Fórmula de Euler

- Expansão em série de Taylor ao redor de zero (MacLaurin)

$$f(x) = \sum_{n=0}^{\infty} x^n \frac{f^{(n)}(0)}{n!},$$

onde $f^{(n)}(x)$ é a derivada de ordem n da função f em x

- Expansão da função $f(x) = e^x$

$$\frac{d(e^x)}{dx} = e^x$$

Para essa função, $f^{(1)}(x) = f^{(2)}(x) = \ldots = f^{(n)}(x) = e^x$

$$e^x = \frac{x^0 e^0}{0!} + \frac{x^1 e^0}{1!} + \frac{x^2 e^0}{2!} + \frac{x^3 e^0}{3!} + \frac{x^4 e^0}{4!} + \frac{x^5 e^0}{5!} + \frac{x^6 e^0}{6!} + \frac{x^7 e^0}{7!} + \ldots =$$

$$= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \frac{x^7}{7!} + \ldots$$

$$e^\theta = \frac{\theta^0 e^0}{0!} + \frac{\theta^1 e^0}{1!} + \frac{\theta^2 e^0}{2!} +$$

$$\frac{\theta^3 e^0}{3!} + \frac{\theta^4 e^0}{4!} + \frac{\theta^5 e^0}{5!} + \frac{\theta^6 e^0}{6!} + \frac{\theta^7 e^0}{7!} + \ldots =$$

$$= 1 + \theta + \frac{\theta^2}{2!} + \frac{\theta^3}{3!} + \frac{\theta^4}{4!} + \frac{\theta^5}{5!} + \frac{\theta^6}{6!} + \frac{\theta^7}{7!} + \ldots$$

- Expansão da função $f(\theta) = \cos\theta$

$$\frac{d(\cos\theta)}{d\theta} = -\text{sen}\,\theta\,, \quad \frac{d(\text{sen}\,\theta)}{d\theta} = \cos\theta$$

Desta forma,

$$\cos\theta = \frac{\theta^0 \cos 0}{0!} + \frac{\theta^1(-\text{sen}\,0)}{1!} + \frac{\theta^2(-\cos 0)}{2!} + \frac{\theta^3(\text{sen}\,0)}{3!} + \frac{\theta^4 \cos 0}{4!} + \frac{\theta^5(-\text{sen}\,0)}{5!} + \frac{\theta^6(-\cos 0)}{6!} + \frac{\theta^7(\text{sen}\,0)}{7!} + \ldots =$$

$$= 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \ldots$$

- Expansão da função $f(\theta) = \text{sen}\,\theta$

$$\text{sen}\,\theta = \frac{\theta^0\,\text{sen}\,0}{0!} + \frac{\theta^1(\cos 0)}{1!} + \frac{\theta^2(-\text{sen}\,0)}{2!} + \frac{\theta^3(-\cos 0)}{3!} +$$

$$\frac{\theta^4\,\text{sen}\,0}{4!} + \frac{\theta^5(-\cos 0)}{5!} + \frac{\theta^6(\text{sen}\,0)}{6!} + \frac{\theta^7(-\cos 0)}{7!} + \ldots =$$

$$= \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \ldots$$

- Voltando à função exponencial, com expoente imaginário: $f(\theta) = e^{j\theta}$

$$\frac{d(e^{j\theta})}{d\theta} = j\,e^{j\theta}$$

Como $j = \sqrt{-1}\,,\ j^2 = -1\,,\ j^3 = -j\,,\ j^4 = 1$,

$$e^{j\theta} = \frac{\theta^0 e^0}{0!} + \frac{\theta^1 j e^0}{1!} + \frac{\theta^2 j^2 e^0}{2!} + \frac{\theta^3 j^3 e^0}{3!} + \frac{\theta^4 j^4 e^0}{4!} + \frac{\theta^5 j^5 e^0}{5!} + \frac{\theta^6 j^6 e^0}{6!} + \frac{\theta^7 j^7 e^0}{7!} + \ldots =$$

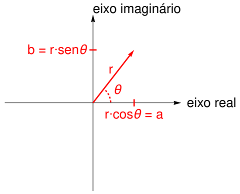
$$= 1 + j\theta - \frac{\theta^2}{2!} - j\frac{\theta^3}{3!} + \frac{\theta^4}{4!} + j\frac{\theta^5}{5!} - \frac{\theta^6}{6!} - j\frac{\theta^7}{7!} + \ldots =$$

$$\left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \ldots\right) + j\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \ldots\right) =$$

$$\boxed{e^{j\theta} = \cos\theta + j\,\text{sen}\,\theta}$$

- Representando o número complexo $\bar{z} = r \cdot e^{j\theta}$ nos eixos real e imaginário:

Out[44]=



Representação polar: $\bar{z} = r\,e^{j\theta}$ ou $\bar{z} = r\angle\theta$

Representação retangular: $\bar{z} = a + jb$

r : módulo ou valor absoluto de $\bar{z}$;
θ : ângulo, fase ou argumento de $\bar{z}$;
a : parte real de $\bar{z}$;
b: parte imaginária de $\bar{z}$.

- Conversão entre a representação retangular e polar

$$\boxed{\begin{array}{l} r = \sqrt{a^2 + b^2} \\ \text{tg}\,\theta = \frac{b}{a} \end{array}} \iff \boxed{\begin{array}{l} a = r \cdot \cos\theta \\ b = r \cdot \text{sen}\,\theta \end{array}}$$