International Journal of Fatigue 33 (2011) 19–22

Contents lists available at ScienceDirect

International Journal of Fatigue

journal homepage: www.elsevier.com/locate/ijfatigue

Evaluation of S–N curves with more than one failure mode

Hubert Bomas *, Klaus Burkart, Hans-Werner Zoch

Stiftung Institut fuer Werkstofftechnik, Badgasteiner Str. 3, D-28359 Bremen, Germany

ARTICLE INFO

Article history:
Received 23 February 2010
Received in revised form 20 April 2010
Accepted 23 April 2010
Available online 28 April 2010

Keywords:
Very high cycle fatigue (VHCF)
S–N curve
Crack initiation
Inclusions

ABSTRACT

In very high cycle fatigue, researchers and engineers are often confronted with S–N curves which exhibit more than one type of failure. Generally, it is assumed that every specific crack-initiation type has its own S–N curve, so that in this case two or more S–N curves can be drawn in the S–N diagrams. This paper will show an S–N diagram with two failure types and will provide a generalised method of drawing correct S–N curves for every single type of failure.

1. Introduction

In very high cycle fatigue, researchers are often confronted with multiple-flaw S–N curves that exhibit more than one type of failure (e.g. [1–5]). Very typical are combinations of surface crack initiation and crack initiations at inclusions. Generally, it is assumed that every crack-initiation type has its own S–N curve, so that in this case two S–N curves are drawn in the S–N diagrams. Established and generally accepted methods for the analysis of multiple-flaw S–N curves are not available.

For two failure modes Weixing and Shenjie offer the "duplex peak distribution model" which assumes that the life-time of each failure type is logarithmic normal distributed and obeys a distribution function F_1 or F_2, respectively [3]. For the life-time distribution function F of examined specimens which show both failure modes they suppose an addition of the weighted single-flaw distribution functions to be a good approach. With t as weighting coefficient the function F appears as

$$F = t \cdot F_1 + (1 - t) \cdot F_2 \quad (1)$$

Sakai et al. published a similar solution for the same problem [5]. The difference to the model of Weixing and Shenjie concerns only the type of the distribution functions F_1 or F_2, which they assume to be Weibull distributions.

This paper shows another approach for life-time distribution functions which is based on the assumption that the probability of crack initiation at a certain flaw-type is independent from that at another flaw-type. The model is demonstrated at an S–N diagram with two failure modes: crack initiation at inclusions and so-called "non-defect" crack initiation. The latter type of crack initiation is rather seldom and will be shown more detailed in chapter 4. The basic calculation of the life-time distribution function F for two failure modes can be directly compared with Eq. (1):

$$F = 1 - (1 - F_1) \cdot (1 - F_2) \quad (2)$$

2. Life-time distribution functions in constant-amplitude test

An S–N curve is the visualisation of a function $N_f(S_a)$ that describes the number of cycles to failure depending on the stress amplitude S_a. It is well known, that N_f, which is also called life-time, is a random variable and obeys a distribution function $F(N, S_a)$ describing the probability P of the life-time to be smaller than N:

$$F(N, S_a) = P(N_f(S_a) < N) \quad (3)$$

Since $F(N, S_a)$ describes the probability of the event "$N_f < N$", its physical sense is that of a failure probability. This means, the distribution function $F(N, S_a)$ is identical with the probability, that the specimen cycled with the stress amplitude S_a fails before reaching the number of cycles N. In some cases, it is more convenient to work with survival probabilities P_s. The relation between the survival probability and the distribution function of the life-time is the following:

$$P_s(N, S_a) = 1 - F(N, S_a) \quad (4)$$

If the specimen exhibits more than one failure mode it can be assumed that every failure mode i has its own life-time N_{fi} which obeys its own dependence on the stress amplitude: $N_{f1}(S_a)$, $N_{f2}(S_a)$,

* Corresponding author. Tel.: +49 421 5350; fax: +49 421 5333.
E-mail address: bomas@iwt-bremen.de (H. Bomas).

0142-1123/$ - see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijfatigue.2010.04.010

etc. Again, the life-times N_{fi} are random variables obeying distribution functions $F_i(N, S_a)$ describing their probabilities to be smaller than N when cycled with the stress amplitude S_a:

$$F_1(N, S_a) = P(N_{f1}(S_a) < N) \tag{5}$$

$$F_2(N, S_a) = P(N_{f2}(S_a) < N) \quad \text{etc.} \tag{6}$$

Now, a specimen is regarded that exhibits, under fatigue conditions, n failure modes. These can be imagined, for example, as failure from the surface, from type-1 inclusions, from type-2 inclusion, etc. It is assumed that these failure modes do not influence each other, a circumstance that is often valid in high-strength materials. This specimen is loaded with a stress amplitude S_a and fails after a number of cycles N_f. By repeating this test, the experimenter gets finally the distribution function of the specimen life-time N_f. If he wants to conclude the distribution functions of the failure-mode specific life-times, he must know the relation between these distribution functions F_i and that of the specimen's life-time F. What is the relation? The answer is easily formulated with survival probabilities: The specimen will survive, if all failure modes survive. According to the multiplication rule of probabilities of independent events, this means that the survival probability P_s of the specimen is the product of the failure-mode specific survival probabilities P_{si}:

$$P_s(N, S_a) = \prod_{i=1}^{n} P_{si}(N, S_a) \tag{7}$$

Translated to the distribution functions, it means the following:

$$F(N, S_a) = 1 - \prod_{i}^{n}(1 - F_i(N, S_a)) \tag{8}$$

This equation is the generalisation of Eq. (2) for the case of n failure modes. Eqs. (7) and (8) show the relation between survival and failure probabilities. In the case of two failure modes a simple geometrical interpretation of these equations is possible and shown in Fig. 1: The area P_s represents the survival probability of a specimen or component with two failure modes whereas the area F represents its failure probability.

The method of combining the failure-mode specific life-time distributions to a specimen life-time distribution which is described by Eq. (8) is quite different from the method Weixing and Shenjie [3] suggested. The next chapters show an application of the proposed model.

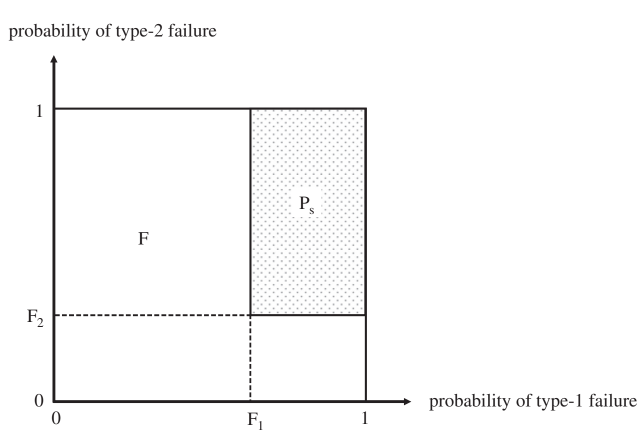

Fig. 1. Geometric interpretation of the Eqs. (7) and (8) for the case of two failure modes.

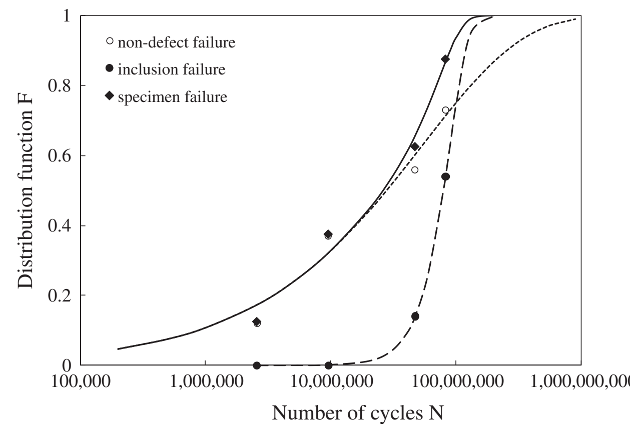

Fig. 2. Experimental values and calculated curves of the life-time distribution functions of the flaws and of the specimen, S_a = 540 MPa, $R = -1$.

3. Constant-amplitude test with single-flaw Weibull-type life-time distribution functions

Slightly notched specimens were machined of the steel SAE 5115 (DIN 20MnCr5), carburised, quenched and tempered. After that, the notches were ground resulting in a smooth surface with compressive residual stresses. Four specimens were stressed at a resonance pulsator with a nominal amplitude S_a = 540 MPa, a stress ratio $R = -1$ and a frequency f = 190 Hz. Due to grinding and the hardness profile resulting from heat treatment, two failure modes were analysed in the broken specimens:

(1) failure from inclusions at N = 47,126,000 and N = 82,456,000
(2) non-defect failure at N = 2,578,200 and N = 9,582,300

Fatigue failure from inclusions is very common for high-strength steels. Fig. 3 shows a typical crack initiation site of this failure type.

The non-defect failure is a less common failure mode which is typical for very high cycle fatigue and was described by different authors [6,7]. Fig. 4 shows a typical crack initiation site of this failure type.

The life-time distribution function of these specimens can be calculated according to Stepnov's method [8]: The experimental life-times are ordered and indexed with a running number j from 1 to 4 in the way that $N_j \leqslant N_{j+1}$ is valid. Now, the experiments yield

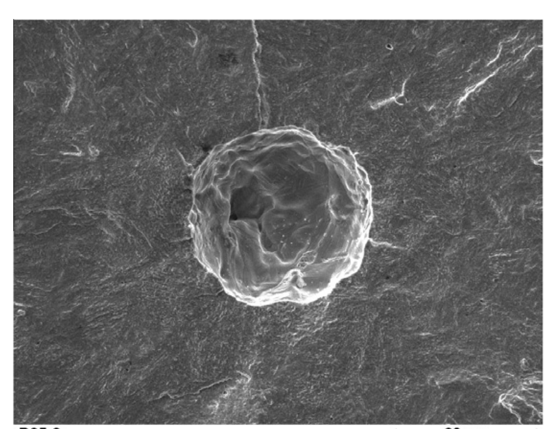

Fig. 3. Crack initiation at an inclusion in specimen D35, SEM micrograph, S_a = 500 MPa, N_f = 31,582,000. The picture shows part of a cavity which has been the home of an inclusion.

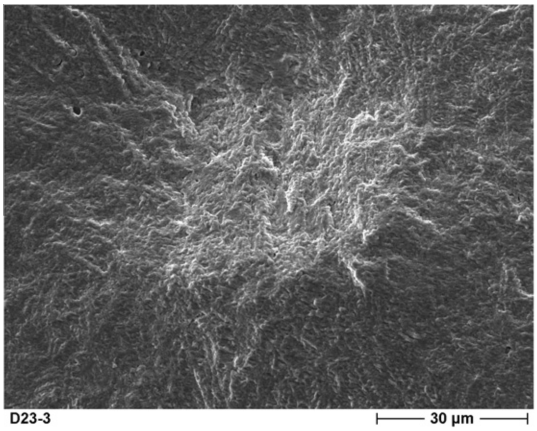

Fig. 4. Non-defect crack initiation in specimen D23, SEM micrograph, S_a = 580 MPa, N_f = 13,462,000.

four experimental values for the distribution function of the specimen life-time:

$$F_{\exp}(N_j) = \frac{j - 0.5}{4} \tag{9}$$

The second information which can be drawn from the experiments is the ratio between inclusion and non-defect crack initiation $(F_{\text{inclusion}}/F_{\text{non-defect}})_{\exp}$. These ratios were calculated according to this instruction: $(F_{\text{inclusion}}/F_{\text{non-defect}})_{\exp}$ = (number of inclusion failures before N_j)/(number of non-defect failures before N_j). The failure type at N_j is according to Stepnov's principle counted with the number ½.

The experimental value $F_{\text{inclusion,exp}}$ for the inclusion life-time distribution can be calculated from the experimental values $F_{\exp}$ and $(F_{\text{inclusion}}/F_{\text{non-defect}})_{\exp}$ with Eq. (8). This conducts to the following equation:

$$F_{\text{inclusion, exp}} = \frac{1}{2}\cdot\left(1+\left(\frac{F_{\text{inclusion}}}{F_{\text{non-defect}}}\right)_{\exp}\right) - \sqrt{\frac{1}{4}\cdot\left(1+\left(\frac{F_{\text{inclusion}}}{F_{\text{non-defect}}}\right)_{\exp}\right)^2 - \left(\frac{F_{\text{inclusion}}}{F_{\text{non-defect}}}\right)_{\exp}\cdot F_{\exp}} \tag{10}$$

The experimental values for the inclusion life-time distribution $F_{\text{inclusion,exp}}$ can be calculated as follows:

$$F_{\text{non-defect}} = F_{\text{inclusion, exp}} \Big/ \left(\frac{F_{\text{inclusion}}}{F_{\text{non-defect}}}\right)_{\exp} \tag{11}$$

With this concept, it is possible to get experimental values for all life-time distributions. Table 1 shows the result. In the case $j = 1$ which describes the earliest failure the application of Eq. (11) is not yet possible since the ratio $(F_{\text{inclusion}}/F_{\text{non-defect}})_{\exp}$ equals zero. In this case Stepnov's formula (Eq. (9)) was applied.

It is now assumed that the failure-mode specific life-times obey two-parametric Weibull distributions [9]. Under this assumption these distributions can be written as:

$$F_i(N, S_a) = 1 - 2^{-\left(\frac{N}{N_{mi}(S_a)}\right)^{m_i}} \tag{12}$$

In this notation, N_{mi} is the median of the life-time in failure mode i, whereas m_i is the Weibull exponent, describing the slope of the distribution function. The distribution function of the specimen life-time which contains n failure modes has now to be written as:

$$F(N, S_a) = 1 - 2^{-\sum_i^n \left(\frac{N}{N_{mi}(S_a)}\right)^{m_i}} \tag{13}$$

By use of the least-squares method applied to the differences $F_{\text{inclusion,exp}} - F_{\text{inclusion}}$ and $F_{\text{non-defect,exp}} - F_{\text{non-defect}}$ between the experimental and the calculated values of the distribution functions the best fitting model parameters $N_{m,\text{non-defect}}$, $m_{\text{non-defect}}$, $N_{m,\text{inclusion}}$ and $m_{\text{inclusion}}$ have been calculated from the experimental values in Table 1. Table 2 shows these parameters.

Fig. 2 gives a graphic presentation of the distribution function values from Table 1 and the calculated curves of the distribution functions. By the described method it is possible to conclude the life-time distribution function of the non-defect and the inclusion failure, respectively, from the experiment with specimens showing both crack initiation mechanisms.

4. Single-flaw Basquin-type S–N diagrams with Weibull-type life-time distribution functions

Eq. (12) describes the failure probability in an S–N diagram for a certain failure type i. The term $N_{mi}(S_a)$ specifies the S–N curve for a failure probability $F_i = 0.5$. Accordingly, Eq. (13) describes the failure probability in an S–N diagram with n failure types. In the

Table 1
Experimental values for the life-time distribution functions.

j	N_j	$F_{\exp}$	$(F_{\text{inclusion}}/F_{\text{non-defect}})_{\exp}$	$F_{\text{inclusion,exp}}$	$F_{\text{non-defect,exp}}$
1	2,578,200	0.13	0.00	0.00	0.13
2	9,582,300	0.38	0.00	0.00	0.38
3	47,126,000	0.63	0.25	0.14	0.56
4	82,456,000	0.88	0.75	0.54	0.73

Table 2
Parameters of the single-flaw life-time distribution functions.

i	N_{mi}	m_i
Inclusion	79,000,000	2.94
Non-defect	28,000,000	0.54

Table 3
Results of constant-amplitude experiments.

Specimen number	S_a MPa	N_f	Crack initiation
D13	460	119,890,000	Non-defect
D14	480	>200,000,000	
D16		>200,000,000	
D49		>200,000,000	
D35[a]	500	31,582,000	Inclusion
D57		77,631,500	Inclusion
D01		94,334,000	Inclusion
D04		>200,000,000	
D30	520	3,695,000	Non-defect
D25		19,598,000	Non-defect
D12		25,593,000	Non-defect
D38		>200,000,000	
D51	540	2,578,200	Non-defect
D45		9,582,200	Non-defect
D15		47,126,000	Inclusion
D59		82,456,000	Inclusion
D58	580	2,617,900	Non-defect
D03		3,596,000	Non-defect
D29		10,969,000	Non-defect
D23[b]		13,462,000	Non-defect
D60	620	467,600	Non-defect
D10		726,000	Non-defect
D18		1,112,200	Inclusion
D07		1,420,000	Non-defect
D32	700	327,800	Non-defect
D26		353,365	Non-defect

[a] The crack initiation site of specimen D35 is shown in Fig. 3.
[b] The crack initiation site of specimen D23 is shown in Fig. 4.

Table 4
Parameters of the failure-specific S–N diagrams.

i	S_{fi} MPa	k_i	m_i
Inclusion	12,658	6	1.55
Non-defect	1647	15	0.73

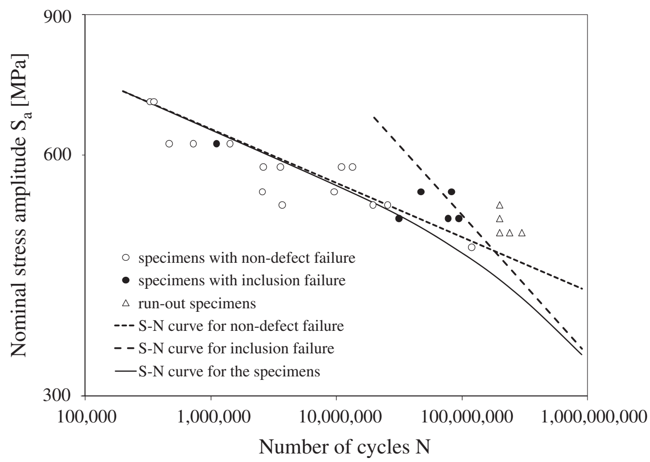

Fig. 5. Experimental S–N values of the specimens with indication of the crack-initiation type and calculated S–N curves for non-defect crack initiation, for crack initiation from inclusions and for the specimens that contain both failure types. The S–N curves represent a fracture probability of 50%. $R = -1$.

following description, it is assumed that the failure-mode specific S–N curves $N_{mi}(S_a)$ can be described according to Basquin [10]:

$$N_{mi}(S_a) = \left(\frac{S_{fi}}{S_a}\right)^{k_i} \tag{14}$$

In this equation, S_{fi} represents the stress amplitudes corresponding to the life-time $N_{mi} = 1$ whereas k_i describes the slopes of the S–N curves. With the Basquin specification of Eq. (14) the Eqs. (11) and (12) turn to:

$$F_i(N, S_a) = 1 - 2^{-\left(N \cdot \left(\frac{S_a}{S_{fi}}\right)^{k_i}\right)^{m_i}} \tag{15}$$

$$F(N, S_a) = 1 - 2^{-\sum_{i=1}^{n}\left(N \cdot \left(\frac{S_a}{S_{fi}}\right)^{k_i}\right)^{m_i}} \tag{16}$$

Additional specimens of the type described in chapter 3 were cycled at different stress amplitudes in order to gather data for an S–N diagram. Table 3 shows the results.

The best fitting parameters $S_{f,\text{inclusion}}$, $k_{\text{inclusion}}$, $m_{\text{inclusion}}$, $S_{f,\text{non-defect}}$, $k_{\text{non-defect}}$, and $m_{\text{non-defect}}$ have been calculated from the experimental values in Table 3. Table 4 shows the result.

Fig. 5 shows the S–N diagram including the experimental values differentiated according to their failure type and the S–N curves for non-defect failure, for inclusion failure and for the specimens. The S–N curves represent a fracture probability of 50%. Since the specimens contain both flaw types their life-time is always shorter than that of the single flaws. By the described method it is possible to conclude the S–N curves for non-defect and inclusion failure from the experiments on specimens with both failure possibilities. The assumption of Basquin-type S–N curves for inclusion and non-defect failure leads to a non-linear curve of the specimen's S–N curve if a double-logarithmic scale is chosen for the S–N diagram.

5. Conclusions

Simple provisions for the theory of probabilities lead to the result that survival probabilities of different crack-initiation modes have to be multiplied in order to get the survival probability of a specimen which exhibits the regarded crack-initiation modes. This leads to a derivation of a specimen life-time distribution function on the base of the crack-initiation specific life-time distributions. An introduction of the dependence of the life-time on the stress amplitude opens the possibility to evaluate crack-initiation specific S–N curves of parts with multiple-flaw crack-initiation mechanisms.

Acknowledgement

Financial support of this work by the Deutsche Forschungsgemeinschaft (DFG) under Contract Number ZO140/3 is gratefully acknowledged.

References

[1] Mayer H, Haydn W, Schuller R, Issler S, Furtner B, Bacher-Hoechst M. Very high cycle fatigue properties of bainitic high carbon–chromium steel. Int J Fatigue 2009;31:242–9.
[2] Shiozawa K, Hasegawa T, Kashiwagi Y, Lu L. Very high cycle fatigue properties of bearing steel under axial loading condition. Int J Fatigue 2009;31:880–8.
[3] Weixing Y, Shenjie G. VHCF test and life distribution of aluminum alloy LC4CS. Int J Fatigue 2008;30:172–7.
[4] Xu X, Yu Y, Cui W, Bai B, Gu J. Ultra-high cycle fatigue behavior of high strength steel with carbide-free bainite/martensite complex microstructure. Int J Miner, Metall Mater 2009;16:285–92.
[5] Sakai T, Lian B, Takeda M, Shiozawa K, Oguma N, Ochi Y, et al. Statistical duplex S–N characteristics of high carbon chromium bearing steel in rotating bending in very high cycle regime. Int J Fatigue 2010;32:497–504.
[6] Bayraktar E, Garcias IM, Bathias C. Failure mechanisms of automotive metallic alloys in very high cycle fatigue range. Int J Fatigue 2006;28:1590–602.
[7] Chai G. The formation of subsurface non-defect fatigue crack origins. Int J Fatigue 2006;28:1533–9.
[8] Stepnov MN. Evaluating the probability of failure in fatigue tests. Ind Lab 1962;28:886–8.
[9] Weibull W. A statistical distribution function of wide applicability. J Appl Mech 1951:293–7.
[10] Basquin OH. The exponential law of endurance tests. Proc Ann Meet, Am Soc Test Mater 1910;10:625–30.