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MIXED MODE FATIGUE CRACK GROWTH: A LITERATURE SURVEY

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Abstract—The applications of fracture mechanics have traditionally concentrated on crack growth problems under an opening or mode I mechanism. However, many service failures occur from growth of cracks subjected to mixed mode loadings. This paper reviews the various criteria and parameters proposed in the literature for predictions of mixed mode crack growth directions and rates. The physical basis and limitations for each criterion are briefly reviewed, and the corresponding experimental supports are discussed. Results from experimental studies using different specimen geometries and loading conditions are presented and discussed. The loading conditions discussed consist of crack growth under mode II, mode III, mixed mode I and II, and mixed mode I and III loads. The effects of important variables such as load magnitudes, material strength, initial crack tip condition, mean stress, load non-proportionality, overloads and crack closure on mixed mode crack growth directions and/or rates are also discussed. Copyright © 1996 Elsevier Science Ltd

Key words—mixed mode fatigue crack growth, mode II crack growth, mode III crack growth, multiaxial fatigue crack growth, mixed mode loading.

INTRODUCTION

THE ADOPTION of the damage tolerance design concept along with increased demand for accurate residual structure and component life predictions have provided growing demand for the study of fatigue crack growth in mechanical components. Traditional applications of fracture mechanics have been concentrated on cracks growing under an opening or mode I mechanism. However, many service failures occur from cracks subjected to mixed mode loadings. Examples for a crack loaded under mixed mode loadings can be found in various engineering applications. Many uniaxially loaded materials, structures and components often contain randomly oriented defects and cracks which develop a mixed mode state by virtue of their orientation with respect to the loading axis. A crack initiated in a transverse plane from a tubing shaft surface under bending and torsion is loaded under a combination of mode I, II and III loadings. Another example of mixed mode loading is the rolling contact problem in high speed rotating bearing[1]. A crack or crack-like defect in the radial direction propagates under combined mode I (due to the centrifugal forces) and mode II (due to the rolling contact forces) loadings.

A characteristic of mixed mode fatigue cracks is that they usually propagate in a non-self similar manner. In other words, a crack changes its growth direction when subjected to mixed mode loadings. Therefore, under mixed mode loading conditions, not only the fatigue crack growth rate is of importance, but also the crack growth direction. Several criteria have been proposed regarding the crack growth direction under mixed mode loadings. Also, several parameters have been suggested to correlate mixed mode fatigue crack growth rates. Some of these criteria are first reviewed. Since mixed mode crack growth tests are not standardized, many different specimen geometries and loadings have been used. Therefore, a review of different types of specimens and loadings is also provided. A review on experimental observations under mixed mode I and II, mode II, mode III, and mixed mode I and III loadings is then presented and experimental results which are difficult to explain are discussed. Some of the factors which can greatly affect the mixed mode crack growth behavior are also briefly reviewed.

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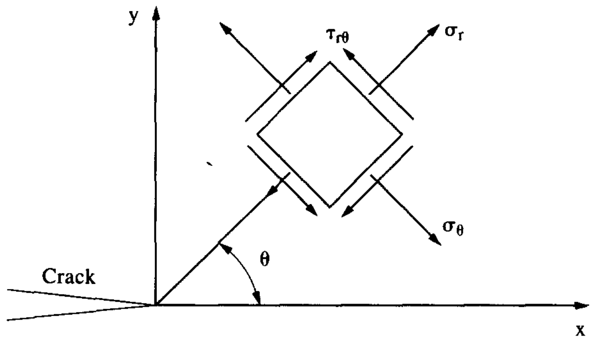

Fig. 1. Polar stress components in a stress element near a crack tip.

PREDICTION OF MIXED MODE CRACK GROWTH DIRECTIONS

Since the first study of mixed mode I and II fatigue crack growth by Iida and Kobayashi[2] much progress has been made in this area. As already indicated, there are two major aspects in mixed mode fatigue crack growth: crack growth direction and crack growth rate. Various criteria for the crack growth direction under mixed mode loadings have been proposed. Some of these criteria are reviewed in this section.

Maximum tangential stress criterion (MTS criterion)

This is one of the widely used theories for mixed mode crack growth, proposed by Erdogan and Sih[3]. This criterion states that: (i) crack propagation starts from the crack tip along the radial direction, $\theta = \theta_c$, on which the tangential stress, σ_θ, becomes maximum (see Fig. 1); and (ii) fracture starts when the maximum tangential stress, σ_θ, reaches a critical stress, σ_c, equal to fracture stress in uniaxial tension. Mathematically, condition (i) for the crack growth direction can be expressed as:

$$\frac{\partial \sigma_\theta}{\partial \theta} = 0 \text{ and } \frac{\partial^2 \sigma_\theta}{\partial \theta^2} < 0. \tag{1}$$

If Westergaard expressions are used to describe the stress field near the crack tip for a mixed mode I and II loading case, it can be shown[4] that the above criterion for the crack growth direction is the solution of the following equation:

$$K_I \sin\theta + K_{II}(3\cos\theta - 1) = 0. \tag{2}$$

This criterion has been widely used because of its simplicity and support by many experimental observations. The application of this criterion can be found from the works by several authors, including Gdoutos[5]; Yokobori *et al.*[6]; Pook[7]; Louah *et al.*[8]; Nayeb-Hashemi *et al.*[9]; Hyde and Chambers[10]; Mahajan and Ravi-Chandar[11]; Brown[12]; Abdel-Mageed and Pandey[13] and Chambers *et al.*[14]. Tian *et al.*[15] extended this criterion to the three-dimensional case. However, several works by Tanaka [16]; Smith and Pascoe [17]; Royer [18] and Abdel-Mageed and Pandey[19] can also be found in the literature that do not support this criterion.

Minimum strain energy density criterion (S-criterion)

This criterion was proposed by Sih[20, 21] and is based on the local density of the energy field in the crack tip region. The crack is assumed to grow in a direction along which the strain energy density factor, S, reaches a minimum value and fracture occurs when this factor reaches a critical value, S_c. The strain energy density factor, S, can be written as:

$$S = a_{11}k_1^2 + 2a_{12}k_1k_2 + a_{22}k_2^2 + a_{33}k_3^2, \tag{3}$$

where a_{ij} are coefficients relating the polar angle, θ, elastic modulus, E, and Poisson's ratio, ν. The k's are defined as:

$$k_i = K_i/\sqrt{\pi} \ (i = \text{I, II, III}), \tag{4}$$

where K_i are stress intensity factors for modes I, II and III, respectively. The necessary and sufficient conditions for the determination of crack growth directions are:

$$\frac{\partial S}{\partial \theta} = 0 \text{ and } \frac{\partial^2 S}{\partial \theta^2} > 0. \tag{5}$$

The S-criterion has been experimentally investigated by a number of researchers, including Sih [20, 22]; Sih and Barthelemy [23]; Badaliance [24]; Patel and Pandey [25] and Gao *et al.* [26]. A main advantage of this criterion is its ease and simplicity, and its ability to handle various combined loading situations. However, some contradictory observations have also been reported by Tanaka [16], and Abdel-Mageed and Pandey [19, 27]. Also, several authors have questioned the theoretical basis of this criterion. Theocaris and Andrianopoulos [28] argued that since S is a summation of distortional and dilatational strain energy densities, these two fundamentally different physical quantities should not be added together. Also, S is defined along the boundary of the so-called core region, which is assumed to be circular. This assumption has not been justified yet. Yan *et al.* [29] suggest that the Nadai elastic–plastic boundary, which considers the different yield strengths in tension and compression, should be used as the boundary of this core region, rather than the assumed circle. Wu [30] pointed out that S is constant under pure anti-plane loading condition (mode III) and the S-criterion fails to yield a preferred direction for this case. Wong [31] suggested that more terms in the Westergaard expressions of the stress field around the crack tip should be included in the S expression.

*The **J**-criterion*

This criterion was proposed by Hellen and Blackburn [32] in an attempt to use path-independent line integrals to study the problem of crack growth under mixed mode loadings. In a two-dimensional problem, a vector, **J**, is defined as:

$$\mathbf{J} = J_{\text{I}}\bar{i} + J_{\text{II}}\bar{j}, \tag{6}$$

with

$$J_k = \int_\Gamma (wn_k - u_{i,k}T_i)\,\mathrm{d}l \quad k = \text{I, II}, \tag{7}$$

where Γ is the contour of integration, w is the strain energy density, n_k is the k component of the unit outward normal to the contour of integration, u_i is the displacement, T_i is the traction and $\mathrm{d}l$ is the arc length element for the integration contour. This criterion states that: (i) a crack extends along the direction of vector **J** and (ii) fracture occurs when this vector **J** reaches a critical value.

This criterion was used by Dai and Zheng [33] for a simple plate specimen under uniaxial load with an inclined crack located at the center. The load conditions were mode I dominating cases. Under this condition, predictions based on the J-criterion were satisfactory. However, as pointed out by Gdoutos [4], for mode II load dominating cases, predictions based on the J-criterion deviate significantly from most experimental data.

*Dilatational strain energy density criterion (*T*-criterion)*

Theocaris and Andrianopoulos [34] and Theocaris *et al.* [35] proposed this criterion. As in the S-criterion, strain energy density is the basis of this criterion. However, the authors suggest separation of the total strain energy density into its distortional and dilatational components. This criterion states that: (i) a crack starts to propagate when the dilatational strain energy, T_{v}, at a point in the vicinity of its tip reaches a critical value, $T_{\text{v,cr}}$; and (ii) the elastic–plastic boundary obtained from the Mises yield condition should be used to evaluate T_{v} around the crack tip.

Condition (i) implies that a crack propagates along the maximum dilatational strain energy direction. According to condition (ii), by definition, the distortional part of the total energy is a constant along the elastic–plastic boundary. Therefore, as stated by Theocaris and Andrianopoulos[34], searching for maximum values of T (total strain energy defined in the same manner as S in the S-criterion) is equivalent to searching for maximum values of T_v. Thus the difference between S and T criteria is not that they are defined in different formats, but that they are defined at different locations. As indicated by Theocaris and Andrianopoulos[28], if the material is highly brittle so that the elastic–plastic interface is negligible, the boundary defined in the T-criterion can be assumed to be a small circle as suggested in the S-criterion. Under this condition, the difference between these two criteria is very small. Lam[36] pointed out another situation for which the difference is negligible, which is that of a long crack under low cyclic stress level. For this case, the relevant elastic–plastic interface for the T-criterion is the reversed plastic zone boundary, which is much smaller than the monotonic plastic zone size. In addition, the magnitude of the applied load under stable fatigue crack growth is usually much smaller than the fracture load.

Vector crack tip displacement (CTD) criterion

This criterion was proposed by Li[37], based on the concept that the vector crack tip displacement (CTD) is the "driving force" for fatigue crack growth. Here, the vector CTD is the vector summation of CTOD and CTSD, which are the crack tip opening displacement vector corresponding to mode I load (CTOD) and effective crack tip sliding displacement vector from mode II load (CTSD). The crack is assumed to propagate along the direction of this vector. This criterion was used by Li[37] to predict one of the available test results by Yokobori *et al.* [6]. Good agreements between predictions and experimental data were reported.

Tangential stress factor and tangential strain factor criteria

Based on the analyses of several previous criteria, Wu and Li[38] defined "tangential stress factor" and "tangential strain factor" as $r^{1/2}\sigma_\theta$ and $r^{1/2}\epsilon_\theta$, respectively. Coordinate variables, r and θ, are defined in Fig. 1. They proposed the so-called "maximum tangential stress factor criterion" and "maximum tangential strain factor criterion". The basic assumption is that a crack will propagate along the direction of $(r^{1/2}\sigma_\theta)_{max}$ based on the first criterion, or $(r^{1/2}\epsilon_\theta)_{max}$ based on the second criterion, if they reach the corresponding critical values. These quantities are evaluated along elastic–plastic boundaries. One of the advantages of these criteria claimed by the authors is that the lateral stress effect can be accounted for by including more terms in the stress field expressions near a crack tip. Also, the authors experimentally showed that the maximum tangential strain factor criterion can give much better predictions than the S-criterion for the case of a simple plate specimen under uniaxial compressive load, with an inclined crack located at the center.

Maximum tangential strain criterion

This criterion was proposed by Chambers *et al.* [14] based on the plastic blunting crack growth mechanism concept. The magnitude of blunting is suggested by the authors to be related to the plastic tangential strain. Small scale yielding was assumed in the development of this criterion. Consequently, the strains within the plastic region were thought to be compatible with those further away from the crack tip (i.e. the elastic field). Thus, tangential strains near the crack tip determined on the basis of linear elastic behavior are assumed to be approximately the same as those occurring in the plastic zone, even though the stresses are beyond the yield stress. Following similar procedures to those for the maximum tangential stress criterion, the direction of fatigue crack growth can be predicted.

Discussion of criteria for crack growth direction prediction

Among the aforementioned criteria, the maximum tangential stress (MTS) and the minimum strain energy density (S) criteria are widely used in mixed mode crack growth studies. The applications of these two criteria have been extended to mixed mode I, II and III loadings by Chen *et al.* [39], where it was reported that the minimum strain energy density criterion results in better predictions than the maximum tangential stress criterion. However, there is no single criterion which gives satisfactory predictions under all loading conditions. Experimental results which do not agree with the predictions can often be found. For example, all the aforementioned criteria, except for the J-criterion, predict that a crack under mode II loading propagates along about a 70 degree direction with respect to the original crack line. However, several authors have reported that a crack under mode II load either propagates in mode I or mode II, depending on conditions such as material properties and load magnitudes. Detailed experimental results are discussed in another section.

It can be noticed that most of the conventional fracture mechanics parameters have been used to predict the crack growth direction under mixed mode loadings, including stress intensity factors, strain energy densities, J-integrals and crack tip opening displacements. However, the use of these parameters have mainly been limited to linear elastic fracture mechanics, since they are usually represented by stress intensity factors in applications. Criteria are not yet available which include detailed considerations from the point of view of elastic–plastic fracture mechanics. Also, the aforementioned criteria do not distinguish between static and cyclic loadings, although the micromechanics of crack initiation and growth may be significantly different for the two loading conditions. Several authors have suggested some justifications for this. For example, Bold *et al.*[40] commented that under proportional loading, the maximum value of the stress around a crack tip which drives brittle fracture is directly proportional to the maximum range of that stress which is the controlling parameter for fatigue crack growth.

MIXED MODE FATIGUE CRACK GROWTH RATE PREDICTIONS

The following parameters are often found in the literature to correlate fatigue crack growth rates under mixed mode loadings.

Effective stress intensity factors

The fatigue crack growth rate has been expressed by Tanaka [16] using a Paris type equation as a function of an effective stress intensity factor:

$$\frac{da}{dN} = C(\Delta K_{eff})^m, \tag{8}$$

where ΔK_{eff} for combined mode I and II loadings is expressed by:

$$\Delta K_{eff} = \left[\Delta K_I^4 + 8\Delta K_{II}^4\right]^{0.25}. \tag{9}$$

This model is based on the assumption that a fatigue crack grows when the sum of the absolute values of the displacements in a plastic strip reaches a critical value. Under mixed mode conditions, it is assumed that deformations due to mode I and mode II loads are not interactive. Although other forms of effective stress intensity factors were also proposed by Tanaka [16], he found the correlation obtained from the parameter expressed by eq. (9) to provide the best fit for his experimental data. Tanaka suggests that this criterion can be extended to the combination of three mode loadings.

Another effective stress intensity factor suggested by Yan *et al.*[41] has the following form for mixed mode I and II loadings:

$$\Delta K_{eff} = \frac{1}{2}\cos\frac{\theta_0}{2}[\Delta K_I(1 + \cos\theta_0) - 3\Delta K_{II}\sin\theta_0], \tag{10}$$

where θ_0 is the crack growth direction obtained from the maximum tangential stress criterion. This parameter is a simple extension of the maximum tangential stress criterion to the case of mixed mode fatigue crack growth. This parameter, however, lacks experimental verification.

Strain energy density factors

The strain energy density factor has been used to correlate mode I fatigue crack growth rates by several researchers, including Sih and Barthelemy [23]; Badaliance [24] and Lam [36, 42]. A Paris type equation has been used in the analyses, except that the ΔK is replaced by ΔS in eq. (8). The mean stress effect has also been investigated by Badaliance [24] using this parameter.

Patel and Pandey[25] used the strain energy density factor to correlate fatigue crack growth rate under mixed mode loadings. The authors suggest that parameters such as K, COD and J are unsuitable to handle the mixed mode crack growth problem, because a crack does not grow in a self-similar manner under mixed mode loads. Fatigue crack growth rate was then suggested to be expressed by the following equation[25]:

$$\frac{da}{dN} = C_s(\Delta S)^{n_s}, \tag{11}$$

where ΔS is the strain energy density factor range, and C_s and n_s are material constants. By equating this equation with the Paris equation for mode I loading, the constants C_s and n_s in eq. (11) can be found. The material constants thus obtained from the Paris equation are assumed not to be sensitive to the modes of loading in the regime of linear elastic fracture mechanics.

Gao's model

A fatigue crack growth rate equation was proposed by Gao *et al.*[43] based on the assumptions that: (i) fatigue crack growth is a function of crack tip reversed plastic deformation and (ii) fatigue crack growth rate is inversely proportional to the true fracture ductility, γ_f. This equation is given as:

$$\frac{da}{dN} = k\frac{r_p(\theta^*)}{\gamma_f}, \tag{12}$$

where $r_p(\theta^*)$ is the maximum extent of crack tip reversed plastic zone, and θ^* is the crack angle which gives the maximum value for r_p and can be expressed as a function of K_I and K_{II}. A so-called "true plastic zone size" obtained by considering the effect of T-stress on the plastic zone size was used in correlating the experimental fatigue crack growth rate data by Gao *et al.* [43] (T-stress is the second term in the series expansion of σ_x after the singular term, where σ_x is the stress parallel to the crack). Based on the experimental mixed mode fatigue crack growth rate analyses using a plate type specimen under biaxial loading, as well as four-point bending and shear specimens made of four materials, the authors concluded that the true plastic zone size is a useful parameter for correlating mixed mode crack growth rate data.

ΔJ as a parameter

The J-integral approach has been suggested to be used to correlate fatigue crack growth rates under mode I loading by Dowling and Begley[44]; Wuthrich[45]; Srivastava[46] and Chow and Lu[47]. This concept was extended to fatigue crack growth rate analyses of small cracks under mixed mode loadings by Hoshide and Socie[48]. The authors state that it is desirable to directly correlate the crack growth rate with crack tip displacements. However, alternative parameters are needed for such a correlation due to the difficulty of crack tip displacement measurements. The J-integral is thought to be a candidate parameter since analytical relationships between the J-integral and displacements along the crack plane exist. In this model, it was assumed that under mixed mode loading situations, the crack opening or sliding displacements (COD or CSD) at a specific distance behind the crack tip are the parameters governing the crack growth, which are expressed as a Paris type equation. The fatigue crack growth rates contributed by mode I and mode II were assumed to be linearly additive. Another assumption was that the material constants in the equations for mode I and II crack growth rate expressions are the same. This parameter was used by Hoshide and Socie[48] to correlate small crack growth rate data from Inconel 718 thin-wall tubular specimens under mixed mode loads. The estimated relation resulted in higher crack growth rates than the experimental data.

Equivalent strain intensity factors

An equivalent strain intensity factor range has been proposed by Socie *et al.*[49] which is given by:

$$\Delta k_{\mathrm{eq}}(\varepsilon) = \left[(Y_2 G \Delta\gamma_{\mathrm{m}}/\pi c)^2 + (Y_1 E \Delta\varepsilon_{\mathrm{n}}/\pi c)^2\right]^{0.5}, \tag{13}$$

where $\Delta\gamma_m$ and $\Delta\varepsilon_n$ are maximum shear strain and normal strain ranges acting on the plane of maximum shear strain range, respectively, c is half the crack length, G and E are shear and Young's modulus, respectively, and Y_1 and Y_2 are geometry factors. The equivalent strain intensity factor was found useful by the authors for correlating mixed mode small crack growth data of SAE 1045 steel and Inconel 718. The applicability of this parameter for long cracks remains to be examined.

Another form of effective strain-based intensity factor range was proposed by Reddy and Fatemi [50] and was used to correlate small fatigue crack growth data of Inconel 718 and SAE 1045 steel under a wide variety of biaxial loading conditions. This parameter is based on a critical plane approach for multiaxial fatigue proposed by Fatemi and Socie [51]. This approach postulates that cracks initiate and grow on the maximum shear stress plane and that the normal stress to this plane assists in the fatigue crack growth process. The components of this model consist of the maximum shear strain range, $\Delta\gamma_{max}$, and the maximum normal stress σ_n^{max}, acting on $\Delta\gamma_{max}$ plane. The mean stress effect is accounted for since the maximum normal stress consists of the alternating stress and the mean normal stress. This parameter can be expressed in the following form:

$$\Delta K_{\mathrm{CPA}} = G \Delta\gamma_{\max}\left(1 + k\sigma_{\mathrm{n}}^{\max}/\sigma_{\mathrm{y}}\right)(\pi c)^{1/2}, \tag{14}$$

where G is the shear modulus, c is half the surface crack length, σ_y is the yield strength and k is a material constant. Satisfactory correlations of surface crack growth rate data were obtained by using this parameter, and the correlations were as good as those obtained using the J-integral. It was postulated that this parameter can be used for materials in which crack initiation and small crack growth occur along the maximum shear strain amplitude planes [50].

Chen and Keer's model

Based on Dugdale's model, the fatigue crack growth rate was related to the accumulated crack opening and sliding plastic displacements by Chen and Keer [52]. The following assumptions were made: (i) the crack closure and the crack branching effects can be neglected; (ii) the total accumulated plastic displacement is the vector sum of the accumulated crack opening and crack sliding displacements; and (iii) the tensile and shear stresses in the yield zone satisfy the von Mises criterion. Based on these assumptions as well as the relationship between stress intensity factors and displacements, and the relationship between ΔJ and displacements under small scale yielding condition, the following expressions were derived for mixed mode I and II loadings:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = \frac{\pi \Delta K_{\mathrm{eff}}^4}{96 E \gamma \sigma_{\mathrm{yc}}^2}, \tag{15}$$

where

$$\Delta K_{\mathrm{eff}} = \left[(\Delta K_{\mathrm{I}}^2 + 3\Delta K_{\mathrm{II}}^2)^3 (\Delta K_{\mathrm{I}}^2 + \Delta K_{\mathrm{II}}^2)\right]^{\frac{1}{8}} \tag{16}$$

and

$$\frac{\mathrm{d}a}{\mathrm{d}N} = \frac{\pi E}{96\gamma}\left(\frac{1 + 3R_\sigma^2}{1 + R_\sigma^2}\right)^{1.5} \frac{\Delta J^2}{\sigma_{\mathrm{yc}}^2}. \tag{17}$$

In these equations, γ is considered as the effective surface energy for fatigue crack growth, R_σ is the ratio of the applied shear stress to tensile stress range and σ_{yc} is the cyclic yield strength. Compared with experimental data, the results predicted by this model were thought to be reasonable by Chen and Keer [52].

EXPERIMENTAL OBSERVATIONS

Specimen configurations used for mixed mode fatigue crack growth studies

Many different specimen geometries have been used to produce different combinations of mixed mode loadings under different test conditions. Since no standardized specimen geometries exist, it is difficult to compare experimental results from different specimen geometries to draw general conclusions with regards to mixed mode crack growth. Richard [53] has presented nine different specimens which are often used in mixed mode fracture and fatigue studies (see Fig. 2). These include: plate specimen with inclined central crack under tension (S1), plate specimen with inclined edge crack under tension (S2), disc specimen with inclined central crack (S3), cruciform specimen with inclined central crack (S4), shear specimen with inclined central crack (S5), tubular specimen with inclined crack under torsion (S6), tubular specimen with transverse crack under combined tensile and torsional stresses (S7), three or four point bending and shear specimen with an offset edge crack (S8), and compact tension and shear specimen (S9). Critical criteria and evaluation of suitable specimens for mixed mode crack growth studies have been made by Richard [53]. These mainly include: ability to apply full range of mixed mode load combinations, compactness, ease of manufacture, ability to form fatigue precracks under mode I loading, and ease of clamping and loading.

Other specimen geometries have also been used in mixed mode crack growth studies, as shown in Fig. 3. The double oblique edge crack specimen [Fig. 3(a)] was used to study the fatigue crack growth behavior of HY80 and HY130 steels under mixed mode loadings at various stress levels by Lal [54]. A horizontal force can also be applied to this specimen. The compact

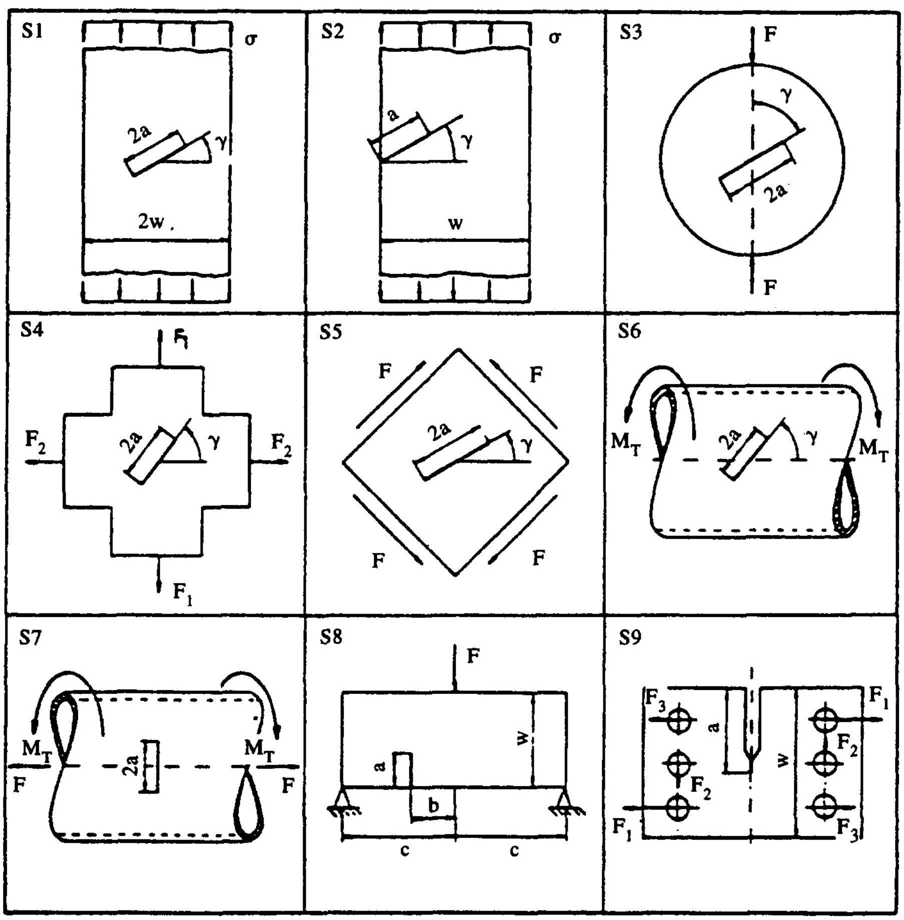

Fig. 2. Specimens commonly used in mixed mode crack growth studies [53].

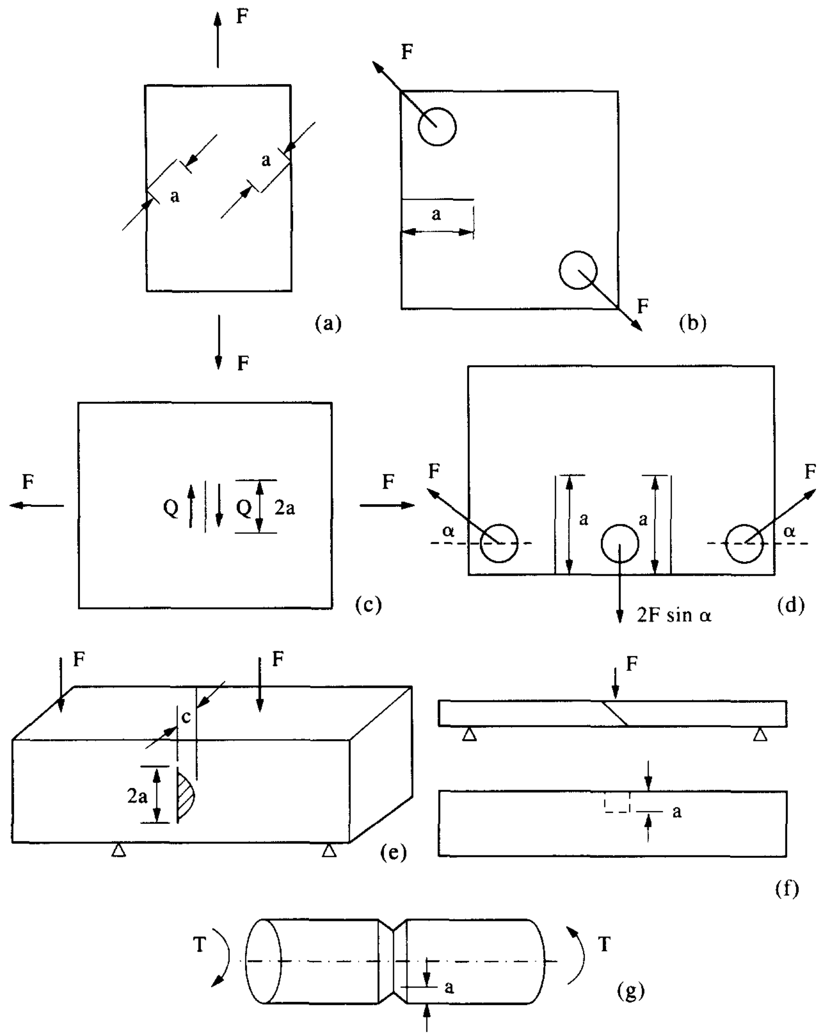

Fig. 3. Additional specimen configurations used in mixed mode crack growth studies: (a) double oblique edge crack specimen, (b) general CT specimen with an inclined load, (c) general CCT specimen loaded with special grips, (d) double CT specimen with inclined loads, (e) four-point bending specimen with penny shaped precrack on side surface, (f) three-point bending specimen with through-thickness precrack on side surface and (g) solid round specimen with circumferential crack under torsion.

tension (CT) specimen with inclined loads [Fig. 3(b)] was used by Mahanty and Maiti [55] to study stable crack growth under mixed mode I and II loadings. A CT type specimen for mode II loading was also suggested by Buzzard *et al.* [56]. The center cracked tension (CCT) specimen loaded in tension and shear [Fig. 3(c)] was used by Otsuka and Tohgo [57] and Otsuka *et al.* [58] to study the fatigue crack growth under static mode I and cyclic mode II loadings. Specially designed grips have to be used for this loading. The double CT specimen with inclined loads [Fig. 3(d)] was used by Chambers *et al.* [14] to produce mixed mode I and II loadings. Pook and Greenman [59] also used this specimen to study crack growth under mode II loading [$\alpha = 90°$ in Fig. 3(d)]. The four-point bending and shear specimen with penny shaped surface crack [Fig. 3(e)] was used by Tohgo *et al.* [60] to produce mixed mode II and III loads. Pook [7] used the specimen shown in Fig. 3(f) to study crack growth behavior under mixed mode I and III loads. The solid round specimen with a circumferential notch [Fig. 3(g)] under torsion is often used to produce pure mode III loading [61].

Crack growth under mixed mode I and II loadings

Most of the experimental studies of crack growth under mixed mode I and II loadings have been conducted using a plate type specimen with an inclined central crack under tension

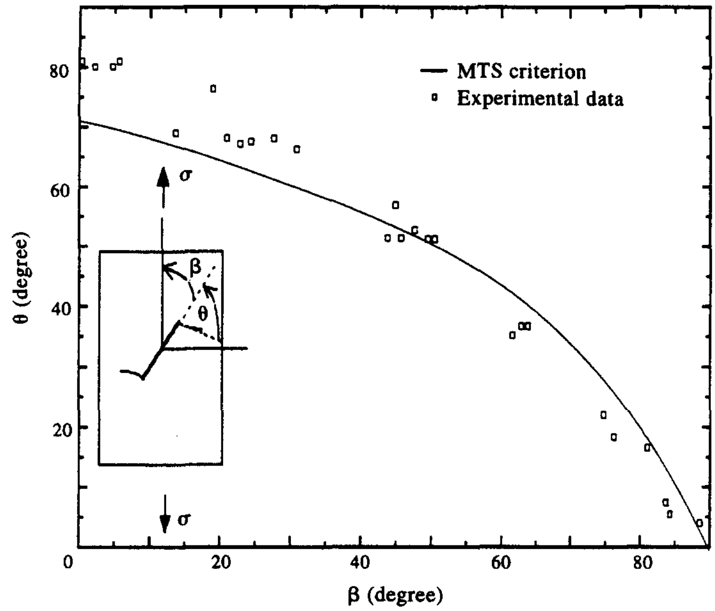

Fig. 4. Typical experimental results from specimens with inclined center crack under monotonic uniaxial tension load [62].

(specimen S1 in Fig. 2). For this case, most of the crack growth direction criteria provide satisfactory predictions. Under monotonic loading, typical experimental results are shown in Fig. 4 [62]. Significant discrepancies occur when the inclined crack line is close to the tensile loading axis (mode II load dominating case), as reported by the authors. It has been reported by Wu and Li [38] that when a compressive load is applied to this type of specimen, there are significant deviations between the predicted and the experimentally observed crack growth directions over a wide range of inclined crack angles. Liu and Allison [63] performed experiments using cruciform specimens made of 2024-T351 and 7075-T7351 aluminum alloys under biaxial monotonic loading. The initial cracks were perpendicular to one of the loading axes. It was found that a crack would turn to the direction perpendicular to the higher tensile load if it was initially perpendicular to the lower tensile load. Under shear only loading, the crack turned to the direction perpendicular to the maximum principal stress.

Under cyclic mixed mode loadings, a crack does not necessarily propagate in mode I when a plate type specimen with inclined central crack is used. Tanaka [16] used this type of specimen made of pure aluminum under uniaxial cyclic load, precracked under mode I loading. Initial cracks were intentionally put in the rolling direction. The crack growth direction was observed to significantly deviate from the predicted directions based on either the MTS or S-criteria. The crack growth rate becomes higher when the mode I component of cyclic stress is accompanied by a mode II component, as compared with the mode I growth rate at the same ΔK_I level. The fatigue crack growth direction was observed to be approximately perpendicular to the applied tensile load axis (i.e. in mode I) at stress ranges just above the threshold values. However, the crack grew in the direction of the initial inclined crack, if a stress range about 60% larger than the threshold range was applied. Based on the analysis of plastic zone size at the crack tip, Tanaka concluded that the transition from mode I growth to mixed mode growth occurs when the monotonic yield zone size is larger than the specimen thickness. Other than mode I crack growth has also been reported using other specimen configurations. In the case of the rolling contact fatigue problem, it appears that cracks grow by a mode II mechanism, as reported by Bold *et al.* [40].

Experimental evidence suggests that the crack growth mode is material and load magnitude dependent. Center cracked tension specimens were used by Otsuka and Tohgo [57] made of aluminum alloys 2017-T3 and 7075-T6. A specially designed grip was used to apply cyclic mixed mode I and II loadings, in addition to a static mode I load. Two kinds of fatigue crack growth

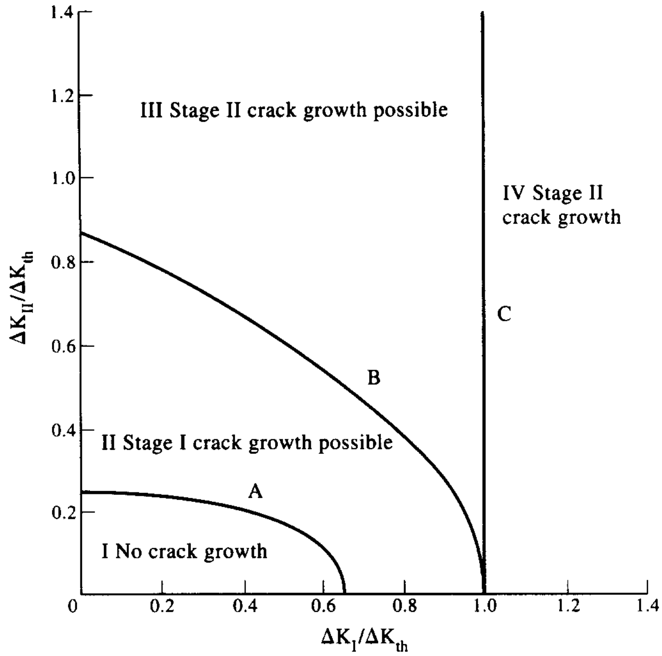

Fig. 5. Relationships between fatigue crack growth modes and the relative applied load magnitudes[65].

features were observed for 2017-T3 alloy. When $\Delta K_{II} > 3$ MPa$\sqrt{m}$ and $K_{II}/K_I > 1.6$, cracks grew in shear mode (mode II) which was further divided into two cases. In one case, cracks grew along the initial inclined crack direction (therefore, in this case, K_{II}/K_I is fixed), and in another case the crack grew in a direction in which K_{II}/K_I increases with crack growth. When K_{II}/K_I was less than 1.6, the crack would grow in mode I. For 7075-T6 alloy, the conditions for shear mode fatigue crack growth were: $\Delta K_{II} > 8$ MPa$\sqrt{m}$ and $K_{II}/K_I > 1.6$. If these conditions were not satisfied, the crack would grow in mode I. In this case, a fatigue crack grew along its initial direction for a short distance before it turned to mode I. The idea of different crack growth modes corresponding to different combinations of mixed mode loads has been extended by Pook [65] to form a "failure mechanism map" in ΔK_I–ΔK_{II} plane, as shown in Fig. 5. In this figure, stage I crack refers to mode II crack growth and stage II crack refers to mode I crack growth. The differences in fatigue crack growth under pure tension, combined tension and torsion, and pure torsion between high cycle and low-cycle fatigue were studied by Hua and Socie [66] using tubular specimens made of 1045 steel. Under tension and combined tension and torsion, cracks propagated in mode I. Under pure torsion, mode II crack propagation was observed and a dominant single crack initiated and continued to grow until final failure in the high cycle fatigue regime. However, in low cycle fatigue, a number of damage nuclei were observed to form and the surface length of microcracks which appeared in the early stages remained almost unchanged during the fatigue life. But the intensity of the microcracks substantially increased with the number of cycles. This study by Hua and Socie [66] further indicates that the crack growth mode depends on both type, as well as magnitude of loading.

Crack growth mode also depends on the initial crack tip condition. Gao *et al.* [26, 43, 67] performed fatigue crack growth tests under mixed mode I and II loadings using four-point bending and shear specimens made of AISI 316 stainless steel, DTD 5120 aluminum alloy, Ti–6Al–4V and 1Cr–Mo–V steels, and cruciform specimens made of AISI 316 stainless steel with an inclined crack under biaxial loading. The specimens were either precracked in mode I or a slit was inserted by spark erosion. Mixed mode I and II, and pure mode II loadings were applied. Experimental results are shown in Fig. 6. It can be seen that if the specimens are fatigue precracked, when the applied load is just above the threshold level, a crack grows in mode II for a short distance (this is expressed as "shear mode growth" in Fig. 6). If the applied load is relatively large, a crack grows in mode I. For the specimens with initial slit, cracks grow in mode I. This observation is contradictory to what Tanaka [16] reported, as already discussed.

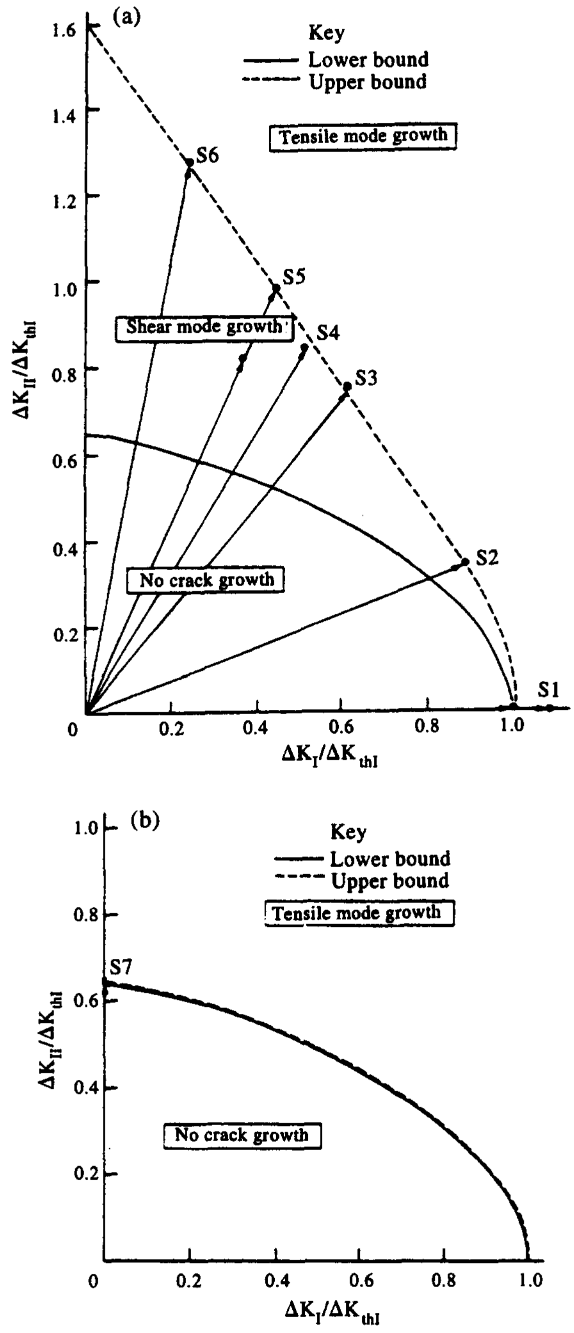

Fig. 6. The upper and lower bound curves of fatigue crack thresholds under mixed mode I and mode II loadings (S refers to specimen number): (a) fatigue precracked specimens and (b) slit specimens with slit root radius ρ = 0.08 mm[67].

Gao *et al.* [26, 43] suggested that the lower and upper bands of threshold values (solid and dashed lines in Fig. 6, respectively) corresponded to surface stress free (for a slit crack) and non-zero contact surface (for fatigue precracked) conditions, respectively. By increasing the tensile mean stress, the contact and rubbing of crack surfaces decreases. In this case, a mode I branch crack easily forms and the upper band approaches the lower band. This was experimentally shown by Gao *et al.* The mean stress effect on mixed mode threshold behavior was also studied by Tong *et al.* [68]. The mode I crack growth threshold decreased as the R ratio increased. Chambers *et al.* [14] also reported that under relatively low load level, using the specimen configuration shown in Fig. 3(d) made of a high chromium steel, a mode II crack always grew for a short distance before it turned to mode I direction. However, when the applied load was relatively large, this short mode II crack growth behavior was found to disappear. These observations were made under both mixed mode and pure mode II loadings. Pook [69] used a similar specimen made of mild steel under mode II loading at different levels. However, no mode II crack growth, even for a short distance, was observed, although the cracks were also precracked under mode I loading. Pook reported that in all cases, fatigue crack growth was at an angle of about 70 degrees to the precrack (i.e. nearly mode I).

From the aforementioned experimental observations, it can be concluded that the crack growth mode depends on many factors including the material, load magnitude, the ratio among different mode loads, initial crack tip condition, mean stress and anisotropy. Bold *et al.* [40] suggested that a residual stress field left by mode I loading would favor a mode II crack growth. It would be interesting to design special experiments to verify this suggestion. Fatigue crack growth maps showing different crack growth modes similar to that proposed by Pook [65] would be helpful to understand the relationship between load magnitudes and growth modes for each material. Quantifying the mode II friction effect on fatigue crack growth is far from clear yet. Iida and Kabayashi [2] and Tanaka [16] observed that fatigue crack growth rates under mixed mode loads are significantly higher than those under mode I load at the same magnitude. However, different loading methods make it difficult to draw general conclusions.

Crack growth under mode II loading

Tables 1 and 2 provide a summary of fatigue crack growth studies under mode II loading. The following remarks can be made based on a review of these studies:

(1) Experimental results using a plate specimen with an inclined center crack show that mode I growth is dominating, even under almost pure shear loading. However, mode II crack growth has been reported with this type of specimen made of aluminum, if the applied shear load is large enough (ref. [57] in Table 1). With other types of specimens, mode II crack growth is easier to produce. These include four-point shear, fixed-grip shear, Y shaped and flat cruciform specimens. However, it appears that the easiest way to produce mode II crack growth is by using thin-walled tubular specimens under completely reversed torsional load, as reported by Hua and Socie [66]; Hu *et al.* [70]; Yokobori *et al.* [71]; and Tanaka *et al.* [72].

(2) The load magnitude can have a significant effect on the fatigue crack growth direction. Socie [73] indicated that a shear mode growth (mode II) generally occurs at high strain ranges, while tensile growth (mode I) occurs at low loads in torsional fatigue tests. Similar observations have been made by Otsuka and Tohgo [57]; and Otsuka *et al.* [58]. From a static mode I superimposed on cyclic mode II test, Otsuka *et al.* [58] observed that if the applied mode II load was above a mode I branch crack threshold, branch crack growth occurred. If the mode II load was between the branch crack threshold and mode II threshold, then mode II growth occurred. If the static mode I load was relatively high, the mode II growth would continue without arrest, otherwise it would arrest. Crack growth direction dependence on the load magnitude has also been reported by Brown and Miller [74] using tubular specimens under combined tensile and torsional loading. When the shear strain to tensile strain ratio was less than 1.5, fatigue cracks which initiated in mode II would turn to mode I. However, when this ratio was larger than 1.5, cracks would continue to grow in mode II.

(3) In general, mode II crack growth occurs more often in aluminum alloys than in steels. Also, based on the limited data listed in Tables 1 and 2 reported by Otsuka and Tohgo [57]; Otsuka *et al.* [58] and Hu *et al.* [70], it seems the lower the material strength, the easier for mode II cracks to grow. In addition, Otsuka *et al.* [58] showed that the sharper the crack tip, the easier for a mode II crack to grow. Therefore, it can be concluded that crack growth directions are material, specimen geometry, as well as load magnitude, dependent.

(4) Several recent experiments were performed by Hu *et al.* [70] using solid round bar specimens with two longitudinal notches located in the opposite directions. The notches had

Table 1. Summary of experimental results from mode II fatigue crack growth studies

Reference	Material	σ_u/σ_y (MPa)	Specimen type	Loading conditions	Crack growth characteristics
Otsuka *et al.*[75]	7N01-T4 high-strength Al alloy	345/(N/A)	Four-point shear	Asymmetrically loaded ($R = 0$), static mode I load was also applied	For specimens: 1) all show continuous steady mode II growth 2) $(da/dN)_{II}$ is 2 to 3 times faster than $(da/dN)_I$ under the same ΔK magnitude
	5083-O Al alloy	306/(N/A)			For specimens used to model a component: 1) If ΔK_{II} is relatively small, crack growth at both crack tips are in mode I 2) If ΔK_{II} is relatively large, at one end of the crack where positive K_I exists, mode II growth was observed; at the other end with negative K_I, mode I growth was observed
Otsuka and Tohgo[57]	7075-T6 Al alloy	587/516	Center-cracked plate	Cyclic tensile and shear loads with static mode I load	1) If $\Delta K_{II} > 8$ MPa(m)$^{0.5}$ and $K_{II}/K_I > 1.6$, mode II initiation was observed, followed by mode I growth to failure. 2) If $\Delta K_{II} > 8$ MPa(m)$^{0.5}$, $K_{II}/K_I > 1.6$, and $\Delta K_I/\Delta K_{II} \ll 1$, mode II growth to final failure was observed. 3) In other cases, mode I growth to final failure was observed
	2017-T3 Al alloy	443/274			If $\Delta K_{II} > 3$ MPa(m)$^{0.5}$ and $K_{II}/K_I > 1.6$, mode II growth; otherwise, mode I growth
Pook[69]	Mild steel		Double-cracked CT specimen [Fig. 3(d)]	N/A	In all cases, mode I crack growth was observed
Otsuka *et al.*[58]	7075-T6 Al alloy	N/A	Fixed-grip shear [Fig. 3(c)]	Alternating shear ($R = -1$) and static mode I	For specimen with fatigue precrack: (static mode I SIF: $K_{IS} = 7$ MPa(m)$^{0.5}$) 1) $\Delta K_{II} = 9.3$ MPa(m)$^{0.5}$, mode II growth; 2) $\Delta K_{II} = 6.2$ MPa(m)$^{0.5}$, mode I growth; For specimen without precrack and $K_{IS} = 7$ MPa(m)$^{0.5}$: 1) $\Delta K_{II} = 18.6$ MPa(m)$^{0.5}$, mode II growth 2)$\Delta K_{II} = 10.9$ MPa(m)$^{0.5}$, mode II crack grows to fracture, with non-propagating mode I cracks developing 3) $\Delta K_{II} = 6.2$ MPa(m)$^{0.5}$, mode I crack grows to fracture, with non-propagating mode II cracks developing 4) $\Delta K_{II} = 3.1$ MPa(m)$^{0.5}$, mode I growth at the notch tip, cracks initiate at different locations on the periphery of the notch Effects of notch tip radius and ΔK_{II}: 1) The larger the notch radius, the easier for mode I crack to grow 2) The larger the magnitude of mode II load, the easier for a mode II crack to grow

Table 1 continued

Royer[18]	22M5 steel	626/407	"Y" shaped specimen	Pulsating load with $R = 0.05$	Under small mode II load, mode I crack growth was observed. When mode II load is large, mode II crack growth was observed
Smith and Pascoe[17]	HY100 steel	N/A	Cruciform specimen (Fig. 2, S4)	Stress biaxiality with $\zeta = -1$ (out-of-phase load)	Both mode I and mode II crack growths were observed under identical loading cases. The authors suggest that this might be due to the fact that the steel has a high sulfide inclusion count
Hua and Socie[66]	1045 HR steel	380/150	Thin-walled tubular specimen	Alternating loads with $R = -1$ (strain-controlled)	1) In HCF region, mode II growth along longitudinal direction was observed. 2) In LCF region, mode II cracks on both planes along longitudinal and transverse directions were observed
Hu *et al.*[70]	40 Cr steel tempered at: 240°C 360°C 460°C 550°C	 1840/1560 1500/1420 1250/1160 1030/960	Solid round specimen with longitudinal semi-circular notch	Alternating torsion with $R = -1$	1) Under the same tempering condition and with an increase of stress, crack growth mode changes from mode I, to longitudinal mode III, to transverse mode II. 2) Under the same stress level and with an increase of strength, growth mode changes from transverse mode II, to longitudinal mode III, to mode I
Yokobori *et al.*[71]	5083P-O Al alloy	78.5/(N/A)	Thin-walled tubular specimen with a transverse notch	a) Alternating torsion, $R = -1$. b) Pulsating torsion, $R = 0$	1) In small scale, stepwise crack growth was observed, and each step consisted of two incremental growths, one in the axial direction and the other in the circumferential direction. 2) In large scale, the overall crack propagation inclines 0 to 15 degrees from the specimen axis at early stage, and 35 to 40 degrees at later stage. 3) For loading case a, four cracks at the four notch corners were found; for loading case b, only two cracks were found
Tanaka *et al.*[72]	7075-T6 Al alloy SNCM8 steel	627/559 1076/971	Thin-walled and solid tubular specimens	Axial and torsional loads ($R = -1$)	All cases show mode II crack growth. In LCF region, cracks grow along the longitudinal direction; in HCF region, they grow in the circumferential direction
Yokobori *et al.*[6]	5083P-O Al alloy	322/152	Thin-walled tubular specimen with a transverse slit (Fig. 2, S7)	Tension and torsion	Four different mixed mode loads were tested, with the largest $\Delta K_{II}/\Delta K_{I} = 2.16$. In all cases, cracks propagated in the direction nearly perpendicular to the direction of principal tensile stress. However, with an increase of shear loads, stepwise cracking was observed (the steps were in the applied shear stress directions)

semicircular cross-section areas. A fully reversed torque was applied to the specimen. Three different kinds of cracks were observed. In the first type, a crack with normal tensile fracture (NF) formed in the direction of 45 degrees to the specimen axis (or mode I). A second crack type was formed on longitudinal shear fracture plane (LS) or mode III. The third crack type formed in the transverse shear fracture plane (TS) or mode II. It was observed that with an increase in the stress level, the fatigue crack growth mode transforms from NF to LS to TS. Generally, these observations are in accordance with the results from Hua and Socie[66] and Otsuka *et al.*[58]. Under the same stress level, but with an increase in material strength, the fatigue crack growth mode transformed from TS to LS to NF. Therefore, a ductile material (or lower strength material) tends to fail by mode II crack growth mechanism, whereas a brittle material (usually with high strength) tends to fail by a mode I crack growth mechanism. This is in accordance with the usual experimental observations. The NF type (mode I crack) fracture provides the longest life time and the TS type (mode II crack) fracture provides the shortest life time.

(5) Mode II crack growth can be realized by adding a positive static mode I load (tensile load) to the crack when it is cycled under mode II loads, as can be seen for the cases reported by Otsuka *et al.*[58] in Table 1 and Yokobori *et al.*[71] in Table 2. Otsuka *et al.* [75] argued that static K_I can prevent the friction between the mating crack surfaces. They also suggested the effect of static K_I on mode II crack growth to be negligible if the magnitude of K_I is large enough to prevent friction between crack surfaces, but not so large as to produce large scale yielding. However, Liu[76] suggested that mode II crack growth can be dominating when the cyclic stress has a strong mode I compression component. He indicated that the compressive stress suppresses mode I crack growth. This argument can be supported by the fact that in contact surface fatigue, usually a crack tends to grow in mode II, as reported by Ghosn[1].

(6) Yokobori *et al.* [71] used thin-walled tubular specimens with a transverse slot, cycled under torsional loading. Stepwise crack growth behavior was observed, with each step consisting of two straight crack increments, one in the longitudinal direction and the other in the circumferential direction. These directions are along the maximum shear planes. However, the crack increment sizes in each direction were different, resulting in the overall crack growth direction being 0 to 15 degrees with respect to the specimen axis at the early growth stage (mode II dominating), and 35 to 40 degrees at later stages (mode I dominating). This suggests that although a crack may propagate in mode I macroscopically, the growth mechanism can be by shear.

(7) A common definition of threshold for mode II crack growth does not yet exist. Some authors define the threshold value for mode II crack growth only under mode II crack growth condition. Yet others define mode II threshold value even when the crack is propagating in mode I, but is under mode II loading. The threshold value obtained according to the later definition is less than that obtained according to the former definition.

Crack growth under mode III loading

Circumferentially notched cylindrical specimens are often used to study fatigue crack growth behavior involving mode III loading [Fig. 3(g)]. Under mode III loads, a crack usually grows in its initial plane, macroscopically. In a finer scale, mode I facets are sometimes observed which are usually aligned in the overall crack growth direction. Different facet widths can be observed depending on the material and loading magnitude. However, mode I facets do not always appear under mode III loads. In this case, it is still not clear whether mode I facets do not form, or they form and are subsequently destroyed by rubbing. Another explanation for lack of mode I facets by Pook[61] is that the mechanism may be similar to that for stage I (mode II) fatigue crack growth which takes place on planes of maximum shear stress under conditions of localized general yielding. In other words, crack growth occurs due to mode II shear, but the macroscopic appearance of the fatigue crack growth is mode I. The type of fracture surface under the presence of mode III displacements usually depends on the material, magnitude of mode III loads and the stress ratio. Pook [61], after reviewing several authors' works, concluded that mode I facets were more commonly observed as the strength level of the tested material increased, ΔK_{III} decreased and R ratio changed from -1 to 0.08. This conclusion is consistent with observations by Brown [77], who found pure mode III cracking without mode I facets to form only in the presence of large scale plasticity. For this case, elastic–plastic fracture mechanics parameters were suggested to be used to characterize fatigue crack growth rates.

Hourlier *et al.* [78] also used circumferentially notched round specimens made of 26NCDV14 steel under torsional cyclic loads in their studies. They observed mode I facets on the fracture surface for long cracks under low torque levels. These facets contained fatigue crack growth features. However, for short cracks under large torques, the fracture surfaces appeared flat and smeared with severe abrasion. These fracture surfaces were formed by ductile rupture features. The flat surface could be interpreted as the results of either macroscopic mode III crack extension, or the consequence of the rubbing effect of a previously existing mode I facet morphology. The existence of ductile rupture features suggests that under mode III loading, the crack front morphology is very complex. The crack growth process regarding the mode I facets can be interpreted as follows [78]: relatively long facets are developed radially in mode I, leaving in between them uncracked ligaments. These uncracked ligaments are then subjected to very high stresses, giving rise to ductile rupture far behind facet crack front. Pook [61] suggested that the threshold stress intensity factor for mode III crack growth is substantially higher than that for mode I facet growth.

Crack growth under mixed mode I and III loadings

Fatigue crack growth behavior under mixed mode I and III loadings has been studied by Pook [7] using three-point bend specimens, with an inclined crack on the specimen lateral surface [Fig. 3(f)]. The crack turned to mode I growth direction, which occurred at two distinct scales. At a fine scale (i.e. 0.1 mm), initial crack growth was by formation of mode I branch cracks which developed into a twisted fracture surface consisting of narrow mode I facets separated by cliffs. The facets eventually grew out and the fracture surface became smooth. At a larger scale (i.e. 1 mm), the shift to mode I crack growth occurred as an initial directional discontinuity. A smooth rotation of the crack front then followed, until it became a pure mode I crack. Unexpected crack arrest was reported to sometimes occur. Among some of the crack arrested specimens, crack growth before the arrest was found to be confined to the specimen interior, resulting in a tongue-shaped crack front at arrest. Upon cycling a crack arrested specimen under a higher load, a crack front first straightened out and then remained reasonably straight for the remainder of the test. Crack arrests took place before the mode I facets had grown out. These results, therefore, represent the growth behavior of a twisted crack.

Static mode III and cyclic mode I tests were conducted by Hourlier and Pineau [79] using circumferentially notched specimen [Fig. 3(g)] made of two titanium alloys and two steels. A steady torque was applied to the specimen by two horizontal arms which were connected to dead weights by a pulley system. Their experimental results show that a strong decrease (as much as two orders of magnitude) in fatigue crack growth rate was caused by the addition of static mode III load, as compared to pure mode I behavior. By measuring the load line displacement as a function of the axial load, it was found that this growth rate reduction was caused by a strong mode I crack closure effect due to the applied static mode III load. Furthermore, tests with a high R-ratio revealed that this crack closure is significantly reduced by a mean mode I stress. Fracture surfaces with mode I facets were observed in one of the steels and one titanium alloy. This phenomenon, however, was not observed in the other two materials. However, in all materials, the fracture surfaces exhibited a strong rubbing effect. The explanation of the mode I facet formations by the authors was that the superimposition of mode III loading rotates the principal stress axes in the plane perpendicular to the direction of crack propagation. Continuous adjustment of the crack plane along the entire crack front was not possible and consequently, the crack breaks into partial fronts which can adjust the new maximum stress direction.

Tschegg *et al.* [64] also studied crack face interactions between mode I and III loads using a solid round specimen with circumferential notches [Fig. 3(g)]. All the specimens were precracked. For the "cyclic mode III and static mode I" (type A) tests, specimens made of AISI C1018 steel were fatigue precracked under cyclic torque in order to obtain a continuous transition from precracking to the measuring procedure. For the "cyclic mode I and static mode III" (type B) tests, specimens were made of a ferrite chromium steel. Static mode III load was produced by dead weights and precracking was conducted under mode I loading. From type A tests, they found fatigue crack growth rates to increase with increasing mode I static load. This was attributed to a reduction in crack face interactions. Fatigue crack growth rates in type B tests reduced with an increase of mode III static load. The ΔK_{Ith} values increased steadily with increasing the static K_{III}.

Akhurst *et al.* [80] attempted to calculate the fatigue crack growth rate in turbo generator rotors using mixed mode fatigue crack growth concept. The rotor's self weight was thought to be the main source of the bending (mode I) cyclic stresses. However, a high torque was applied to the shaft during operation, subjecting any crack normal to the rotor's axis to mode III loading. Fatigue crack growth rate was reported to reduce by a factor of 10 to 20 due to the presence of mode III load. This result was verified by a laboratory test.

Circumferentially notched solid round specimens made of PMMA (polymethyl methacrylate) were used by Davenport and Smith [81] to study the fracture behavior under superimposed monotonic mode I and mode III loadings. The applied loadings were torque (mode III) and tensile load (mode I). The loading sequence consists of tension first followed by torsion, torsion first followed by tension, and torsion and tension being applied simultaneously. Simultaneous loading resulted in the lowest toughness, and torsion preload followed by tensile fracture resulted in the greatest toughness. The authors suggested that plastic flow is relatively unconstrained in torsion, causing the plastic zone at the crack tip to be large prior to fracture. Therefore, the subsequent tensile fracture toughness is considerably higher than that for mode I load alone. For simultaneous tension and torsion loading, plastic flow is more constrained and brittle fracture occurs in preference to continued yielding. The higher hydrostatic stress component associated with the simultaneous loading inhibits the formation of a plastic zone, resulting in the lowest toughness. Therefore, the strong influence of non-linear deformation causes the mixed mode fracture toughness to be dependent on the sequence and type of loading.

Some factors affecting mixed mode fatigue crack growth behavior

Some of the factors which can greatly influence the fatigue crack growth behavior under mixed mode loadings include load non-proportionality, overloads, crack closure and the T-term. Extensive studies of the effects of these factors on mixed mode crack growth have not been conducted. It is, therefore, difficult to generalize the findings from the few available experimental studies which are briefly discussed below.

In non-proportional mixed mode loadings, the ratios among different mode loads vary during each cycle. The most widely studied feature of non-proportional loading concerns the effect of superimposing one static mode load on another cyclic mode load [40]. Results from some of these experiments have already been reviewed in the previous subsections. Using cruciform specimens with inclined cracks made of 35NCD16 steel and AU4G aluminum alloy, Hourlier *et al.* [78] conducted non-proportional fatigue crack growth tests under mixed mode loadings. The non-proportional loading gave rise to crack bifurcation. The crack growth rates measured after crack bifurcation were very similar to those corresponding to pure mode I loading. Since tests with similar loading conditions applied to two different materials gave rise to different crack paths, the authors concluded that a criterion based solely on nominal values of stress intensity factors is unable to correctly predict the crack path behavior for a wide range of materials.

Nayeb-Hashemi *et al.* [9] and Nayeb-Hashemi [82] studied the effect of mode II overloading on subsequent mode I crack growth using four-point and three-point bending and shear specimens made of AISI 4340 steel. No retardation was observed during the mode I load cycles, after a single prior mode II overload had been applied. In fact, the crack growth rate accelerated for a very short distance, much smaller than the mode II plastic zone size. However, when prior mode II overloads were cycled enough times (i.e. 10^4 cycles), a branched crack was formed, with crack growth retardation reported during the following mode I loading.

A plate type specimen made of 2024-T3 aluminum alloy with a central inclined crack under uniaxial tensile cyclic loading was used by Abdel-Mageed and Pandey [13] to study the closure effect on mixed mode crack growth. The crack opening stress intensity factor was measured using a clip gauge technique. The predicted crack path incorporating the closure effect resulted in a better agreement between the experimental and predicted crack growth paths using the maximum tangential stress criterion. Hourlier *et al.* [78] suggest that the crack closure phenomenon can have significant effect on crack growth directions.

Usually, only the first term in the series expansion of stress intensity factors is used to describe the crack tip stress field. The second term in the series expansion represents a stress, T, parallel to the crack. It was indicated by Karihaloo *et al.* [83]; Pook [7] and Smith and Pascoe [17] that a crack is directionally stable if the T-term was negative. Pook [7] also suggested that cracks are not necessarily unstable in the growth direction when the T-term becomes positive. Gao *et al.* [43] showed that at moderate stress levels, the T-term becomes significant in the crack tip stress field and should not be omitted arbitrarily. They also concluded that the T-stress has no obvious effect on mixed mode threshold behavior.

SUMMARY AND CONCLUSIONS

(1) The use of fracture mechanics has traditionally concentrated on crack growth under an opening or mode I mechanism. However, many service failures occur from cracks subjected to mixed mode loadings. A characteristic of mixed mode fatigue cracks is that they usually propagate in a non-self similar manner. Therefore, under mixed mode loading conditions, not only the fatigue crack growth rate is of importance, but also the crack growth direction.

(2) Various criteria for crack growth direction prediction under mixed mode loadings have been proposed. The maximum tangential stress and the minimum strain energy density criteria have been widely used in mixed mode crack growth studies. However, experimental results which do not agree with the predictions can often be found. Also, the use of these parameters has mainly been limited to linear elastic fracture mechanics regime, since they are usually represented by stress intensity factors in applications. Criteria are not yet available which include detailed considerations from the point of view of elastic–plastic fracture mechanics.

(3) Parameters which have been used to correlate fatigue crack growth rates under mixed mode loadings include effective stress intensity factors, strain energy density factors, J-integral and equivalent strain intensity factors. Even though good correlations of mixed mode crack growth rate data have been obtained by some of the aforementioned parameters, there is no single parameter which gives satisfactory correlations under all loading conditions.

(4) Many different specimen geometries have been used to produce different combinations of mixed mode loadings under different test conditions. These include plate specimens with inclined edge or central crack, cruciform specimen with central inclined crack, tubular specimen with inclined or transverse crack, and three or four point bending and shear specimens. The main considerations in design of the specimen are ability to apply full range of mixed mode load combinations, compactness, ease of manufacture, ability to form fatigue precracks under mode I loading, and ease of clamping and loading.

(5) Most of the experimental studies of crack growth under mixed mode I and II loadings have been conducted using a plate type specimen with an inclined central crack under tension. Experimental evidence suggests that the crack growth mode depends on material, load magnitude, initial crack tip condition, ratio among different mode loads and mean stress. Fatigue crack growth maps showing different crack growth modes corresponding to different combinations of mixed mode loads have been utilized to understand the relationship between load magnitudes and growth modes for each material. Quantifying the mode II friction effect on mixed mode fatigue crack growth is far from clear yet.

(6) Experimental results using a plate specimen with an inclined center crack show that mode I growth is dominating, even under almost pure shear loading. The easiest way to produce mode II crack growth appears to be by using thin-walled tubular specimens under completely reversed torsional load. The load magnitude can have a significant effect on the fatigue crack growth direction. Using tubular specimen in torsion, a ductile material tends to fail by mode II crack growth mechanism, whereas a brittle material tends to fail by a mode I crack growth mechanism. Also, although a crack may propagate in mode I macroscopically, the growth mechanism can be by shear.

(7) Circumferentially notched cylindrical specimens are often used to study fatigue crack growth behavior involving mode III loading. Under mode III loads, mode I facets are sometimes observed which are usually aligned in the overall crack growth direction. Mode I facets are more commonly observed as the strength level of the tested material increases, ΔK_{III} decreases and R ratio increases. The threshold stress intensity factor for mode III crack growth has been found to be substantially higher than that for mode I facet growth.

(8) Under mixed mode I and III loadings, crack growth behavior represents the growth behavior of a twisted crack. A significant decrease in mode I fatigue crack growth rate can result from the addition of a static mode III load, which has been attributed to a strong mode I crack closure effect due to the applied static mode III load. Also, material nonlinear deformation has a strong influence on mixed mode I and III crack growth behavior, causing the fracture toughness to be dependent on the sequence and type of loading.

(9) Fatigue crack growth behavior under mixed mode loadings can be strongly influenced by factors such as load non-proportionality, overloads, crack closure and T-stress term. The number of studies dealing with these effects is very limited, and the roles and contributions of these factors in mixed mode crack growth are far from clear yet.

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