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Fatigue crack growth simulation of aluminium alloy under spectrum loadings

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ABSTRACT

Fatigue crack growth in structure components, which is subjected to variable amplitude loading, is a very complex subject. Studying of fatigue crack growth rate and fatigue life calculation under spectrum loading is vital in life prediction of engineering structures at higher reliability. The main aim of this paper is to address how to characterize the load sequence effects in fatigue crack propagation under variable amplitude loading. Thus, a fatigue life under various load spectra, which was predicted, based on the Austen, Forman and NASGRO models. The findings were then compared to the similar results using FASTRAN and AFGROW codes. These models are validated with the literature-based fatigue crack growth test data in 2024-T3 Aluminium alloys under various overload, underload, and spectrum loadings. With the consideration of the load cycle interactions, finally, the results show a good agreement in the behaviour with small differences in fatigue life compare to the test data.

1. Introduction

Nowadays, the study of fatigue crack growth rate and fatigue life calculation under spectrum loading is very important for the reliable life prediction of engineering structures. A number of load interaction models have been developed to correlate fatigue crack growth rates and predict crack growth under variable amplitude loading over the past three decades.

It is difficult to model all the parameters influence fatigue crack growth correctly due to the random nature of variable amplitude loading. Overloads are known to retard crack growth, while underloads accelerate crack growth relative to the background rate. These interactions, which are highly dependent upon the loading sequence, make the prediction of fatigue life under variable amplitude loading more complex than under constant amplitude loading. Many models have been developed to predict the fatigue lives of components subjected to variable amplitude loadings [1–4]. The earliest of these are based on calculations of the yield zone size ahead of the crack tip and they are still widely used in many applications and research. The Wheeler model [5] and Willenborg et al. model [6], for example, both fall into this category. The Willenborg model [4–9], on the other hand, does not incorporate any empirical parameters but it uses the material yield stress to give a plastic zone size. The amount of retardation is determined as a function of the stress intensity factor necessary to cancel the effect of the overload plastic zone. The model computes an effective stress intensity factor that is being reduced by the compressive residual stress. However, the Willenborg model was found to be not reliable for predicting the overload retardation [4].

The second main category of retardation models, known as the crack closure models, is based on Elber's experimental observation [10], which used to model crack growth rates under variable amplitude loadings [11–16]. As a result of the tensile plastic deformation left in the wake of a fatigue crack, a partial closure of the crack faces occurs during part of a fatigue load cycle. Since crack propagation can only occur during the time for which the crack is fully open, the formation of crack closure reduces the range of the applied stress that is effective for crack propagation. In addition, the magnitude of stress required for the crack to be fully open, i.e., the crack opening stress, depends on the previous load history. As a crack propagates through an overload plastic zone, the residual stresses in the zone increase the load required to open the crack and cause crack growth retardation. Thus, the use of the crack closure models required the crack opening stress to be determined throughout the load history. This is accomplished either by direct experimental measurements [17–19] or by finite element computations [20–22]. However, the major drawback to using crack closure models is that measuring the crack opening stress under variable amplitude loading is very difficult and the magnitude and the precision might depend on the measuring techniques, while the finite element analysis for computing the crack opening stress is often complicated and relatively time consuming. More recent proposals include combinations of the Wheeler model with the Newman crack closure model [23] and model based on the strain energy density factor [24]. However, due to the number and complexity of the mechanisms involved in this problem, no universal model exists yet. Kujawski [25] clearly indicated that

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there is no general agreement among researchers regarding the significance of closure concept on fatigue crack Behaviour.

With respect to the continuity information between this study and the available literature, the purpose of this paper is to characterize the effects of load sequence on fatigue crack propagation under variable amplitude loading. For that reason, a feasible study towards the crack propagation model under various spectra loading has been carried out based on the Austen, Interpolation Forman and NASGRO models. These models are compared to FASTRN and AFGROW codes under various variable amplitude and spectrum loading. One of the aims of the analysis is to show the effect of using different fatigue crack growth (FCG) models with various load sequences. The results showed that a different behaviour has been found for different models with load sequences. The predicted fatigue life based on the Austin model gave the minimum values, while the maximum value were given by NASGRO model and the other models lies in between of them. The results show also the effect of the load sequences on fatigue life.

2. Theoretical background

The reason for building fatigue crack growth models is to link theoretical ideas with the observed data. Modelling of FCG rate data has enhanced the ability to create damage tolerant design philosophies [26]. The influence of the mean stress is probably the most significant and it usually results in closely spaced lines parallel to each other. Region I, which is shown in Fig. 2, represents the early development of a fatigue crack and the crack growth rate, for which da/dN is typically in the order 10^{-6} mm/cycle or smaller of the test data results from ASTM E647. This region is extremely sensitive and it is largely influenced by the microstructure features of the material such as grain size, the mean stress of the applied load. The most important feature of this region is the existence of a stress intensity factor range below which fatigue crack should not propagate. This value is defined as the fatigue crack growth threshold and is represented by the symbol ΔK_{th}. The limitation of the Paris law is that it is only capable of describing data in region II. If the data exhibits a threshold (region I) or an accelerated growth (region III) Paris law cannot adequately describe these regions. Region III represents the fatigue crack growth at very high rate, $da/dN > 10^{-3}$ mm/cycle due to rapid and unstable crack growth just prior to final failure. The da/dN versus ΔK curve becomes steep and asymptotically approaches the fracture toughness K_c for the material.

A number of equations have been developed to describe the sigmoidal da/dN–ΔK relationship. Paris and Erdogan [27] were apparently the first to discover the power law relationship to describe the stable crack growth in region II. Many variations based on the Paris law have been developed to consider the R-ratio effect, the threshold value of the stress intensity factor range (ΔK_{th}), and the facture toughness of the material (K_c) [28–31]. Since the fatigue the stress intensity factor range, ΔK, and the maximum stress intensity factor ($K_{\max}$) mainly control crack growth rate. Another model, the Austen growth model [32], is known as a implicitly model threshold and it is expressed in the following equation:

$$da/dN = C \cdot (\Delta K_{eff})^n \tag{1}$$

where $\Delta K_{eff} = \Delta K_{\max\ eff} - \Delta K_{\min\ eff}$, $K_{\max\ eff} = K_{\max} + K_{SF}$, $K_{\min\ eff} = -\max(K_{\min}, K_{CL})$, K_{SF} is defined as the modification for static fracture and K_{CL} is known as the stress intensity at the crack closure. Furthermore, Austen modelled the onset of fast fracture using the following expression:

$$K_{SF} = K_{\max}/(K_{1C} - K_{\max}) \tag{2}$$

where K_{1C} is the plane strain fracture toughness.

Austen also takes account of the threshold and short cracks by applying a crack closure stress K_{CL} expressed as:

$$K_{CL} = K_{\max} - K_{\max} \cdot \sqrt{(a + I_o)/a} + \Delta K_{th}/(1 - R) \tag{3}$$

I_o is the smallest crack size that will propagate and is given by:

$$I_o = 1/\pi(K_{th}/\Delta\sigma_o)^2 \tag{4}$$

$\Delta\sigma_o$ is the un-notched fatigue strength, while ΔK_{th} is the threshold stress intensity.

The threshold stress intensity is expressed as a bilinear function of mean stress (R-ratio) and the Austen model does not posses an explicit mean stress (R-ratio) correction. Austen argued the irrelevance of this and attributed it to a manifestation of crack closure and retardation.

Forman [33] was concerned with modelling of the fast fracture region (III) and developed the growth law expression given below:

$$da/dN = C \cdot \Delta K^n/[(1 - R) \cdot K_C - \Delta K] \tag{5}$$

where K_C is the plane stress fracture toughness.

The Forman growth law implicitly models mean stress (R-ratio) effects but this facility is often criticised having no independent controlling parameter to 'fine-tune' the fit as 'm' in the Walker model [34]. The R-ratio is constrained between $R_{\min}$ and $R_{\max}$, and the effective ΔK used in the above equation obtained from the expression:

$$\Delta K = K_{\max} \cdot (1 - R) \tag{6}$$

where $K_{\max}$ is the maximum stress intensity of the cycle and R is the constrained R-ratios obtained above.

Another related development has lead NASGRO to extend the generalized Willenborg model by taking into account the reduction of retardation due to underloads [33]. The NASGRO equation represents the most comprehensive growth law formulation comprising mean stress (R-ratio) effect, threshold, the honest of fast fracture and crack closure [32]. The NASGRO formula is expressed as:

$$da/dN = C \cdot [\{(1-f)/(1-R)\}\Delta K]^n [(1 - (\Delta K_{th}/\Delta K)]^p/[1 - (K_{\max}/K_C]^q \tag{7}$$

where C, n, p and q are the empirically derived coefficients from the measured data, and the other parameters such as the crack tip opening function, f are determined from the following formulation:

$$F = \begin{vmatrix} \max\left\{(R), \left(A_0 + A_1 \cdot R + A_2 \cdot R^2 + A_3 \cdot R^3\right)\right\} \\ if (R \geq 0) \\ A_0 + A_1 \cdot R \quad if \quad (-2 \leq R \leq 0) \\ A_0 - 2 \cdot A_1 \quad if \quad (R < -2) \end{vmatrix} \tag{8}$$

where

$$\begin{aligned} A_0 &= (0.825 - 0.34\alpha + 0.05\alpha^2)[\cos(\pi SR/2)]^{1/\alpha} \\ A_1 &= (0.415 - 0.071\alpha)SR \\ A_2 &= 1 - A_0 - A_1 - A_3 \\ A_3 &= 2A_0 + A_1 - 1 \end{aligned} \tag{9}$$

α is the plain stress/strain constraint factor and SR is the ratio of the maximum applied stress to the flow stress. These values are all empirically derived. The threshold stress intensity is obtained from the following equation

$$\Delta K_{th} = \Delta K_o(\sqrt{a/(a + a_o)})/[(1 - f)/(1 - A_o)(1 - R)]^{1 + C_{th}R} \tag{10}$$

where ΔK_o is the threshold stress intensity range at $R = 0$, obtained from test results, a is the crack length, a_o is the intrinsic crack length given as the constant and C_{th} is the threshold coefficient obtained from test results.

3. Methodology

Aluminium alloys are widely used in the design of many engineering application, due to their good mechanical properties and low densities. In this application center-cracked specimen geometry described in ASTM E647 [35] is used with a width of 229 mm, thickness of 4.1 mm and 610 mm in length, while the initial crack size is 12.7 mm, for which E = 71,750 MPa. The chemical composition and mechanical and fatigue properties of this material are shown in Tables 1 and 2, respectively [36–39].

Components and structures that are subjected to quite diverse load histories, their histories may be rather simple and repetitive and at the other extreme, they may be completely random. The cycle-by-cycle analysis can be performed with or without involving the interaction effects, i.e., the effect of a load cycle on crack growth in later cycles. The programmable and variable amplitude load histories given by Ray and Patanker [13] and Huang et al. [24] are used in this analysis with different load sequences from high to low or low to high shown in Fig. 1 (cases 1 to 6). These types of loading represent the load sequencing and spectrum loading in most of the application. To account load ranges and mean of the used load history, the rainflow counting method was then used. For that, towards the crack propagation model under these types of loading carried out based on the Austen, Forman and NASGRO models. In this overview the three main input parameters are geometry, material and loading with different cases. The process proceeds by selection of the fatigue crack growth model to show the Behaviour of the geometry. The results of the previous process predict the fatigue life and fatigue crack growth. At each cycle, to get a new result it is possible to change any of the factors (fatigue crack growth model, geometry, material, loading, initial crack length and stress ratio), which mean the ability to make a new prediction. The detail flow of such process is shown in Fig. 3. The modelling and simulation of the analysis were performed based on the commercial software package.

4. Results and discussion

Many engineering structures are subjected to random loading in service and the fatigue life will be affected by load sequence. However, for design purposes it is particularly difficult to generate an algorithm to quantify these sequence effects on fatigue crack propagation, due to the number and to the complexity of the mechanisms involved in this problem [38]. The presence of interaction effects is always altering the crack growth rate under the application of variable amplitude loading (VAL). For correctly predicting the crack growth under VAL, it is necessary to involve the interaction effects while developing the prediction models as a part of cycle-by-cycle analysis using different models. Hence, one of the purposes of this paper is to address how to characterize the effect of variable amplitude loading in fatigue crack propagation.

Despite the extensive work on crack growth, there is still a need for a satisfactory and generally applicable method to predict the fatigue crack propagation to consider various effects. In the current investigation, systematic crack growth predictions were conducted on an Aluminium alloy. The effects of overload, underload, and sequence loading on fatigue crack growth were studied. Several existing models were evaluated critically based on the experimental results [40].

Thus, the fatigue crack propagation models under VAL are presented in this paper based on the Austen, Interpolation Forman and NASGRO models. For demonstrating the validation of these models predictions are compared with test data, FASTRN and AFGROW codes given in Ray and Patanker [13].

Figs. 4 and 5 exhibited the results of the comparisons under two types of block loading, one with decreasing the minimum stresses and the other with increasing them. The changes of stress ratio related to changing of minimum stress with a constant maximum stress. The data predicted using the Austen, Interpolation Forman and NASGRO models are compared with those models performed by Ray and others [13,23,24]. The maximum differences in life predicting for the load cases for all models are 40% as a maximum compared to the test data. The lowest life has been found using the Austen prediction model, while NASGRO gave the maximum and the others are in between both Auten and NASGRO. Moreover, the fatigue life predicted under the load case 2 is higher than the case 1. The results show clearly the effect of changeable stress ratio and the first block is more effective than others. The results indicate that, when the first value of R is high, it is clearly reduce the life, although the value of R will be decreased later. In the load cases when the stress ratio is low in the first block of the load, it has less effect, although its value will be increased later.

The second group of loading has a changeable stress ratio also with a constant maximum stresses. According to the results in Fig. 6, although, all the models show the same behaviour, the life predicated by the Austen model was found to be the lowest by 40% compared to those obtained from experimental value [13]. However, it was found that the highest value is predicted using NASGRO model with 9% greater than the experimental value.

For the load cases 4 and 5, the stress ratios are changeable either in a decreasing way (case 4) or increasing (case 5) due to the changes in both stresses (maximum and minimum), which differ from the cases 1 and 2. The results in Figs. 7 and 8 show good agreement to the predicted life for all models with a difference range from 17% to 30% related to experimental results for the two load cases (4) and (5), except the results of the Austen model, which are less by more than 50% for the load case 4 compared to load case 5. For the random loading case, i.e., case 6, the results show a good agreement of AFGROW model with experiment values and 10% difference with the Interpolation Forman and NASGRO models, while for the FASTRAN and the Austen models are 30% and 50%, respectively. These results are clearly shown in Fig. 9.

Table 1
Chemical composition of aluminium alloy 2024 T3.

Component	wt.%	Component	wt.%
Al	90.7–94.7	Cr	Max. 0.1
Cu	3.8–4.9	Fe	Max. 0.5
Mg	1.2–1.8	Mn	0.3–0.9
Si	Max. 0.5	Ti	Max. 0.15
Zn	Max. 0.25	Other, each	Max. 0.05
Other, total	Max. 0.15		

Table 2
Mechanical and fatigue properties of aluminium alloy 2024 T3.

Description	Symbols	Values
Yield stress (MPa)	σ_y	345
Ultimate tensile strength (MPa)	σ_u	483
Plane strain fracture toughness (MPa $\sqrt{m}$)	K_{1C}	36.26
Plane stress fracture toughness (MPa $\sqrt{m}$)	K_{1D}	72.53
Part through fracture toughness (MPa $\sqrt{m}$)	K_{1E}	50.55
Forman exponent	m_y	3.29
Forman coefficient (m/MPa) $(m^{1/2})^{(n-1)}$	C	1.55E−10
Walker exponent	m_w	0.3
NASGRO exponent	p	0.5
NASGRO exponent	q	1
Modulus of elasticity (GPa)	E	71.75
Fatigue strength coefficient (MPa)	f	130
Elongation at break (%)		18

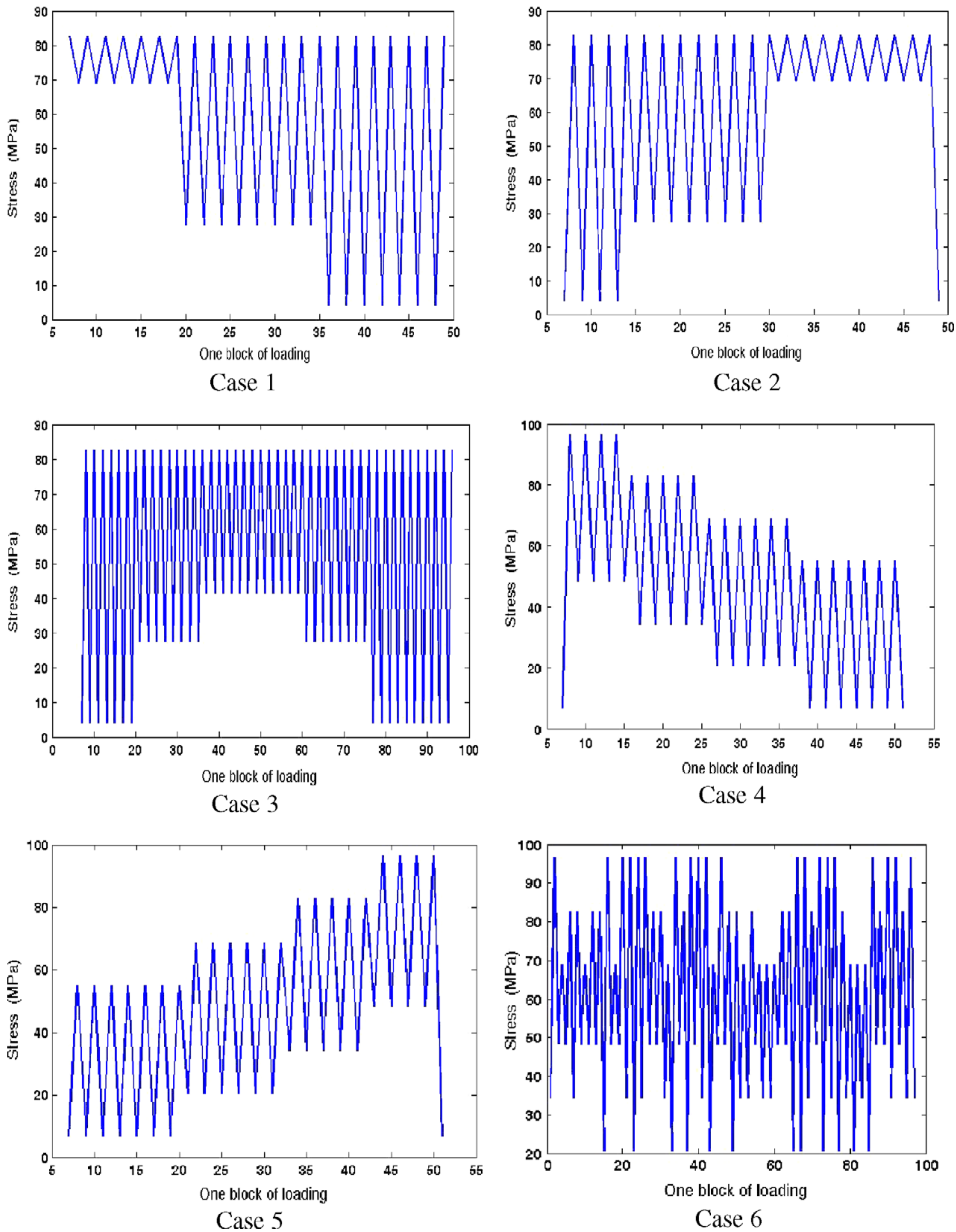

Fig. 1. Display of load histories with different sequences.

Using VAL in practice, the fatigue life is often affected by load or cycle sequences. Neglecting the effect of cycle interaction in fatigue calculations under variable amplitude loading can lead to completely invalid life predictions [26]. The this case study the load ranging from 4.14 to 82.8 MPa for the first three cases, while for the second three cases ranged from 6.9 to 96.6 MPa. Figs. 10–12 show the effect of the load sequence for the six load cases based on the NASGRO model (Fig. 10), the Interpolation Forman model (Fig. 11) and the FASTRAN model (Fig. 11). From these curves the life under load case 3 is the lowest, while the maximum life under

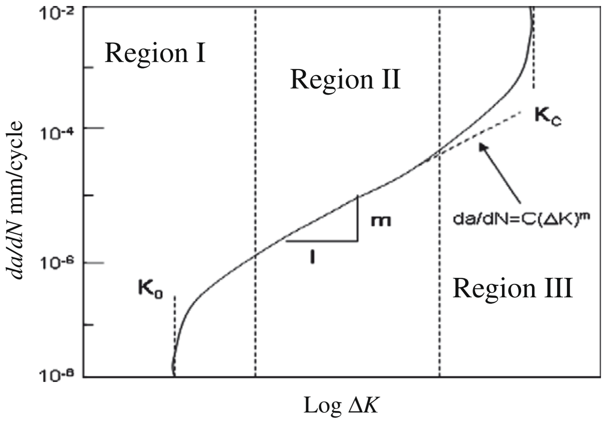

Fig. 2. Typical *da*/*dN* versus *dK* curve.

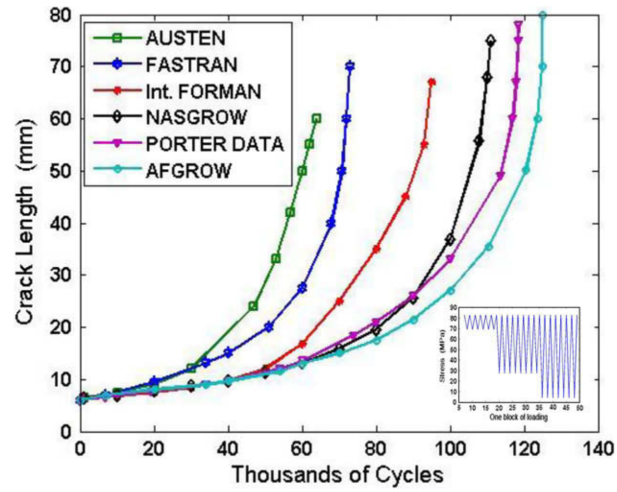

Fig. 4. Comparison of fatigue crack growth with different FCG models under load case 1.

the load case 4 or 5 and the other load cases in lying in between the lowest (case 3) and highest (case 4 and 5).

With reference to load cases 1 and 2, the changing in *R*-ratios due to changing in the minimum stresses only, while for the load cases 4 and 5 the changes due to variability of both maximum and minimum stresses. Fig. 13 shows the effect of stress ratio from high to low for load case 1 and for load case 2 was from low to high, which indicate that the value of stress ratio in the beginning of the load had much effect on the crack growth. For the two load cases 4 and 5, the results clearly show the effect of stress ratio and it can be seen in Fig. 14. Consequently, increasing the stress ratio leads to the fatigue life reduction for the two fatigue crack growth models, i.e., NASGRO and AFGROW models. From the overall findings, therefore, the effect of load sequences on the fatigue life prediction is necessary to involve the interaction effects and neglecting the sequences effect lead to inaccurate results.

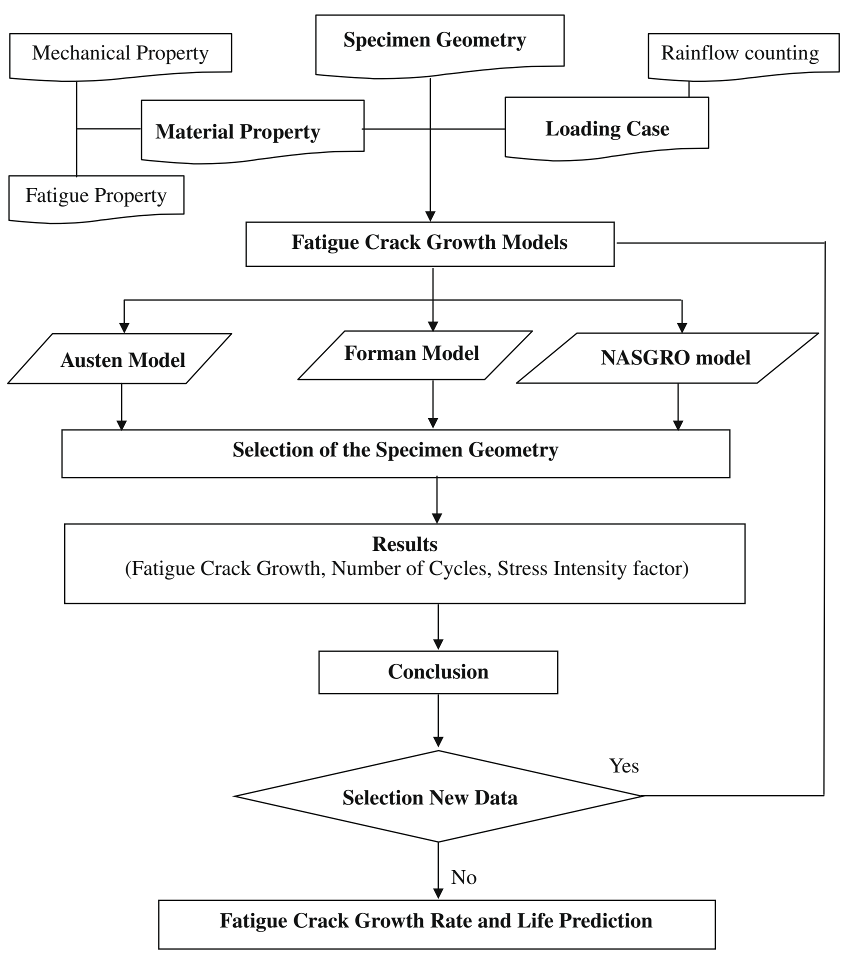

Fig. 3. Flow chart of the process.

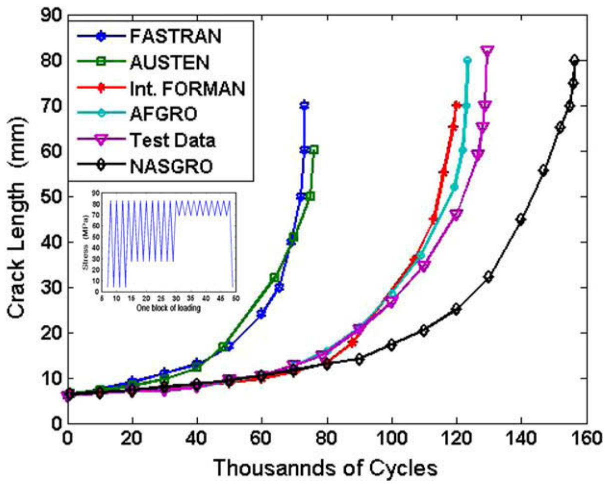

Fig. 5. Comparison of fatigue crack growth with different FCG models under load case 2.

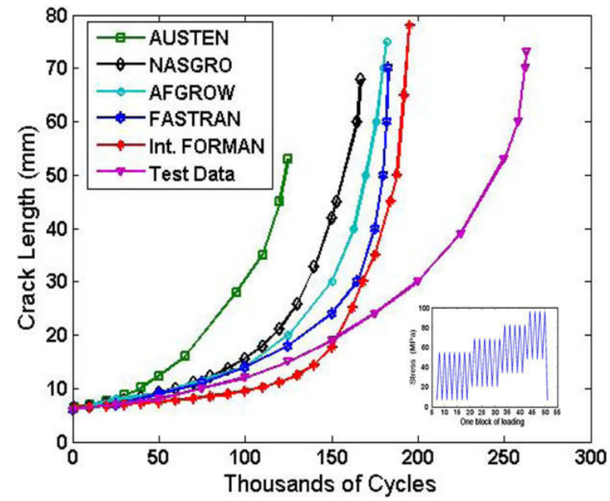

Fig. 8. Comparison of fatigue crack growth with different FCG models under load case 5.

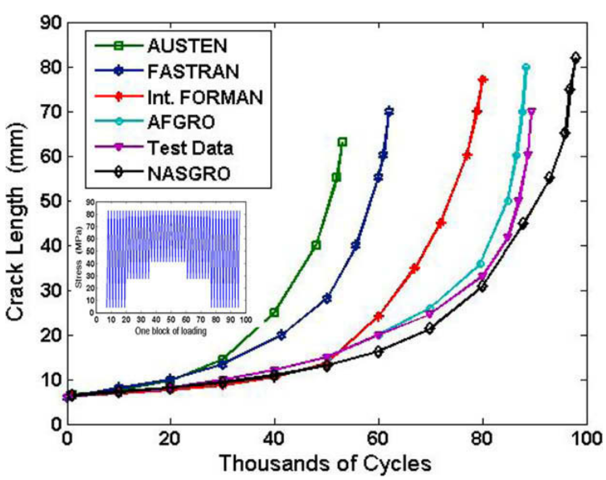

Fig. 6. Comparison of fatigue crack growth with different FCG models under load case 3.

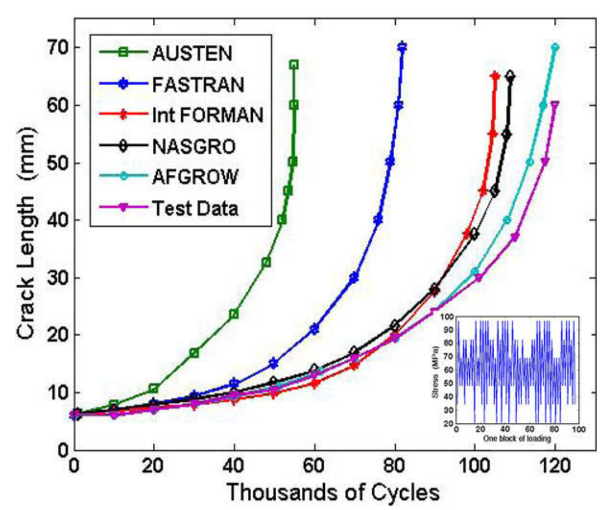

Fig. 9. Comparison of fatigue crack growth with different FCG models under load case 6.

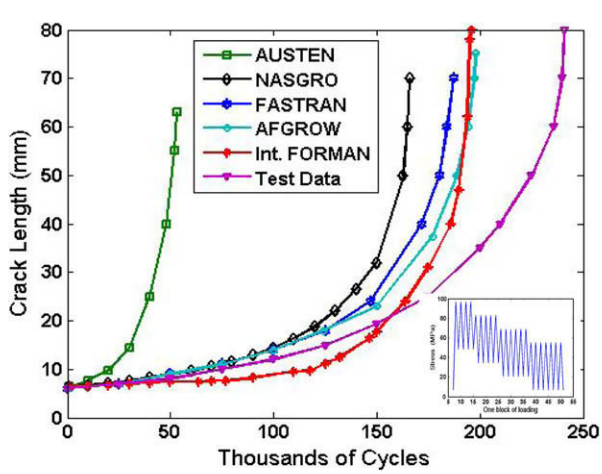

Fig. 7. Comparison of fatigue crack growth with different FCG models under load case 4.

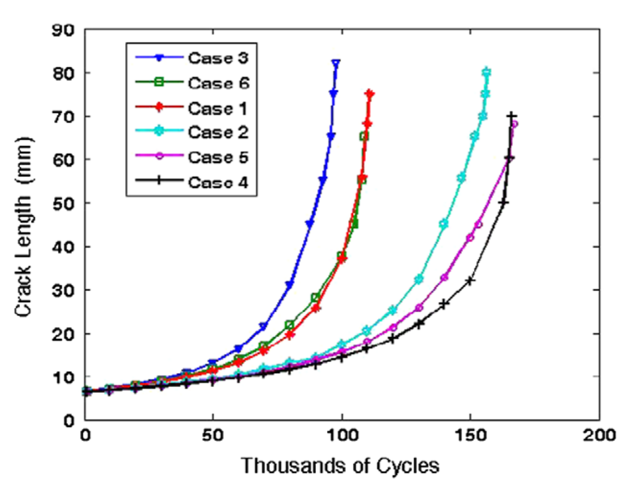

Fig. 10. Fatigue crack growth based on NASGRO model for all load cases.

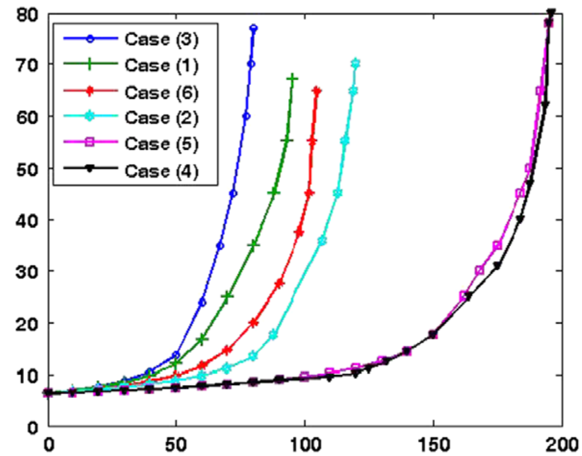

Fig. 11. Fatigue crack growth based on FORMAN model for all load cases.

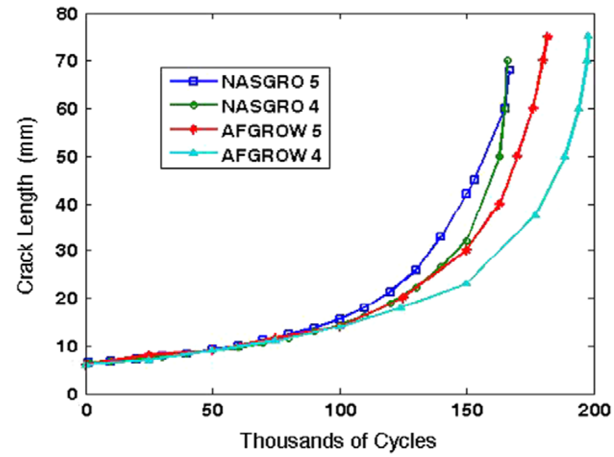

Fig. 14. Fatigue crack growth based on NASGRO and AFGROW models for the load cases 4 and 5.

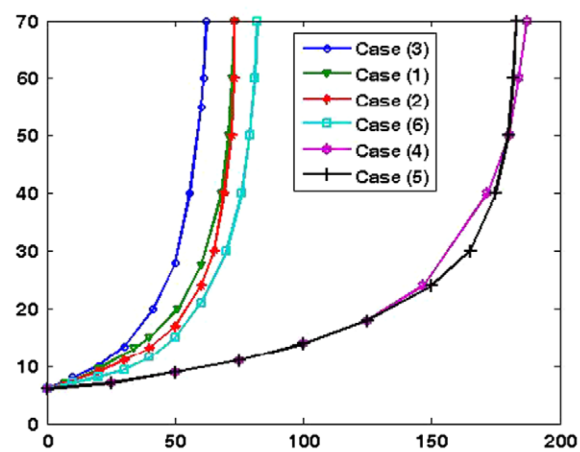

Fig. 12. Fatigue crack growth based on FASTRAN model for all load cases.

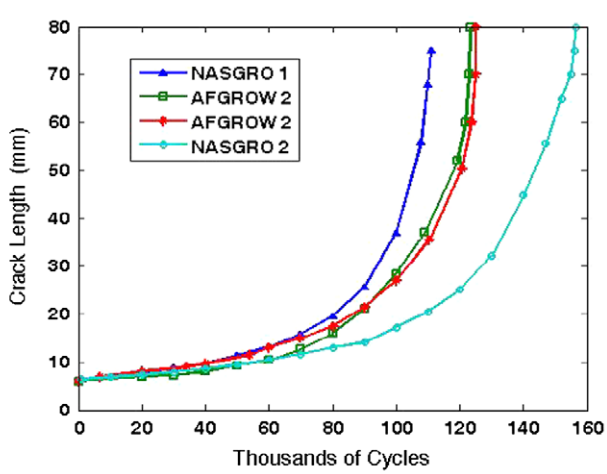

Fig. 13. Fatigue crack growth based on NASGRO and AFGROW models for the load cases 1 and 2.

5. Conclusions

The application of multiple over and under loads can interact with each other, and as a result they could either accelerate or decelerate the overall crack growth retardation depending on the frequency of the overload. Furthermore, the crack growth retardation is reduced during the latter stage of the fatigue life of a structure when the net section stress approaches the yield strength of the material.

Three different models namely, the Austen, Interpolation Forman and NASGRO have been used to predict the fatigue life on center-cracked 2024-T3 Aluminium alloy specimens under several program loadings. The load spectra and the schematic comparisons of predicted values with test data and those of FASTRN and AFGROW codes are compared.

All the findings obtained from the comparisons with the six different program loadings agree with some discrepancies relating to the test data. It is obvious that neglecting the effect of load sequence in fatigue calculations under variable amplitude loading can lead to completely invalid life predictions. Finally, the present models have been proved applicable for crack propagation under variable amplitude loading with different approaches.

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