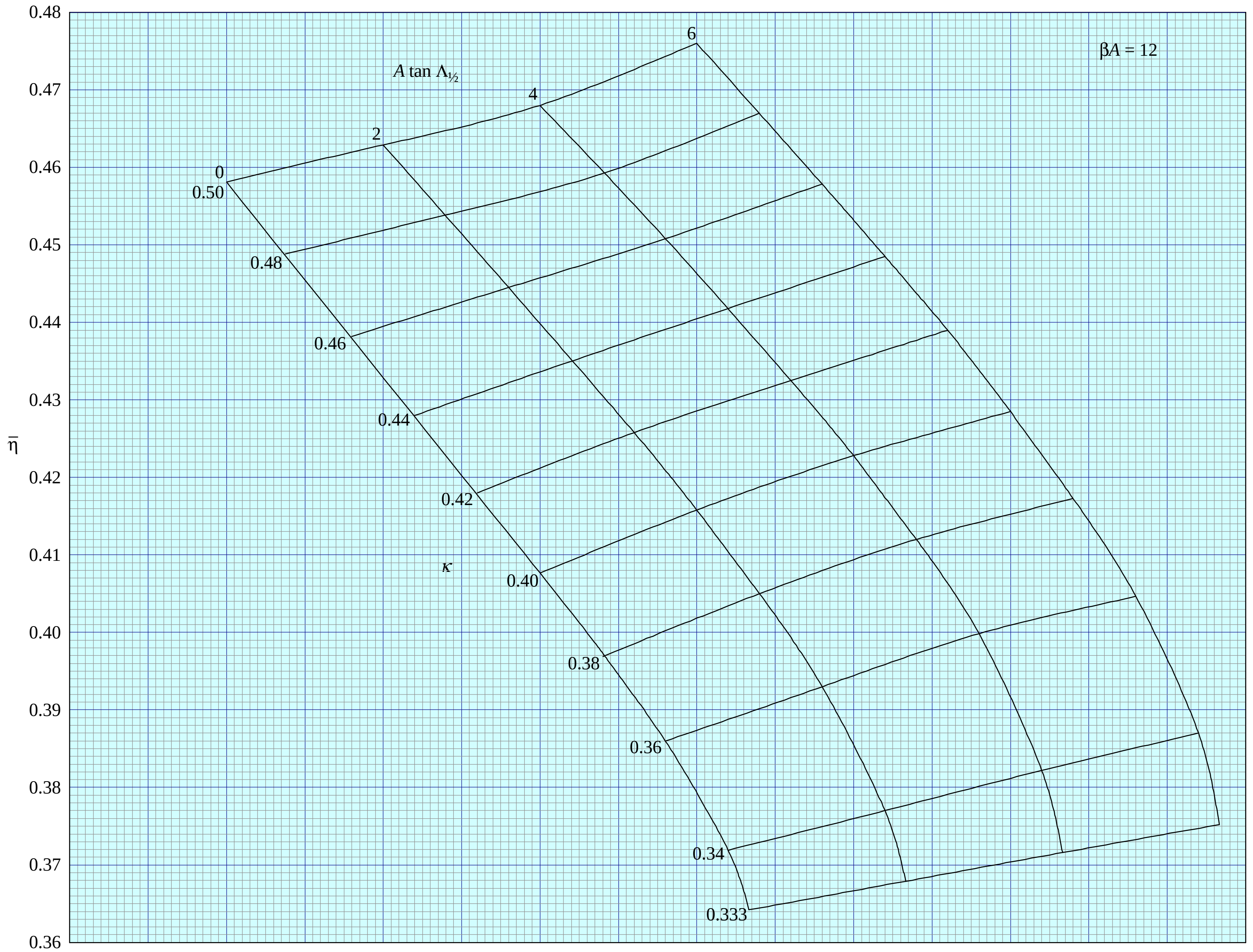


FIGURE 5 THEORETICAL SPANWISE CENTRE OF PRESSURE

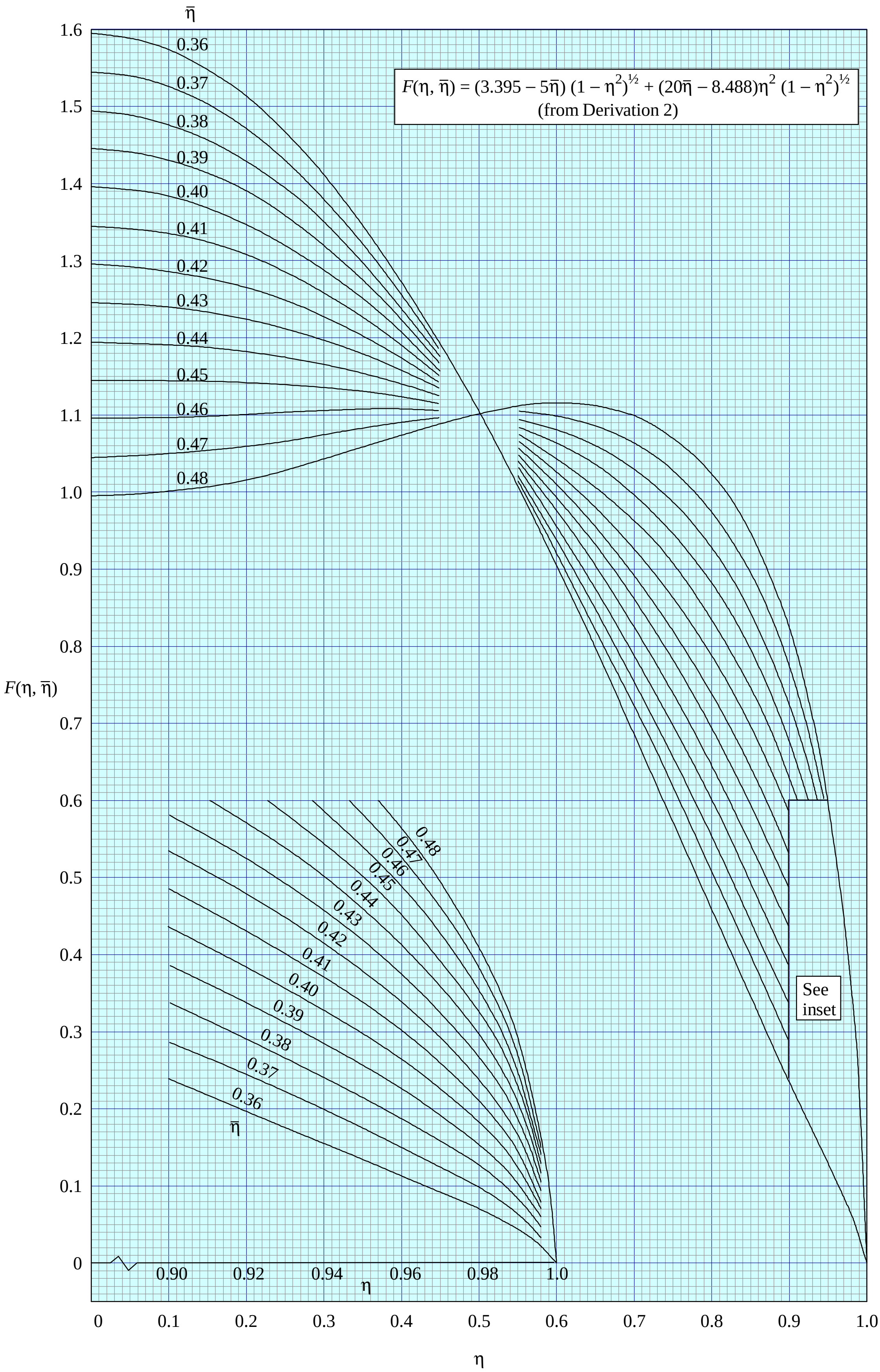


FIGURE 6 THE FUNCTION $F(\eta, \bar{\eta})$

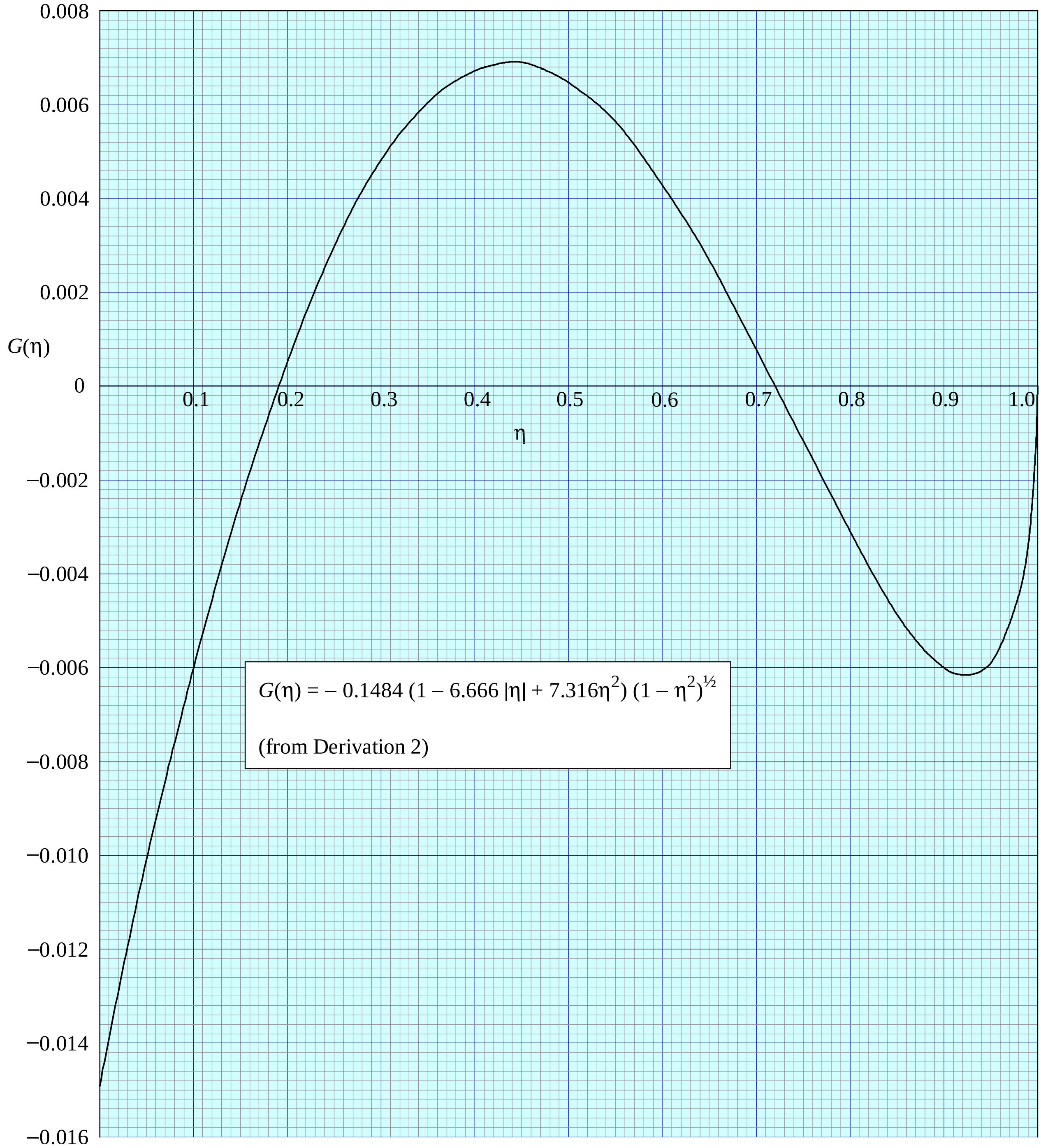


FIGURE 7 THE FUNCTION $G(\eta)$

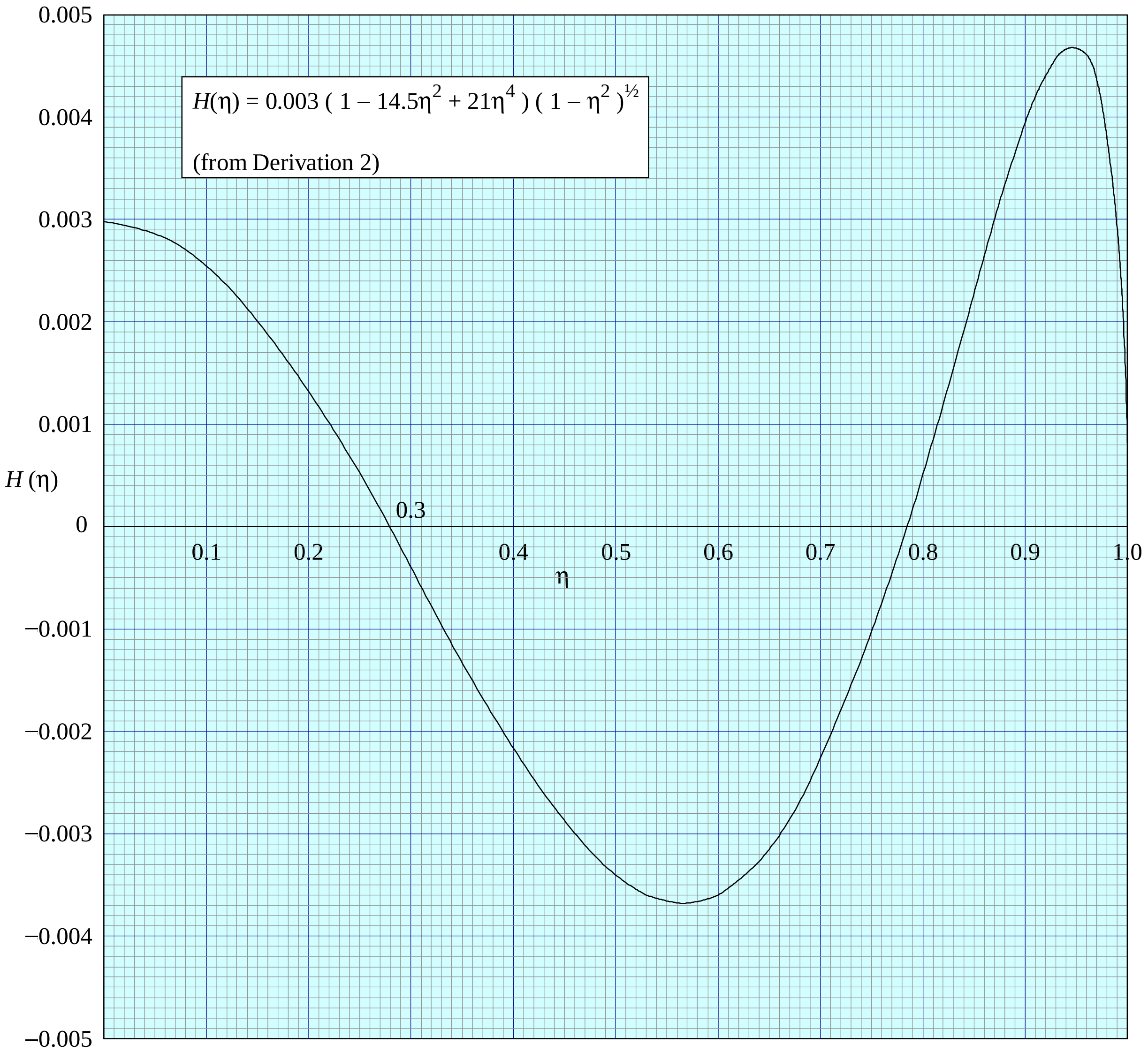


FIGURE 8 THE FUNCTION $H(\eta)$

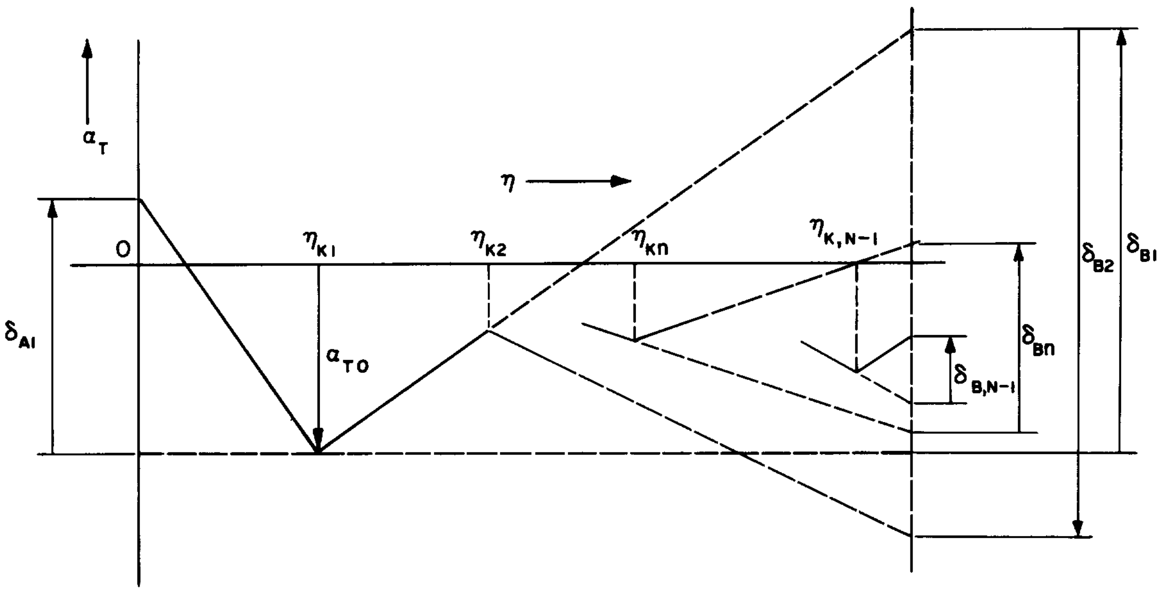
APPENDIX A EXTENSION OF METHOD OF SECTION 5 FOR AN ARBITRARY NUMBER OF LINEAR TWIST SEGMENTS

A1. ADDITIONAL NOTATION

N	number of linear twist segments
Subscripts	
i	denotes i'th linear twist segment of type B (i = 1 to n)
n	denotes n'th linear twist segment of type B (n = 1 to $N - 1$)

A2. TWIST REPRESENTATION BY *N* LINEAR SEGMENTS

Sketch A2.1 illustrates a generalised twist distribution which has been idealised in the form of an arbitrary number (N) of linear segments.



Sketch A2.1

For the twist distribution shown in Sketch A2.1 the effective local twist is given by

$$\alpha_E = \alpha_T + \delta_{A1} K_{A1} + \sum_{n=1}^{N-1} \delta_{Bn} K_{Bn} \tag{A2.1}$$

where

$$\left.\begin{aligned}\alpha_T &= \alpha_{T0} + \delta_{A1}(\eta_{K1} - \eta)/\eta_{K1} && \text{for } 0 \le \eta_{K1} \\ &= \alpha_{T0} + \sum_{i=1}^{n} \delta_{Bi}(\eta - \eta_{Ki})/(1 - \eta_{Ki}) && \text{for } \eta_{Kn} \le \eta \le \eta_{K,\,n+1}\end{aligned}\right\} \qquad \text{(A2.2)}$$

where $n = 1$ to $N - 1$ and $\eta_{KN} = 1$.

The corresponding lift coefficient is given by

$$\bar{C}_{LT} = \alpha_{T0} a_1 + \delta_{A1} \bar{C}_{LTA1} + \sum_{n=1}^{N-1} \delta_{Bn} \bar{C}_{LTBn} \, . \qquad \text{(A2.3)}$$

THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item was monitored and guided by the Aerodynamics Committee, which first met in 1942 and now has the following membership:

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* Corresponding Member

The technical work in the assessment of the available information and the construction and subsequent development of the Data Item was carried out under contract to ESDU by Mr D.C. Greenman of British Aerospace plc, Bristol. Particular assistance in the preparation of this Item was received from Mr R.G. Williams of British Aerospace plc, Bristol.