

# SEM5950 - SEM0586

## Legged Robots

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Aula #1: Princípios de locomoção  
com pernas

**Prof. Dr. Thiago Boaventura**  
[tboaventura@usp.br](mailto:tboaventura@usp.br)

São Carlos, 27/05/19





# Por que pernas?

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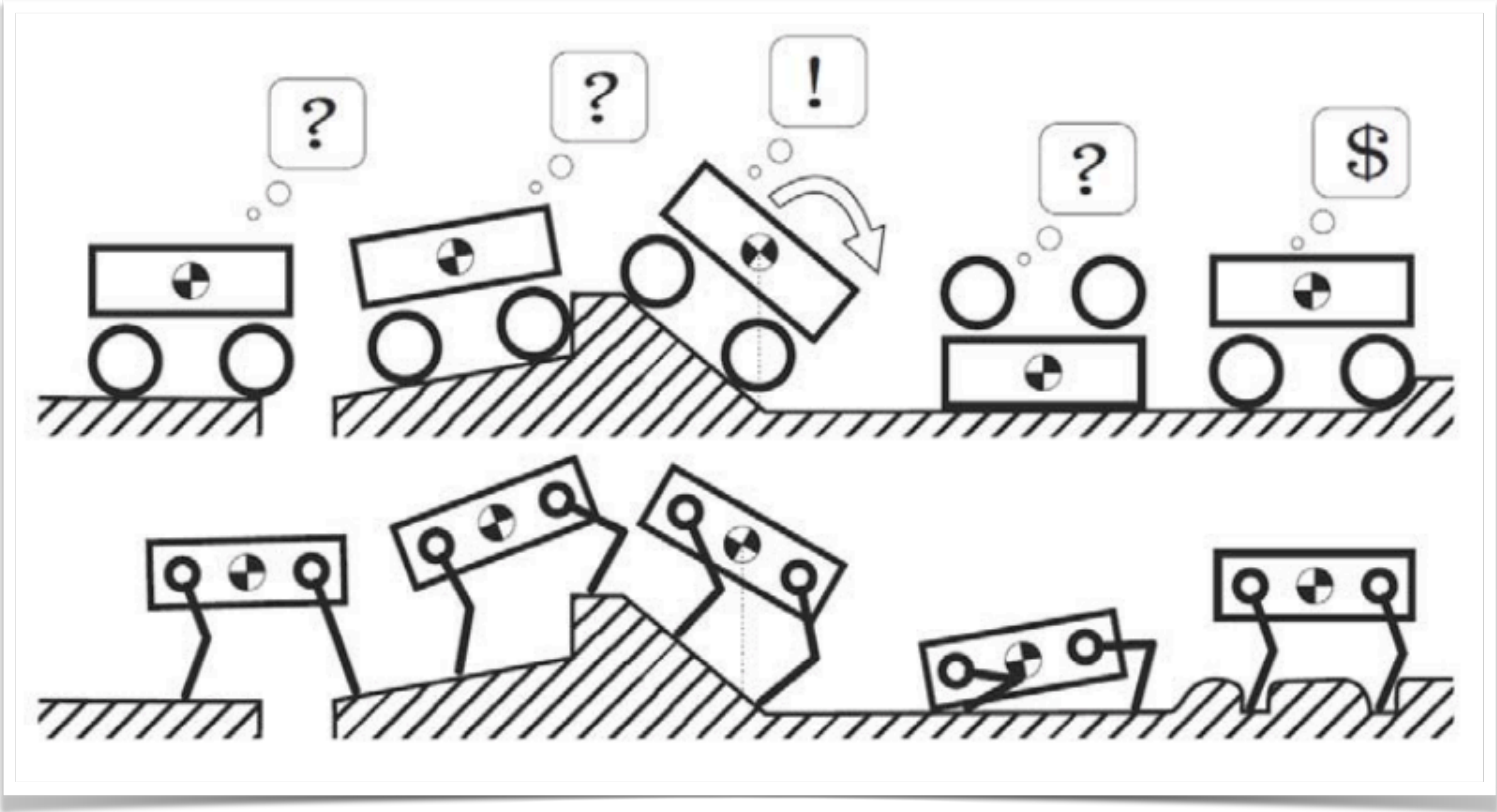
# Por que não rodas?

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# Perna vs. Rodas

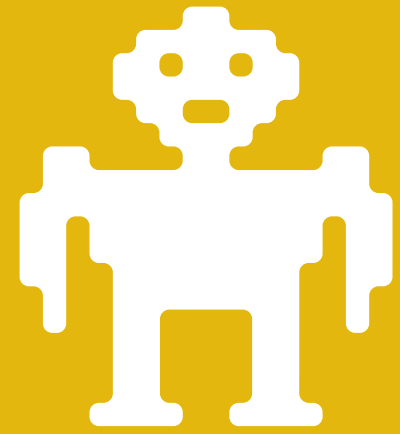






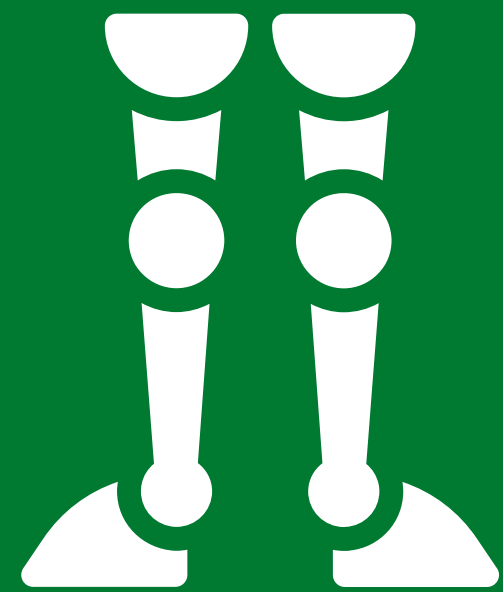


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- Histórico
- Estado da arte

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- Base de suporte
- CoP
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- Capture point

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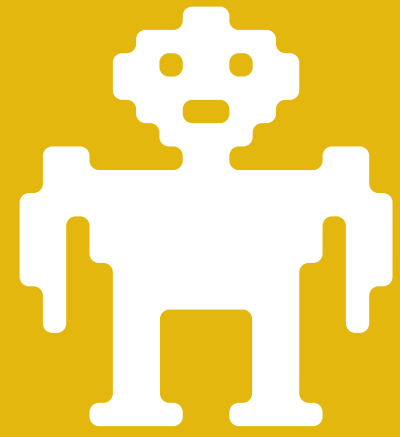


- Bibliografia

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- Histórico
- Estado da arte

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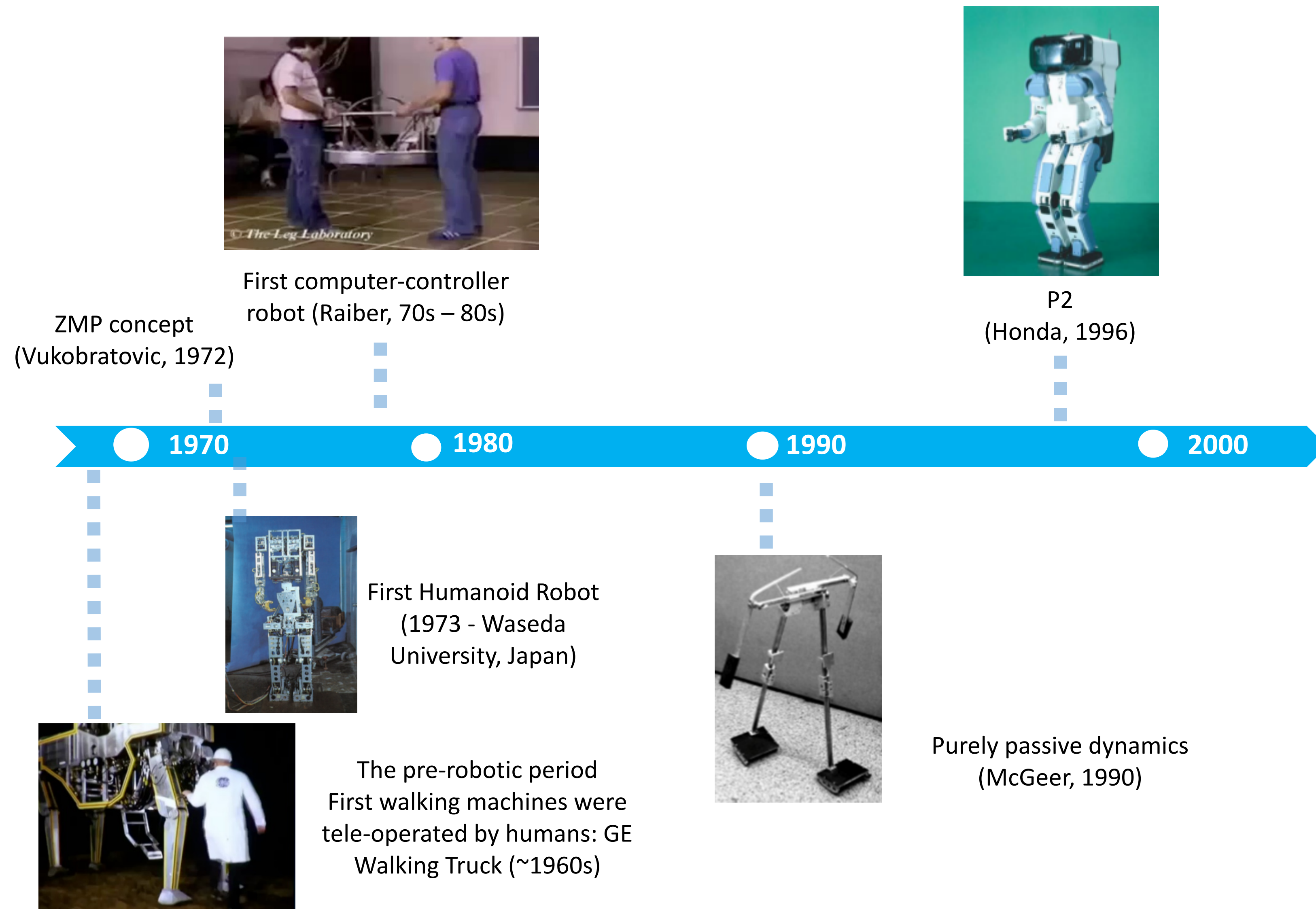


# Histórico (resumido) de robôs com pernas

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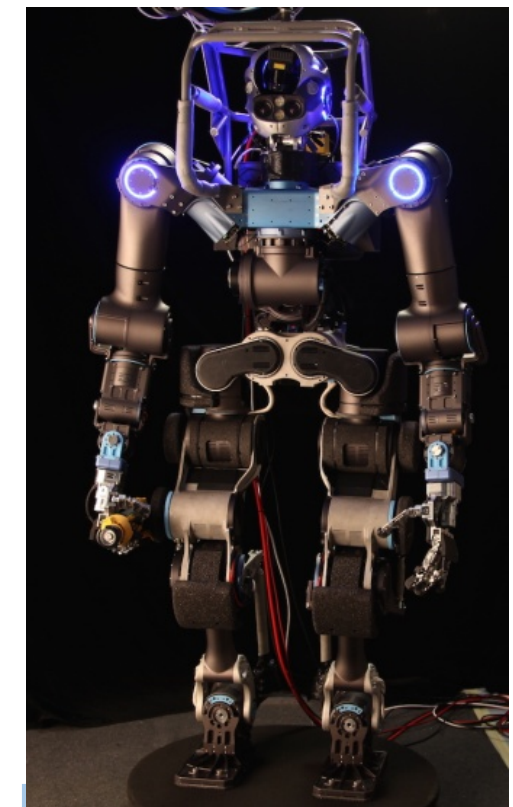
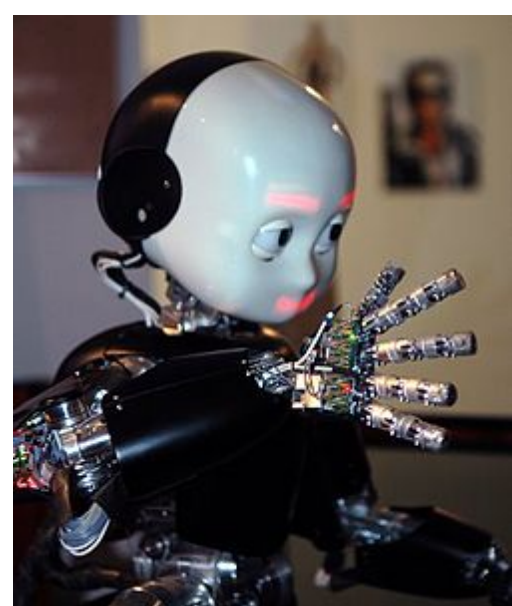
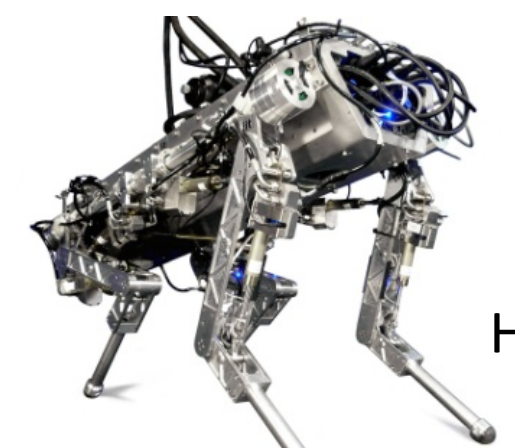
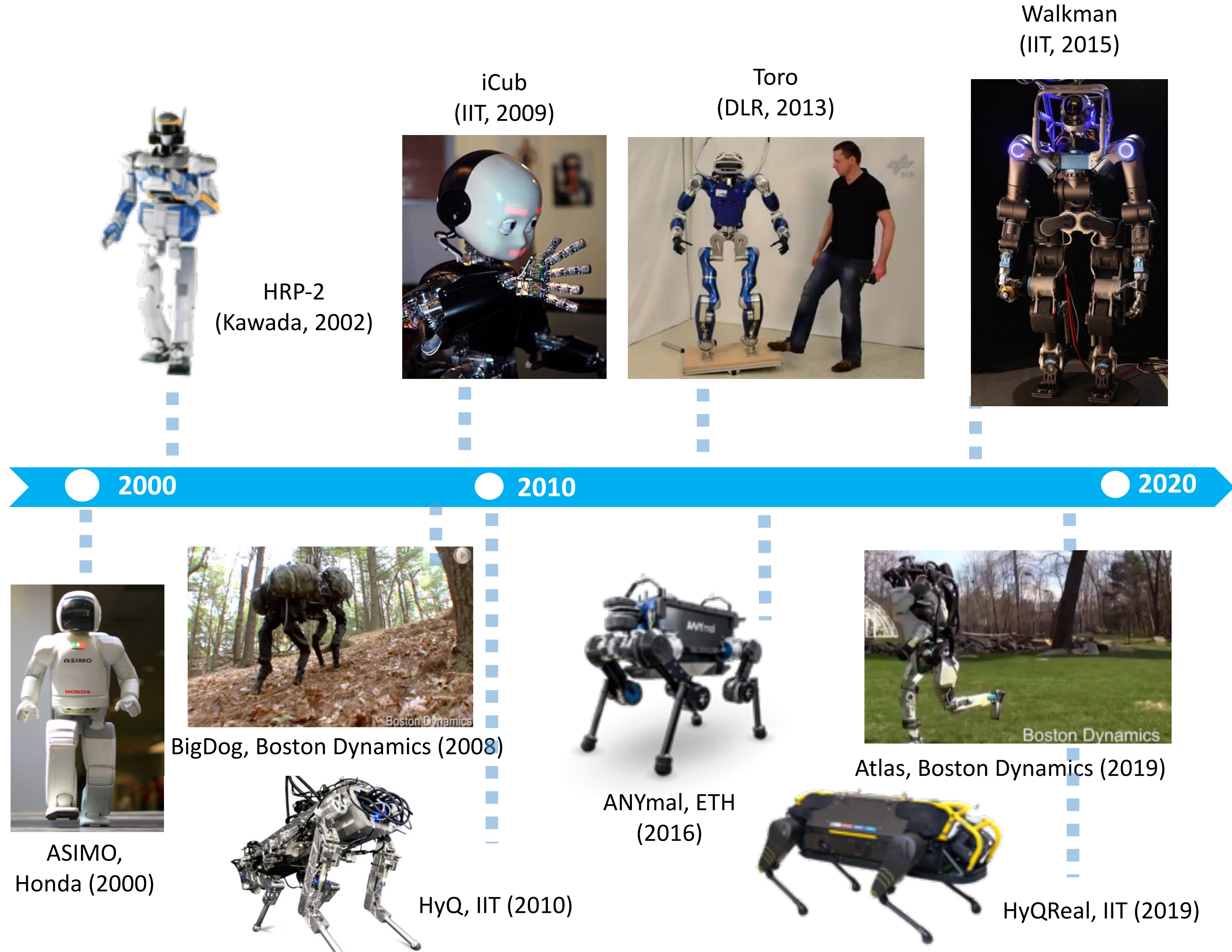


# Histórico (resumido) de robôs com pernas

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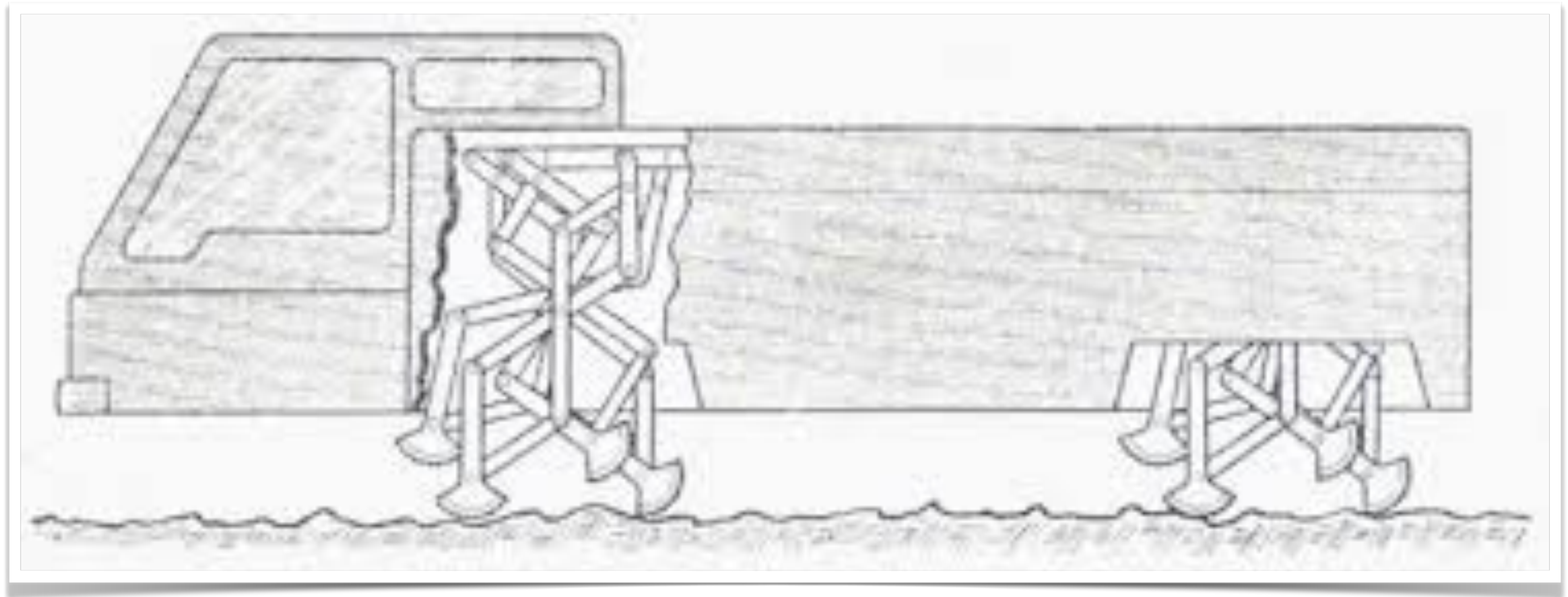


# 1957 — Walking Machines by Joseph Shigley

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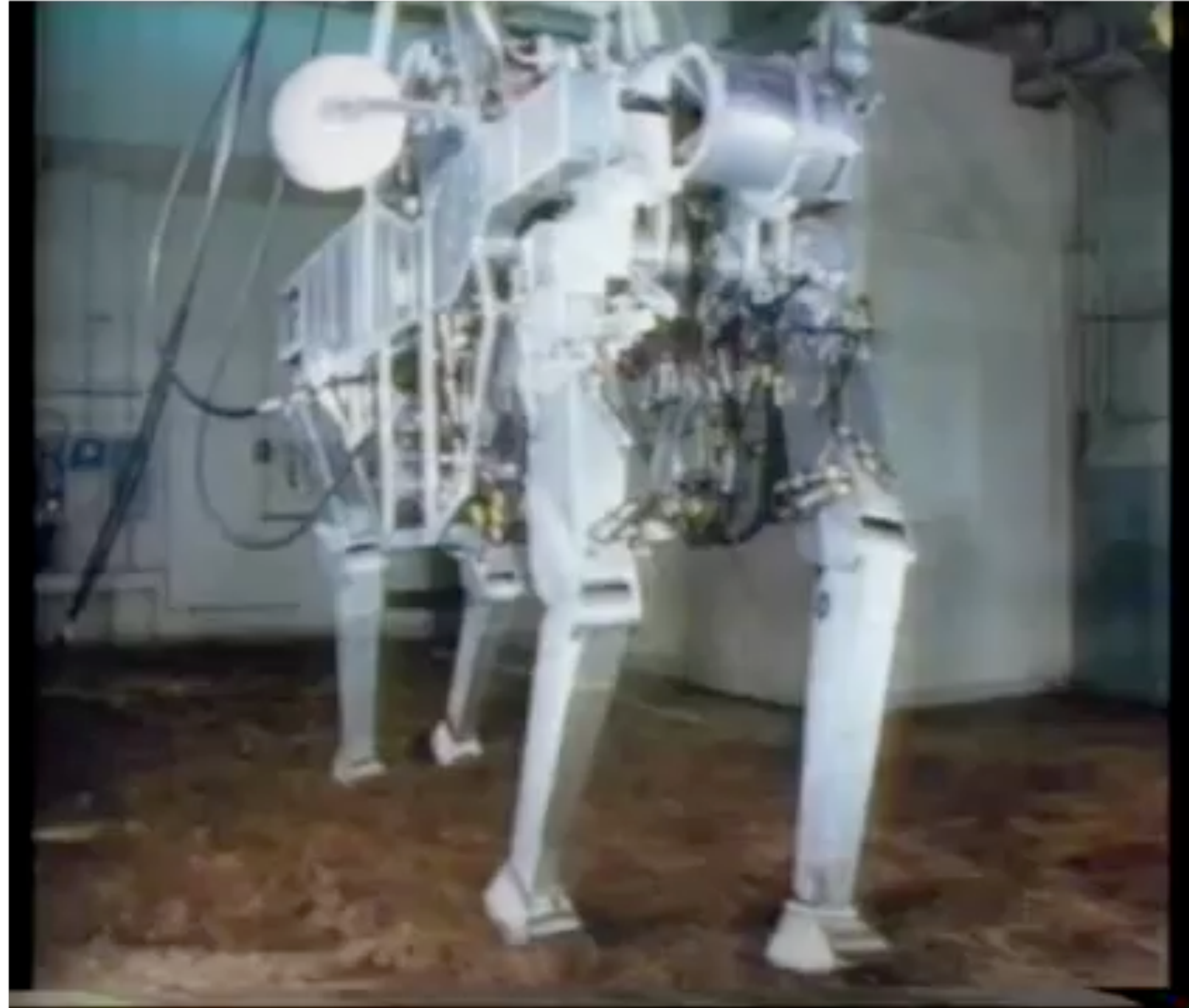


# 1969 — Walking truck by General Electric

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# 1984 — The OSU Adaptive Suspension Vehicle

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# Theo Jansen's walking mechanism

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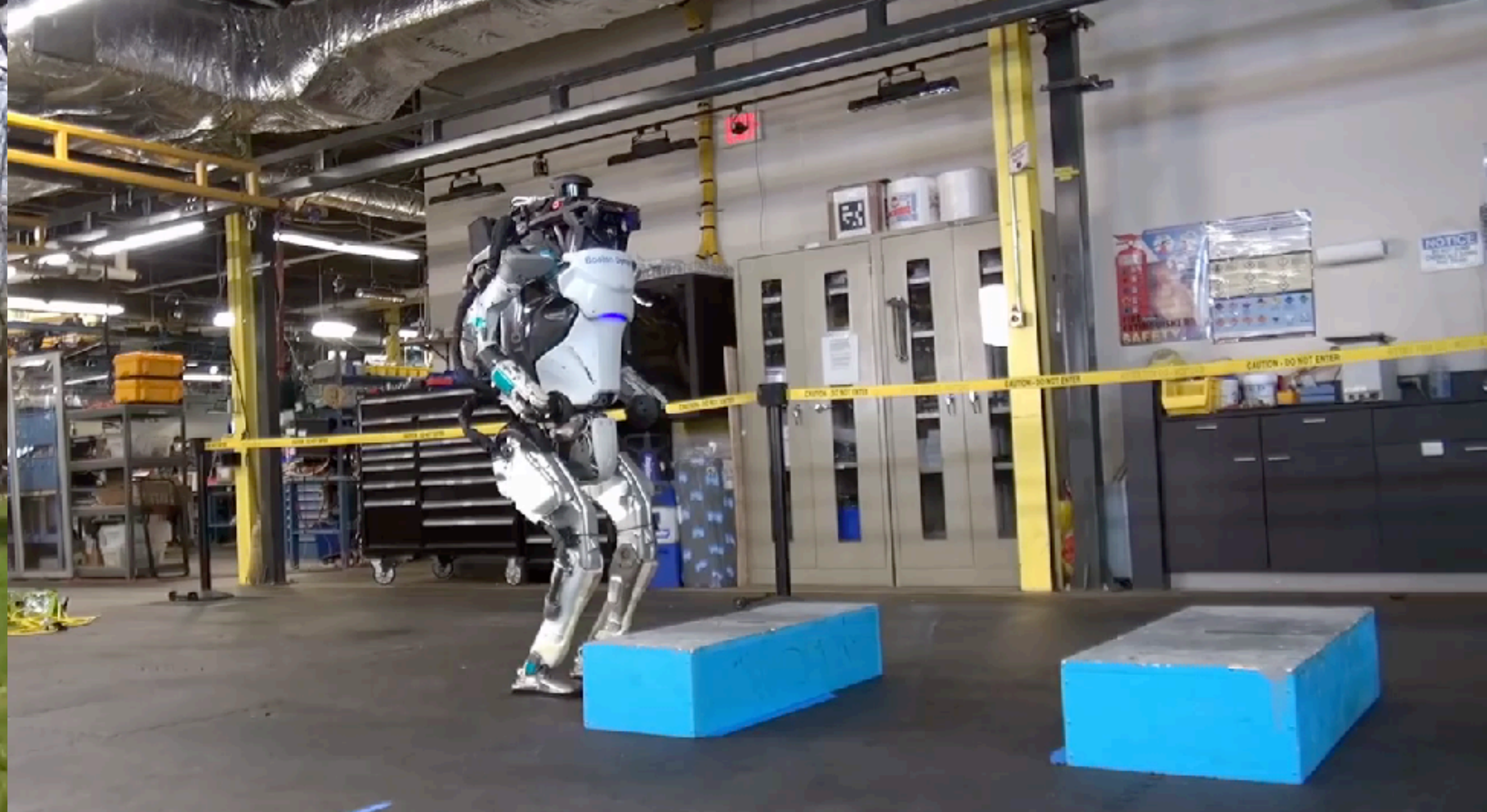






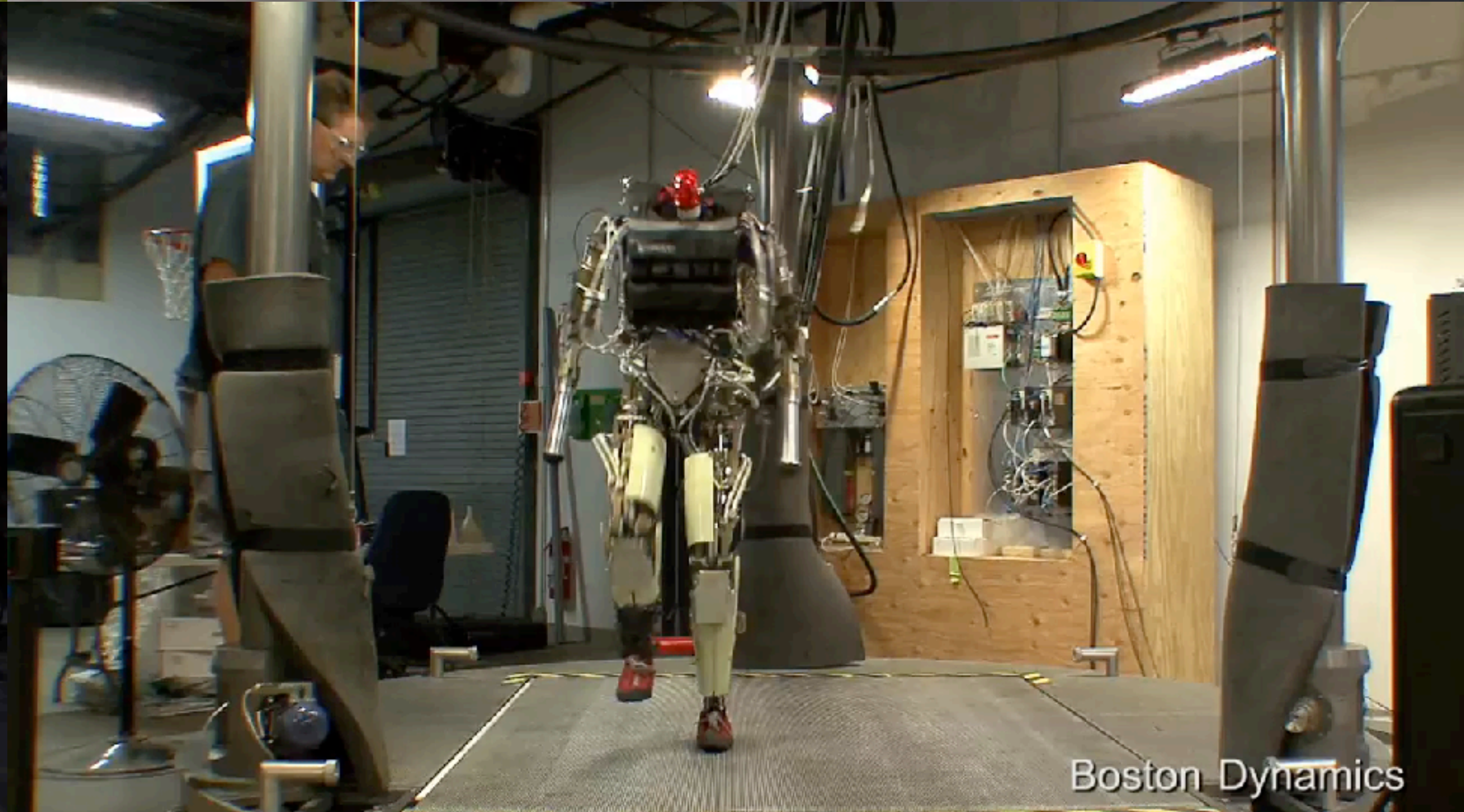


Boston Dynamics



Meet Atlas, the latest humanoid robot from Alphabet Inc.'s Boston Dynamics.

WSJ  
Boston Dynamics



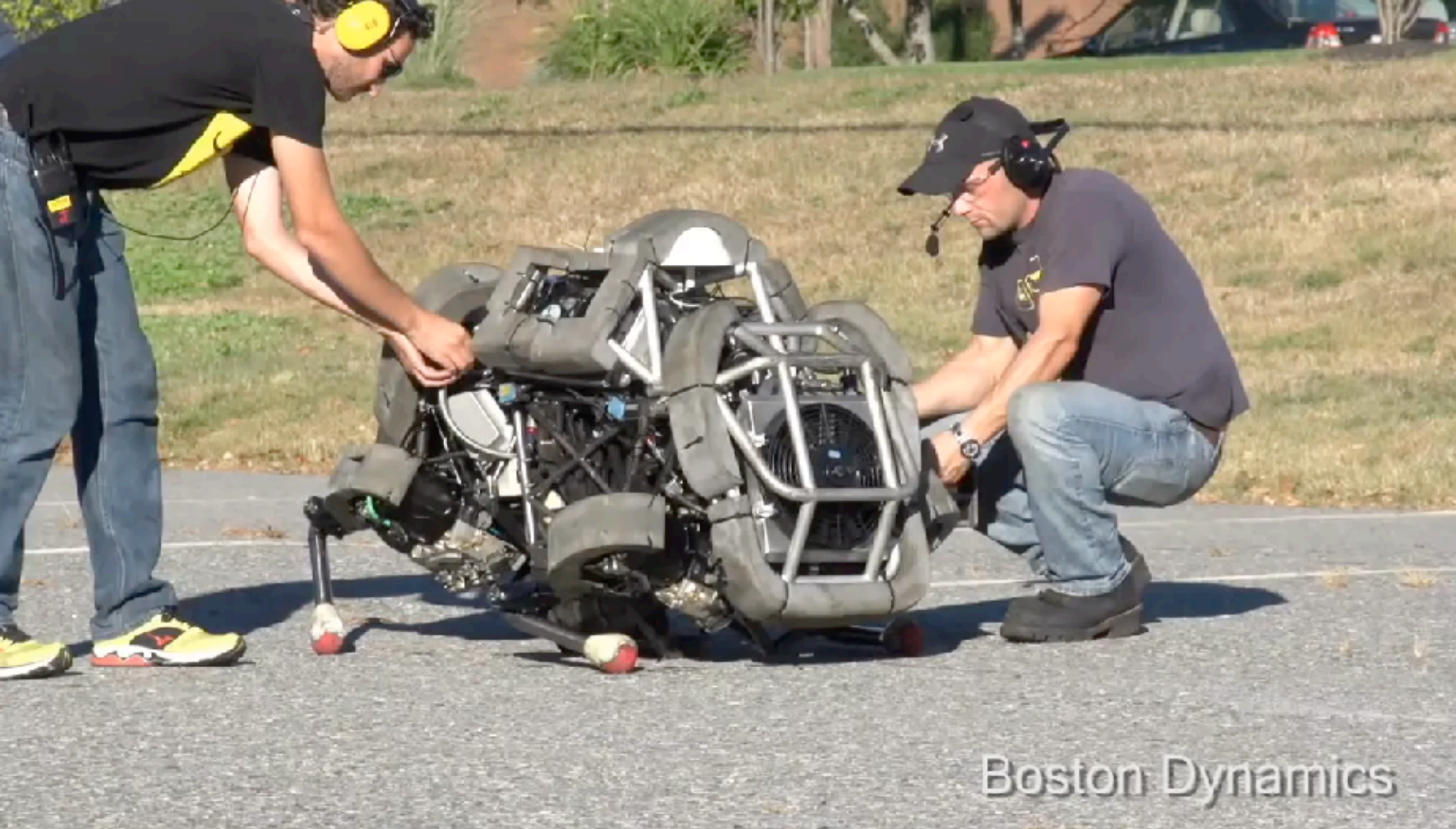
Boston Dynamics





Boston Dynamics









BOSTON DYNAMICS - SEPTEMBER 2019

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Massachusetts Institute of Technology

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# Autonomous Running Jumps Over Obstacles in the MIT Cheetah 2

Hae-Won Park, Patrick Wensing, and Sangbae Kim

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MITMECHE

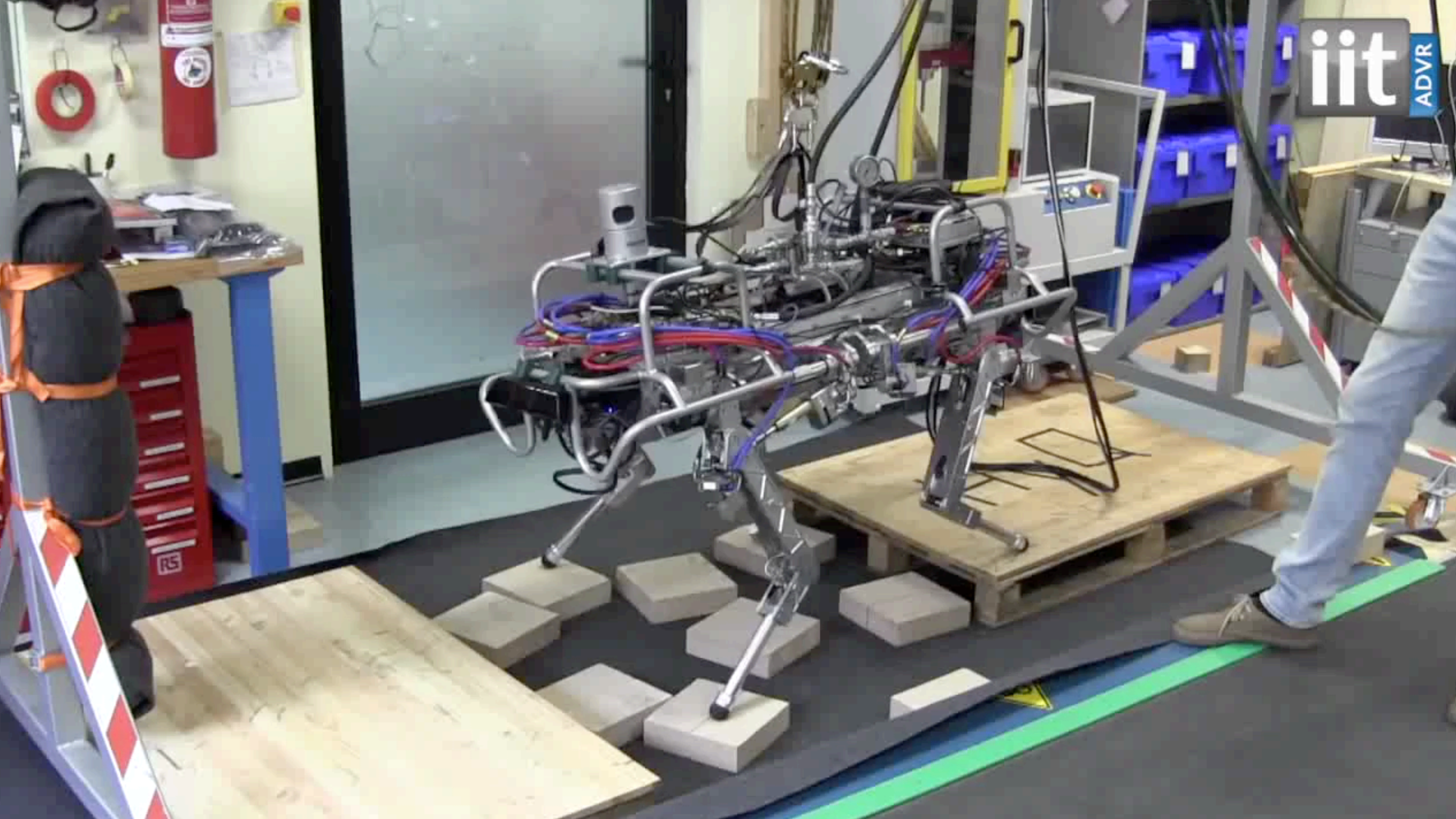
BIOMIMETIC ROBOTICS LAB

















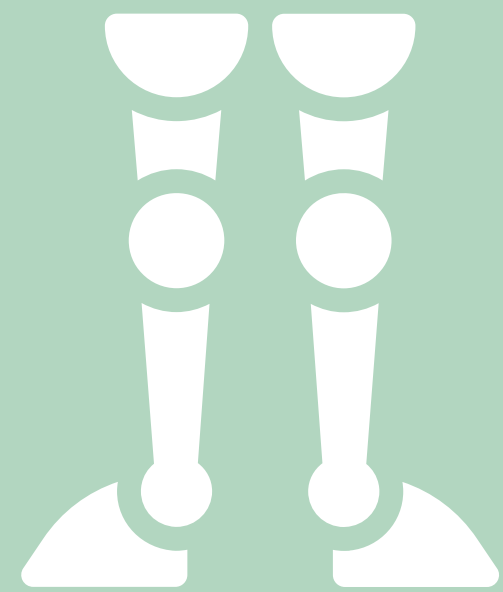


AWYmal



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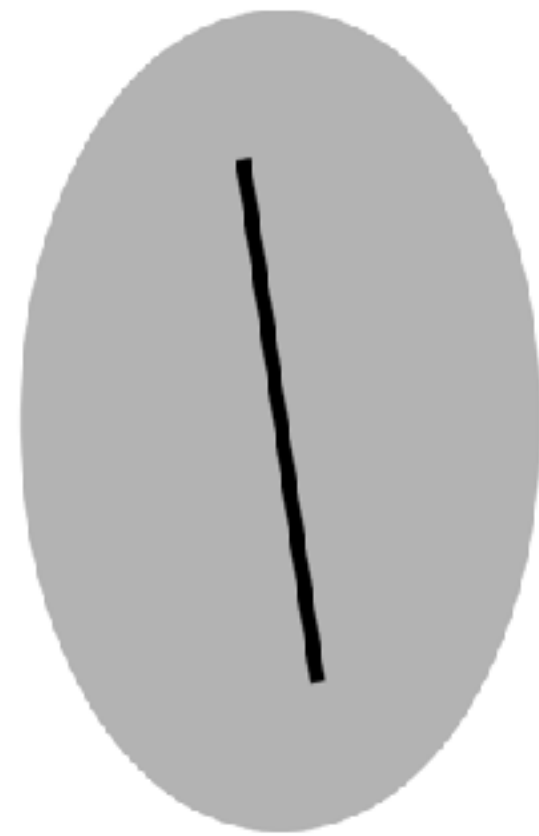


# Envoltória convexa (Complex hull)

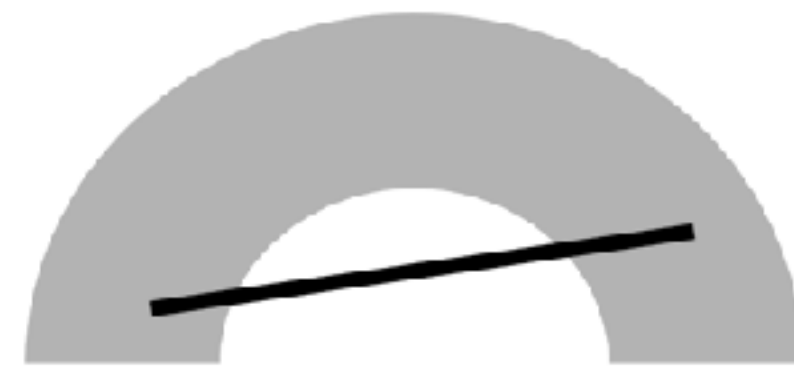
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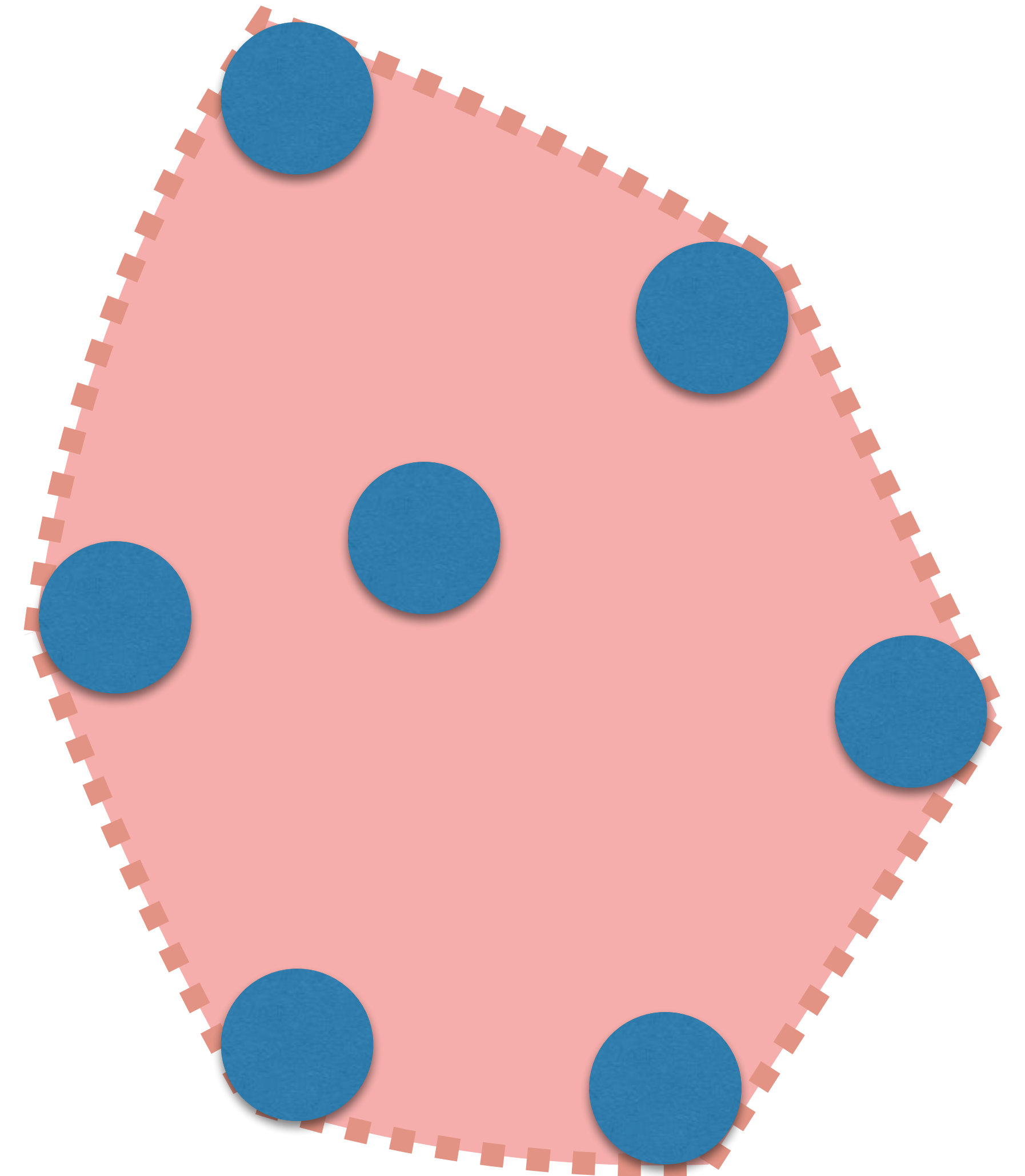


Convexo



Não convexo

**Menor polígono convexo  
que contém uma coleção  
de objetos (pontos)**



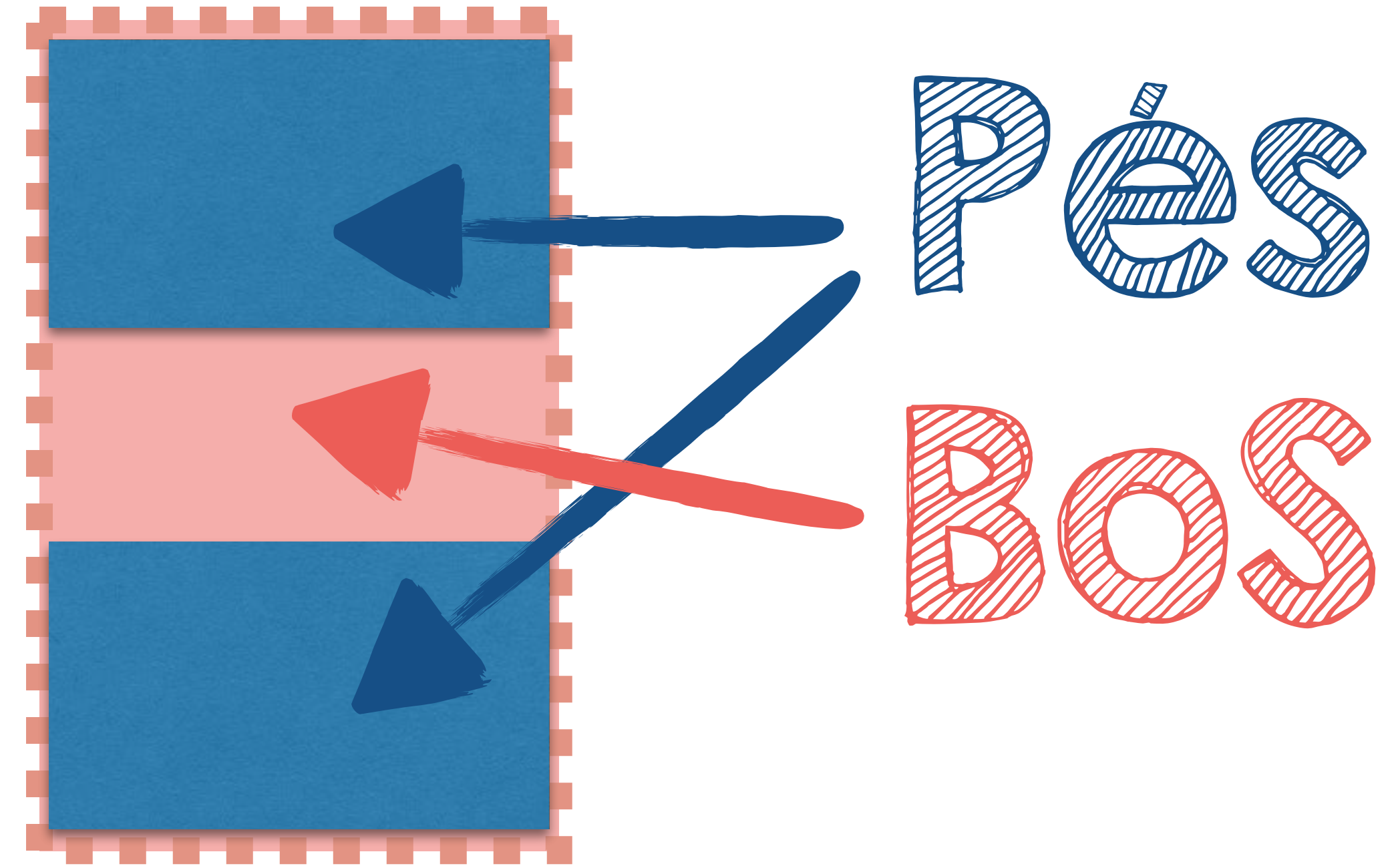


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Polígono formado pela **envoltória convexa** dos **pontos de contato** com o ambiente

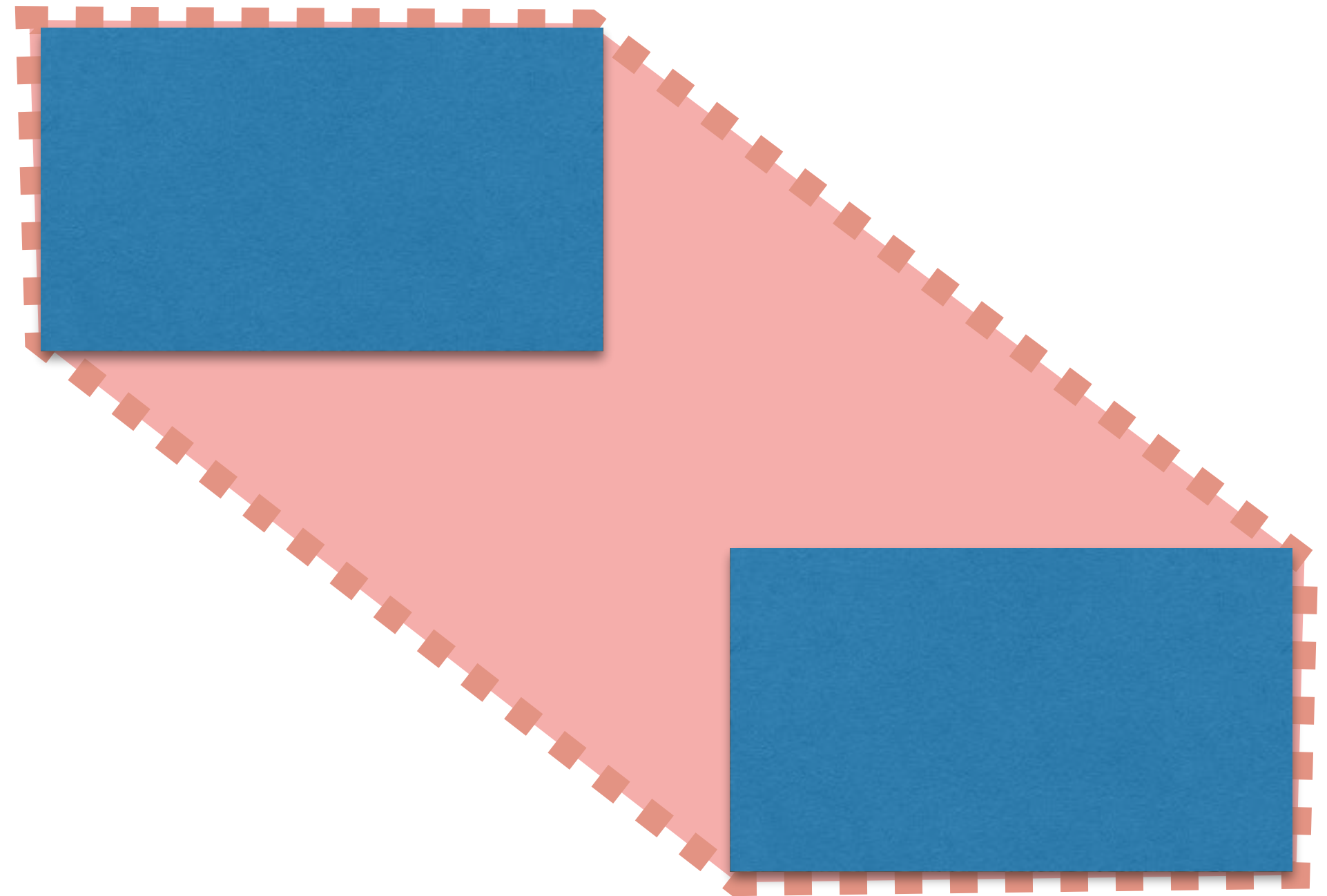


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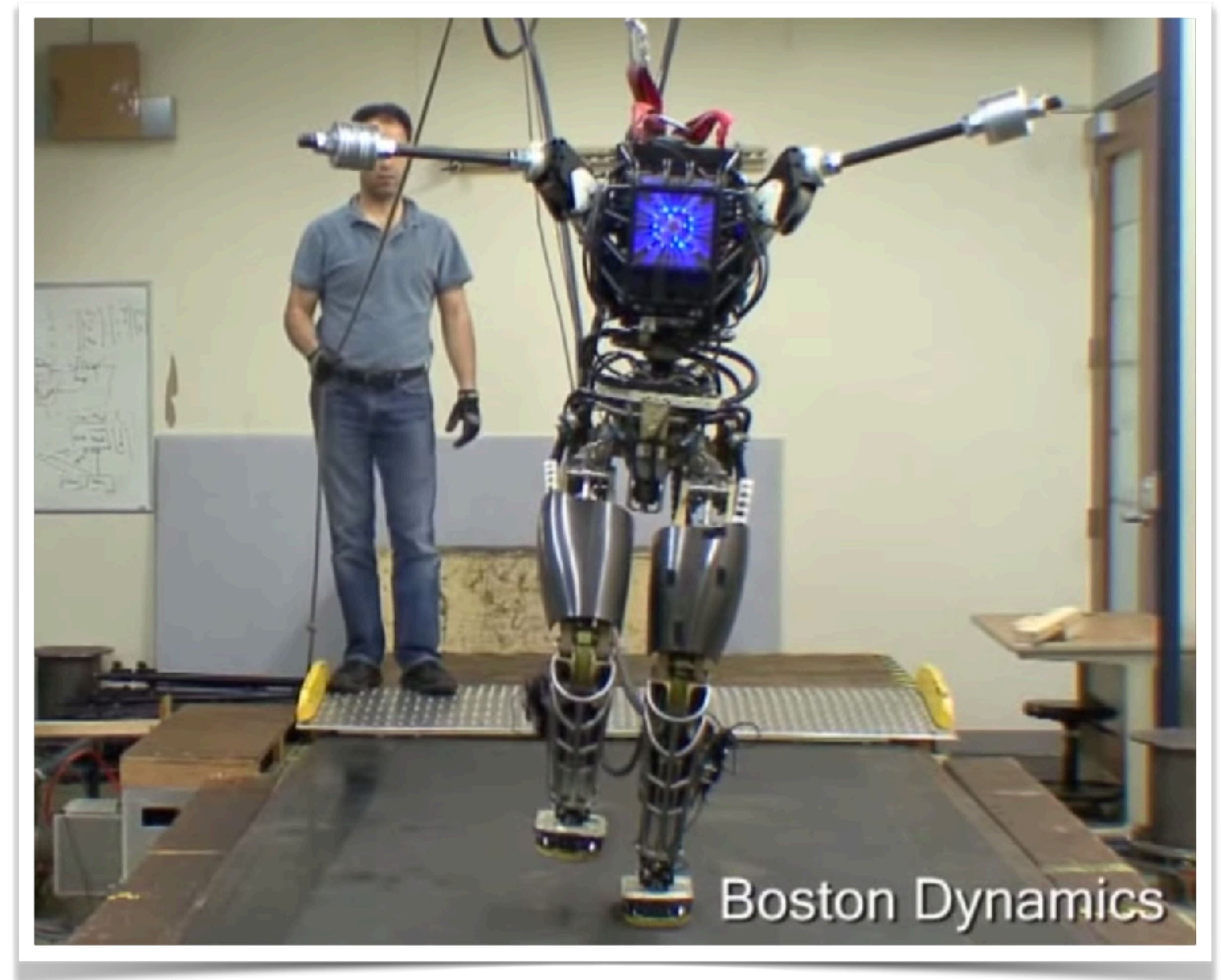
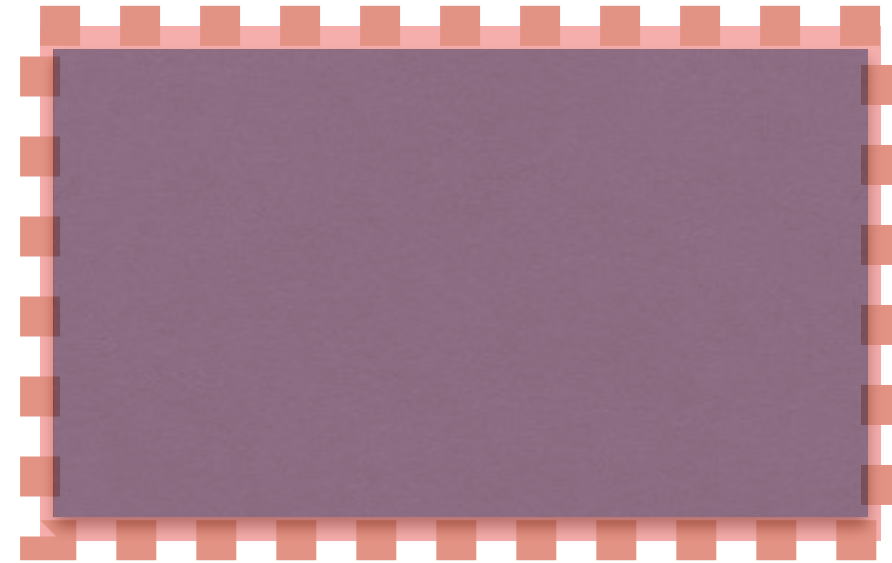


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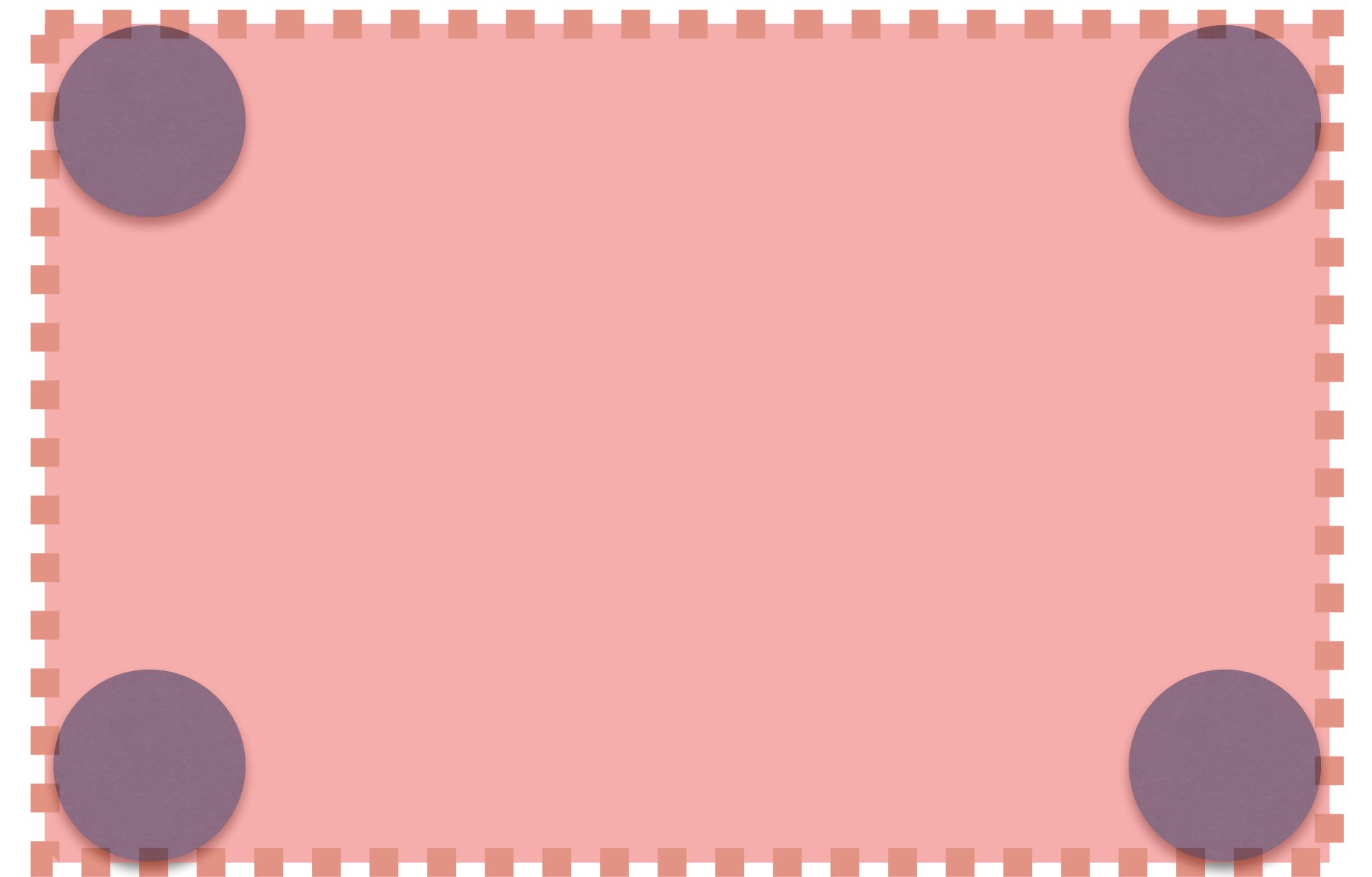
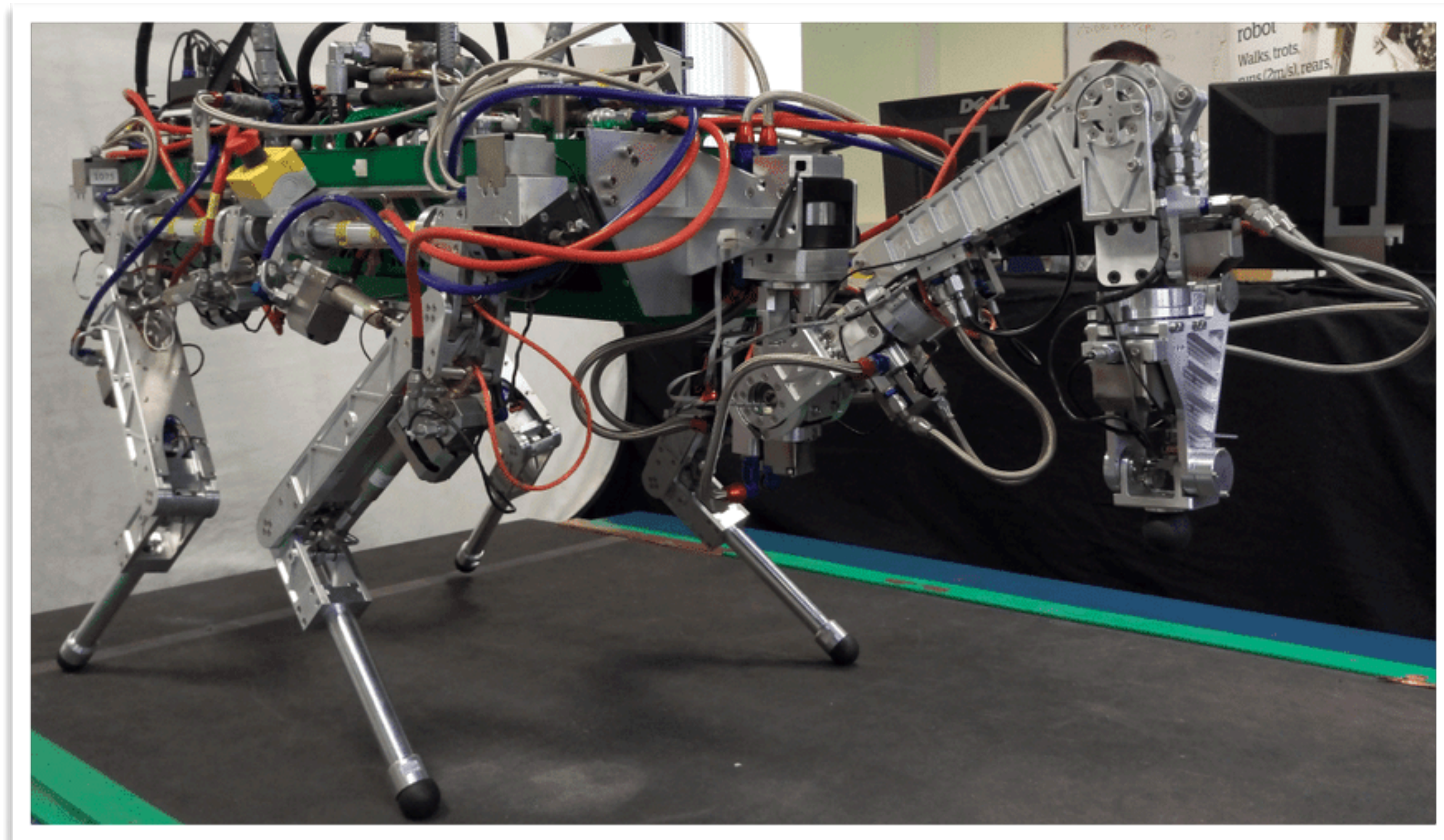


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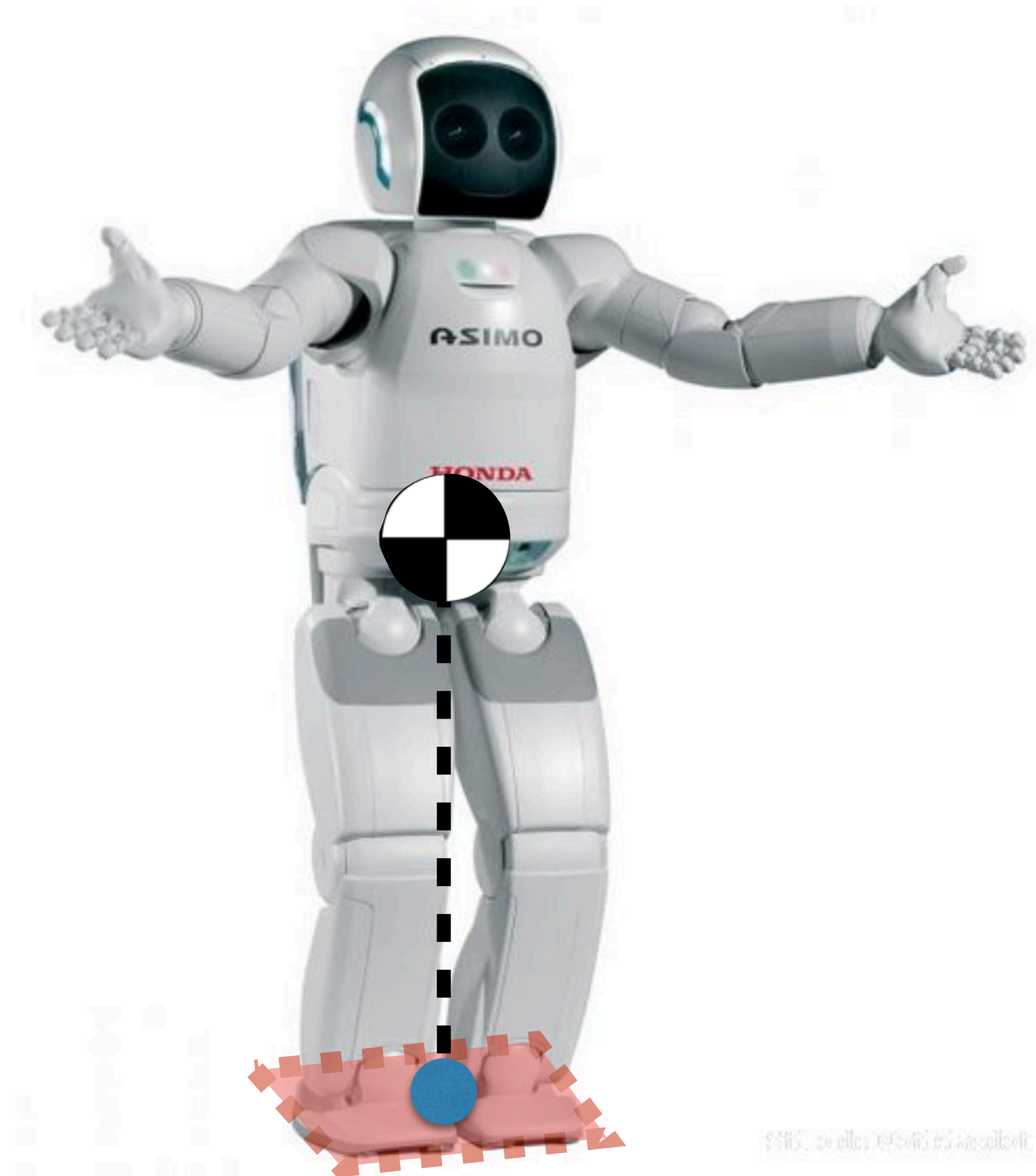


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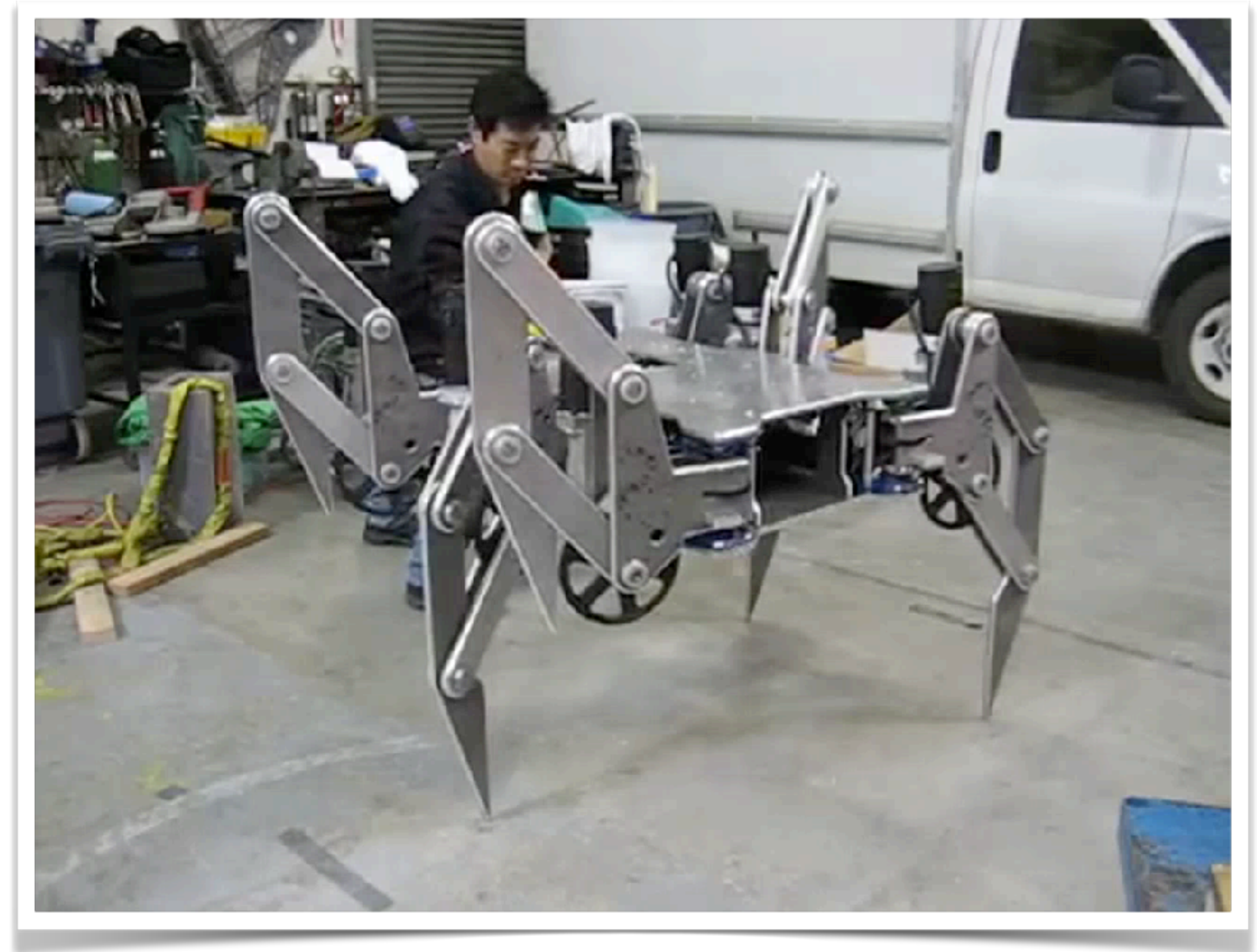
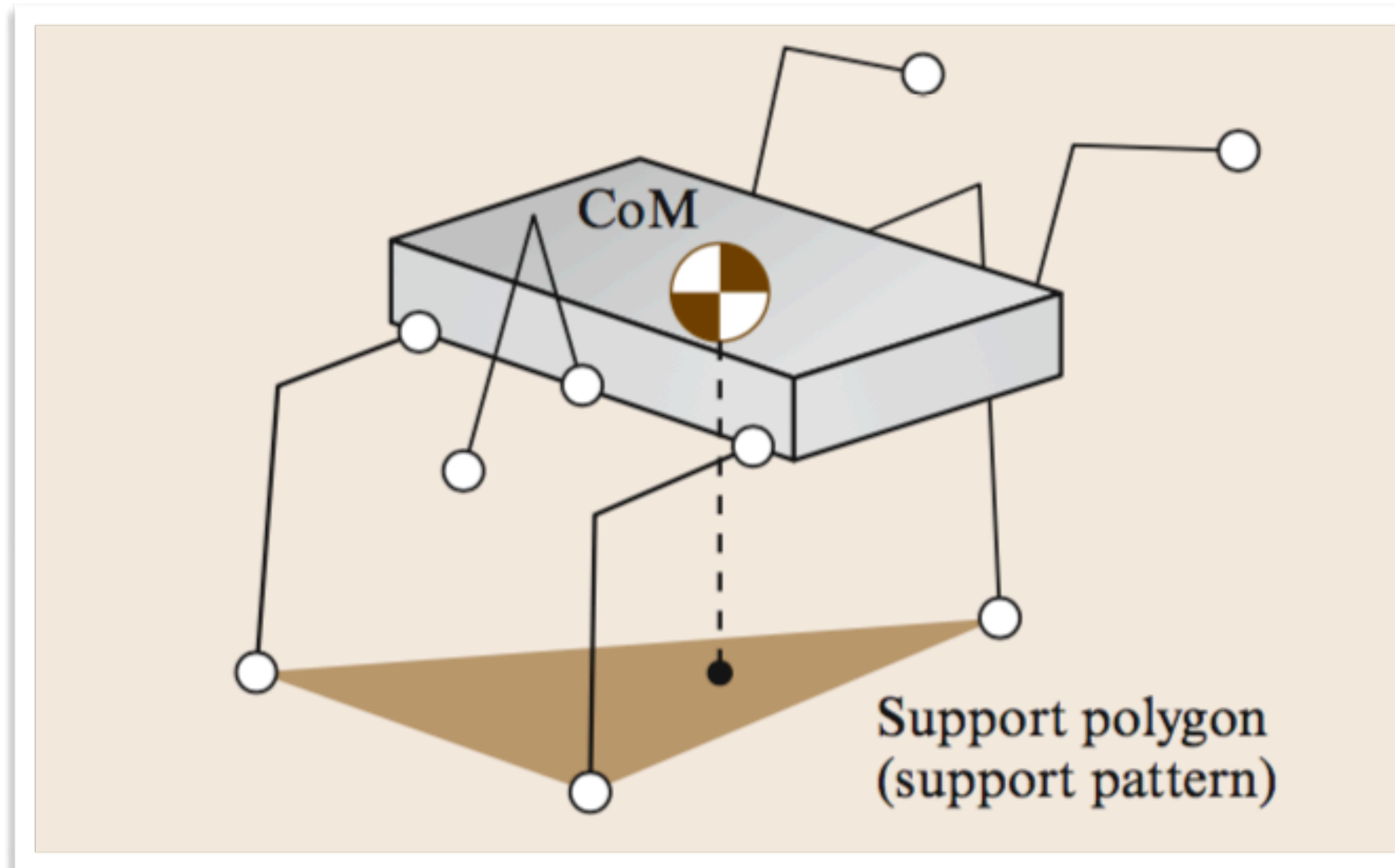


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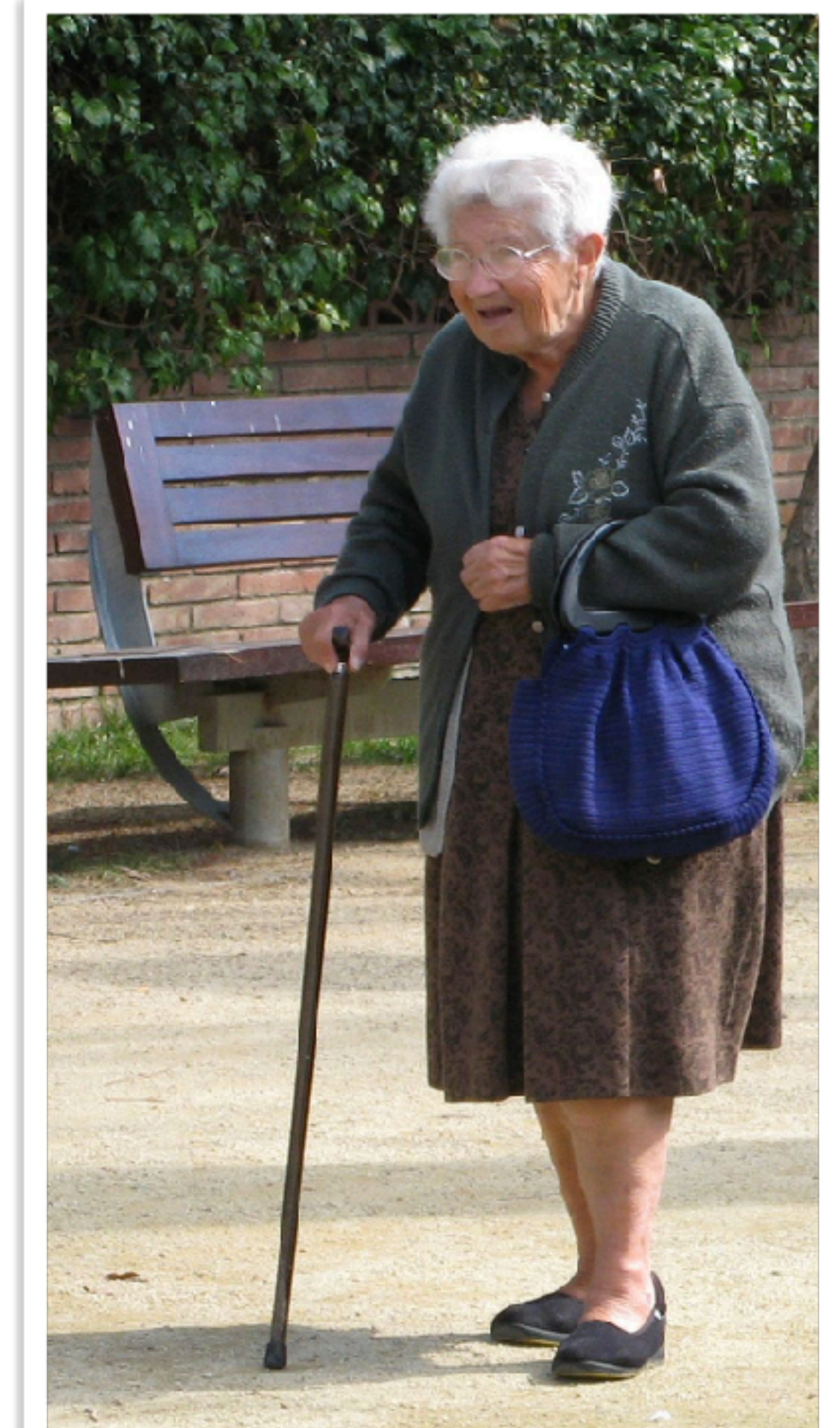


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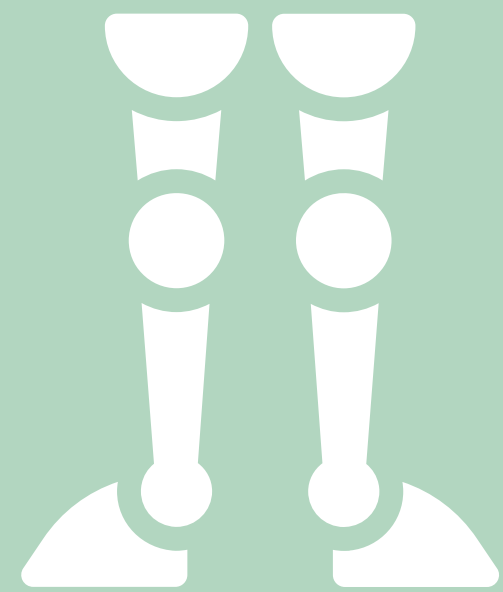
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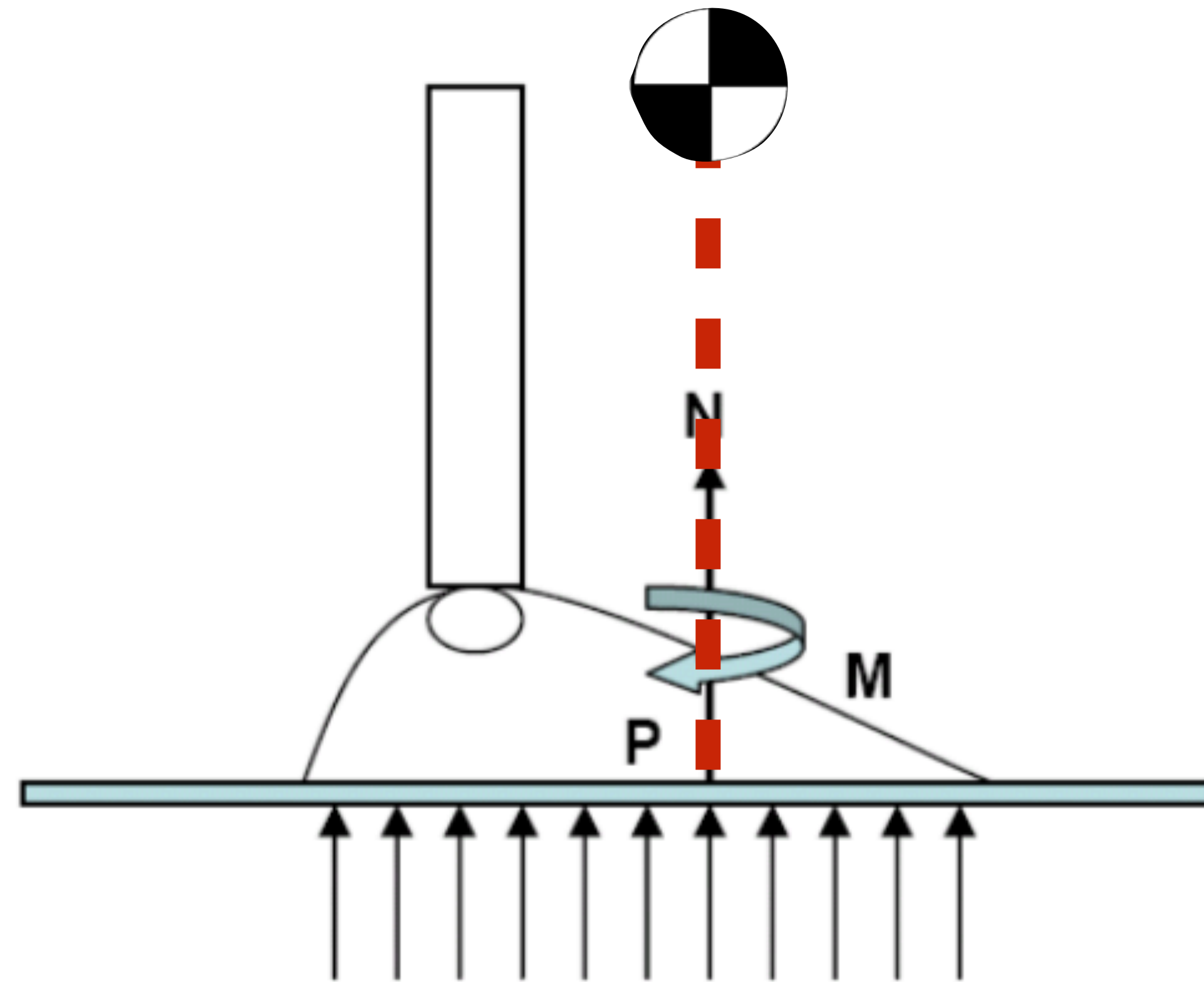


# Centro de Pressão (CoP)

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Ponto por onde  
passa a **força**  
**resultante** das  
forças de interação

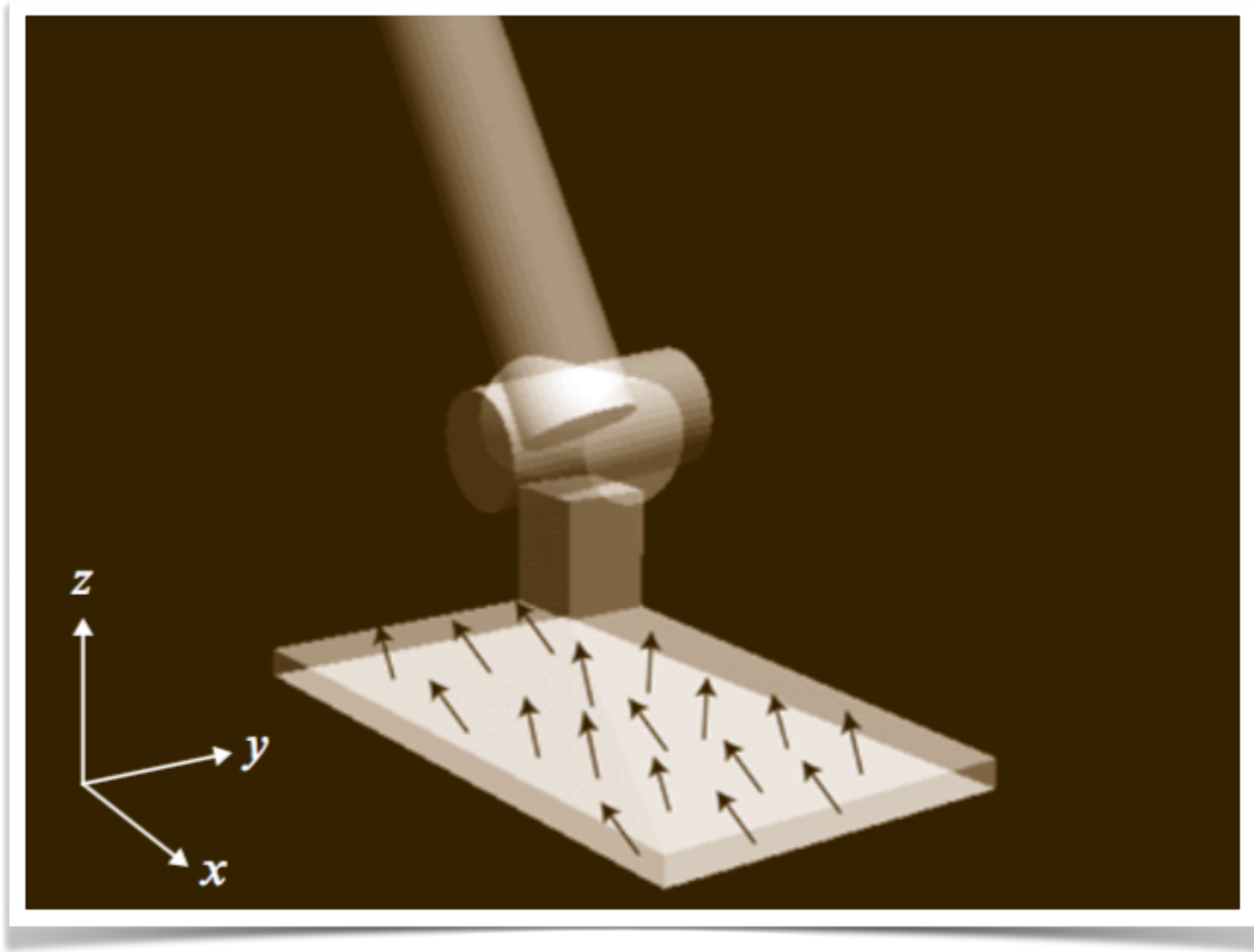


# Centro de Pressão (CoP)

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**N** Pontos de contato:

$$p_i (i = 1, \dots, N)$$

Forças de reação

$$f_i := [f_{ix} \ f_{iy} \ f_{iz}]^T$$

CoP:

$$p := \frac{\sum_{i=1}^N p_i f_{iz}}{\sum_{i=1}^N f_{iz}}$$

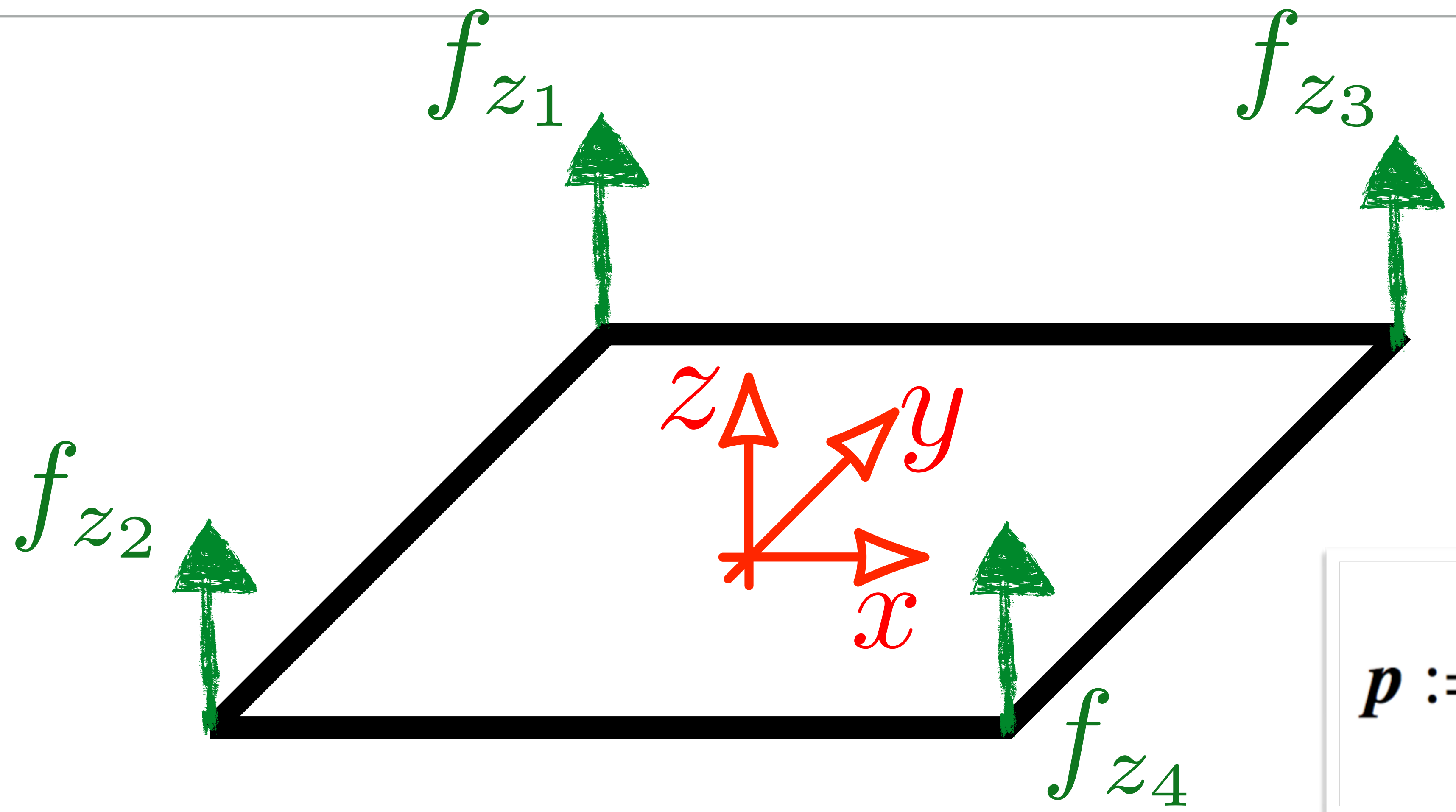


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$$p := \frac{\sum_{i=1}^N p_i f_{iz}}{\sum_{i=1}^N f_{iz}}$$

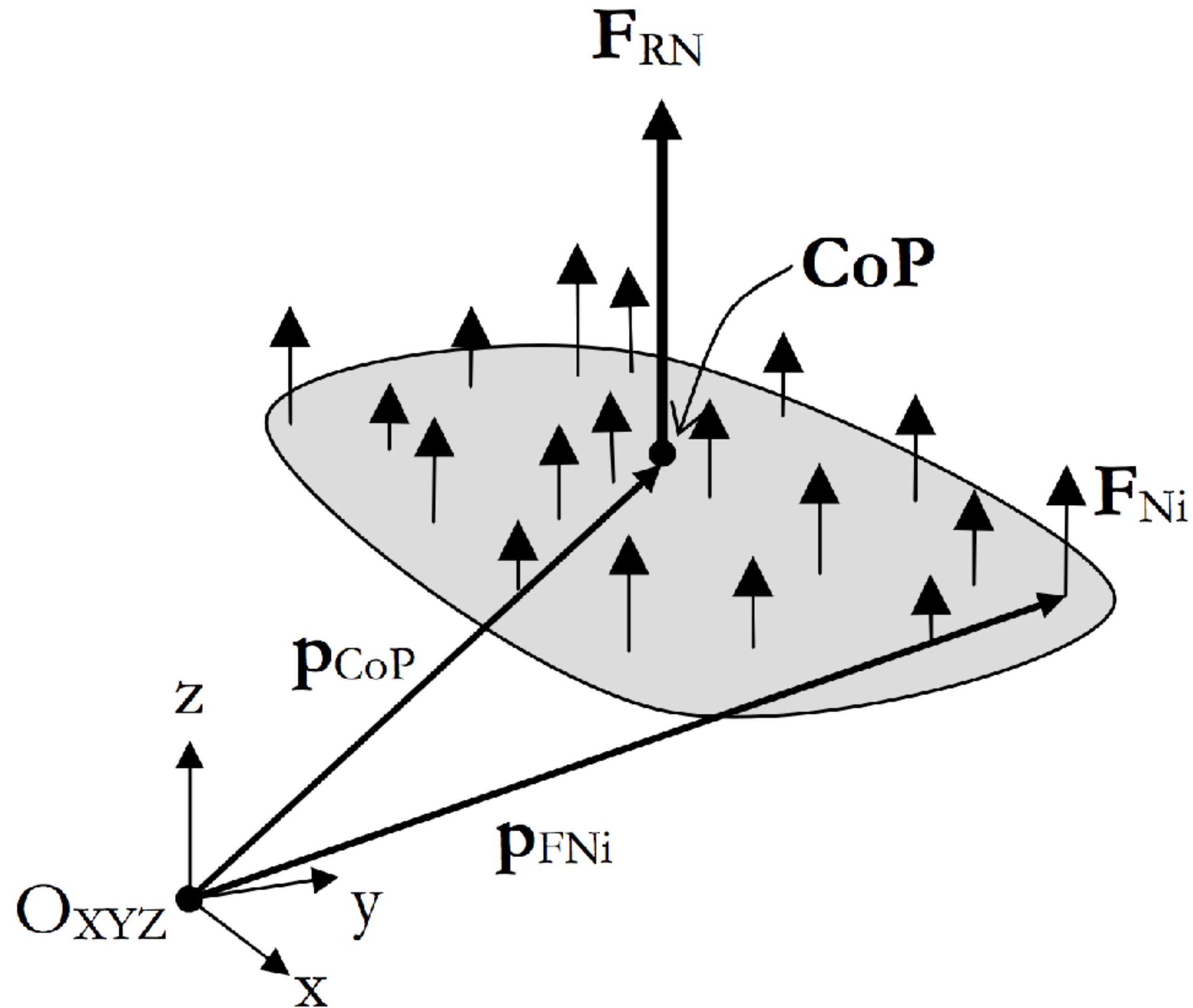


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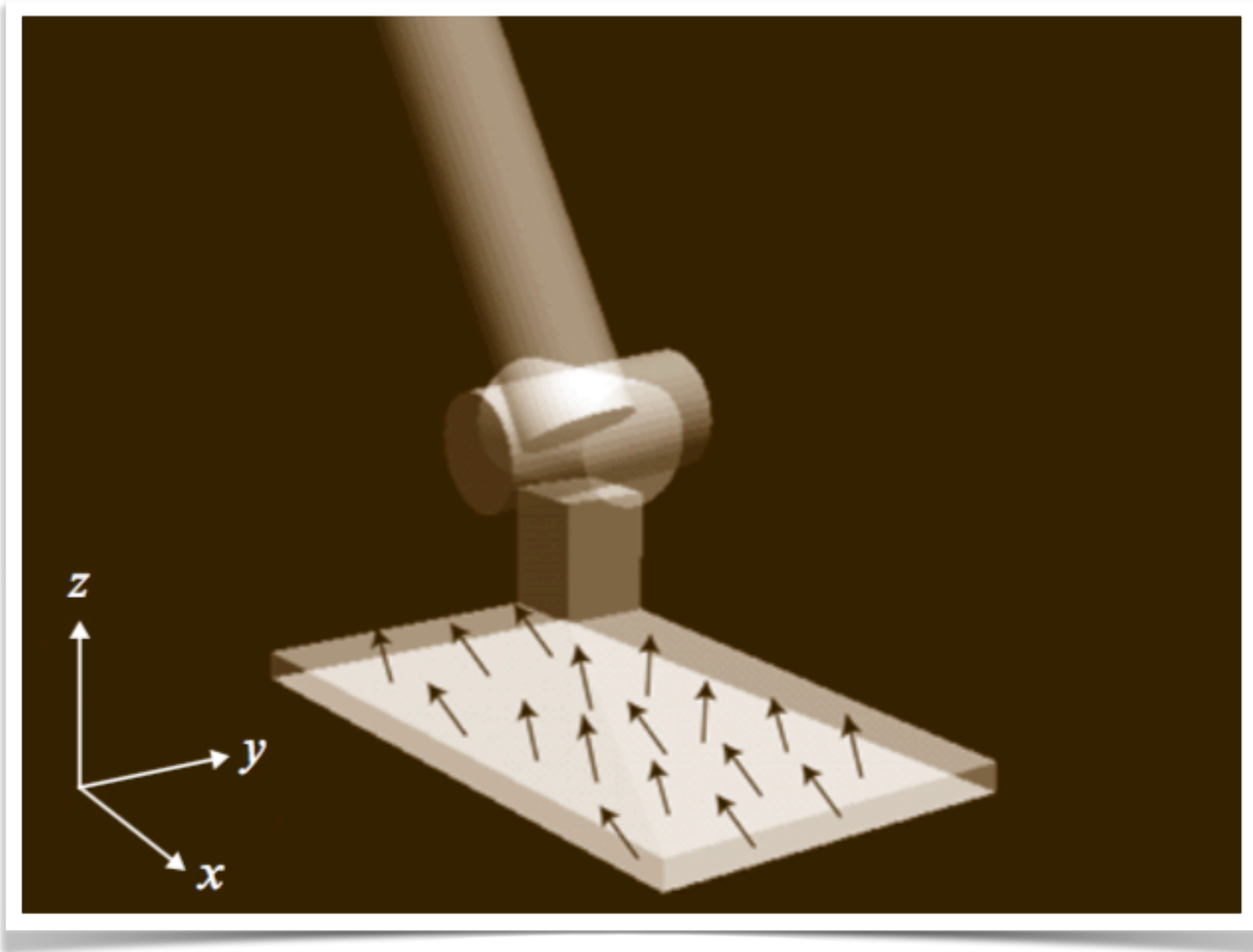


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Também pode ser definido por:

$$p = \sum_{i=1}^N \alpha_i p_i ,$$

$$\alpha_i := f_{iz} / f_z ,$$

$$f_z := \sum_{i=1}^N f_{iz} .$$



# CoP: Center of Pressure

Robô pode somente empurrar  
contra o chão:

$$f_{iz} \geq 0 \quad (i = 1, \dots, N)$$

Então:

$$\begin{cases} \alpha_i \geq 0 & (i = 1, \dots, N) \\ \sum_{i=1}^N \alpha_i = 1. \end{cases}$$

Logo...

**O CoP nunca deixa o  
polígono de suporte!**

$$\begin{aligned} \mathbf{p} &= \sum_{i=1}^N \alpha_i \mathbf{p}_i, \\ \alpha_i &:= f_{iz} / f_z, \\ f_z &:= \sum_{i=1}^N f_{iz}. \end{aligned}$$

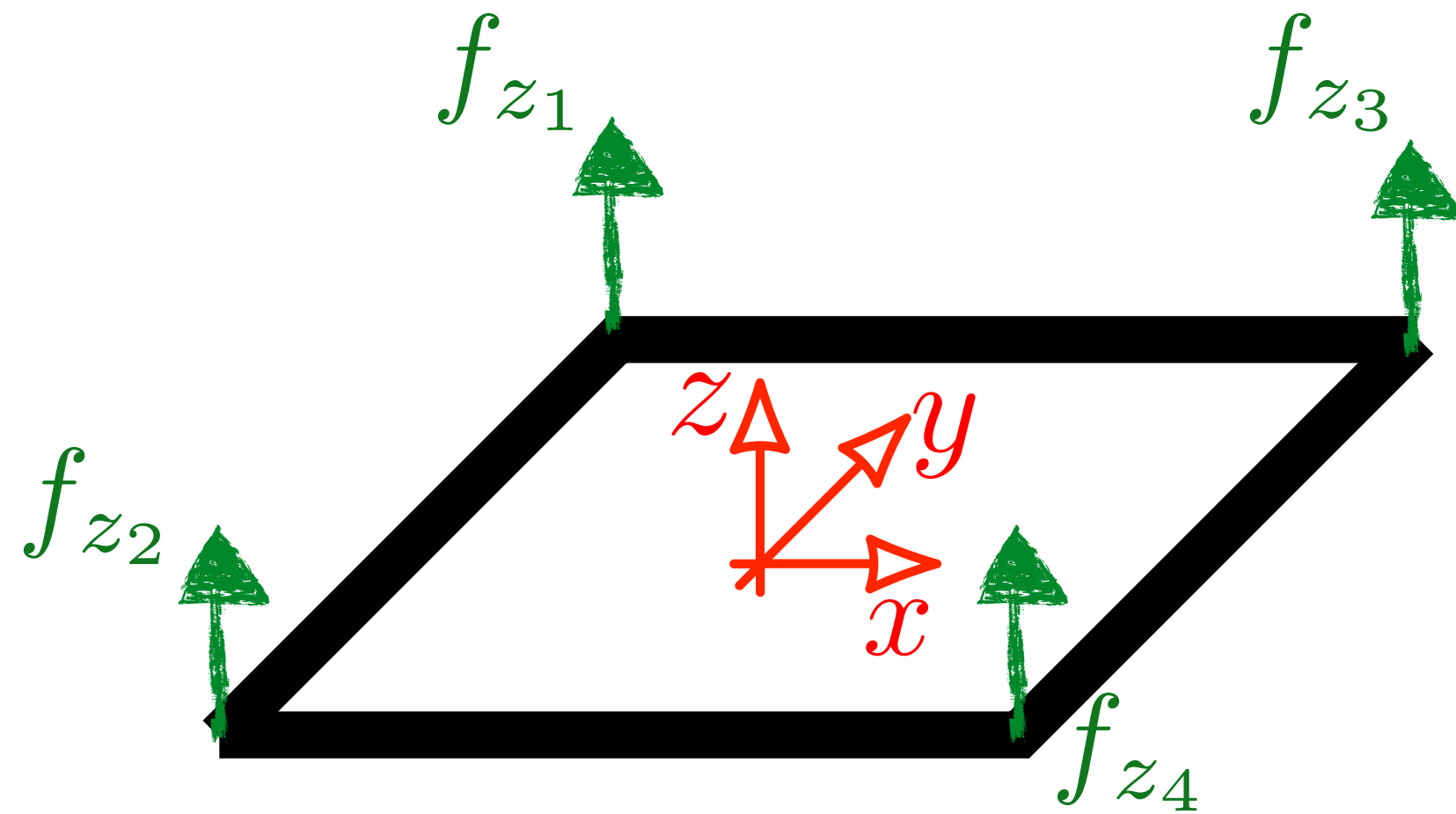


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Superfície horizontal:

$$p_{iz} = p_z$$

$$\mathbf{p} := \frac{\sum_{i=1}^N \mathbf{p}_i f_{iz}}{\sum_{i=1}^N f_{iz}}$$

$$\boldsymbol{\tau}_x = \boldsymbol{\tau}_y = 0$$

$$\boldsymbol{\tau} = \sum_{i=1}^N (\mathbf{p}_i - \mathbf{p}) \times \mathbf{f}_i$$

$$\tau_x = \sum_{i=1}^N (\cancel{p_{iy}} - p_y) f_{iz} - \sum_{i=1}^N (\cancel{p_{iz}} - p_z) f_{iy}$$

$$\tau_y = \sum_{i=1}^N (\cancel{p_{iz}} - p_z) f_{ix} - \sum_{i=1}^N (\cancel{p_{ix}} - p_x) f_{iz}$$

$$\tau_z = \sum_{i=1}^N (p_{ix} - p_x) f_{iy} - \sum_{i=1}^N (p_{iy} - p_y) f_{ix}$$

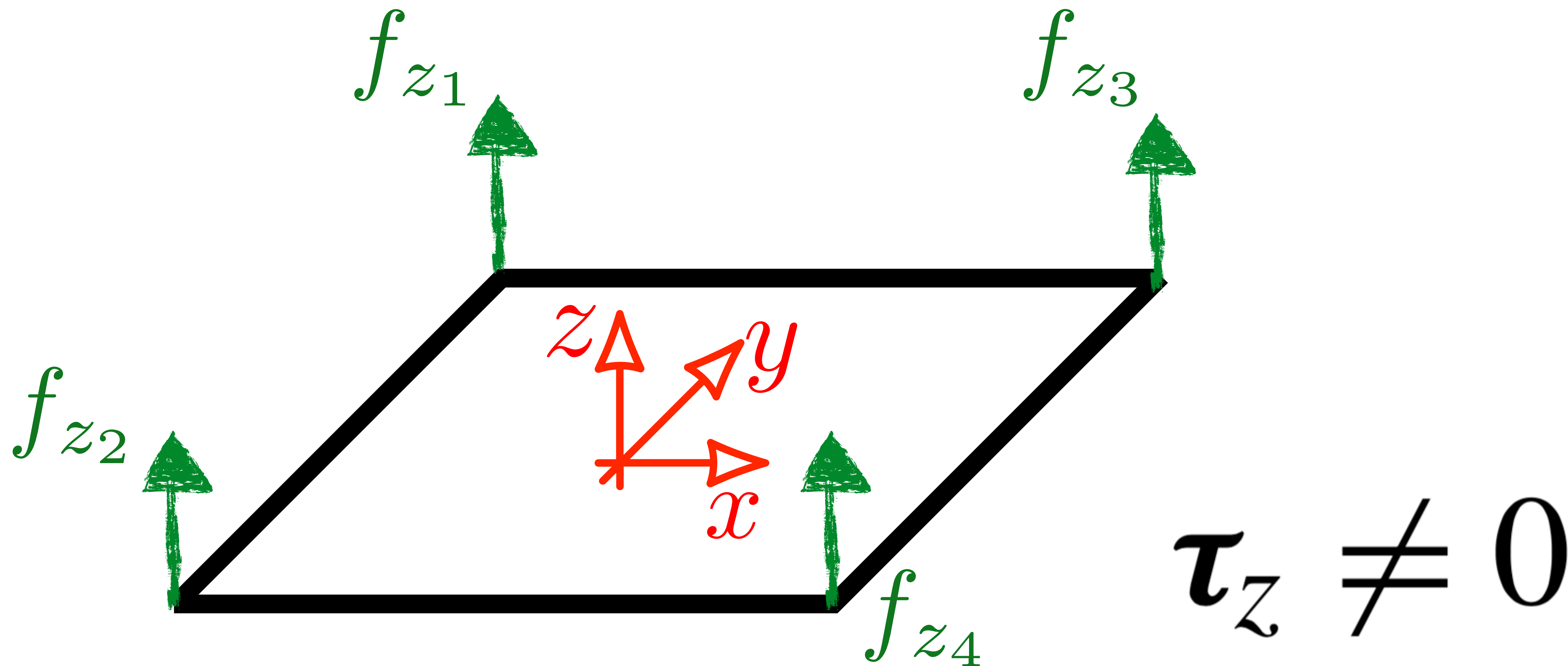


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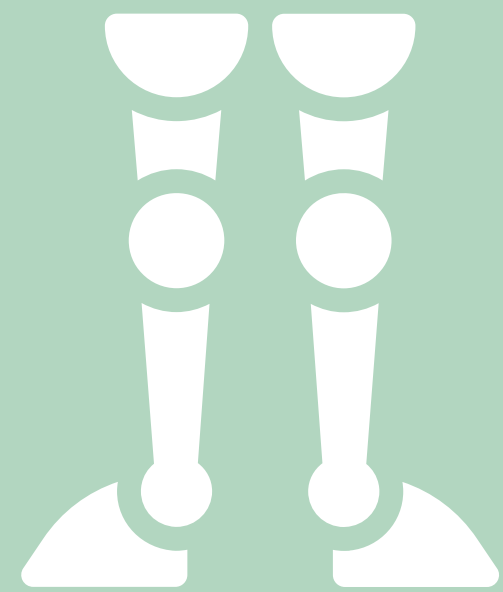


equivalente ao CoP, mas nem sempre



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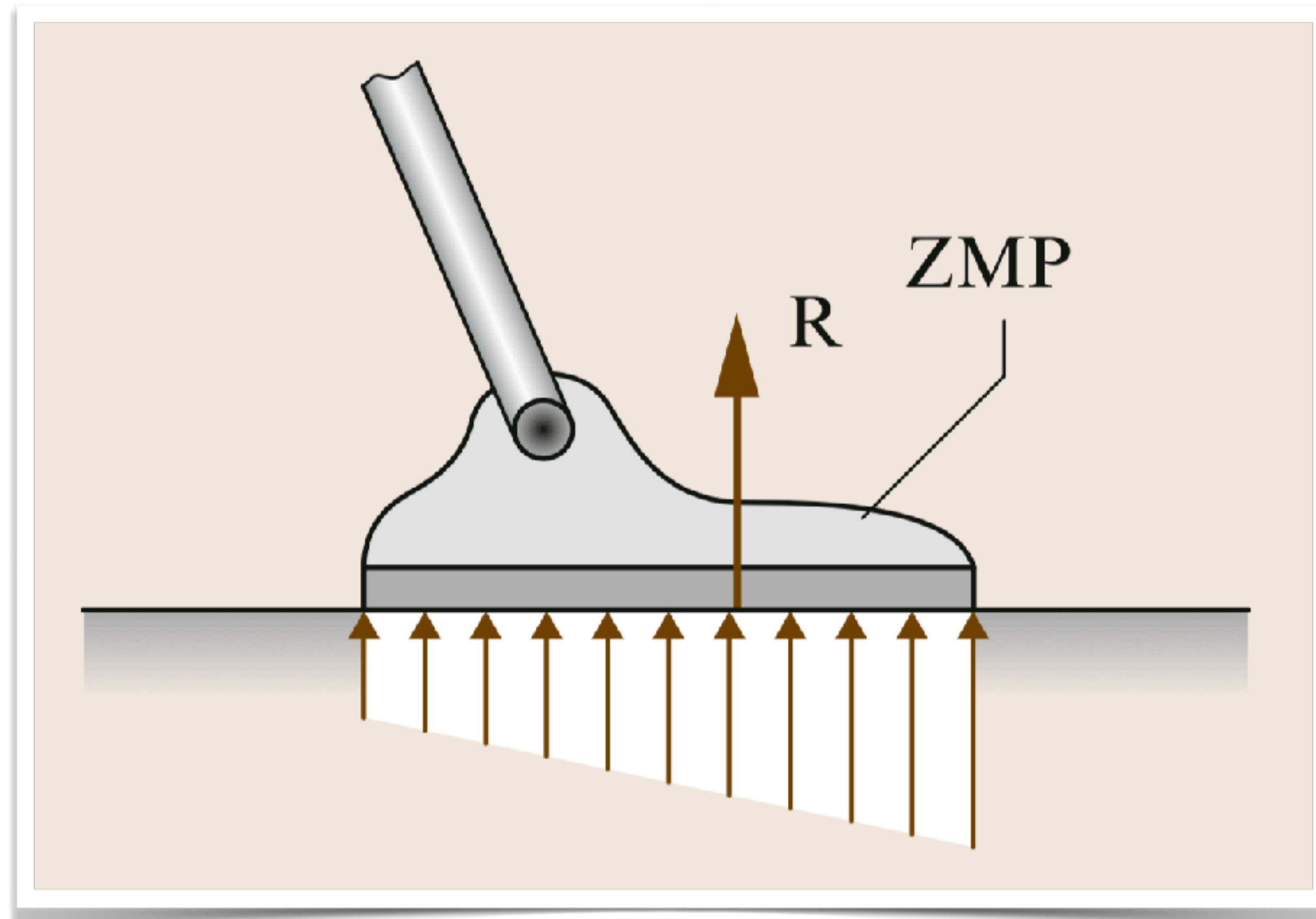
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# ZMP: Zero moment point

(praticamente) **Equivalente ao CoP**



Nomenclatura não é muito precisa:

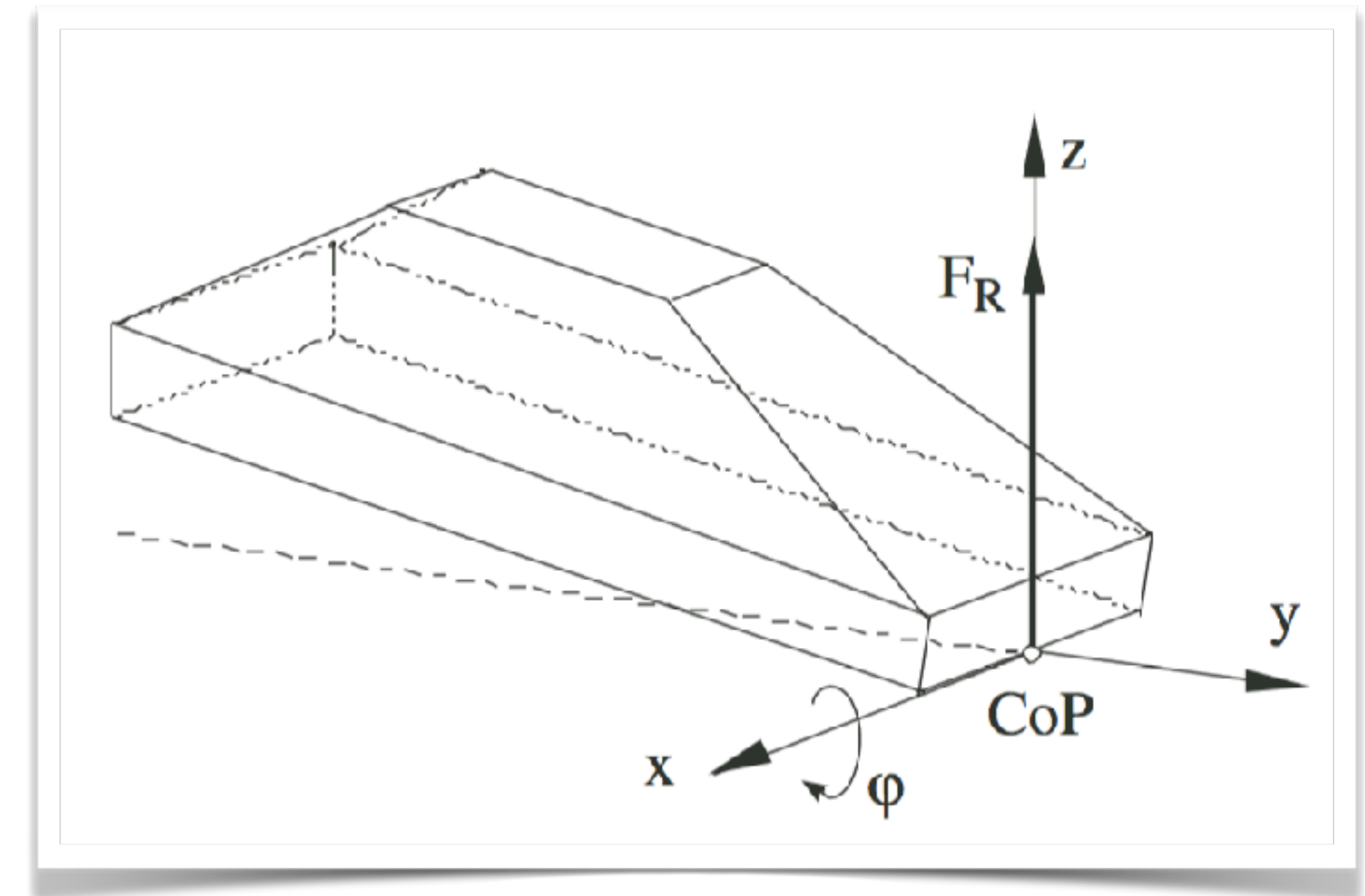
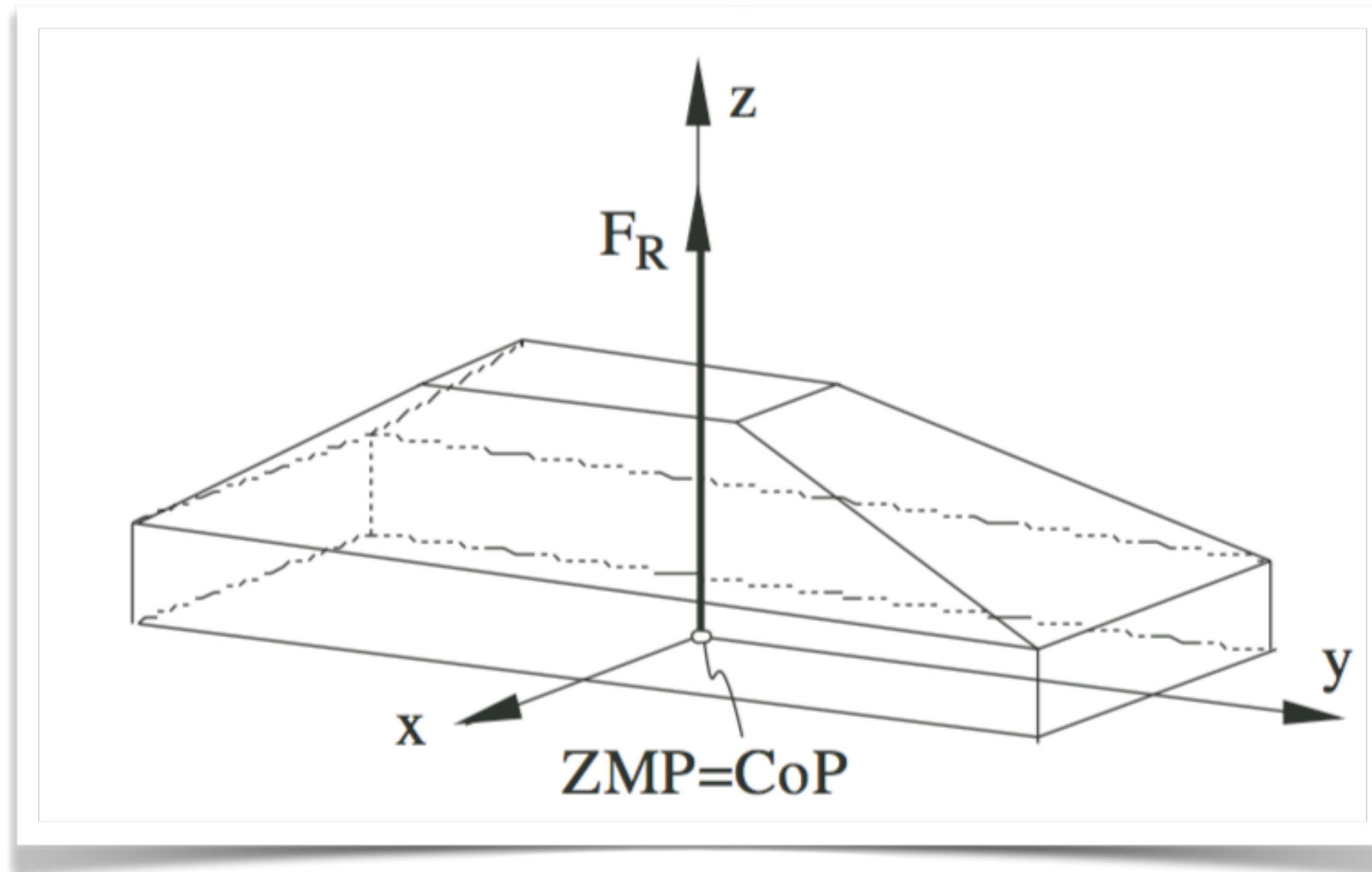
$$\tau_x = \tau_y = 0$$

$$\tau_z \neq 0$$



# ZMP: Zero moment point

(praticamente) **Equivalente ao CoP**



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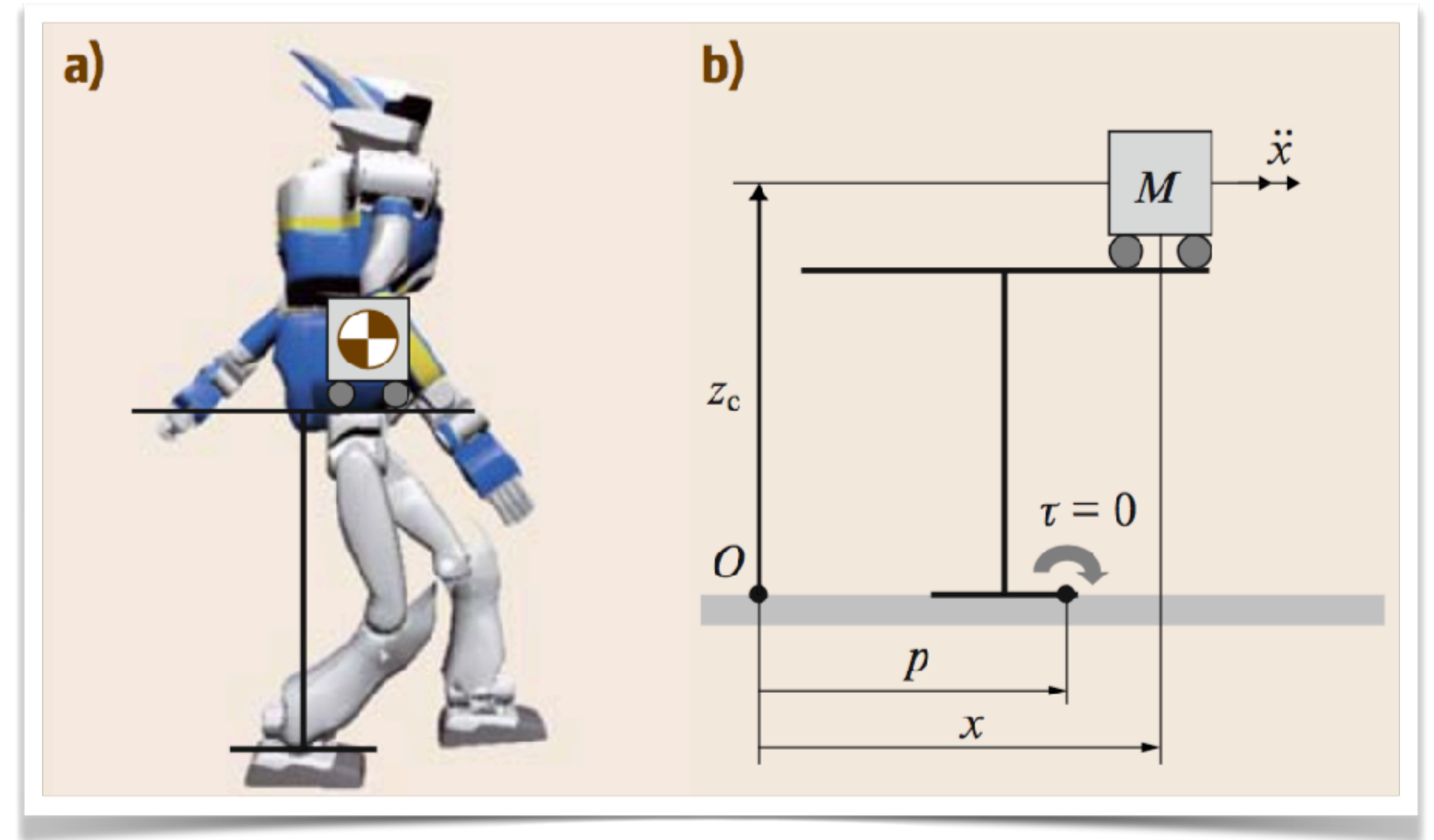
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# Computed ZMP

Leva em  
consideração  
o movimento  
do robô





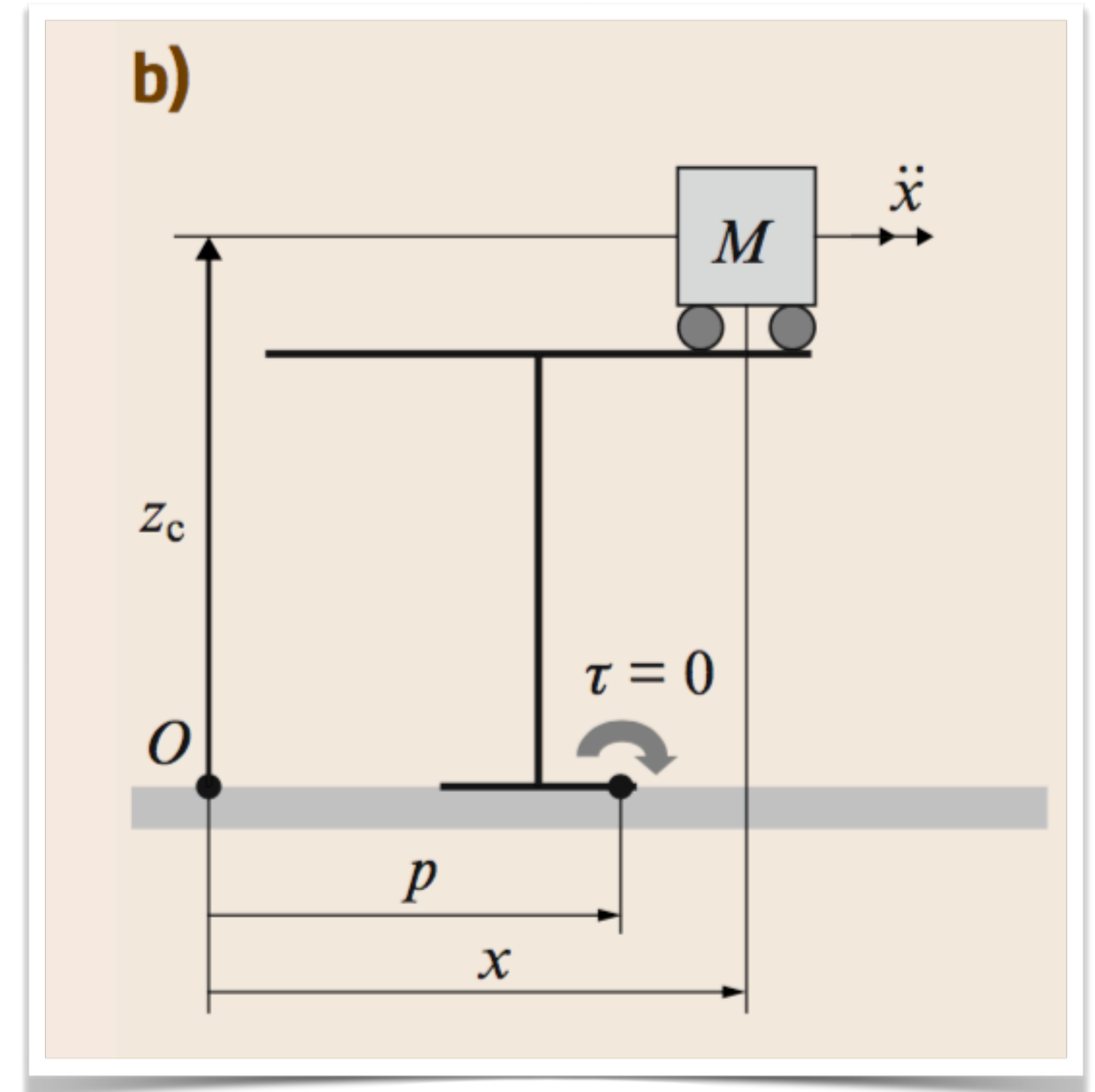
# Computed ZMP

$$\tau = -Mg(x - p) + M\ddot{x}z_c$$

$$\text{ZMP: } \tau = 0$$

$$p = x - \frac{z_c}{g}\ddot{x}$$

$$p := \frac{\sum_{i=1}^N p_i f_{iz}}{\sum_{i=1}^N f_{iz}}$$

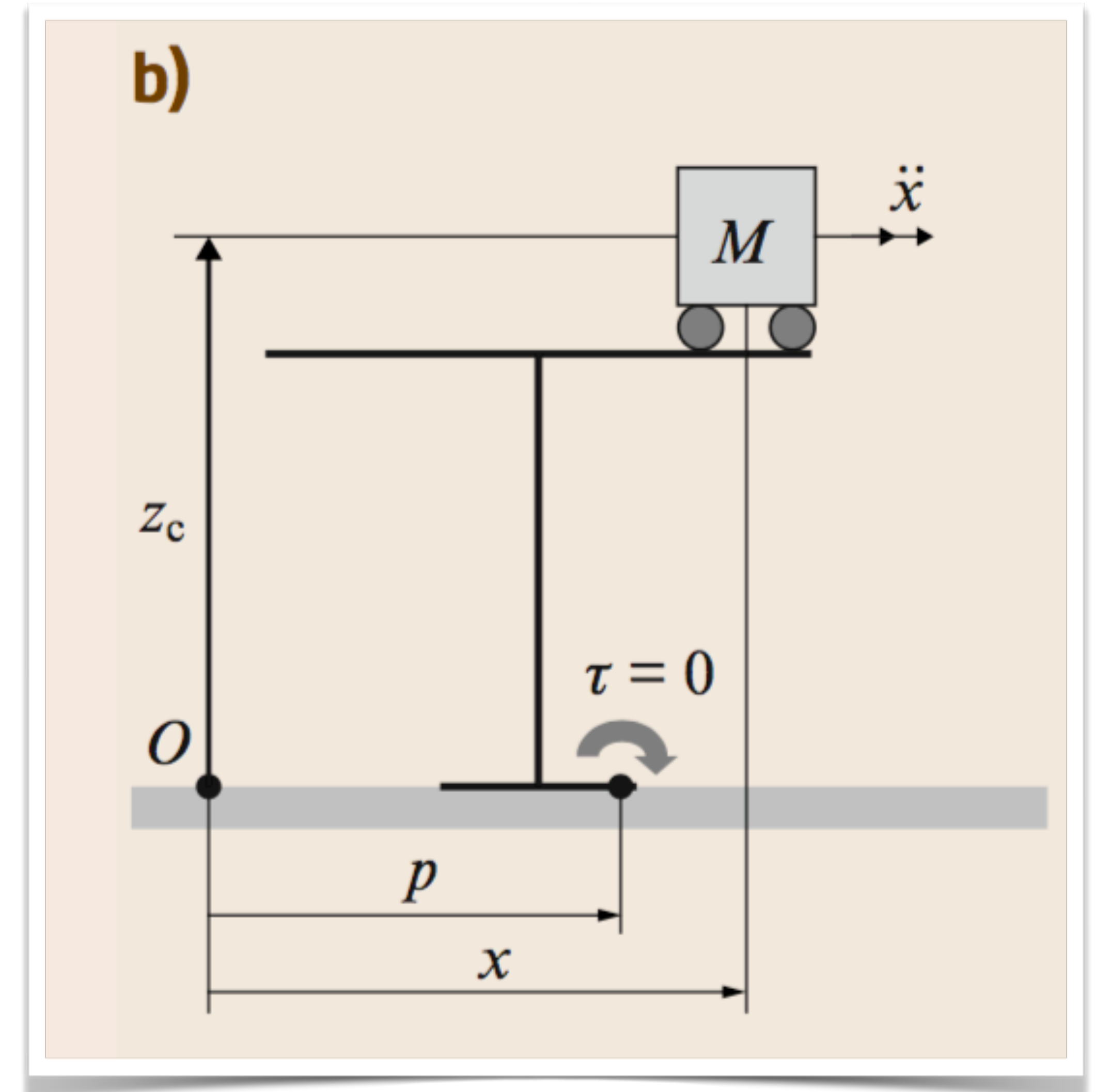




# Computed ZMP

$$p = x - \frac{z_c}{g} \ddot{x}$$

Para acelerações  
nulas, o **ZMP**  
coincide com a  
projecção do **CoM**



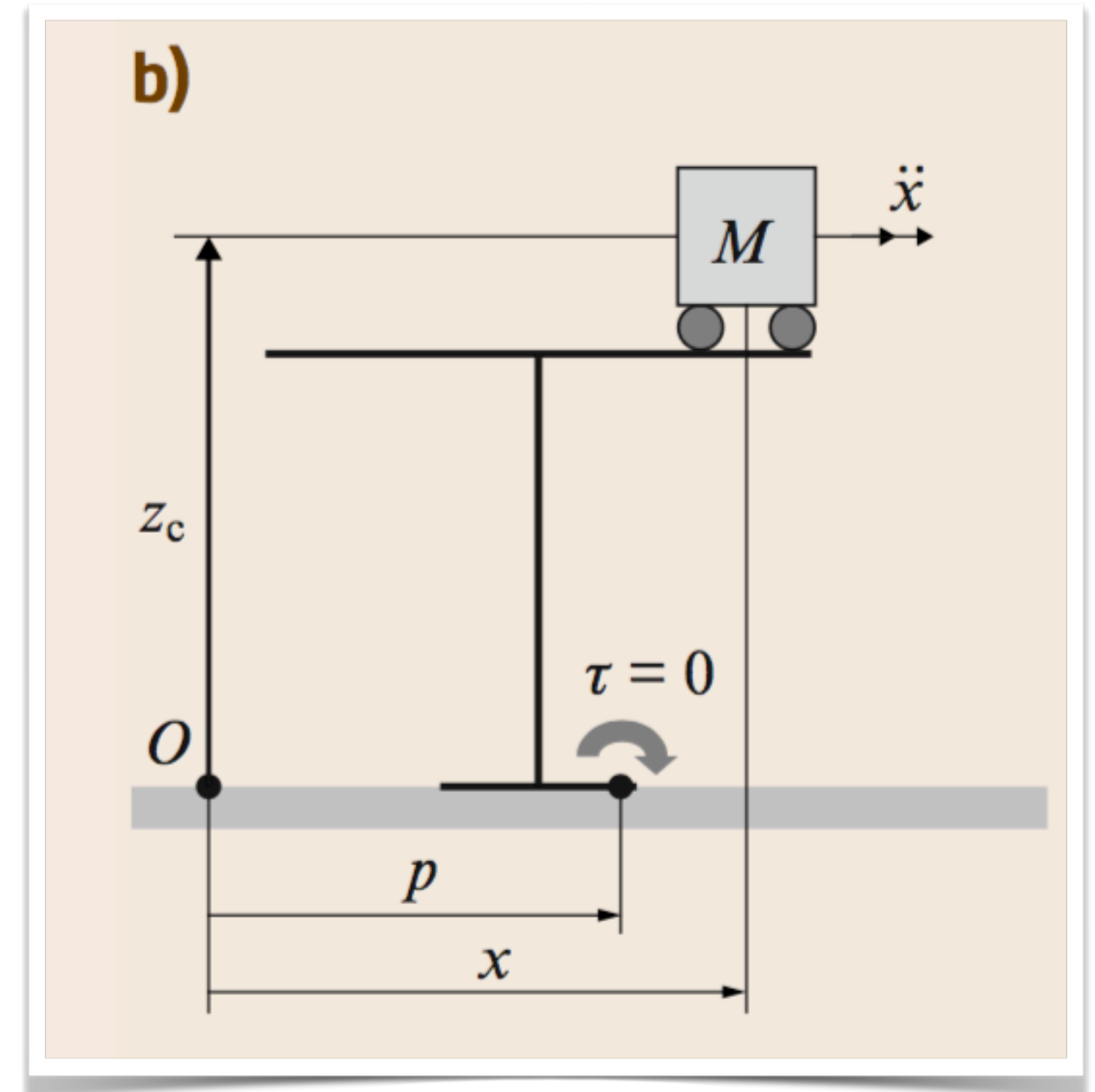


# Computed ZMP

$$p = x - \frac{z_c}{g} \ddot{x}$$

**Não é limitado pela base de suporte**

Caso o ZMP saia da área de suporte, ele é chamado '**Fictitious ZMP**' (FZMP) ou '**Foot Rotation Indicator**' (FRI)



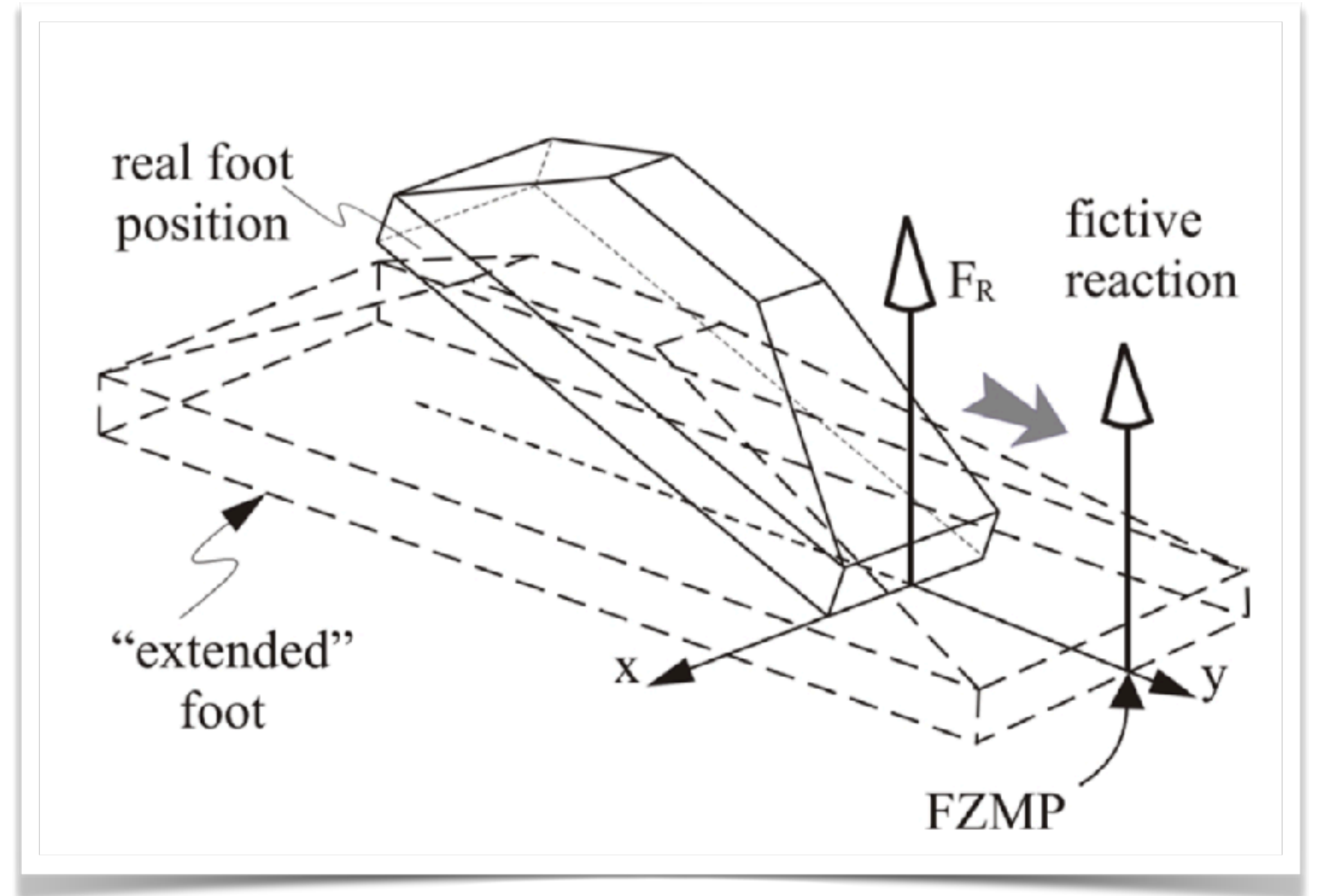
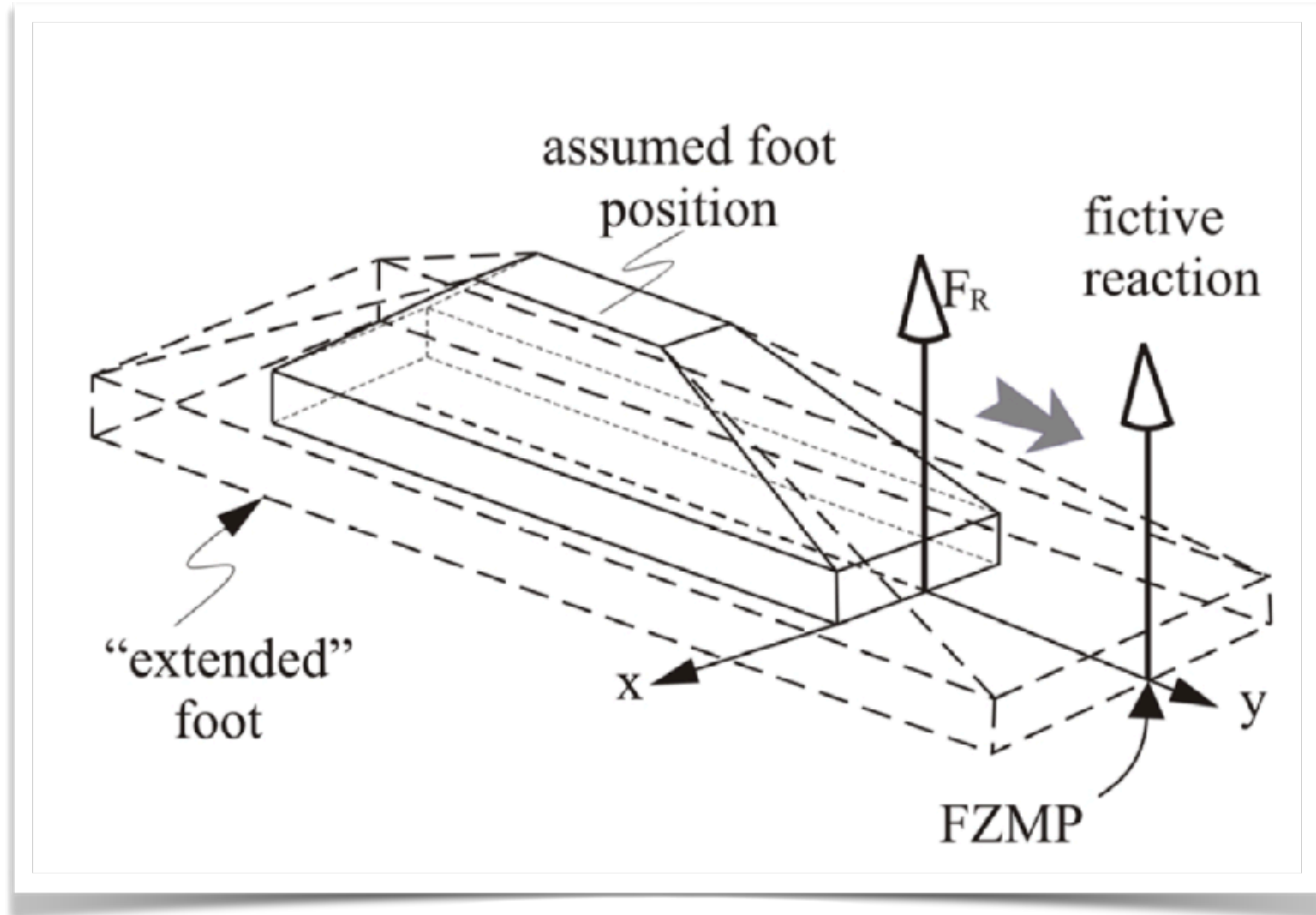




Boston Dynamics

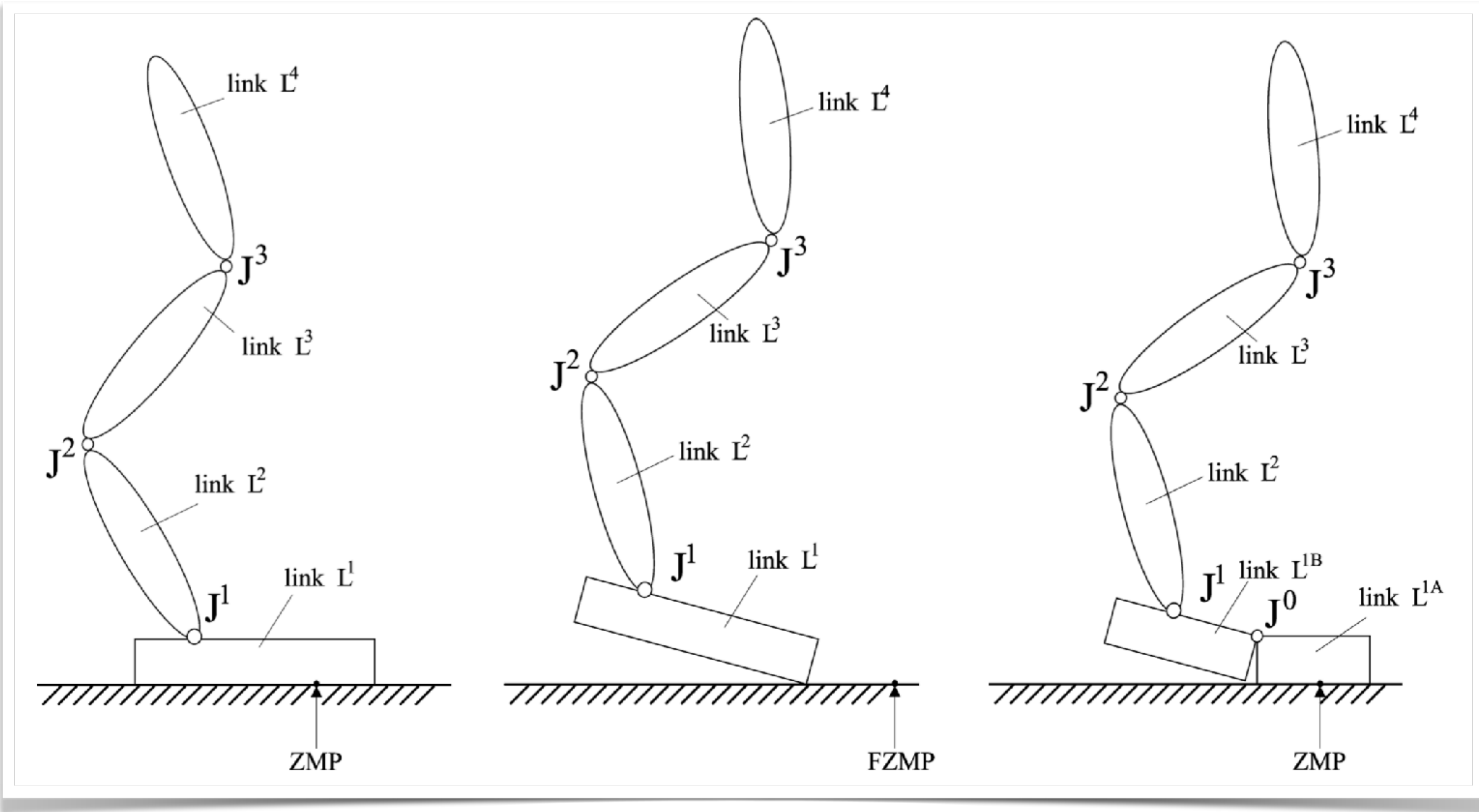


# FZMP



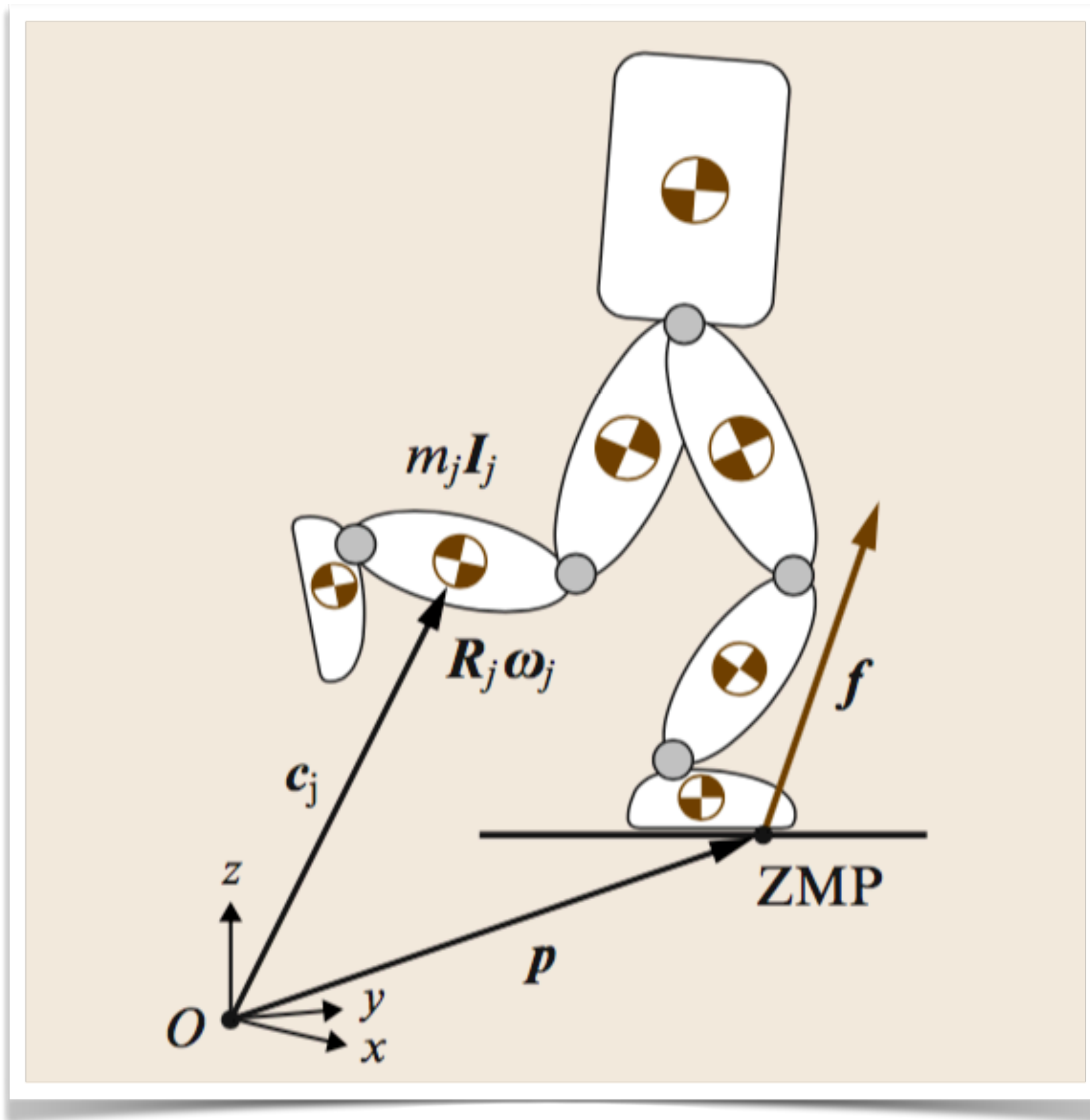


# FZMP





# ZMP em 3D



**Momento linear:**

$$\mathcal{P} = \sum_{j=1}^N m_j \dot{c}_j$$

**Momento angular:**

$$\mathcal{L} = \sum_{j=1}^N [c_j \times (m_j \dot{c}_j) + R_j I_j R_j^T \omega_j]$$

**ZMP:**

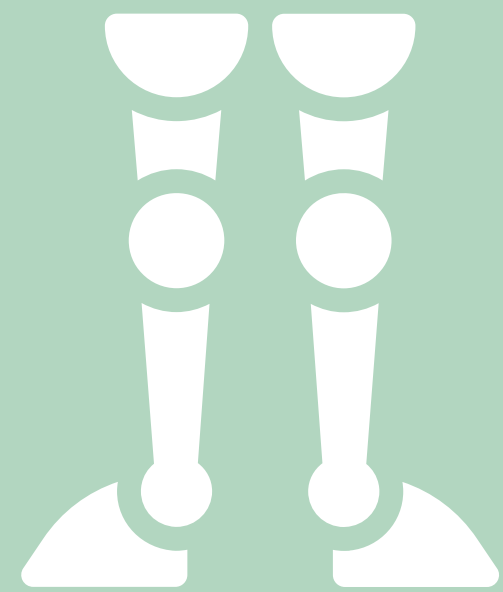
$$p_x = \frac{Mgx + p_z \dot{\mathcal{P}}_x - \dot{\mathcal{L}}_y}{Mg + \dot{\mathcal{P}}_z},$$

$$p_y = \frac{Mgy + p_z \dot{\mathcal{P}}_y + \dot{\mathcal{L}}_x}{Mg + \dot{\mathcal{P}}_z},$$



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# Capture point



Ponto no chão onde o robô deve posicionar o **ZMP** para parar o movimento de seu **CoM**

Também chamado de **Extrapolated Center of Mass (XCoM)**

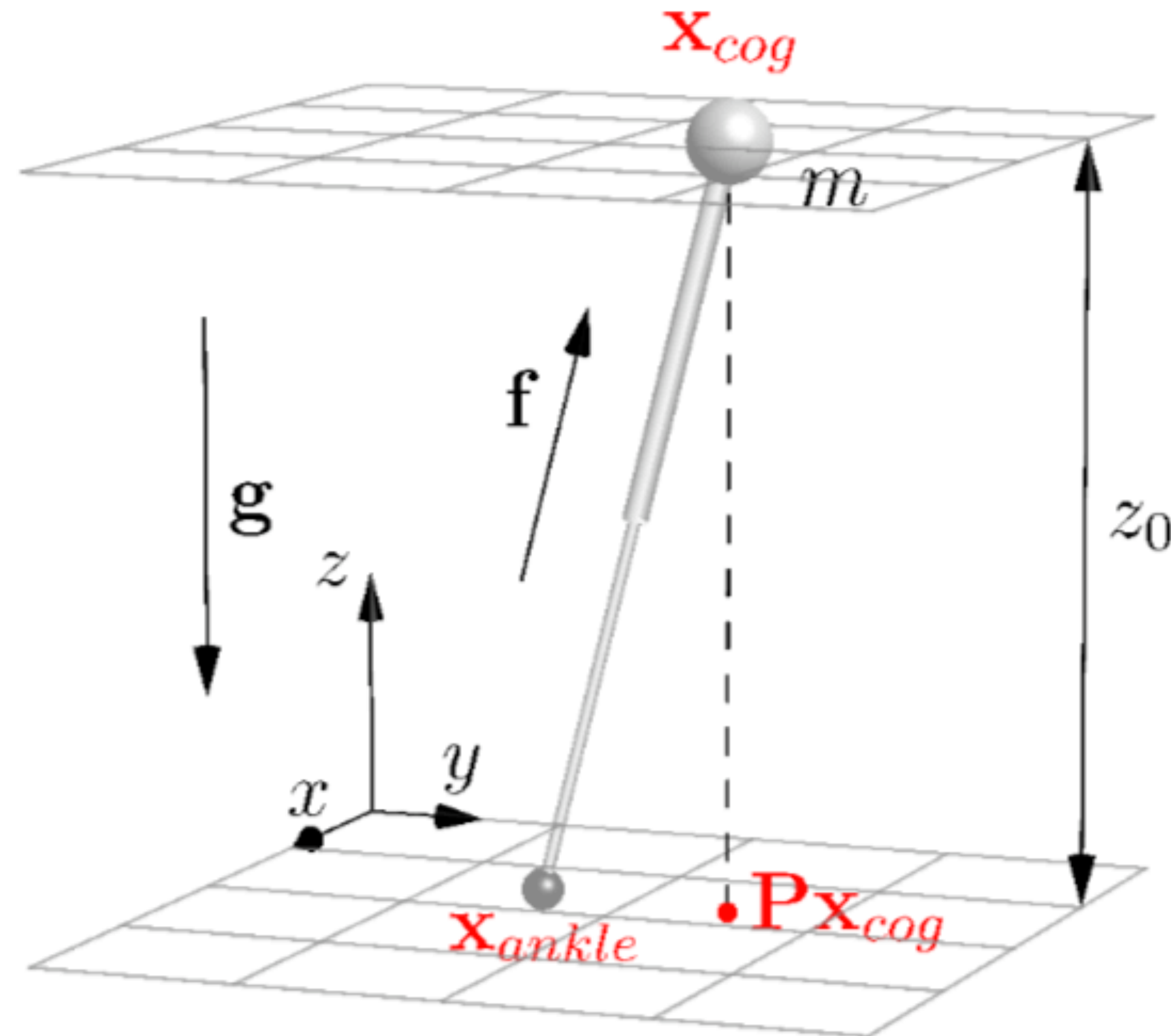
[ Hof, A. L. (2008). The 'extrapolated center of mass' concept suggests a simple control of balance in walking. Human movement science, 27(1), 112-125. ]



# Capture point

## Baseado no modelo do pêndulo invertido linear

[ Pratt, J., et al. (2006). Capture point: A step toward humanoid push recovery ]





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# Referência bibliográfica



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*That's all Folks!*