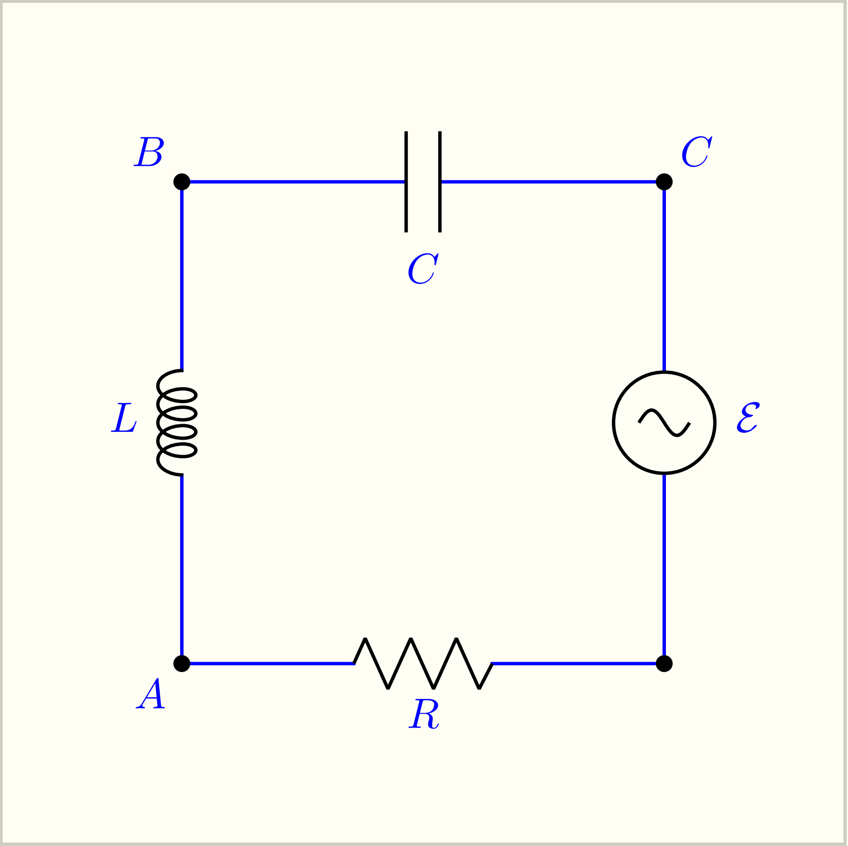
Física IV

25 agosto 2020

Circuitos de Corrente Alternada

Circuitos de corrente alternada

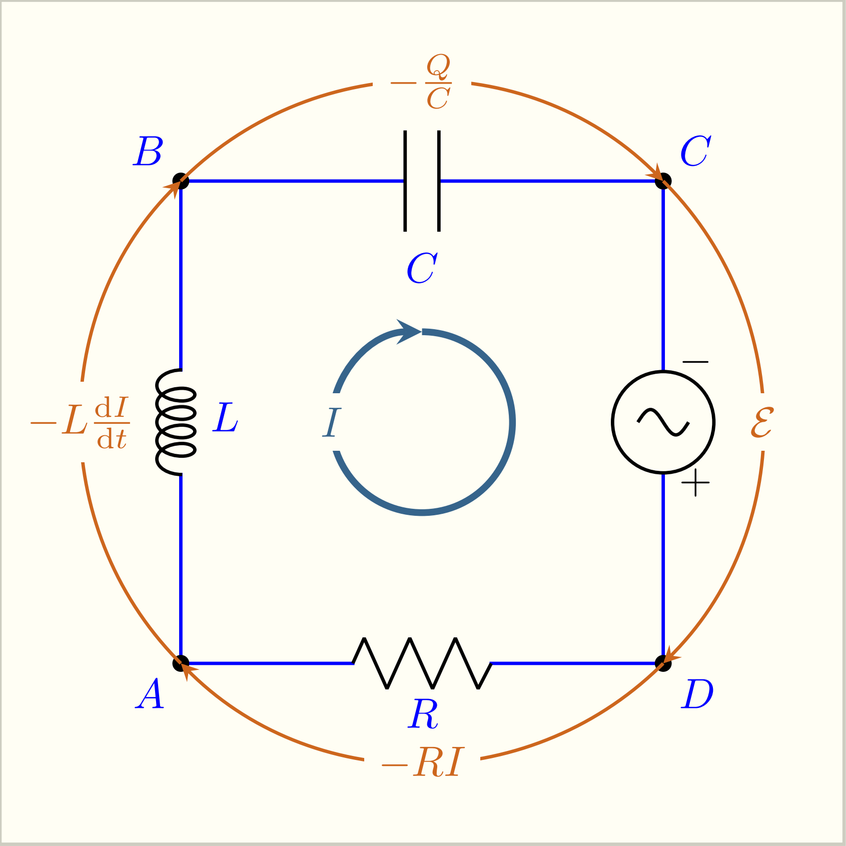
$I(t) = ?$



Circuitos de corrente alternada

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = \mathscr{E}$$

$$-L\frac{dI}{dt} - \frac{Q}{C} + \mathscr{E} - RI = 0$$



$$\mathscr{E} = L\frac{dI}{dt} + \frac{Q}{C} + RI$$

$$\mathscr{E} = L\frac{d^2Q}{dt^2} + \frac{Q}{C} + R\frac{dQ}{dt}$$

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = \mathscr{E}$$

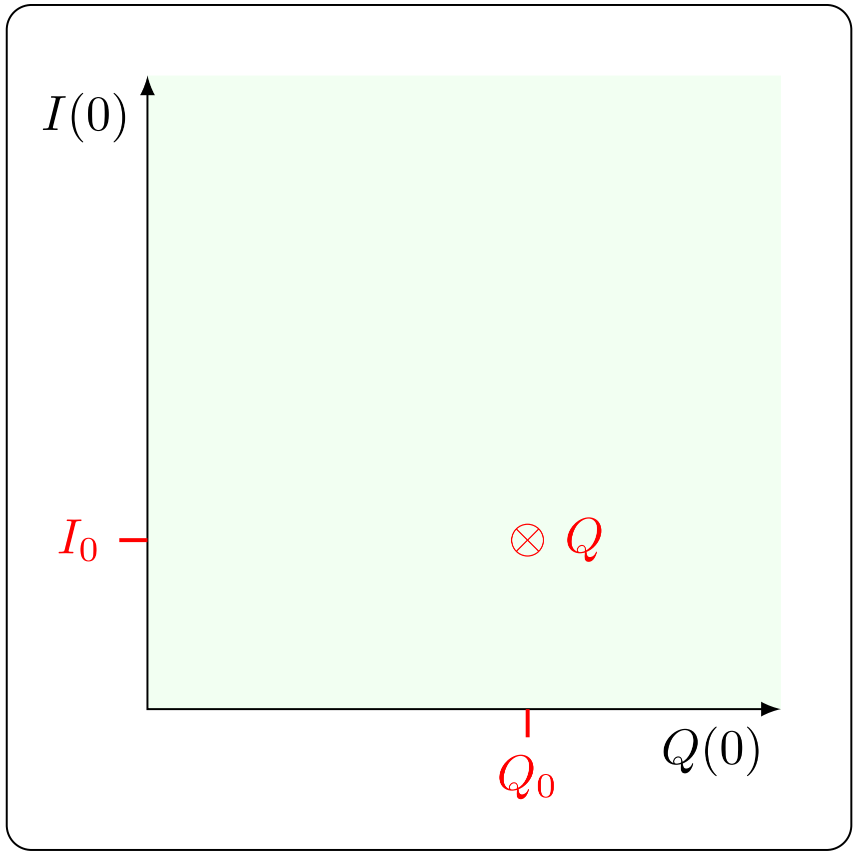
Circuitos
de corrente alternada
Resolver equação diferencial

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = \mathscr{E}$$

$+$ Condições iniciais : $\begin{cases} Q(t=0) = Q_0 \\ I(t=0) = I_0 \end{cases}$

Circuitos
de corrente alternada
Resolver equação diferencial

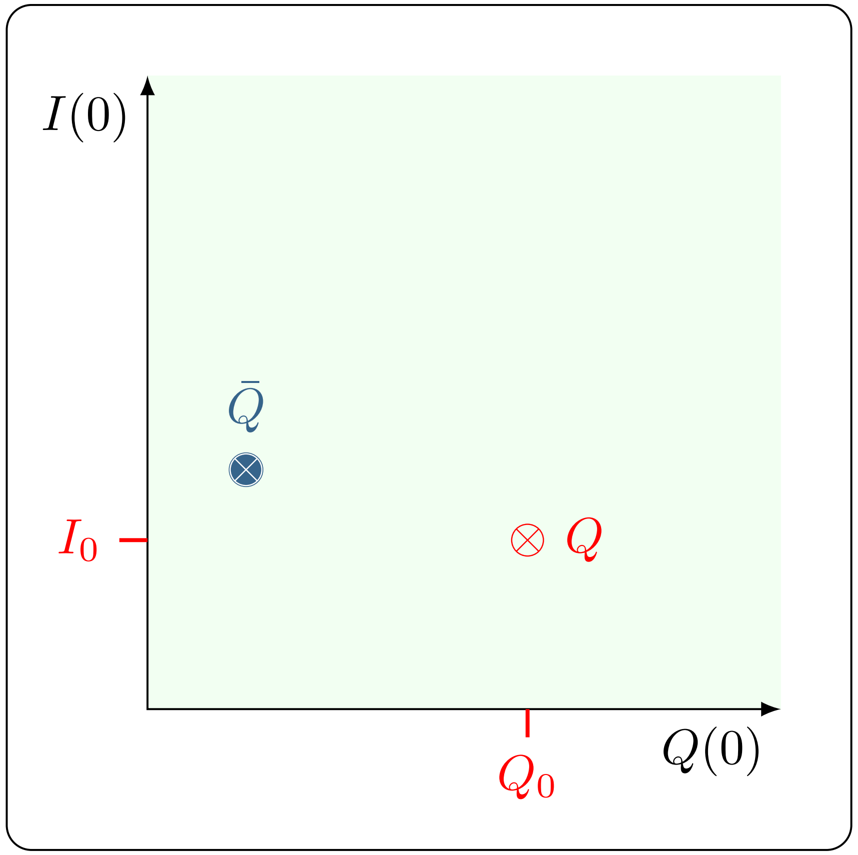
$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = \mathscr{E}$$



Circuitos
de corrente alternada
Resolver equação diferencial

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = \mathscr{E}$$

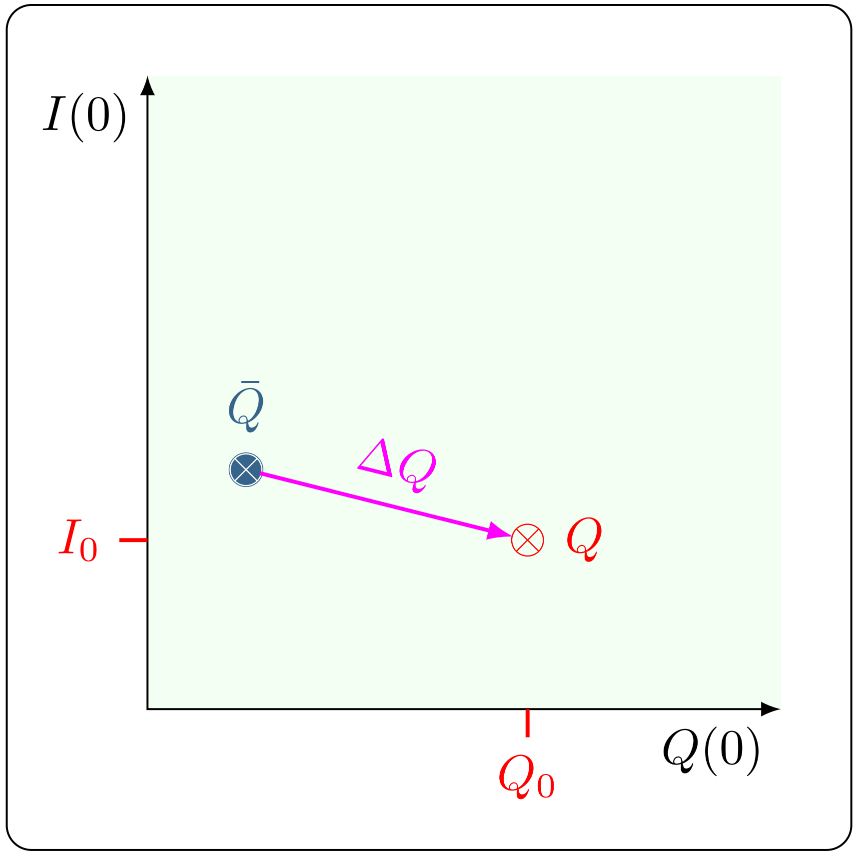
$$L\frac{d^2\bar{Q}}{dt^2} + R\frac{d\bar{Q}}{dt} + \frac{\bar{Q}}{C} = \mathscr{E}$$



Circuitos
de corrente alternada
Resolver equação diferencial

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = \mathscr{E}$$

$$L\frac{d^2\bar{Q}}{dt^2} + R\frac{d\bar{Q}}{dt} + \frac{\bar{Q}}{C} = \mathscr{E}$$

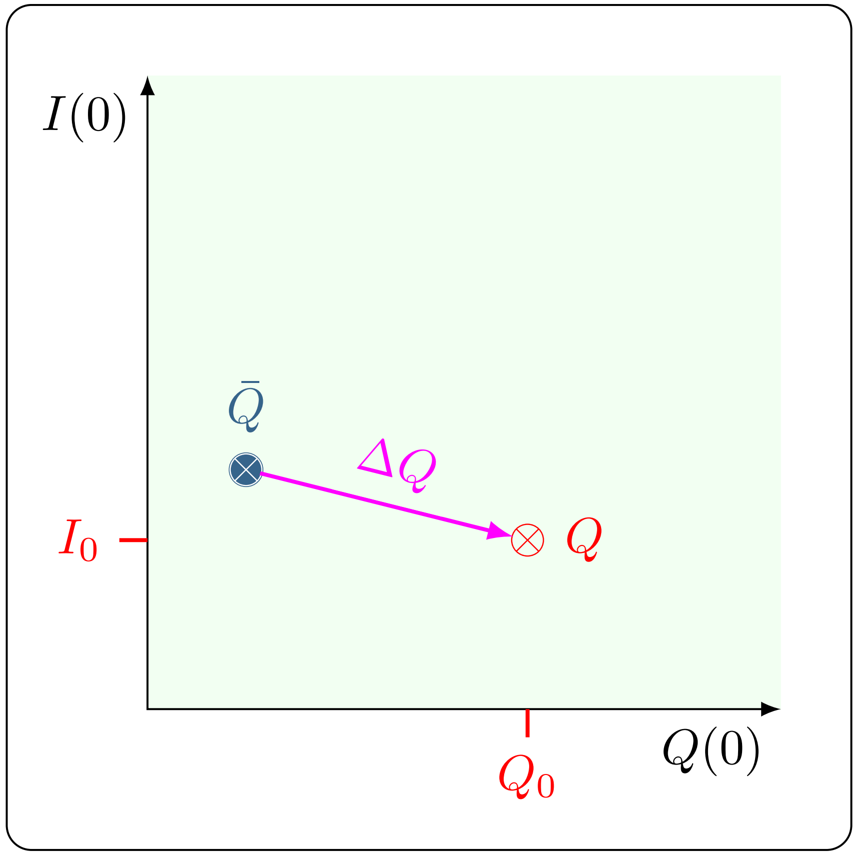


Circuitos
de corrente alternada
Resolver equação diferencial

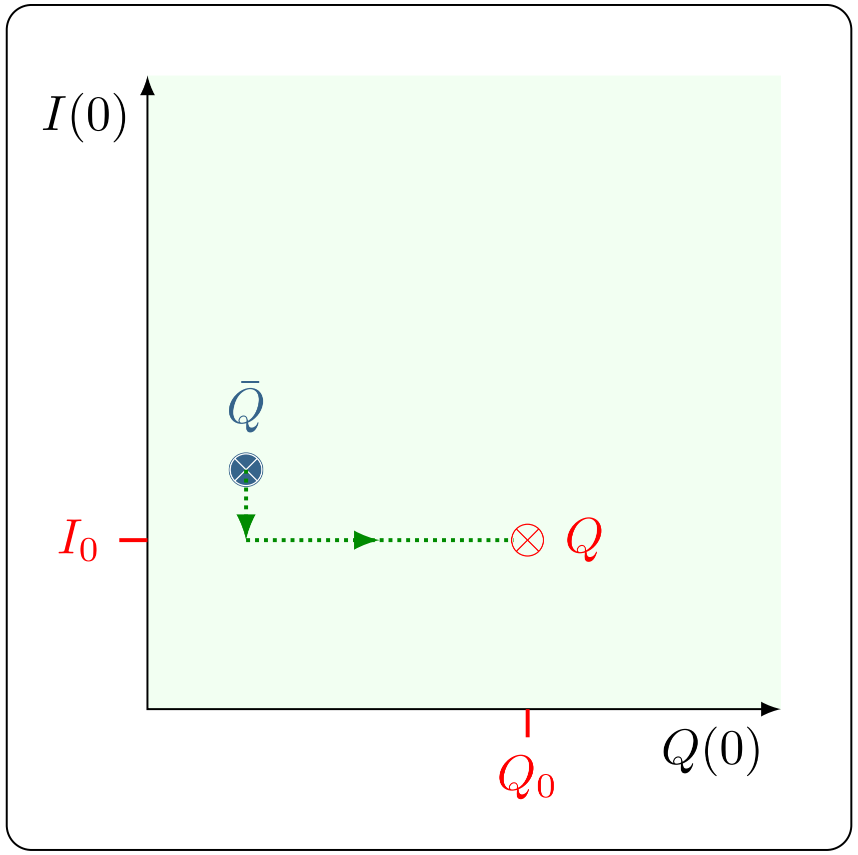
$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = \mathscr{E}$$

$$L\frac{d^2\bar{Q}}{dt^2} + R\frac{d\bar{Q}}{dt} + \frac{\bar{Q}}{C} = \mathscr{E}$$

$$\Rightarrow L\frac{d^2\Delta Q}{dt^2} + R\frac{d\Delta Q}{dt} + \frac{\Delta Q}{C} = 0$$



Circuitos
de corrente alternada
Resolver equação diferencial



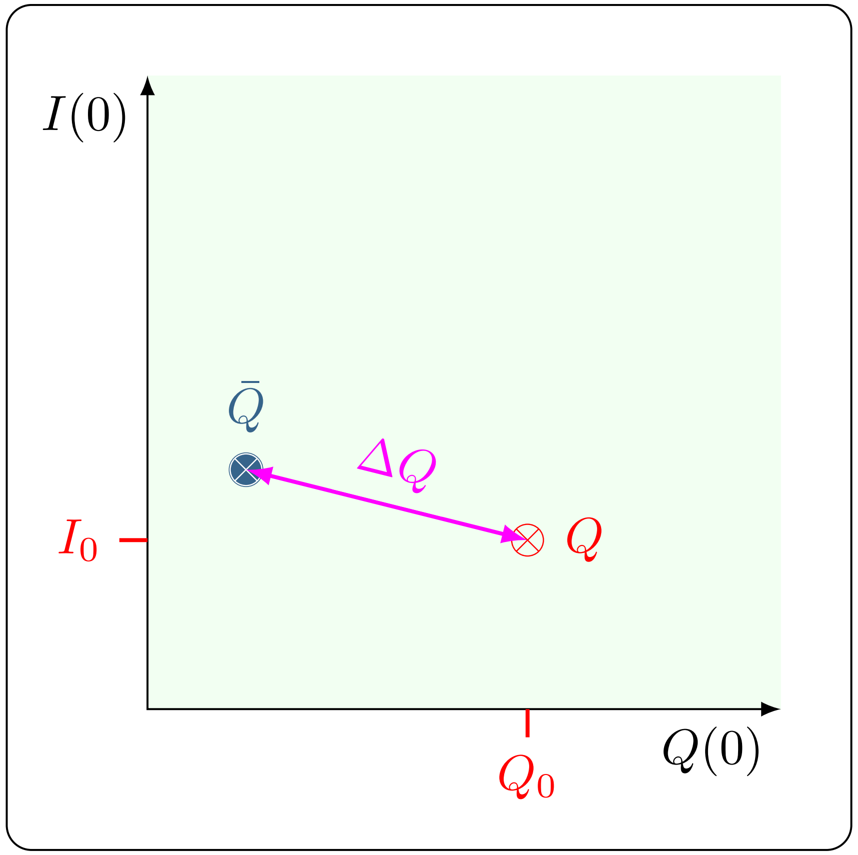
Circuitos de corrente alternada Equação homogênea

$$L\frac{d^2\Delta Q}{dt^2} + R\frac{d\Delta Q}{dt} + \frac{\Delta Q}{C} = 0$$

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = \mathscr{E}$$

$$L\frac{d^2\bar{Q}}{dt^2} + R\frac{d\bar{Q}}{dt} + \frac{\bar{Q}}{C} = \mathscr{E}$$

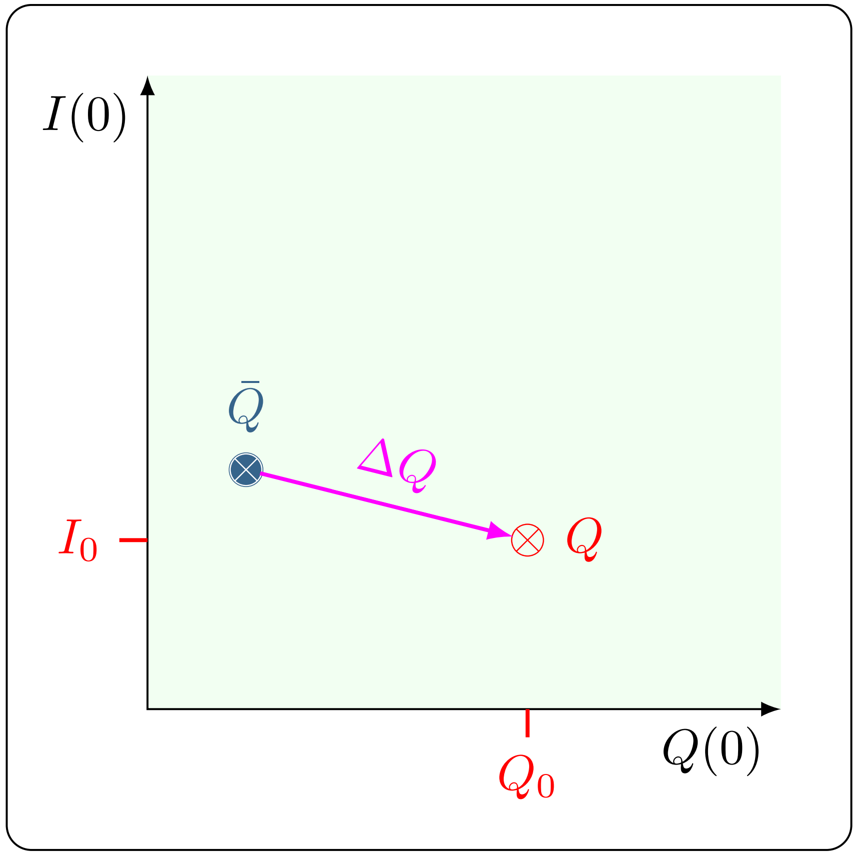
$$\Rightarrow L\frac{d^2\Delta Q}{dt^2} + R\frac{d\Delta Q}{dt} + \frac{\Delta Q}{C} = 0$$



Circuitos de corrente alternada Equação homogênea

$$L\frac{d^2\Delta Q}{dt^2}+R\frac{d\Delta Q}{dt}+\frac{\Delta Q}{C}=0$$

$$\Delta Q = e^{st}$$

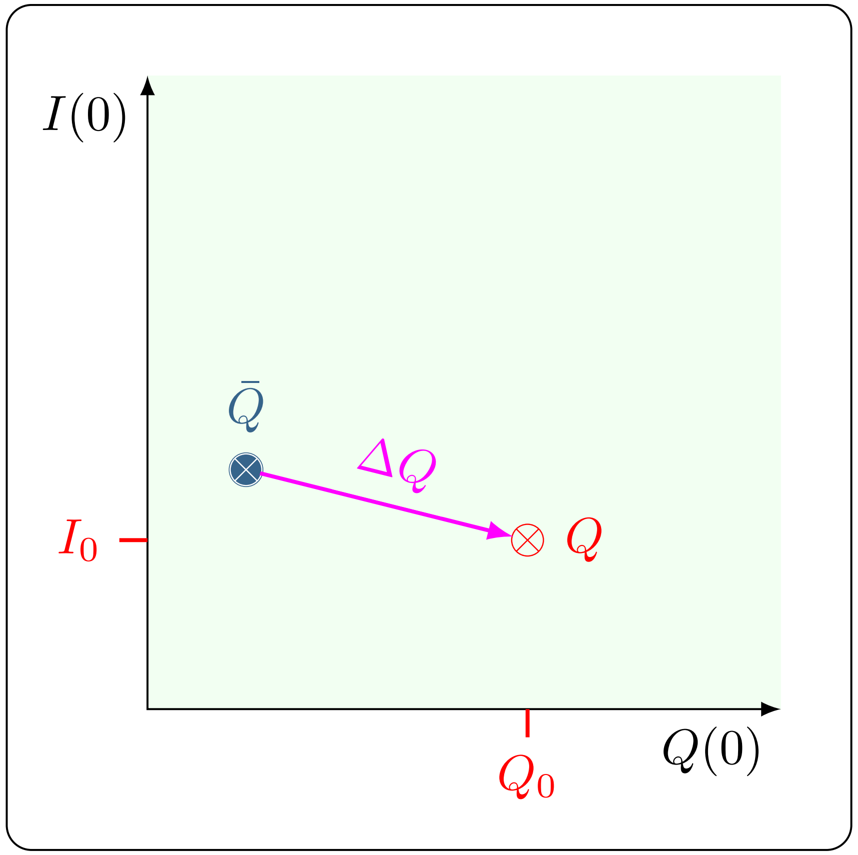


Circuitos de corrente alternada Equação homogênea

$$L\frac{d^2\Delta Q}{dt^2} + R\frac{d\Delta Q}{dt} + \frac{\Delta Q}{C} = 0$$

$$\Delta Q = e^{st}$$

$$\frac{d\Delta Q}{dt} = se^{st} = s\Delta Q$$



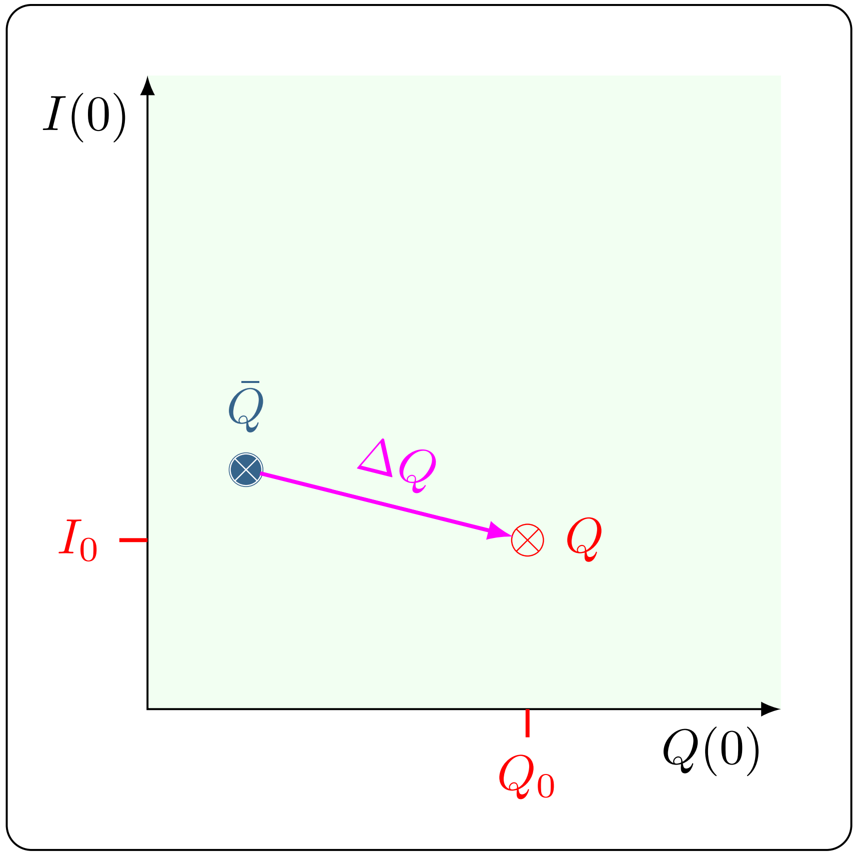
Circuitos de corrente alternada Equação homogênea

$$L\frac{d^2\Delta Q}{dt^2} + R\frac{d\Delta Q}{dt} + \frac{\Delta Q}{C} = 0$$

$$\Delta Q = e^{st}$$

$$\frac{d\Delta Q}{dt} = se^{st} = s\Delta Q$$

$$\frac{d^2\Delta Q}{dt^2} = s^2e^{st} = s^2\Delta Q$$



Circuitos de corrente alternada Equação homogênea

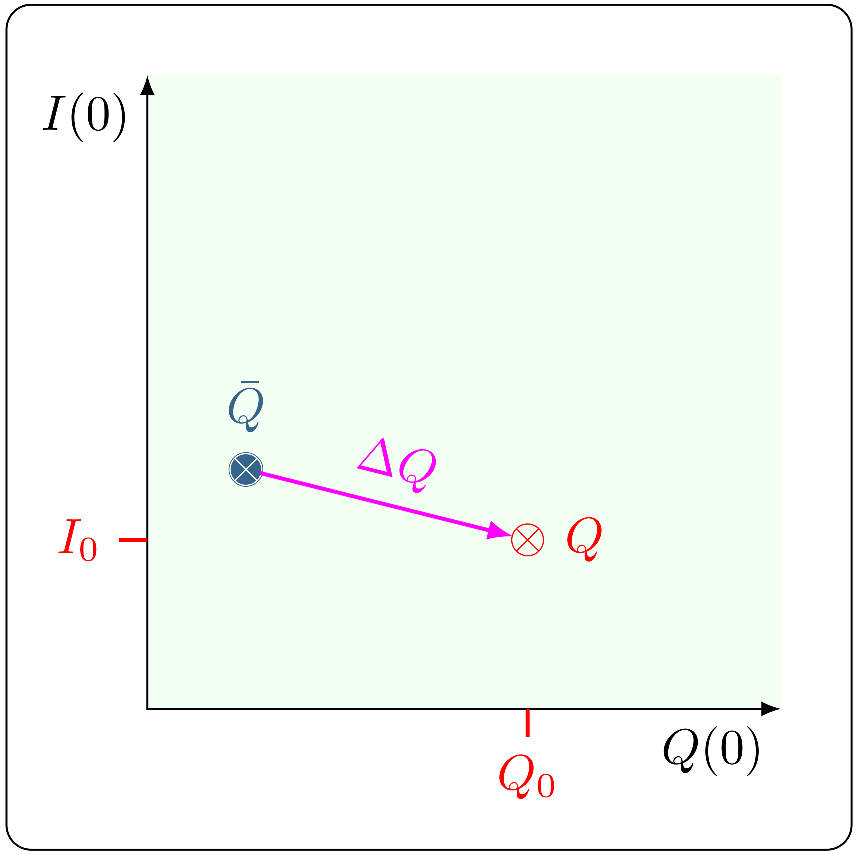
$$L\frac{d^2\Delta Q}{dt^2} + R\frac{d\Delta Q}{dt} + \frac{\Delta Q}{C} = 0$$

$$\Delta Q = e^{st}$$

$$\frac{d\Delta Q}{dt} = se^{st} = s\Delta Q$$

$$\frac{d^2\Delta Q}{dt^2} = s^2e^{st} = s^2\Delta Q$$

$$Ls^2\Delta Q + Rs\Delta Q + \frac{\Delta Q}{C} = 0$$



Circuitos
de corrente alternada
Equação homogênea

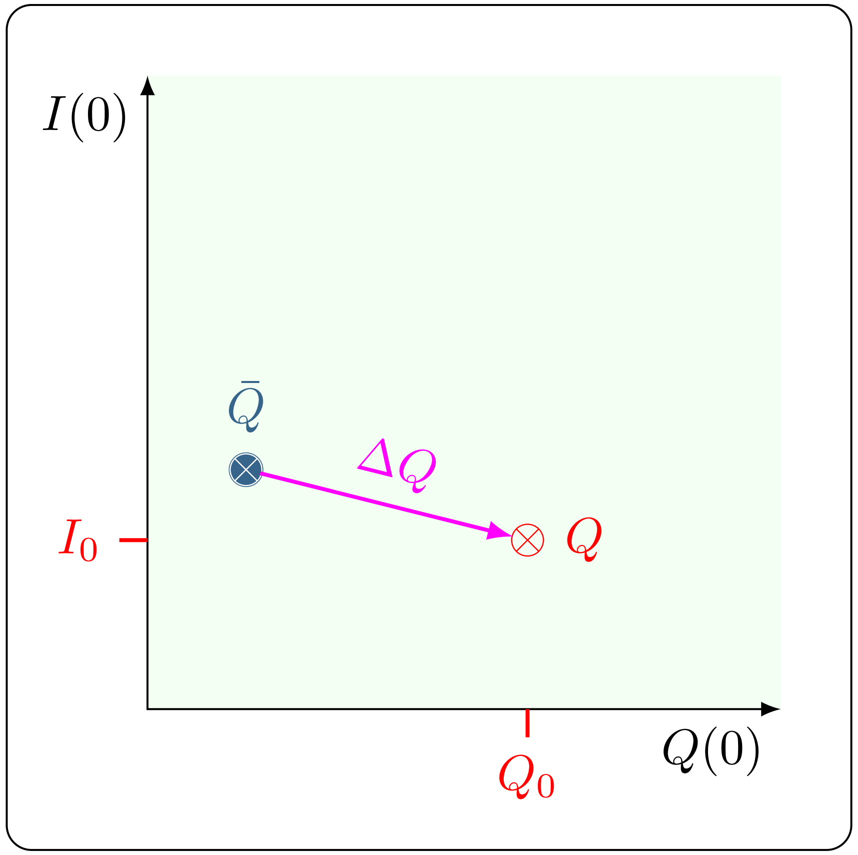
$$L\frac{d^2\Delta Q}{dt^2} + R\frac{d\Delta Q}{dt} + \frac{\Delta Q}{C} = 0$$

$$\Delta Q = e^{st}$$

$$\frac{dQ}{dt} = se^{st} = sQ$$

$$\frac{d^2Q}{dt^2} = s^2e^{st} = s^2Q$$

$$Ls^2\cancel{\Delta Q} + Rs\cancel{\Delta Q} + \frac{\cancel{\Delta Q}}{C} = 0$$



Circuitos de corrente alternada Equação homogênea

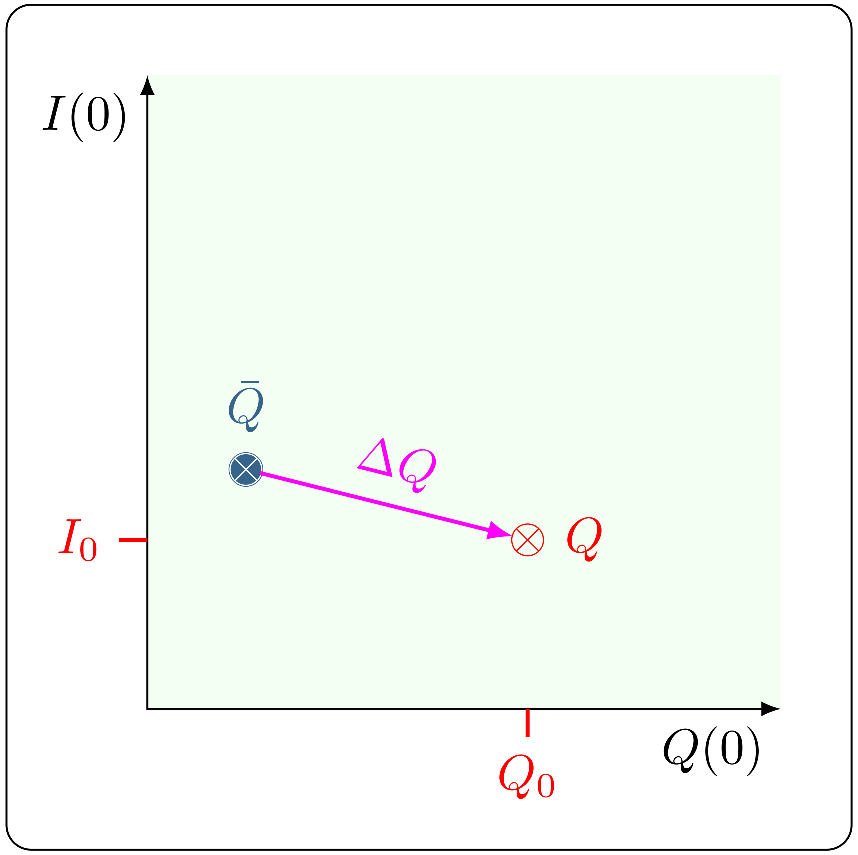
$$L\frac{d^2\Delta Q}{dt^2}+R\frac{d\Delta Q}{dt}+\frac{\Delta Q}{C}=0$$

$$\Delta Q = e^{st}$$

$$\frac{dQ}{dt}=se^{st}=sQ$$

$$\frac{d^2Q}{dt^2}=s^2e^{st}=s^2Q$$

$$Ls^2+Rs+\frac{1}{C}=0$$



Circuitos de corrente alternada Equação homogênea

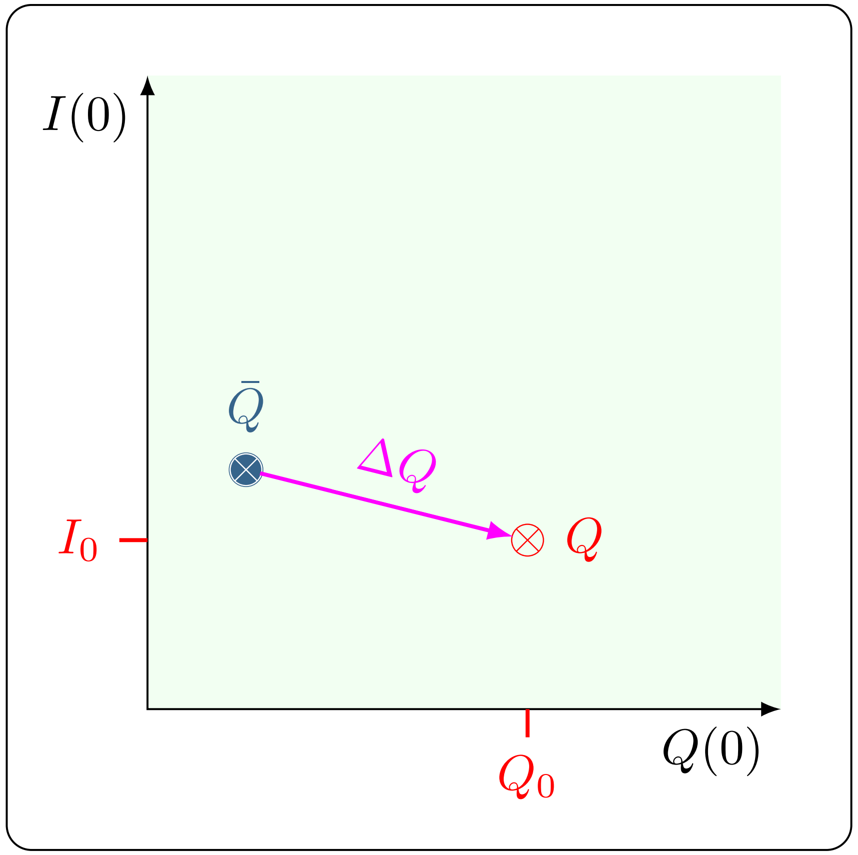
$$L\frac{d^2\Delta Q}{dt^2} + R\frac{d\Delta Q}{dt} + \frac{\Delta Q}{C} = 0$$

$$\Delta Q = e^{st}$$

$$\frac{dQ}{dt} = se^{st} = sQ$$

$$\frac{d^2Q}{dt^2} = s^2e^{st} = s^2Q$$

$$Ls^2 + Rs + \frac{1}{C} = 0$$



$$\Rightarrow s = \frac{-R}{2L} \pm \frac{\sqrt{R^2 - 4\frac{L}{C}}}{2L}$$

Circuitos de corrente alternada Equação homogênea

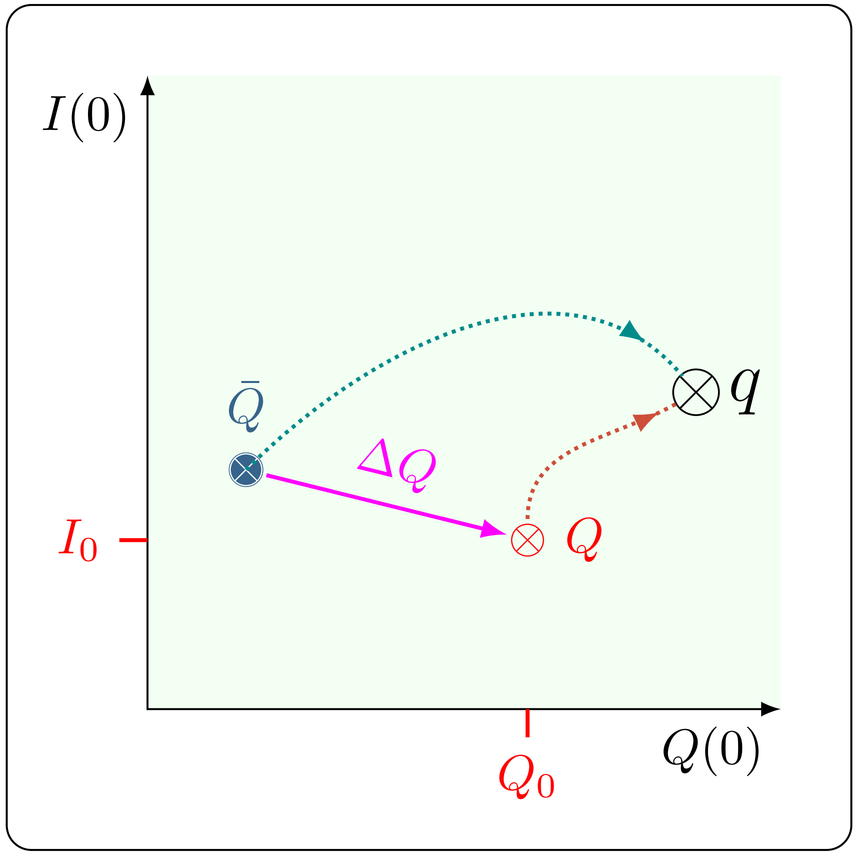
$$L\frac{d^2\Delta Q}{dt^2} + R\frac{d\Delta Q}{dt} + \frac{\Delta Q}{C} = 0$$

$$\Delta Q = e^{st}$$

$$\frac{dQ}{dt} = se^{st} = sQ$$

$$\frac{d^2Q}{dt^2} = s^2e^{st} = s^2Q$$

$$Ls^2 + Rs + \frac{1}{C} = 0$$



$$\Rightarrow s = \frac{-R}{2L} \pm \frac{\sqrt{R^2 - 4\frac{L}{C}}}{2L}$$

Circuitos de corrente alternada Equação homogênea

$$\Delta Q = e^{st}$$

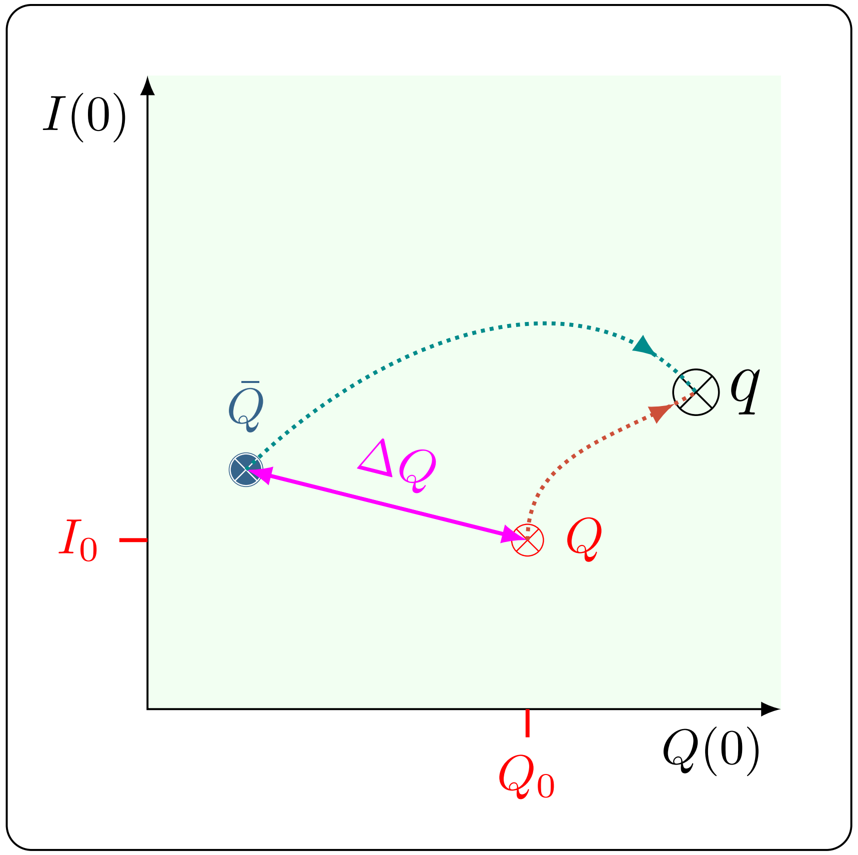
$$s = \frac{-R}{2L} \pm \frac{\sqrt{R^2 - 4\frac{L}{C}}}{2L}$$

$$\Delta Q = e^{st}$$

$$\frac{d\Delta Q}{dt} = se^{st} = sQ$$

$$\frac{d^2\Delta Q}{dt^2} = s^2e^{st} = s^2Q$$

$$Ls^2 + Rs + \frac{1}{C} = 0$$

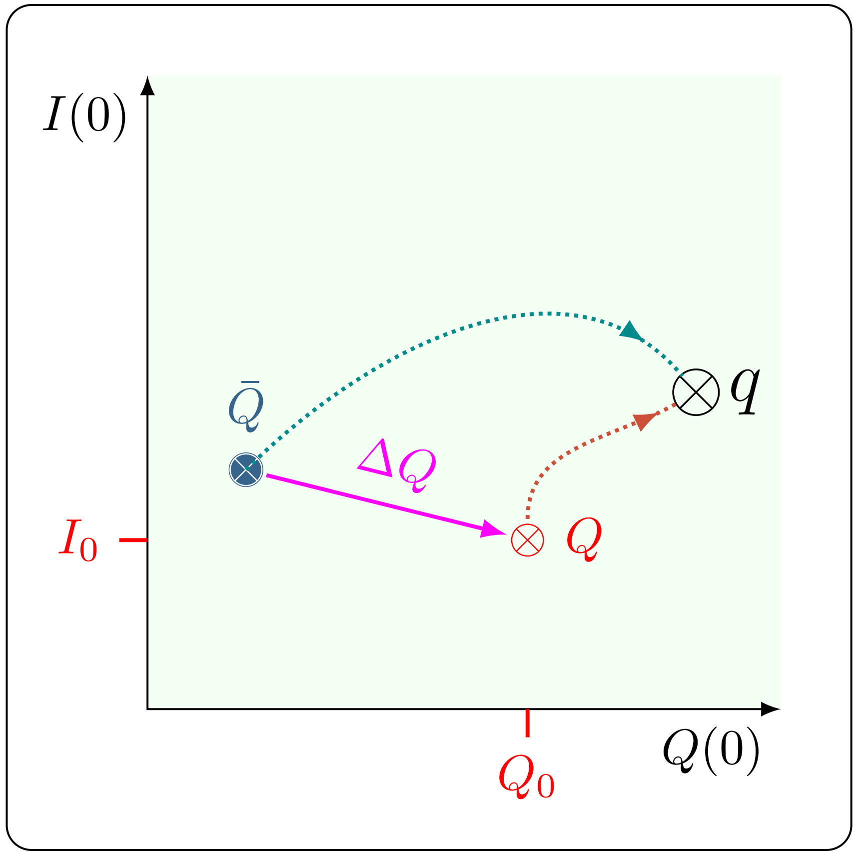


$$\Rightarrow s = \frac{-R}{2L} \pm \frac{\sqrt{R^2 - 4\frac{L}{C}}}{2L}$$

Circuitos de corrente alternada Equação homogênea

$$\Delta Q = e^{st}$$

$$s = \frac{-R}{2L} \pm \frac{\sqrt{R^2 - 4\frac{L}{C}}}{2L}$$



Escalas de tempo

$$\tau_{LC} = \sqrt{LC}$$

$$\tau_{RL} = \frac{2L}{R}$$

$$s = -\frac{1}{\tau_{RL}}\left(1 \mp \sqrt{1 - \frac{\tau_{RL}^2}{\tau_{LC}^2}}\right)$$

Circuitos de corrente alternada Equação homogênea

$$\Delta Q = e^{st}$$

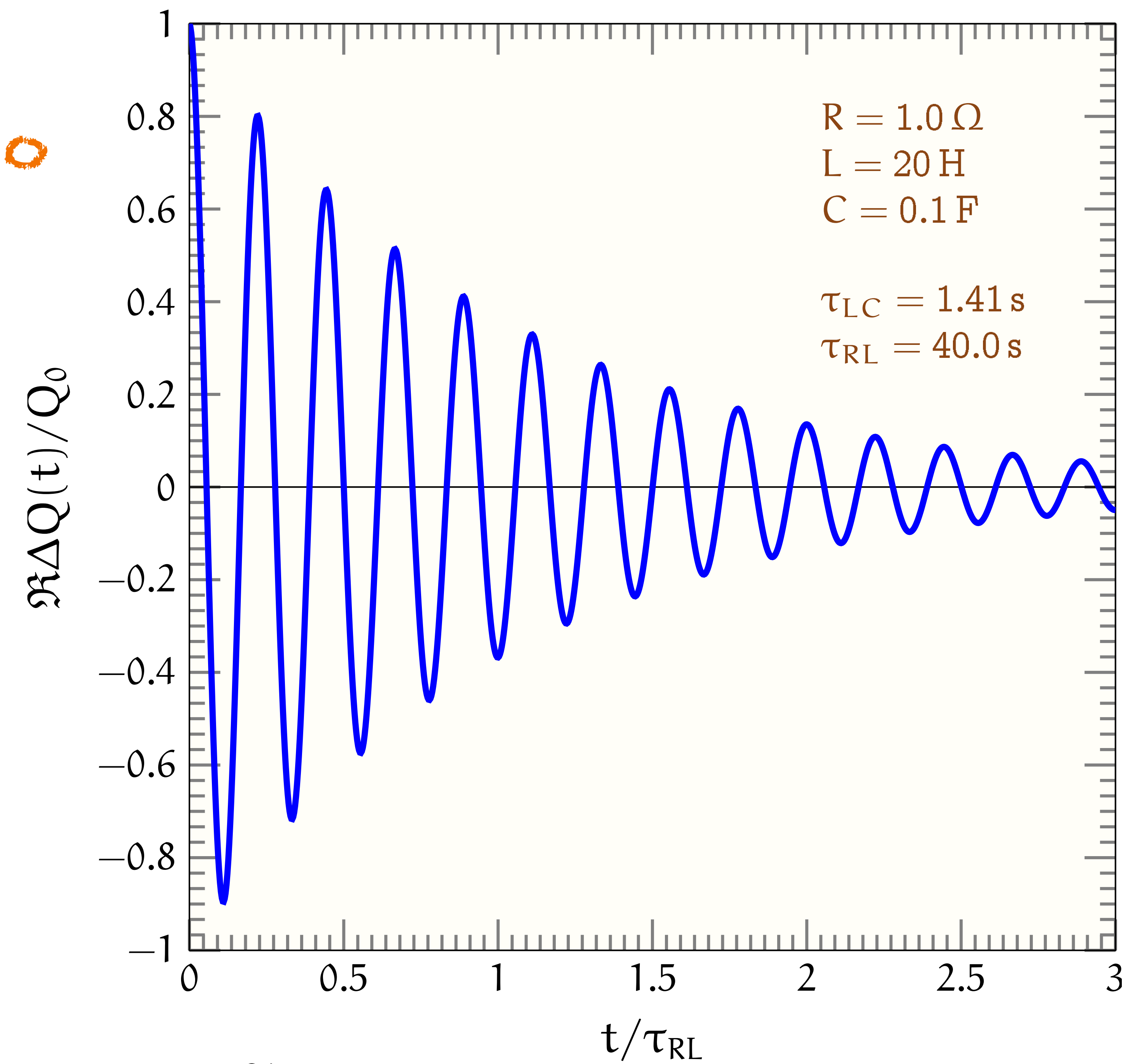
$$s = \frac{-R}{2L} \pm \frac{\sqrt{R^2 - 4\frac{L}{C}}}{2L}$$

Escalas de tempo

$$\tau_{LC} = \sqrt{LC}$$

$$\tau_{RL} = \frac{2L}{R}$$

$$\tau_{LC} < \tau_{RL}$$



Circuitos de corrente alternada Equação homogênea

$$\Delta Q = e^{st}$$

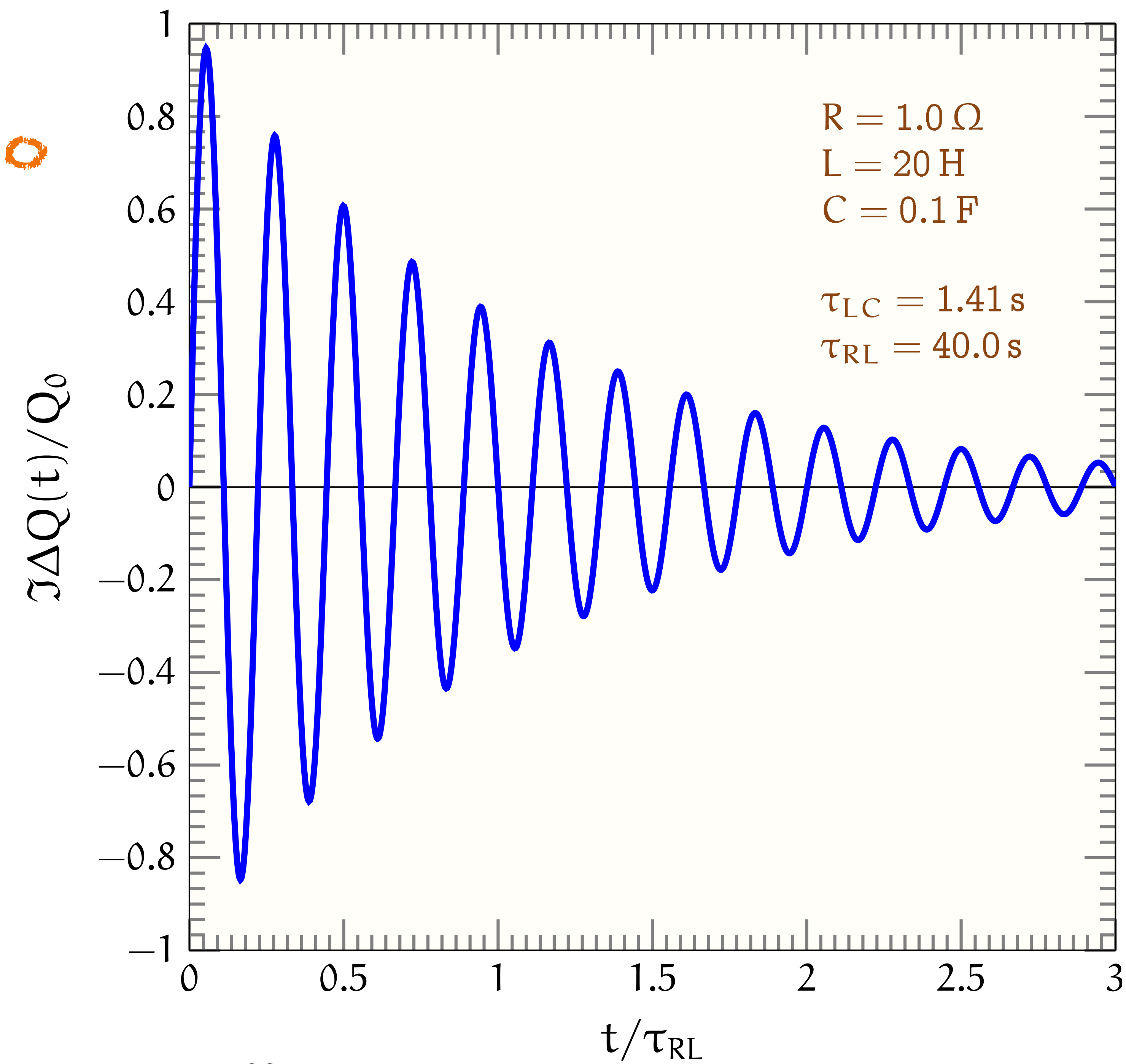
$$s = \frac{-R}{2L} \pm \frac{\sqrt{R^2 - 4\frac{L}{C}}}{2L}$$

Escalas de tempo

$$\tau_{LC} = \sqrt{LC}$$

$$\tau_{RL} = \frac{2L}{R}$$

$$\tau_{LC} < \tau_{RL}$$



Circuitos de corrente alternada Equação homogênea

$$\Delta Q = e^{st}$$

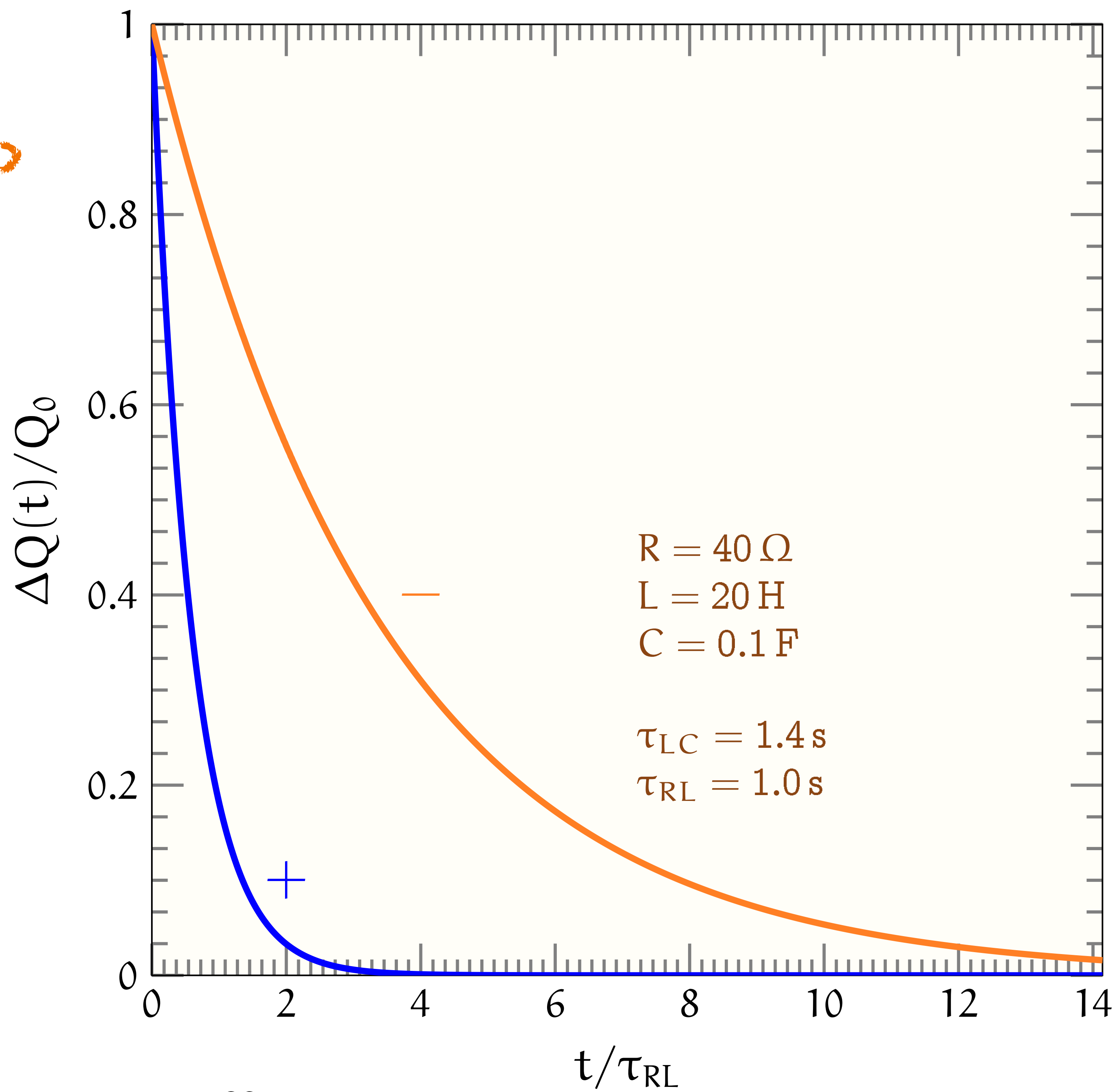
$$s = \frac{-R}{2L} \pm \frac{\sqrt{R^2 - 4\frac{L}{C}}}{2L}$$

Escalas de tempo

$$\tau_{LC} = \sqrt{LC}$$

$$\tau_{RL} = \frac{2L}{R}$$

$$\tau_{RL} < \tau_{LC}$$



Circuitos de corrente alternada Solução estacionária

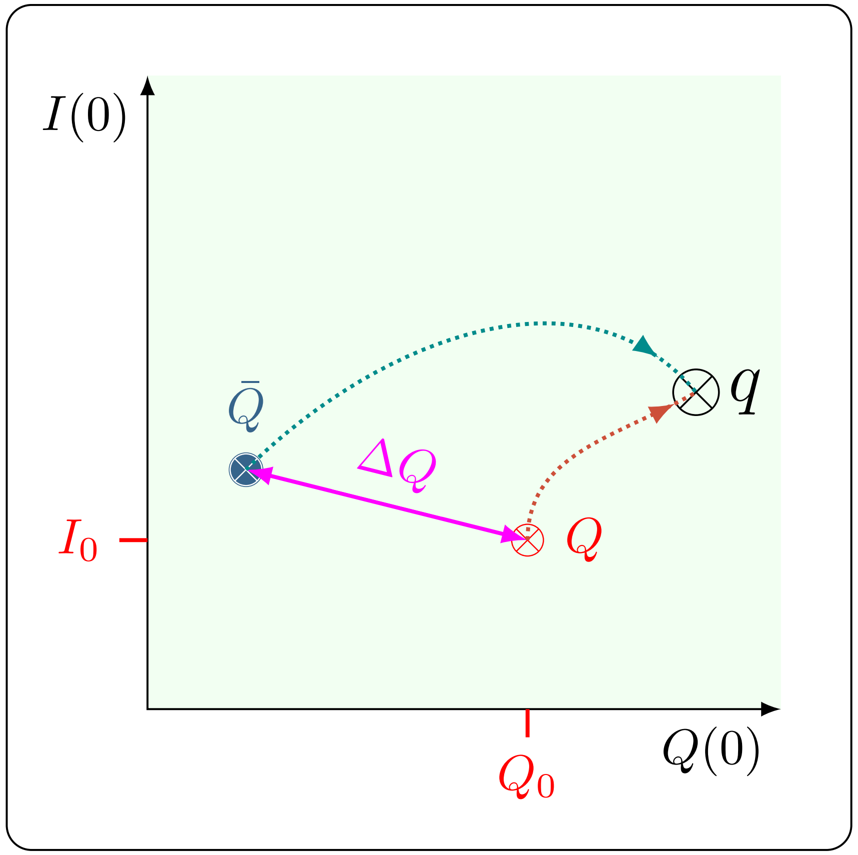
$$L\frac{d^2\Delta Q}{dt^2}+R\frac{d\Delta Q}{dt}+\frac{\Delta Q}{C}=0$$

$$\Delta Q = e^{st}$$

$$\frac{dQ}{dt} = se^{st} = sQ$$

$$\frac{d^2Q}{dt^2} = s^2e^{st} = s^2Q$$

$$Ls^2 + Rs + \frac{1}{C} = 0$$



$$\Rightarrow s = \frac{-R}{2L} \pm \frac{\sqrt{R^2 - 4\frac{L}{C}}}{2L}$$