ELSEVIER

Int. J. Fatigue Vol. 19, No. 7, pp. 529–535, 1997

0142–1123/97/$17.00+.00

PII: S0142-1123(97)00078-9

Modified rainflow counting keeping the load sequence

R.J. Anthes

ABB Daimler–Benz Transportation (Deutschland) GmbH, Am Rathenaupark, 16761 Hennigsdorf, Germany
(Received 25 February 1997; revised 2 June 1997; accepted 11 June 1997)

An algorithm for performing rainflow cycle counting that enables a correct translation of the load sequence into damaging events is presented. By that, fatigue damage accumulation can be carried out more accurately if models are used which consider load sequence effects. A flow chart is presented to enable integration of the algorithm into a fatigue crack growth model. © 1997 Elsevier Science Ltd.

(Keywords: rainflow cycle counting; load sequence effects; damaging event)

INTRODUCTION

Rainflow cycle counting[1] is a procedure for determining damaging events in variable amplitude loadings. Closed hysteresis loops are registered in the stress–strain path. These loops are treated as one cycle of the corresponding constant amplitude loading. Generally. damage of the cycles is quantified by considering Wöhler-curves from constant amplitude tests and by employing an approach that covers the influence of mean stresses.

In fatigue, damage is usually interpreted as crack growth. It is well known that fatigue crack growth depends more on the effective stress or strain range than on its total ranges[2,3]. The effective values result from the crack opening values which, for their part, strongly depend on the load history. A lot of load sequence models have been developed up to now to describe these effects, for example, Refs [4–9]. These models employ closed hysteresis loops as input data if rainflow cycle counting of the load sequence is performed. However, the order in which the load reversals appear in the load sequence is not transferred exactly into the series of closed hysteresis loops. Therefore in many cases, the load sequence effect is considered incorrectly. In this paper, a modification of the classical rainflow cycle counting is proposed, that eliminates this weakness.

CLASSICAL RAINFLOW CYCLE COUNTING

At present, the rainflow method developed by Matsuishi–Endo[1] is regarded as the best procedure to recognize damaging events in a complex loading history. It is based on the cyclic material behaviour that can be described by a simple mathematical model consisting of the two modules 'Masing behaviour'[10] and 'memory rules'. An illustration of this model is given in *Figure 1*. The construction of the stress–strain path resulting from a given stress–time or strain–time history only requires knowledge of the cyclic stress–strain curve (cssc). The path starts with an initial stress and strain value of zero. The segment resulting from the first peak simply follows the cssc. Upon load reversal, a half cycle of a hysteresis loop is formed that can be described by Masing behaviour[10]. The model is completed by the definition of three memory rules:

- memory 1: if a hysteresis loop closes at the cssc, the further stress–strain path follows the cssc.
- memory 2: if a hysteresis loop closes at a half cycle, the further stress–strain path follows this half cycle.
- memory 3: a half cycle that was started at the cssc ends if the stress and strain value of the starting point with opposite sign is reached. The further stress–strain path then follows the cssc. The stress–time path shown in *Figure 1* produces memory 3 at $X = -c$ because $|f| > |c|$, and at $Y = -f$ because $|g| > |f|$. Note that the path between X and f is constructed using the cssc of the third quadrant.

An optimized algorithm for performing rainflow counting as described above was presented by Clormann–Seeger[11]. The paper includes the source code of a simple computer program written in Fortran 77. Knowledge of the whole load history is not required. Thus, the algorithm is suitable for real-time operating. This advantage is as well achieved by the procedure of Downing–Socie[12] that counts the same closed hysteresis loops. However, Clormann–Seeger's approach records them closer to the sequence as they occur in the history. Another approach by Glinka–Kam[13] enables real-time operating by processing the load history in blocks. An approach of Hong[14] is essentially equal to Clormann–Seeger[11]. It only differs in the treatment of unclosed loops of the stress–strain path. After a first

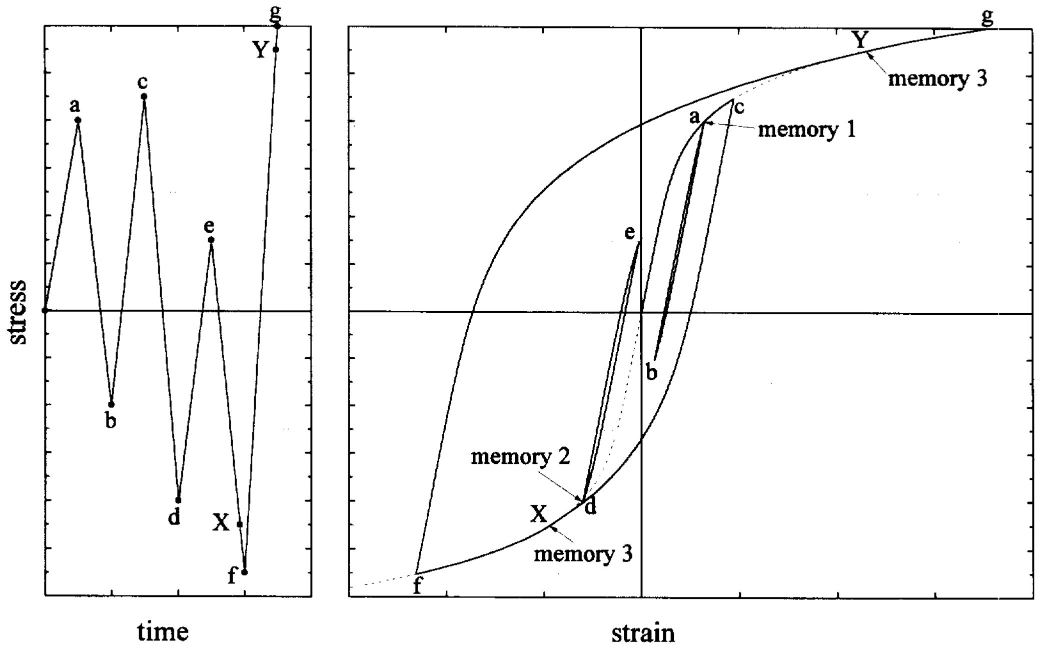

Figure 1 The uniaxial cyclic stress–strain model

rainflow counting of the load history, the remanent part is rearranged to begin and end with the maximum peak or minimum valley. Then again, rainflow counting is performed. However, this rearrangement is not very convenient if load sequence effects have to be taken into account.

The classical rainflow cycle counting registers hysteresis loops only at closing time. However, the most part of the total loop can be formed a long time (regarding load reversals) before final closing. If a hysteresis loop is registered only at closing time, its influence on the loops closed in the meantime since developing is not considered. This is illustrated by the example given in *Figure 2*. The stress–time history at the left leads to the stress–strain path in the middle that forms the closed hysteresis loops shown at the right. The sequence of the reversals **1** to **7** is registered in the order (**4**–**3**), (**6**–**5**), (**2**–**1**). An equivalent form is (**3**–**4**), (**5**–**6**), (**1**–**2**) because the starting point and the ending point of the closed loop are not differentiated when damage is determined. If a typical load sequence model is used, the influence of (**4**–**3**) on (**6**–**5**) and (**2**–**1**) is considered. However, this sequence is obviously not correct because peaks **1** and **2** occur before peaks **3**, **4**, **5** and **6**.

MODIFIED RAINFLOW CYCLE COUNTING

The modification of the classical rainflow method presented now is very simple but decisive. Every rising half cycle is registered and treated as a damaging event. If this half cycle does not close a hysteresis loop, it is called a virtual hysteresis loop. Three different cases have to be distinguished depending on the load sequence:

1. The virtual hysteresis loop will be closed later by

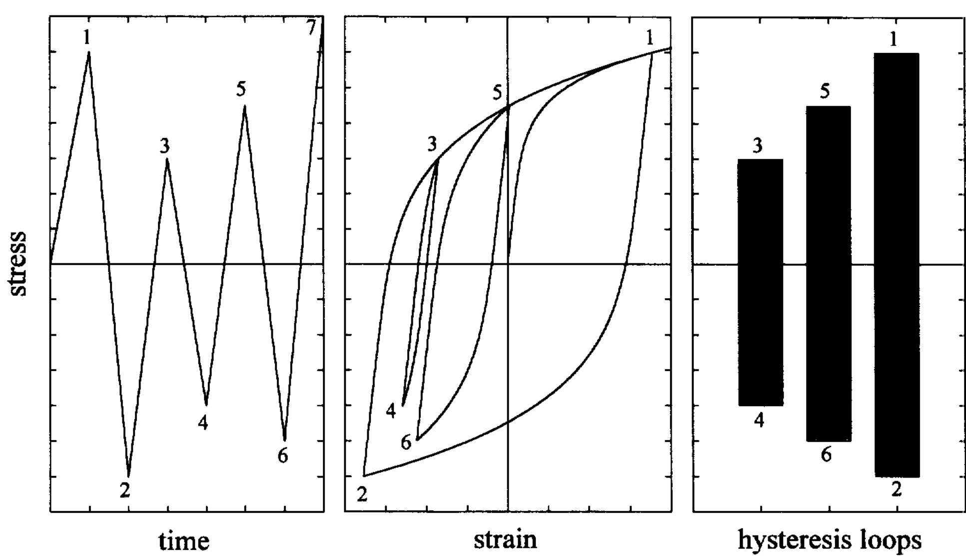

Figure 2 Example of the classical rainflow cycle counting

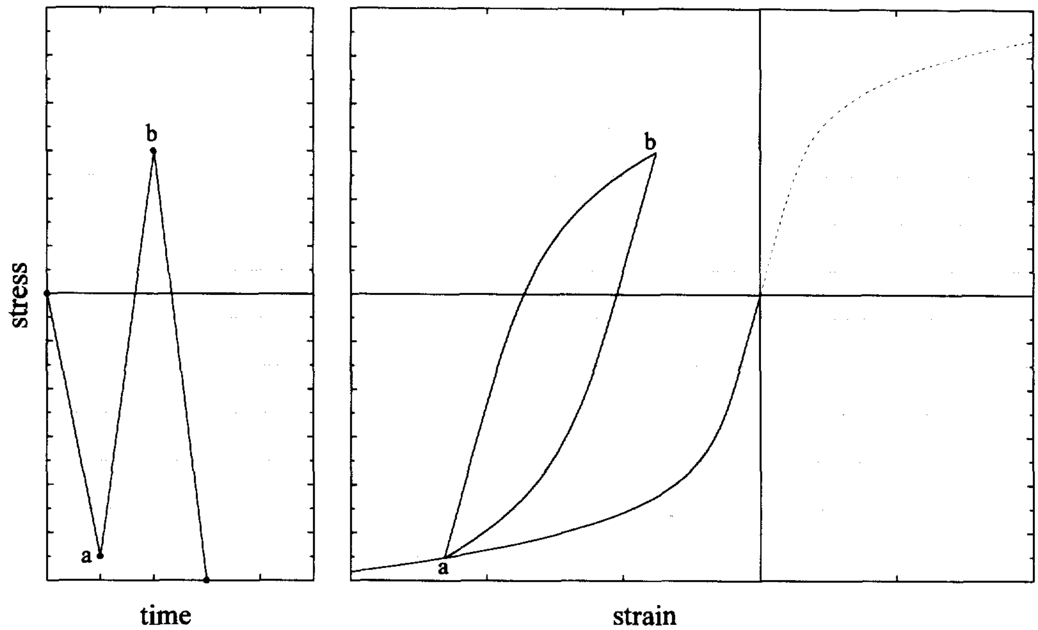

Figure 3 Hysteresis loop closed by a falling half cycle

a falling half cycle, see *Figure 3*. In this case, no further measure is necessary because the damage increment of this loop has been determined previously.

2. The rising half cycle treated as a virtual hysteresis loop will later continue, see *Figure 4*. Then, a new enlarged virtual hysteresis loop is registered. The increase in damage is determined by calculating the damage of the enlarged virtual loop and subtracting the damage of the previous virtual loop. This is further explained by the example given in *Figure 4*. The increase of damage from **b** to **d** is the sum of parts **c–b** and **b–d**. The latter is determined from

$$\Delta D_{\mathbf{b}-\mathbf{d}} = \Delta D_{\mathbf{a}-\mathbf{d}} - \Delta D_{\mathbf{a}-\mathbf{b}} \quad (1)$$

If a damage model is used that describes damage development dependent on terms that might have changed since $\Delta D_{\mathbf{a}-\mathbf{b}}$ was determined, for example, $\Delta\sigma_{\mathrm{eff}}$ or the crack length a, $\Delta D_{\mathbf{a}-\mathbf{b}}$ should be determined again but using the current values of these terms now. If the increased half cycle does not close a hysteresis loop, it is registered as a virtual hysteresis loop again.

3. The virtual hysteresis loop will not close until failure. Nevertheless, its amount of damage remains valid. Note that this treatment of unclosed parts of the cyclic stress–strain path differs from the classical rainflow cycle counting.

Figure 5 shows the flow chart of the modified rainflow cycle counting based on a flow chart from Ref. 11. It is designed expecting a strain–time sequence as input data. However, stress–time input data can be processed

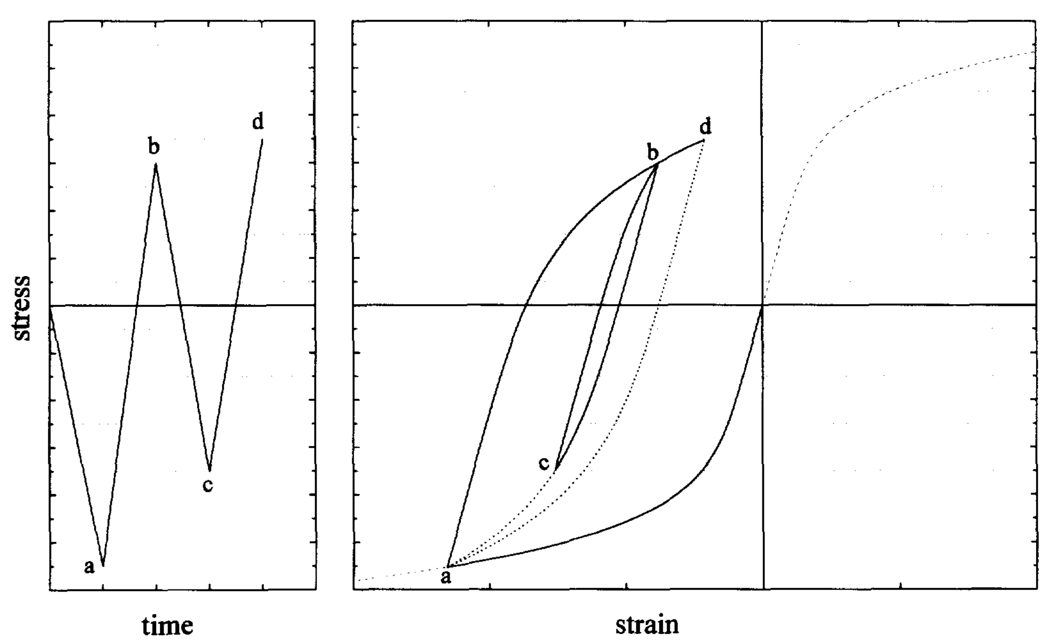

Figure 4 Increased rising half cycle

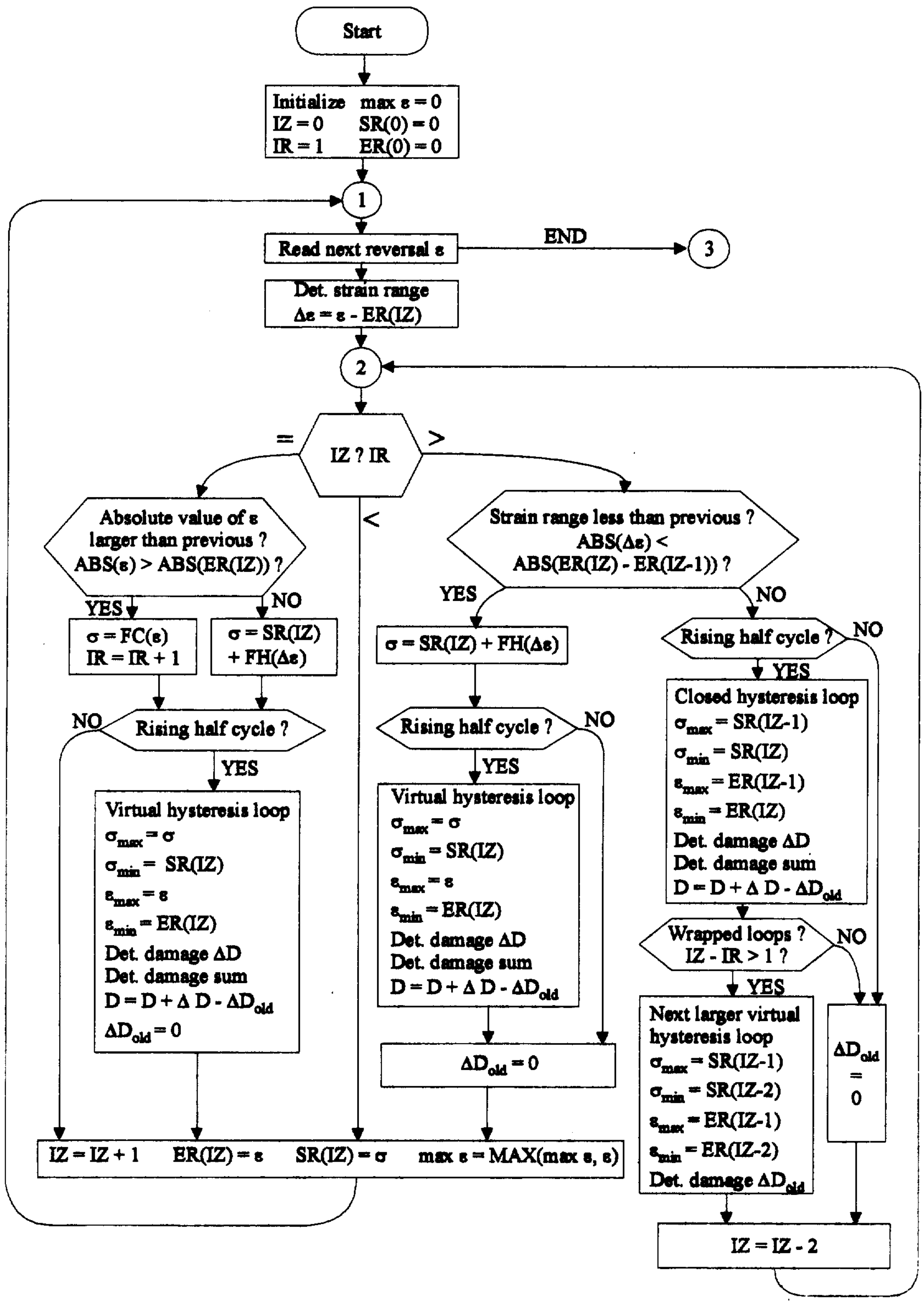

Figure 5 Flow chart of the modified rainflow cycle counting

as well in this algorithm but leading to a slightly different form of the flow chart. Two residual stacks are used containing the starting points of the half cycles which are not closed: ER for the strain values, SR for the stress values. The stacks are initially empty and will be filled (or refilled) when processing the strain history. Two counters are defined to organize the contents of these stacks. The first one, IZ, points at the position where the last starting point of a half cycle is located. It therefore represents the actual size of the stacks. The second one, IR, points at the position where the first starting point of a half cycle is located that is able to form a closed hysteresis loop. The values from position 1 to position (IR-1) belong in half cycles that cannot be closed because memory 1 or memory 3 had occurred. As can be seen from the flow chart, the algorithm is essentially based on a comparison of the two counters. The stress values are determined using an equation for the cssc (referred to as FC in the flow chart) and an equation for the half cycle (referred to as FH), respectively. The latter is obtained by doubling the cssc. Commonly, the Ramberg–Osgood formulation[15]

$$\epsilon = \frac{\sigma}{E} = \left(\frac{\sigma}{K'}\right)^{1/n'} \quad (2)$$

is used to describe the cssc. Herein, E denotes the modulus of elasticity, K' the strain hardening coefficient, and n' the strain hardening exponent. The half cycle then takes the form

$$\Delta\epsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2K'}\right)^{1/n'} \quad (3)$$

See *Figure 6* for definitions of $\Delta\sigma$ and $\Delta\epsilon$. The flow chart includes references to the calculation of the actual damage increment ΔD and the actual damage sum D, respectively. It is supposed that at least the maximum and minimum stress–strain values of the real or virtual hysteresis loop are required to perform this. Furthermore, it seems reasonable that the load sequence model used in the damage model should take into account the maximum strain value of the past history. Otherwise, rising load peaks leading to memory 1 would not be considered. Three different kinds of damaging events are registered by the modified rainflow counting algorithm:

1. closed hysteresis loops;
2. rising half cycles which are able to form a closed hysteresis loop;
3. rising half cycles which are not able to form a closed hysteresis loop because of memory 3.

The whole stress–strain path is taken into account except the part following the rising cssc in case of memory 1. Therefore, the number of damaging events, n_d, detected from a load history consisting of n_r reversals is

$$n_d = \text{INT}((n_r - 1)/2) \text{ if the first peak is positive,} \quad (4)$$

$$n_d = \text{INT}(n_r/2) \text{ if the first peak is negative.} \quad (5)$$

The algorithm presented above was successfully integrated into a newly developed short crack model[16].

EXAMPLE

The modified rainflow cycle counting will be illustrated by the example given in *Figure 7*. The strain–time history considered is shown above leading to the stress–strain path below. The initial values of the counters are IZ = 0 and IR = 1. *Table 1* lists their development when the strain history is processed. The first damaging events are registered for the half cycles **b–c** and **d–e** which are treated as virtual hysteresis loops. Note that up to reversal **f** no closed loop has been formed. From **f** to **g**, a stepwise description according to the flow chart in *Figure 5* will be given to further clarify the algorithm. The counters at reversal **f** are IZ = 6 and IR = 2. The description starts at label (1) in the flow chart.

- read next reversal ϵ(**f**)
- determine strain range $\Delta\epsilon$ = ϵ(**g**)–ER(**g**)

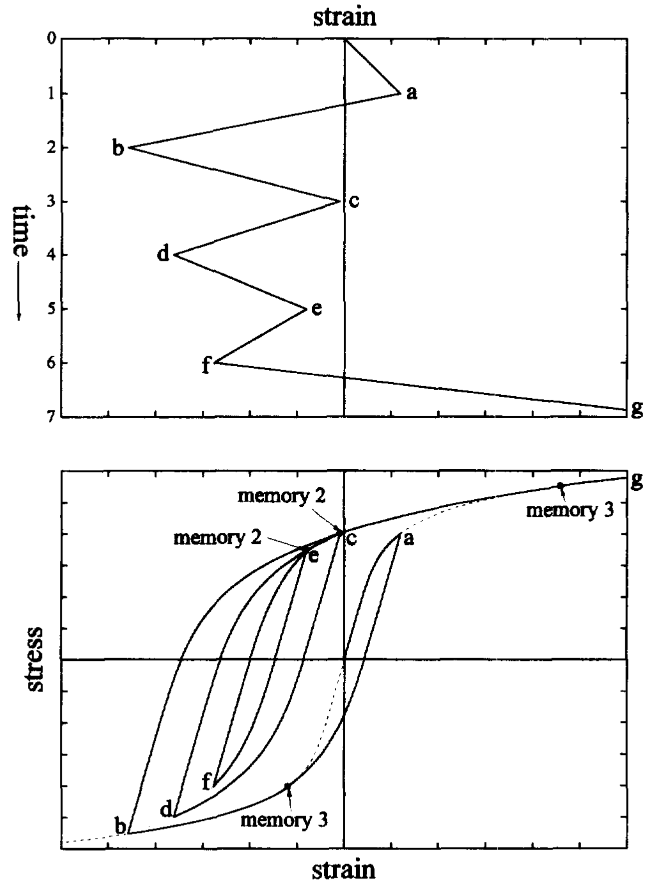

Figure 7 Example of the modified rainflow cycle counting

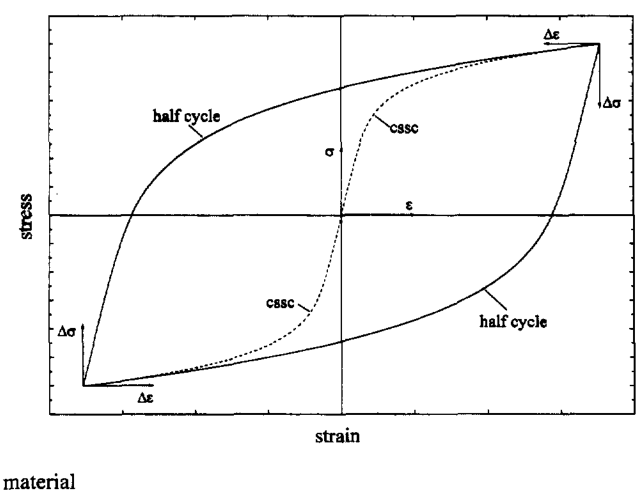

Figure 6 Masing behaviour of the material

Table 1 Counter values IZ and IR according to the example given in *Figure 2*

After reversal	IZ	IR
0	0	1
a	1	1
b	2	2
c	3	2
d	4	2
e	5	2
f	6	2
g	3	3

- compare counters: ⇒IZ = 6 > IR = 2
- current strain range larger than previous strain range!
- rising half cycle!
- closed hysteresis loop between **f** and **e**!
- determine stress and strain values
- determine damage increment $\Delta D = \Delta D_{f-e}$
- determine actual damage sum: $D = D + \Delta D$ ($\Delta D_{old} = 0$!!)
- compare counters: IZ–IR = 6–2 > 1 ⇒ wrapped half cycles
- consider next larger virtual hysteresis loop from **d** to **e**
- determine stress and strain values
- determine damage increment $\Delta D_{old} = \Delta D_{d-e}$
- IZ = IZ–2 ⇒IZ = 4
- back to label (2)!
- compare counters: ⇒IZ = 4 > IR = 2
- current strain range larger than previous strain range!
- rising half cycle!
- consider closed hysteresis loop between **d** and **c**
- determine stress and strain values
- determine damage increment $\Delta D = \Delta D_{d-c}$
- determine actual damage sum: $D = D + \Delta D - \Delta D_{old}$
- compare counters: IZ–IR = 4–2 > 1 ⇒ wrapped half cycles
- consider next larger virtual hysteresis loop from **b** to **c**
- determine stress and strain values
- determine damage increment $\Delta D_{old} = \Delta D_{b-c}$
- IZ = IZ–2 ⇒IZ = 2
- back to label (2)!
- compare counters: ⇒IZ = IR = 2
- absolute strain value of reversal **g** larger than previous absolute strain value!
- determine current stress value from the cssc
- IR = IR + 1 ⇒IR = 3 (FC)
- rising half cycle!
- consider virtual hysteresis loop between **b** and **g**
- determine stress and strain values
- determine damage increment $\Delta D = \Delta D_{b-g}$
- determine actual damage sum: $D = D + \Delta D - \Delta D_{old}$
- IZ = IZ + 1 ⇒IZ = 3
- register reversal: ER(3) = ϵ(**g**); SR(3) = σ(**g**); maxϵ = ϵ(**g**)
- back to label (1)!
- END

PRACTICAL RELEVANCE

Load sequence effects are known to be a dominant factor in fatigue damage accumulation. The reason for these effects can be put down to the influence of the preceding load history on the actual crack opening level. Theoretical load sequence models are usually based on a description of the plastic deformation around the crack tip. The modified rainflow counting algorithm presented in this paper is intended to provide correct input data for the load sequence model. Therefore, it is regarded as an improvement of fatigue damage description. Although the area of application is not limited, improvements of fatigue life prediction are most probable for cases showing significant load sequence effects. The significance essentially depends on three factors:

Load-time sequence

Load sequence effects are decisively determined by the amount of irregularity which is usually increased by high overload or underload peaks.

Crack length

Very short cracks show transient crack opening behaviour even for constant amplitude loading. Crack opening develops with increasing crack advance starting nearly at minimum load. For short cracks, the load sequence effect predominantly lowers the crack opening level because an increase of the opening level requires crack advance without further load peaks. However, in general, this condition is met seldomly because short crack growth is small compared with the plastic zone sizes at the crack tip. Therefore, the load sequence effect of short cracks is generally less significant than that for long cracks because there is less crack closure.

Structure

Notches or other structural inhomogenities cause transient crack opening behaviour even for long cracks under constant amplitude loading. For variable amplitude loading, the crack opening level is influenced not only by the load sequence but also by the actual position of the crack tip inside the notched region. Therefore, the load sequence effect for considerable crack growth through the notched region is generally more significant than that for crack growth in unnotched structures.

From the considerations above it can be deduced that most the important load sequence case is that of physically short or long crack growth through the notched region under highly irregular loading.

CONCLUSION

An algorithm for performing rainflow cycle counting considering the load sequence was proposed. This algorithm is intended to provide correct input data for a load sequence model commonly used in fatigue damage accumulation. Unclosed parts of the stress–strain path are registered by a special approach. By that, an improvement of fatigue life prediction is expected compared with calculations using the classical rainflow method.

REFERENCES

1 Matsuishi, M. and Endo, T., Fatigue of metals subjected to varying stress. In *Proceedings of the Kyushu Branch of Japan*

Society of Mechanics Engineering, Fukuoka, Japan (in Japanese), 1968, pp. 37–40.
2 Elber, W., Fatigue crack closure under cyclic tension. *Engineering Fracture Mechanics*, 1970, **2**, 37–45.
3 Elber, W., The significance of fatigue crack closure. In *Damage Tolerance in Aircraft Structures*, ASTM STP 486. ASTM, Philadelphia, 1971, pp. 230–242.
4 Willenborg, J., Engle, R. M. and Wood, H. A., A crack growth retardation model using an effective stress concept. TM 71-1-FBR. Wright–Patterson Air Force Base, OH, 1971.
5 Wheeler, O. E., Spectrum load and crack growth. *Transactions ASME, Journal of Basic Engineering*, 1972, **94**, 181–186.
6 Eidinoff, H. L. and Bell, P. D., Application of the crack closure concept to aircraft fatigue propagation analysis. In *Proceedings 9th ICAF Symposium on Aeronautical Fatigue*, pp. 5.2–1/58, Darmstadt, LBF (TR-136) 1977.
7 de Koning, A. U., A simple crack closure model for prediction of fatigue crack growth rates under variable amplitude loading. In *Fracture Mechanics: Thirteenth Conference, Special Technical Publication*. ASTM STP 743. ASTM, Philadelphia, 1981, pp. 63–85.
8 Führing, H., Modell zur nichtlinearen Rißfortschrittsvorhersage unter Berücksichtigung von Lastreihenfolge-Einflüssen (LOSEQ), FB-162. LBF, Darmstadt, 1982.
9 Führing, H. and Seeger, T., Fatigue crack growth under variable amplitude loading. In *Subcritical Crack Growth Due to Fatigue Stress Corrosion and Creep*, ed. L. H. Larson, Hrsg. Elsevier Applied Science, London, 1984, pp. 109–133.
10 Masing, G., Eigenspannungen und Verfestigung beim Messing. In *Proceedings of the 2nd International Congress of Applied Mechanics*, Zürich, 1926, pp. 332–335.
11 Clormann, U. H. and Seeger, T., RAINFLOW-HCM, Ein Zählverfahren für Betriebsfestigkeitsnachweise auf werkstoffmechanischer Grundlage. *Stahlbau*, 1986, **55**, 65–71.
12 Downing, S. D. and Socie, D. F., Simple rainflow counting algorithms. *International Journal of Fatigue*, 1982, **4**, 31–40.
13 Glinka, G. and Kam, J. C. P., Rainflow counting algorithm for very long stress histories. *International Journal of Fatigue*, 1987, **9**, 223–228.
14 Hong, N., A modified rainflow counting method. *International Journal of Fatigue*, 1991, **13**, 465–469.
15 Ramberg, W. and Osgood, W. R., Description of stress–strain curves by three parameters. Technical Note No. 902, NACA, 1943.
16 Anthes, R. J., Ein neuartiges Kurzrißfortschrittsmodell zur Anrißlebensdauervorhersage bei wiederholter Beanspruchung. Heft 57, Institut für Stahlbau und Werkstoffmechanik, Technische Hochschule Darmstadt, 1997.