STABILITY DERIVATIVE $(L_v)_\Gamma$. CONTRIBUTION OF FULL-SPAN DIHEDRAL TO ROLLING MOMENT DUE TO SIDESLIP

1. NOTATION AND UNITS

The derivative notation used is that proposed in ARC R&M 3562 (Hopkin, 1970) and described in Item No. 86021. Coefficients and aeronormalised derivatives are evaluated in aerodynamic body axes with origin at the aircraft centre of gravity, assumed to coincide with the aerodynamic centre of the wing, and with the wing span as the characteristic length. The derivative L_v is often written as $\partial C_l/\partial\beta$ or $C_{l\beta}$ in other systems of notation, but attention must be paid to the reference dimensions used and it is to be noted that a constant datum value of V is employed in the Hopkin system.

		SI	*British*
A	aspect ratio, b^2/S		
$(a_{10})_M$	two-dimensional lift-curve slope of wing section at Mach number M	rad^{-1}	rad^{-1}
b	wing span	m	ft
C_l	rolling moment coefficient, $\mathscr{L}/\frac{1}{2}\rho V^2 Sb$		
c_r	wing root chord	m	ft
c_t	wing tip chord	m	ft
$\mathscr{L}$	rolling moment	N m	lbf ft
L_v	aeronormalised rolling moment derivative due to sideslip, $(\partial\mathscr{L}/\partial v)/\frac{1}{2}\rho VSb$		
$(L_v)_\Gamma$	that part of aeronormalised rolling moment derivative due to sideslip arising from dihedral		
M	Mach number		
S	gross wing area	m^2	ft^2
V	velocity of aircraft relative to air	m/s	ft/s
v	sideslip velocity	m/s	ft/s
β	sideslip angle, $\sin^{-1}(v/V) \approx v/V$	radian	radian
Γ	dihedral angle of wing (see sketch on Figure 1)	degree	degree
δ_t	wing tip deflection due to load	m	ft
κ	$(a_{10})_M(1-M^2)^{1/2}/2\pi$		
$\Lambda_{1/4}$	angle of sweepback of quarter-chord line	degree	degree

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Λ_E	equivalent angle of sweepback at Mach number M, $\tan^{-1}[(\tan\Lambda_{1/4})/(1-M^2)^{1/2}]$	degree	degree
λ	wing taper ratio, c_t/c_r		
ρ	density of air	kg/m^3	slug/ft^3

2. NOTES

The curves give the variation of the parameter

$$-\frac{(L_v)_\Gamma(1-M^2)^{1/2}}{\kappa\Gamma}$$

with Λ_E for various values of

$$\frac{A(1-M^2)^{1/2}}{\kappa}$$

for four values of the taper ratio λ for wings having constant dihedral over the entire wing semi-span. The contribution of dihedral to the rolling moment due to sideslip varies linearly with dihedral angle and is independent of the lift coefficient provided that the latter is below about 60 per cent of its stalling value.

The curves have been obtained by the use of Weissinger's extended lifting-line theory as described in Derivation 2 and have been found to give values of $(L_v)_\Gamma$ which generally agree with experimental results to within ± 10 per cent for a wide range of wing planforms. Weissinger's theory normally applies only to thin wings in inviscid incompressible flow, but by the use of the subsonic linearised theory similarity law and the parameter κ it has been extended to cover the case of wings of finite thickness in viscous compressible flow, provided that there is no boundary layer separation from the wing. It has been assumed that there is no significant spanwise variation in κ. In cases where κ does vary significantly a mean value can be taken, as suggested in Derivation 2.

For a wing with part-span dihedral or with spanwise sections of different dihedral, see Item No. Aero A.06.01.09.

For non-rigid wings, allowance must be made for the additional dihedral due to the wing deflection under load. This dihedral is not constant over the wing span and it is suggested in Derivation 1 that the equivalent full-span dihedral angle corresponding to a tip deflection δ_t should be taken as $67.6\delta_t/(b/2)$ degrees.

3. DERIVATION

The Derivation lists selected sources that have assisted in the preparation of this Item.

1.	LEVACIC, I.	Rolling moment due to sideslip. Part I: the effect of dihedral. RAE Rep. Aero. 2028, 1945.
2.	DeYOUNG, J.	Theoretical antisymmetric span loading for wings of arbitrary plan form at subsonic speeds. NACA Rep. 1056, 1949.

4. EXAMPLE

Find the contribution of dihedral to the rolling moment derivative due to sideslip at a Mach number of 0.4 for a wing with full-span dihedral, for which

$$A = 3.0,\ \Lambda_{1/4} = 30°,\ \lambda = 0.5,\ \Gamma = 5° \text{ and } (a_{10})_M = 5.6 \text{ rad}^{-1}.$$

The equivalent angle of sweepback is

$$\Lambda_E = \tan^{-1}\left[\frac{\tan 30°}{(1-0.4^2)^{1/2}}\right] = 32.2°.$$

Also, since

$$\kappa = \frac{5.6}{2\pi}(1-0.4^2)^{1/2} = 0.817,$$

$$\frac{A(1-M^2)^{1/2}}{\kappa} = \frac{3.0(1-0.4^2)^{1/2}}{0.817} = 3.37.$$

From Figure 1c for $\lambda = 0.5$, with $A(1-M^2)^{1/2}/\kappa = 3.37$ and $\Lambda_E = 32.2°$,

$$\frac{(L_v)_\Gamma(1-M^2)^{1/2}}{\kappa\Gamma} = -0.0082,$$

and thus, with $\Gamma = 5°$, $(L_v)_\Gamma = \dfrac{-0.0082 \times 0.817 \times 5}{(1-0.4^2)^{1/2}} = -0.037$.

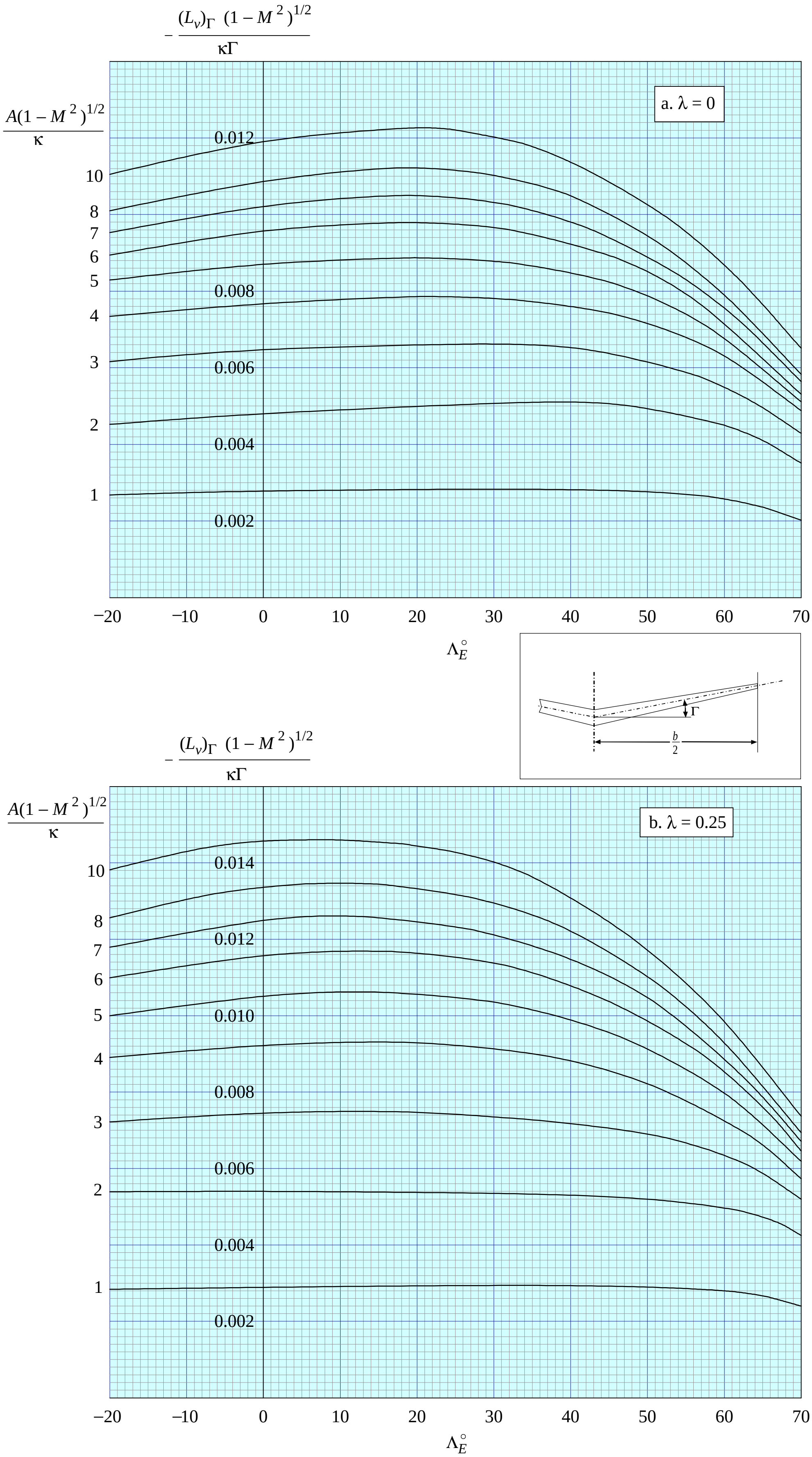
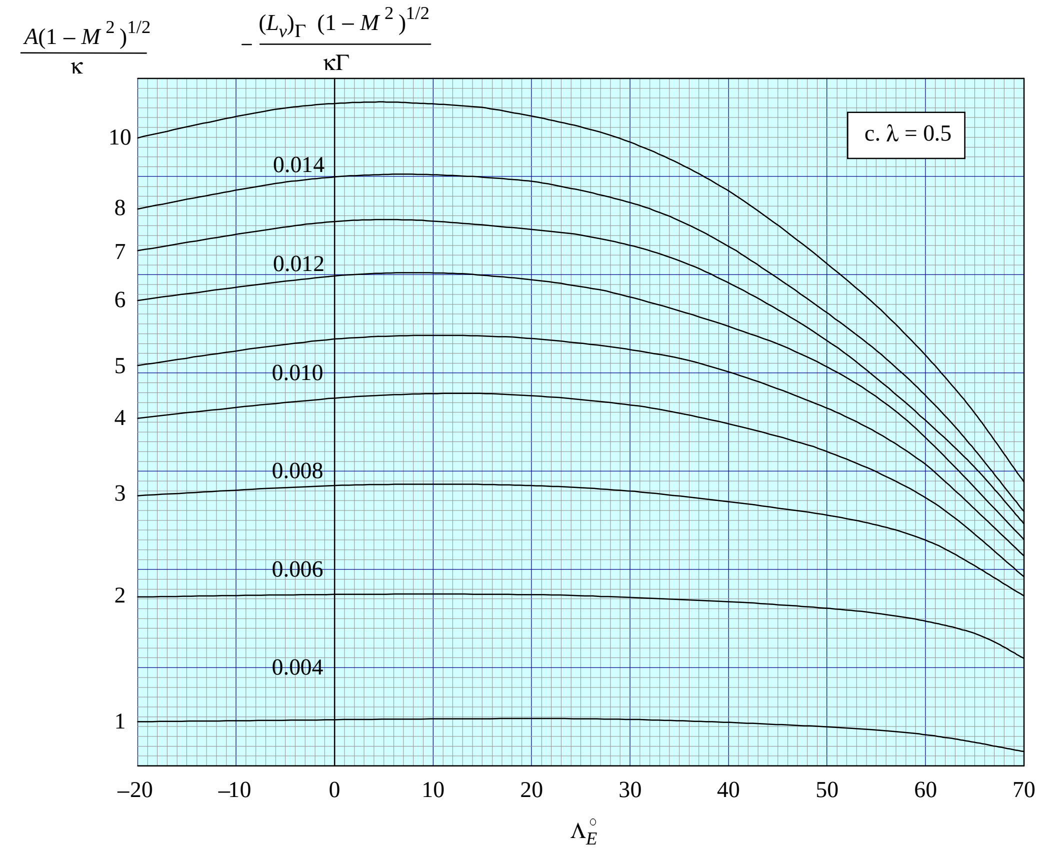

$-\frac{(L_v)_\Gamma\,(1-M^2)^{1/2}}{\kappa\Gamma}$
$\frac{A(1-M^2)^{1/2}}{\kappa}$
a. λ = 0
0.012
0.008
0.006
0.004
0.002
10
8
7
6
5
4
3
2
1
−20 −10 0 10 20 30 40 50 60 70
Λ_E°
Γ
$\frac{b}{2}$
$-\frac{(L_v)_\Gamma\,(1-M^2)^{1/2}}{\kappa\Gamma}$
$\frac{A(1-M^2)^{1/2}}{\kappa}$
b. λ = 0.25
0.014
0.012
0.010
0.008
0.006
0.004
0.002
10
8
7
6
5
4
3
2
1
−20 −10 0 10 20 30 40 50 60 70
Λ_E°

FIGURE 1



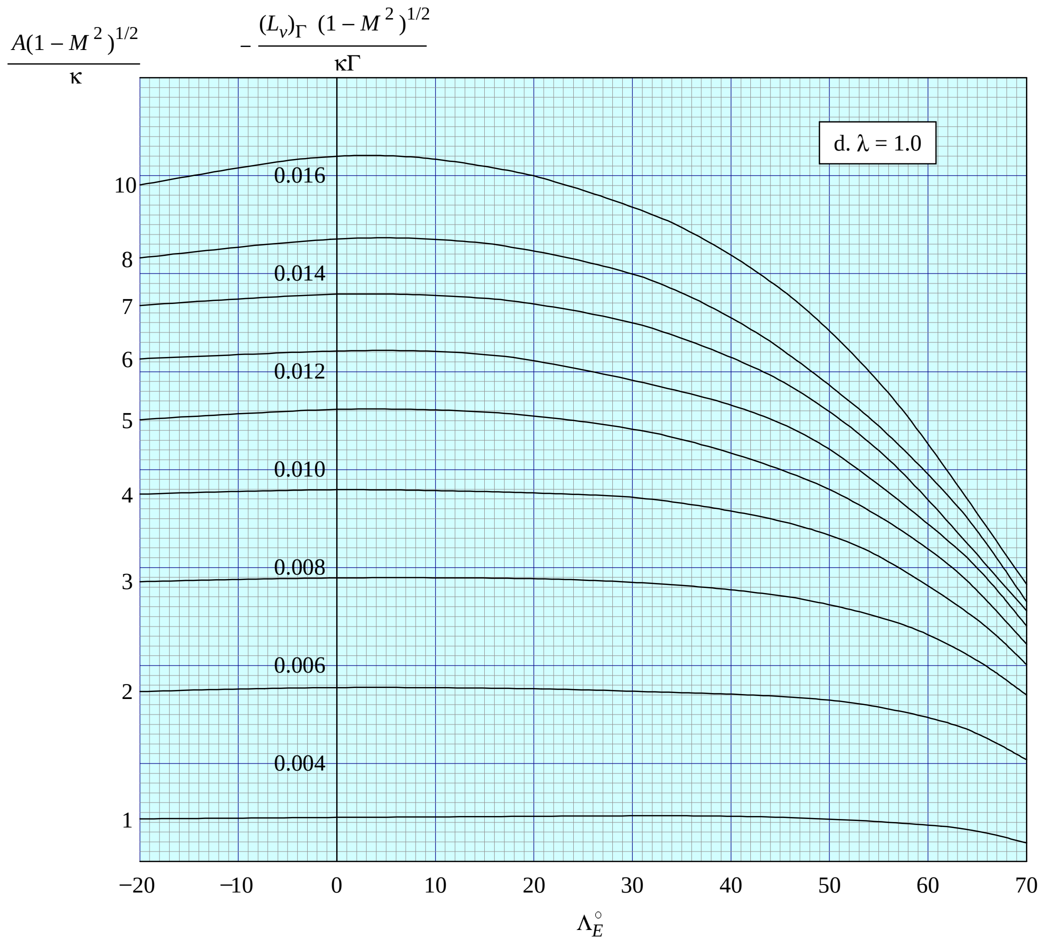


FIGURE 1 (concluded)