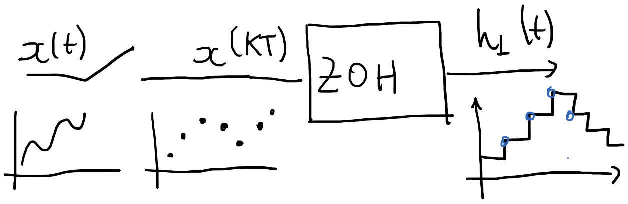
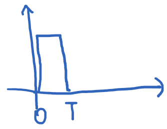
REAL





$$h_1(t) = x(0).\left[1(t) - 1(t-T)\right] + x(T).\left[1(t-T) - 1(t-2T)\right] + \ldots$$

$$h_1(t) = \sum_0^{\infty} x(KT).\left[1(t-KT) - 1(t-(K+1)T)\right]$$

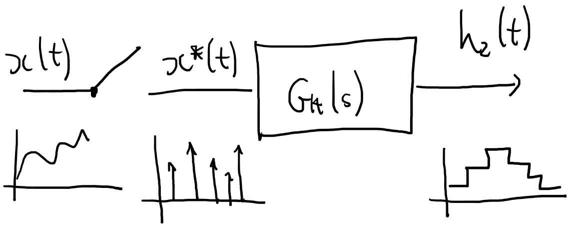
$$\mathcal{L}\left[1(t-KT)\right] = \frac{e^{-KTs}}{s}$$

$$\Downarrow$$

$$H_1(s) = \mathcal{L}\left[h_1(t)\right] = \sum_0^{\infty} x(KT).\frac{e^{-KTs} - e^{-(K+1)Ts}}{s}$$

$$\boxed{H_1(s) = \frac{1-e^{-Ts}}{s}.\sum_0^{\infty} x(KT)\, e^{-KTs}}$$

MODELO MATEMÁTICO



MAS $h_1(t) = h_2(t)$

$$\Rightarrow H_2(s) = \frac{1-e^{-Ts}}{s}.\underbrace{\boxed{\sum_0^{\infty} x(KT).e^{-KTs}}}_{X^*(s)}$$

$$H_2(s) = G_h(s).X^*(s)$$

TREM IMPULSOS

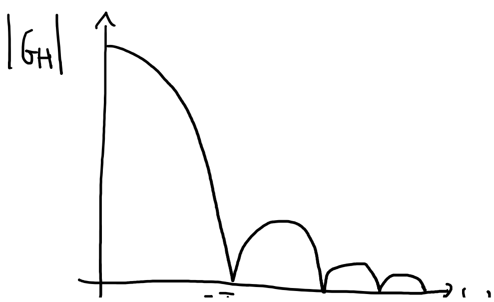
$$\mathcal{L}\left[\delta(t-KT)\right] = e^{-KTs} \Rightarrow X^*(s) = \boxed{\sum_0^{\infty} x(KT)\, e^{-KTs}}$$

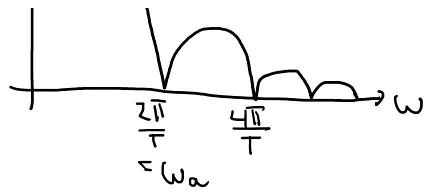
$$\Rightarrow \boxed{G_H(s) = \frac{1-e^{-Ts}}{s}}$$

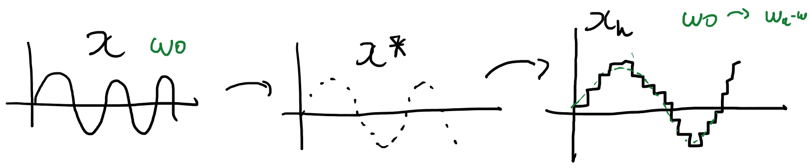
→ FUNÇÃO DE TRANSF. Z.O.H

CONCLUSÃO

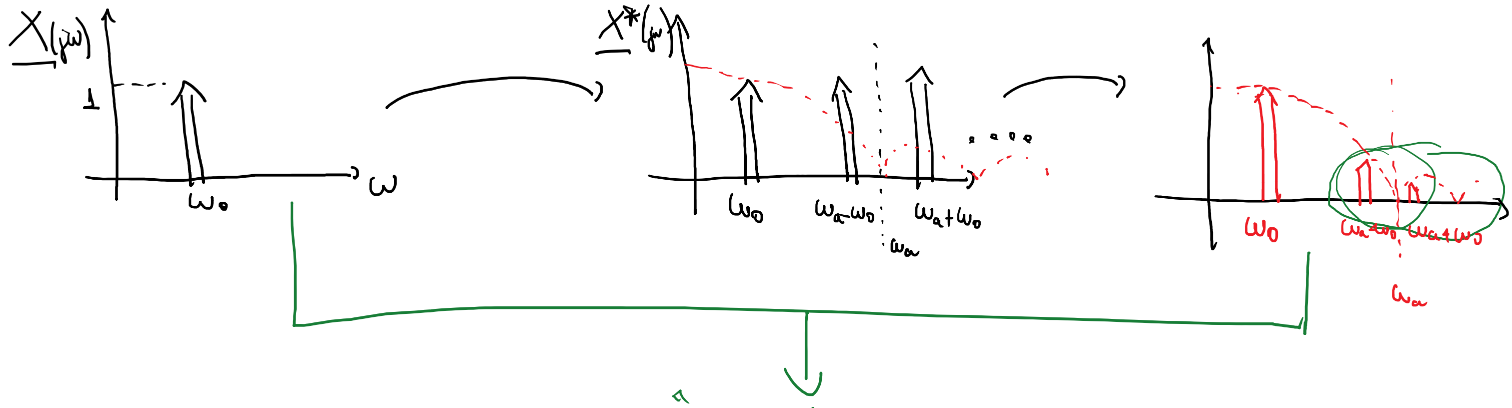
$$|G_H(s)| = \left|\frac{1-e^{-T.j\omega}}{j\omega}\right| = T.\left|\frac{\sin \omega T/2}{\omega T/2}\right|$$







SUPONHA QUE $x(t)$ = HARMÔNICO ω_0



"QUASE" TÃO BOM QUANTO O RECONSTRUTOR IDEAL

Transformada Z

quinta-feira, 27 de agosto de 2020 11:16

teaching the 'Z-TRANSFORM'...

MOTIVAÇÃO

CONTÍNUO

$u(t) \rightarrow$ SIST DINÂMICOS $\rightarrow y(t)$

$x(t)$

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \rightsquigarrow \text{eq. diferencial}$$

$\hookrightarrow \dot{x} = sX$

eq. dif. eq. algébrica

SINAL $\rightarrow$ $\frac{S+S+S}{Z+Z+9}$ $\rightarrow$ SINAL

Z ou S Z ou S

DISCRETO

$u[k] \rightarrow$ SIST DINAMICO $\rightarrow y[k]$

$$\begin{cases} x[k+1] = Ax[k] + Bu[k] \\ y[k] = Cx[k] + Du[k] \end{cases} \rightsquigarrow \text{eq. diferença}$$

ex: $y[k+2] - 2y[k+1] - y[k] = u[k]$

$$\begin{cases} x[k+1] = z.X(z) - z.x(0) \\ \text{ou} \\ x[k-1] = z^{-1}.X(z) \end{cases}$$

$\downarrow$ eq. diferença $\downarrow$ eq. algébrica

A TRANSFORMADA Z

$$X(z) = \mathcal{Z}[x(t)] = \mathcal{Z}[x(kT)] = \underbrace{\sum_{0}^{\infty} x(kT).z^{-k}}_{\text{SÉRIE POTÊNCIAS } z^{-1}}$$

$\hookrightarrow$ COMPLEXO

$$X(z) = x(0) + x(T).z^{-1} + x(2T).z^{-2} + \dots$$

EX 1) DEGRAU UNITÁRIO

$$x(t) = \begin{cases} 1(t) & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$\mathcal{L}[x(t)] = \frac{1}{s}$

$$X(z) = \mathcal{Z}[1(t)] = \sum_{0}^{\infty} 1 \cdot z^{-K} = 1 + z^{-1} + z^{-2} + \ldots$$

$\Rightarrow$ PROG GEOMÉTRICA $\quad S_n = a_1 \cdot \dfrac{q^n - 1}{q - 1}$

$q = z^{-1}$

$n = \infty \Rightarrow S_n = X(z) = 1 \cdot \dfrac{0 - 1}{z^{-1} - 1} = \dfrac{-1}{z^{-1} - 1} = \boxed{\dfrac{1}{1 - z^{-1}} = \dfrac{z}{z - 1} \quad \text{se } |z| > 1}$

$a_1 = 1$

se $|z| > 1 \Rightarrow q^n = 0$

RAMPA $\quad x(t) = \begin{cases} t & \text{se } t \geq 0 \\ 0 & \text{se } t < 0 \end{cases} \quad \ldots \mathcal{L}[x(t)] = \dfrac{1}{s^2}$

$$X(z) = \mathcal{Z}[t] = \sum x(KT) \cdot z^{-K} = \sum KT \cdot z^{-K} = T \cdot \sum K \cdot z^{-K}$$

$$X(z) = T \cdot \left(z^{-1} + 2z^{-2} + 3z^{-3} + 4z^{-4} \ldots \right)$$

SOMA

$$z^{-1} + z^{-2} + z^{-3} + z^{-4} + \ldots = z^{-1} \cdot \frac{-1}{z^{-1} - 1}$$

$$z^{-2} + z^{-3} + z^{-4} + \ldots = z^{-2} \cdot \frac{-1}{z^{-1} - 1}$$

$$z^{-3} + z^{-4} + \ldots = z^{-3} \cdot \frac{-1}{z^{-1} - 1}$$

$$\frac{1}{1 - z^{-1}} \cdot \Big(\underbrace{z^{-1} + z^{-2} + z^{-3} + \ldots}_{z^{-1} \cdot \frac{-1}{z^{-1} - 1}} \Big)$$

$$T \frac{z^{-1}}{(1 - z^{-1})^2} \quad p/ \ |z| > 1$$

Ex) POLINOMIAL

$f = K \quad k \geq 0 \quad z^{1}$

(A) POLINOMIAL

$$x(k) = \begin{cases} a^k & k \geq 0 \\ 0 & k < 0 \end{cases}$$

$$X(z) = \mathcal{Z}(a^k) = \sum a^k z^{-k} = 1 + a\, z^{-1} + a^2 z^{-2} + a^3 z^{-3} \ldots$$

$$X(z) = 1 \cdot \frac{1}{1 - a z^{-1}} = \frac{z}{z - a} \quad \text{Vale p/ } |a \cdot z^{-1}| < 1$$

Ex) $x(t) = \begin{cases} e^{-at} & p/\, t \geq 0 \\ 0 \end{cases}$ $\quad \mathcal{L} = \frac{1}{s+a}$

$$x(kT) = e^{-akT}$$

$$X(z) = \sum e^{-akT} \cdot z^{-k} = 1 + e^{-aT} z^{-1} + e^{-2aT} z^{-2} \ldots\ldots$$

$$= \frac{1}{1 - e^{-aT} \cdot z^{-1}} = \frac{z}{z - e^{-aT}}$$

Ex) $x(t) = \begin{cases} \operatorname{sen} \omega t & p/\, t \geq 0 \\ 0 & p/\, t < 0 \end{cases}$

1) $\operatorname{sen} \omega t = \frac{1}{2j} \left(e^{j\omega t} - e^{-j\omega t} \right)$

2) $\mathcal{Z}\left[e^{-at} \right] = \frac{1}{1 - e^{-aT} z^{-1}}$

...

$$\frac{z^{-1} \cdot \operatorname{sen} \omega T}{1 - 2 z^{-1} \cos \omega T + z^{-2}} = \frac{z \operatorname{sen} \omega T}{z^2 - 2z \cos \omega T + 1}$$

Propriedades

quinta-feira, 27 de agosto de 2020 14:11

1) $\mathcal{Z}[a \cdot x(t)] = a \cdot X(z)$

2) LINEARIDADE

$x(t) = \alpha \cdot f_1(t) + \beta \cdot f_2(t)$

$\Rightarrow X(z) = \alpha F_1(z) + \beta \cdot F_2(z)$

3) $\mathcal{Z}[a^k x(k)] = X(a^{-1} \cdot z)$

4) $\mathcal{Z}[x(t-nT)] = z^{-n} X(z)$

$\mathcal{Z}[x(t+nT)] = z^{n} \cdot \left[X(z) - \sum_{k=0}^{n-1} x(kT) z^{-k}\right]$

PROVA 1)

$\mathcal{Z}[x(t-nT)] = \sum_{k=0}^{\infty} x(kT-nT) \cdot z^{-k} =$

$= z^{-n} \cdot \sum_{k=0}^{\infty} x(kT-nT) \cdot z^{-(k-n)}$

$m = k-n$

$\mathcal{Z}[x(t-nT)] = z^{-n} \cdot \sum_{m=-n}^{\infty} x(mT) \cdot z^{-m}$

p/ $m<0$, $x(mT) = 0$

$\mathcal{Z}[x(t-nT)] = z^{-n} \cdot \underbrace{\sum_{0}^{\infty} x(mT) z^{-m}}_{X(z)}$

$$\boxed{\mathcal{Z}[x(t-nT)] = z^{-n} \cdot X(z)}$$

Multiplicar uma transformada Z por z^-n equivale ao

Multiplicar uma transformada Z por z^-n equivale no tempo a atrasar a função x(t) em nT

PROVA 2)

$x(t+kT)$	$z^k X(z) - z^k x(0) - z^{k-1} x(T) - \ldots - z x(kT - T)$

FAZER!!

5) $\mathcal{Z}[e^{-at} \cdot x(t)] = X(\underline{z \cdot e^{aT}})$

ex: DADO QUE $\mathcal{Z}[\operatorname{sen} \omega t] = \dfrac{z^{-1} \operatorname{sen} \omega T}{1 - 2z^{-1} \cos \omega T + z^{-2}}$

QUAL $\mathcal{Z}[e^{-at} \operatorname{sen} \omega t] = ???$

$\Rightarrow$ PELA PROPR. (5) $\Rightarrow$ $\dfrac{z^{-1} e^{-aT} \cdot \operatorname{sen} \omega T}{1 - 2z^{-1} e^{-aT} \cos \omega T + z^{-2} e^{-2aT}}$

6) VALOR INICIAL

$$\boxed{x(0) = \lim_{z \to \infty} X(z)}$$

PROVA: PELA DEFINIÇÃO $X(z) = \underline{x(0)} + x(1) \cancel{z^{-1}} + x(2) \cancel{z^{-2}} \ldots$

7) TEOREMA V. FINAL

$$\boxed{\lim_{k \to \infty} x(k) = \lim_{z \to 1} (1 - z^{-1}) \cdot X(z)}$$

$$\boxed{\lim_{K\to\infty} x(K) = \lim_{z\to 1} (1-z^{-1}) \cdot X(z)}$$

PROVA)

$$Z(x(K)) = X(z) = \sum x(K)\, z^{-K}$$

$$Z(x(K-1)) = z^{-1}X(z) = \sum x(K-1)\, z^{-K} \quad (-)$$

$$\sum x(K)\, z^{-K} - \sum x(K-1)\, z^{-K} = X(z) - z^{-1}X(z)$$

$z \to 1$

$$\lim_{z\to 1}\left[\left(x(0) - \cancel{x(-1)}^{0}\right) + \left(\cancel{x(1)} - \cancel{x(0)}\right) + \left(x(2) - \cancel{x(1)}\right) + \dots\right] = \lim_{z\to 1} (1-z^{-1}) \cdot X(z)$$

$$\boxed{\lim_{K\to\infty} x(K) = \lim_{z\to 1} (1-z^{-1})\, X(z)}$$