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C. Pictures, slides, film

6.11. **A Meeting with the Universe NASA EP-177**, edited by B. M. French and S. P. Maran (NASA, Washington, DC, 1981). Some nice x-ray related illustrations placed in the broader astronomical context are included in this large-format book. (E)

6.12. **Vision of "Einstein,"** slide collection, Photographic Services, Smithsonian Institution, Washington, DC 20560 (1981). Here are 57 slides covering all x-ray astronomy and including many of the exciting false-color images from *Einstein*. A booklet gives a fairly detailed explanation of each slide. (E)

6.13. **"Cosmic Fire,"** distributed by King Features (235 E. 45 St., New York, NY 10017). This WGBH "Nova" film is a rework of an earlier BBC production. It touches on many of the major problems in x-ray astronomy. Some of the active practitioners in the field explain what they do, how they do it, and why. (E)

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Relativity without light

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The relativistic addition law for parallel velocities is derived directly from the principle of relativity and a few simple assumptions of smoothness and symmetry, without making use of the principle of the constancy of the velocity of light.

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I. INTRODUCTION

The first statement of the special theory of relativity[1] was intimately tied to the first unambiguous assertion that the velocity of light in empty space has the value c, independent of frame of reference. Reflecting this historical circumstance, light has almost always had a central role in subsequent expositions of relativity, playing, for example, an important part in establishing a convention for the synchronization of distant clocks, or determining the rate of moving clocks or the length of moving rigid rods.

Relativity, however, is not a branch of electromagnetism and the subject can be developed without any reference whatever to light. Since this is not the conventional approach, I should emphasize that in asserting that relativity can be developed without light, I do not have in mind the trivial sense in which anything else that moves at the invariant velocity c can serve as well. Nor do I mean merely that the theory can be built out of the fact that the relation of lightlike separation between a pair of events is invariant under change of frame, regardless of whether or not there exists any form of matter or energy that actually can propagate at the invariant velocity.

What I do mean, and shall show below, is that a parallel velocity addition law of the form

$$w = (u + v)/(1 + Kuv), \tag{1.1}$$

with K a universal non-negative constant, is the most general possible compatible with the principle of relativity, supplemented only by certain natural assumptions of homogeneity, isotropy, and smoothness.[2] It follows, of course, from (1.1) that $K^{-1/2}$ is an invariant velocity. Thus Einstein's second postulate is a consequence of his first,[3] if it is stated generally in terms of an invariant velocity rather than specifically in terms of the behavior of light.[4]

From this point of view, experiments establishing the constancy of the velocity of light are only significant because they determine the numerical value of the parameter K. Because that value turned out to be the inverse square of the speed of light in empty space, rather than the expected Galilean value $K = 0$, the impact of such experiments was, of course, revolutionary, and light became an essential part of the new theory of space and time. However, the value of K can be determined from careful measurements of the speed of any moving object from two inertial frames in relative motion.[5] Measurements on light offer an especially elegant and precise route to determining the parameter K appearing in the addition law (1.1), but the law itself, as I shall show, follows from the principle of relativity and the fundamental relations between distance, time, and velocity, without the need of any additional facts or postulates.

There are pedagogical as well as conceptual advantages to eliminating light from its central role in relativity theory.

By starting only with the principle of relativity and refraining from trying to relate temporal judgments made in different frames of reference, one is led directly to (1.1) which lies, of course, at the heart of special relativity, without ever having to face the distracting sense of paradox that bedevils more conventional attempts from the very first steps. The price is a somewhat higher level of analysis: a little elementary calculus is unavoidable. The approach is therefore unavailable for a general education physics course, but as an introduction to special relativity for physics majors I believe it has much to recommend it.

In what follows I shall use without critical analysis the concept of an inertial frame of reference and, within any given inertial frame, the concepts of distance, time, and velocity. I shall use these last three notions only through the relation that the distance covered by a uniformly moving object in a given time is the product of its velocity with the time, a connection that any more rigorous development must surely preserve. What logical rigor I bring to the argument will be entirely negative: I shall scrupulously avoid making any assumptions of how distances, times, and velocities in one inertial frame are related to those in another. It is possible to be more systematic and more economical in basic concepts, but to do so here would distract from the central point that the existence of an invariant velocity is not required as an independent assumption.

My derivation of the addition law (1.1) from the principle of relativity blends *Gedankenexperiment* and analysis. The *Gedankenexperimente* are of the usual sort, except that no light signals or particles ("photons") moving at special invariant velocities ever appear. The analysis is rather less familiar and more intricate; without the second postulate one has to work harder.

I present in Sec. II that part of the analysis that is of a general character, independent of any *Gedankenexperiment*, reducing the problem of finding the function of two variables specifying the general addition law to that of finding a function of a single variable. The *Gedankenexperimente* are designed to determine that unknown function.

I describe two such *Gedankenexperimente* in Secs. III and IV, each of a somewhat different character and leading by different routes to the same conclusion (1.1). There is, of course, no logical need for two independent arguments. I include the first because it is a direct generalization of an earlier argument[6] that very simply and economically extracts the addition law from first principles *using* the principle of the constancy of the velocity of light. The second argument is, I suspect, a better starting point to give the whole procedure a tighter logical structure, since it deals directly with the relation between periodic processes in different frames, and thus involves in a direct and fundamental way those aspects of phenomena that are encompassed with such deceptive simplicity in the general notion of time.

The arguments in Secs. III and IV are carried to the point where they can be concluded by the single discussion I give in Sec. V.

II. THE GENERAL FORM FOR A VELOCITY ADDITION LAW

We consider various objects and frames of reference that move uniformly along a single common direction. An addition law is a relation

$$w = f(u,v) \tag{2.1}$$

between the velocity w of an object in frame A, its velocity u in frame B, and the velocity v of frame B in frame A.

We can identify the velocity of the uniformly moving object with that of its proper frame C, and regard (2.1) more systematically as a relation between the relative velocities of various frames of reference, writing it in the form

$$v_{CA} = f(v_{CB}, v_{BA}). \tag{2.2}$$

The principle of relativity is, of course, implicit in the assumption that f depends only on the relative velocities v_{CB} and v_{BA}.

In the absence of any distinction between the two directions of motion, three velocities related by (2.2) must continue to be related if all three signs are changed, so f must be an odd function:

$$f(-x,-y) = -f(x,y). \tag{2.3}$$

The same symmetry also requires that

$$v_{XY} = -v_{YX}. \tag{2.4}$$

In view of the last two relations we find, interchanging A and C in (2.2), that

$$\begin{aligned} f(v_{AB}, v_{BC}) = v_{AC} = -v_{CA} &= -f(v_{CB}, v_{BA}) \\ &= f(-v_{CB}, -v_{BA}) = f(v_{BC}, v_{AB}); \end{aligned} \tag{2.5}$$

i.e., f must be symmetric in its arguments:

$$f(x,y) = f(y,x). \tag{2.6}$$

Another important property of f follows from introducing a fourth frame and noting that v_{DA} can be expressed in two different ways:

$$f(v_{DB}, v_{BA}) = v_{DA} = f(v_{DC}, v_{CA}). \tag{2.7}$$

Expanding v_{DB} on the left and v_{CA} on the right,

$$\begin{aligned} v_{DB} &= f(v_{DC}, v_{CB}), \\ v_{CA} &= f(v_{CB}, v_{BA}), \end{aligned} \tag{2.8}$$

we find the general condition that

$$f(f(x,y),z) = f(x, f(y,z)). \tag{2.9}$$

All of the above relations follow from simple considerations of symmetry. I now also assume that the addition law we seek is expressed by a *smooth* function f. But if f is continuous and differentiable, then we can express it in terms of a function of a single variable, as follows.

If we define

$$f_2(x,y) = \frac{\partial f(x,y)}{\partial y}, \tag{2.10}$$

then differentiating (2.9) with respect to z gives

$$f_2(f(x,y),z) = f_2(x, f(y,z)) f_2(y,z). \tag{2.11}$$

Setting z to zero in (2.11) gives

$$f_2(f(x,y),0) = f_2(x,y) f_2(y,0), \tag{2.12}$$

where we use the fact, evident from the original definition of f, that

$$f(y,0) = y. \tag{2.13}$$

Let us now fix x and regard $f(x,y)$ as a function f of the single variable y, depending parametrically on x. Let us regard $f_2(y,0)$ as a second function of y. The structure of (2.12) is then that of the ordinary differential equation

$$f_2(f,0) = \frac{df}{dy} f_2(y,0), \tag{2.14}$$

or

$$dy/f_2(y,0) = df/f_2(f,0). \tag{2.15}$$

Defining a new function $h(z)$ by

$$h(z) = \int dz/f_2(z,0), \tag{2.16}$$

we see that (2.15) requires that

$$h(f) = h(y) + \text{const}, \tag{2.17}$$

where the constant is independent of y but can depend on the parameter x. The symmetry (2.6) now requires that the constant be precisely $h(x)$ (plus a genuine constant that can be absorbed into a redefinition of h).[7] We conclude that there must be a function h of a single variable such that

$$h(f(x,y)) = h(x) + h(y), \tag{2.18}$$

so that the form of the addition law is

$$f(x,y) = h^{-1}(h(x) + h(y)). \tag{2.19}$$

To determine the addition law it therefore suffices to determine the function h. For this purpose it is enough to know the function $f(x,y)$ in the neighborhood of $y = 0$, since (2.16) gives

$$h'(z) = \frac{1}{\partial f(z,y)/\partial y}\bigg|_{y=0}. \tag{2.20}$$

The condition (2.13) is consistent with (2.18) only if

$$h(0) = 0, \tag{2.21}$$

which provides the boundary condition necessary to determine h from (2.20) by integration.

We turn now to some *Gedankenexperimente* which enable us to determine the form of $h(z)$ to within a single universal constant K.

III. PRECISE FORM OF THE ADDITION LAW: A GEDANKEN RACE

We consider a race taking place within a long straight train (Fig. 1). A tortoise and a hare start at the rear of the train toward the front. The hare gets there first, turns immediately around, and, racing back towards the rear, encounters the tortoise still making its way toward the front.[8] Let u be the speed of the tortoise in the train frame and s, the speed (in either direction) of the hare.

The part of the train where the two meet again is behind the front end by some fraction r of the full length of the train. That fraction is a frame-independent invariant, since there can be no disputing where on the train the meeting occurred. (They might, for example, meet in the 73rd car from the front of a train consisting of 100 identical cars, giving the value $r = 0.73$. Only passengers in car 73 would testify to having observed the encounter, and this testimony would be acceptable to observers in any frame of reference, even though they might have quite different ideas about the lengths of the cars.)

Let us now calculate r in a frame (the "v frame") in which the train moves with speed v (Fig. 2) and examine the consequences of the fact that r cannot depend on v. It will suffice to consider v less than u, so we can take the direction of motion of the hare on its return trip to be opposite to that of the train in the v frame. Let w be the v-frame speed of the tortoise, and let s_1 and s_2 be the v-frame speeds of the hare on its way toward and back from the front of the train. These speeds are related to the train-frame speeds by the addition law

$$w = f(u,v), \quad s_1 = f(s,v), \quad s_2 = f(s,-v). \tag{3.1}$$

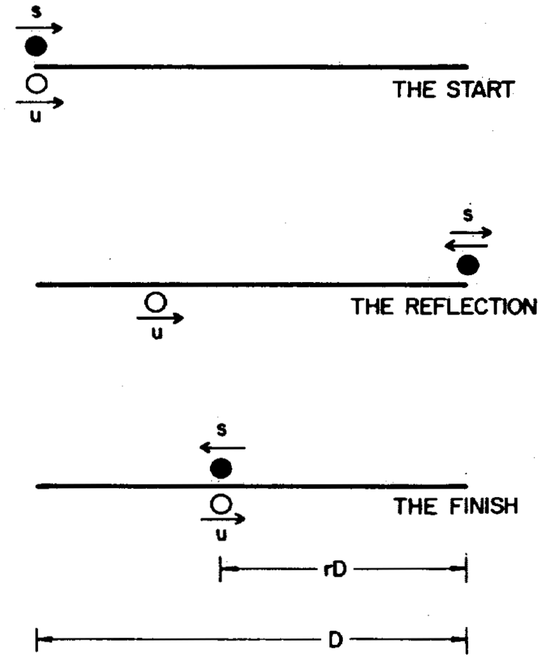

Fig. 1. The race between the hare (black sphere) and tortoise (white sphere) as described in the train frame. The speed of the hare (in either direction) is s, and the speed of the tortoise is u. The distance between the front (right) end of the train and the final meeting place of hare and tortoise is a fraction r of the total length D of the train.

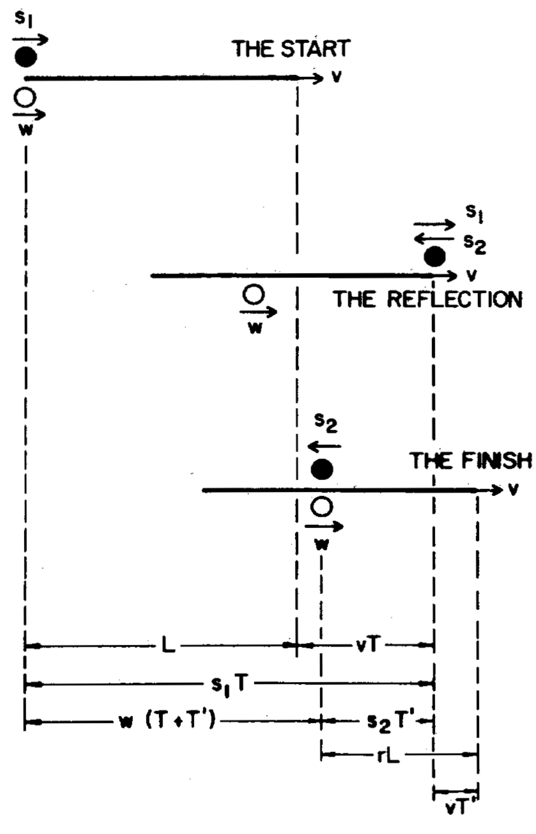

Fig. 2. The race between the hare (black sphere) and tortoise (white sphere) as described in the v frame, in which the train moves to the right with speed v. The speed of the hare is s_1 to the right and s_2 to the left; the speed of the tortoise is w. The length of the train is L. The time between the start and the reflection is T, and the time between the reflection and the finish, T'. Various lengths appearing in Eqs. (3.2)–(3.4) are indicated.

[The last of these follows from noting that the *velocities* of the hare in the two frames are $-s$ and $-s_2$, so we have $-s_2 = f(-s,v) = -f(s,-v)$, in view of the oddness of f.]

Let T be the time (in the v frame) it takes the hare to get from the rear of the train to the front, and T' the time it takes the hare to get from the front back to the tortoise. Let L be the (v-frame) length of the train. We do not know the values of T, T', or L, but it does not matter—they will drop out of the final result. To express the quantity r entirely in terms of the speeds w, s_1, s_2, and v, we need only note the following:

(a) The total distance the tortoise covers from the start of the race to its reencounter with the hare is $w(T+T')$. This, however, is equal to the distance the hare covers in going from the rear to the front, s_1T, minus the distance the hare goes back towards the rear before meeting the tortoise, s_2T':

$$w(T+T') = s_1T - s_2T'. \tag{3.2}$$

(b) The distance the hare moves in going from the rear to the front is just the length of the train augmented by the distance the train moves while the hare is so moving:

$$s_1T = L + vT. \tag{3.3}$$

(c) The distance the hare moves in going from the front back to the tortoise is just the length of the train from the front to the meeting point on the train, rL, diminished by the distance the train moves while the hare is so moving:

$$s_2T' = rL - vT'. \tag{3.4}$$

We can solve (3.3) and (3.4) for T and T', substitute the resulting expressions in (3.2), cancel the factor L common to all terms, and solve the resulting expression for r to find[9]

$$r = (s_1 - w)(s_2 + v)/(s_2 + w)(s_1 - v). \tag{3.5}$$

Pause to note that the derivation of (3.5) is entirely free of relativistic notions, the entire computation having been done in a single frame (the v frame). Relativity has informed the argument only through the absence of any attempts to make naive identifications of the v-frame speeds appearing in (3.5) with the train frame speeds s and u, and through the care taken to eliminate from the final expression any v-frame lengths or times.

To find the form of the addition law we must determine the function h appearing in (2.19). A comparison of (3.1) with the expression (2.20) for h shows that

$$\left.\frac{\partial s_1}{\partial v}\right|_{v=0} = 1/h'(s),$$

$$\left.\frac{\partial s_2}{\partial v}\right|_{v=0} = -1/h'(s),$$

$$\left.\frac{\partial w}{\partial v}\right|_{v=0} = 1/h'(u). \tag{3.6}$$

Also, of course, as $v \to 0$,

$$s_1 \to s, \quad s_2 \to s, \quad w \to u. \tag{3.7}$$

Because r is independent of v, $\partial \ln(r)/\partial v$ must vanish. But using (3.6) and (3.7) we can evaluate this quantity from (3.5) at $v=0$:

$$\left.\frac{\partial \ln(r)}{\partial v}\right|_{v=0} = \frac{2su^2}{s^2-u^2}\left[\frac{1}{s^2}\left(\frac{1}{h'(s)} - 1\right) - \frac{1}{u^2}\left(\frac{1}{h'(u)} - 1\right)\right]. \tag{3.8}$$

For the quantity in (3.8) to vanish it is necessary that

$$(1/s^2)[1 - 1/h'(s)] = (1/u^2)[1 - 1/h'(u)]. \tag{3.9}$$

Since the left side of (3.9) depends only on s and the right side only on u, each side must be equal to a constant K independent of s or u, and we conclude that

$$h'(u) = 1/(1 - Ku^2). \tag{3.10}$$

Integration of (3.10) leads directly to the addition law (3.1). I defer this to Sec. V, turning in Sec. IV to a somewhat different way of arriving at (3.10).

IV. PRECISE FORM OF THE ADDITION LAW: A GEDANKEN OSCILLATOR

Consider a ball rolling back and forth between the rear and front of the train at a fixed (train frame) speed u. We examine this "u oscillator" in a frame (the "v frame") in which the train moves with a speed v less than u. Let $t_1(u,v)$ and $t_2(u,v)$ be the v-frame times for the back-to-front and front-to-back parts of each cycle. We first establish that the difference between these times is independent of the train frame speed u of the ball:

$$t_1(u,v) - t_2(u,v) \text{ independent of } u. \tag{4.1}$$

This is, of course, obvious in the train frame, where the difference is, in fact, identically equal to zero. Since we make no assumptions about how statements of time in different frames are related, however, we must establish (4.1) directly in the v frame.

Since the time difference in (4.1) must be a continuous function of u, it is enough to establish that it is the same for two values of u and u' whose ratio is the ratio of two odd integers[10]:

$$u = (2m+1)u_0, \quad u' = (2n+1)u_0. \tag{4.2}$$

Let the balls start together at the rear of the train. Since u and u' are train-frame speeds, it is evident in the train frame that after exactly $m + \frac{1}{2}$ complete cycles of the first ball and $n + \frac{1}{2}$ of the second, the two will arrive together at the front of the train; and after another $m + \frac{1}{2}$ cycles of the first and $n + \frac{1}{2}$ of the second, they will again arrive together at the rear. Since these facts can be verified simply by counting round trips and noting the presence of two balls in the same place at the same time, they must be valid in any frame. Hence the v-frame times T_1 and T_1' that it takes the balls to complete their first $m + \frac{1}{2}$ and $n + \frac{1}{2}$ respective cycles must be equal, as must the corresponding times T_2 and T_2' for the next $m + \frac{1}{2}$ and $n + \frac{1}{2}$ cycles. We then certainly have

$$T_1 - T_2 = T_1' - T_2'. \tag{4.3}$$

But T_1 is the v-frame time for m round trips plus a back-to-front trip, while T_2 is the time for m round trips plus a front-to-back trip. The round-trip times drop out of the difference, leaving

$$T_1 - T_2 = t_1(u,v) - t_2(u,v). \tag{4.4}$$

In the same way

$$T_1' - T_2' = t_1(u',v) - t_2(u',v). \tag{4.5}$$

Collectively (4.3)–(4.5) assert that

$$t_1(u,v) - t_2(u,v) = t_1(u',v) - t_2(u',v), \tag{4.6}$$

which is precisely the content of (4.1).

We now express the time difference appearing in (4.1) in terms of the function f appearing in the addition law. Note

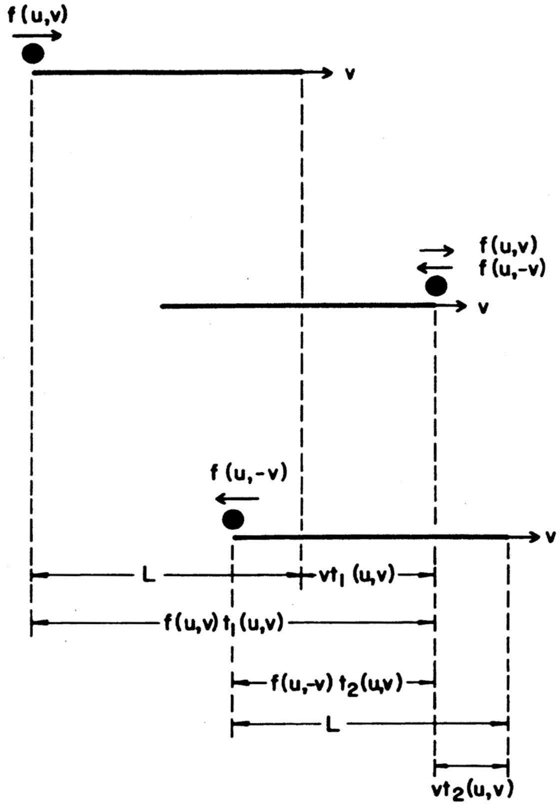

Fig. 3. A Gedanken oscillator, as described in the v frame, in which the train moves to the right with speed v. The speed of the ball is $f(u,v)$ to the right and $f(u,-v)$ to the left. The length of the train is L. The time between the top and middle pictures is $t_1(u,v)$; the time between the middle and bottom pictures is $t_2(u,v)$. Various lengths appearing in Eqs. (4.7) and (4.8) are indicated.

first that in the v frame the ball moves from rear to front with speed $f(u,v)$ taking a time $t_1(u,v)$ to cover a distance equal to the (v-frame) length L of the train plus the distance $vt_1(u,v)$ moved by the front of the train in that time (Fig. 3):

$$f(u,v)t_1(u,v) = L + vt_1(u,v). \tag{4.7}$$

The ball next moves from front to rear with speed $f(u,-v)$, taking a time $t_2(u,v)$ to cover a distance equal to the length L of the train minus the distance $vt_2(u,v)$ moved by the rear of the train in that time:

$$f(u,-v)t_2(u,v) = L - vt_2(u,v). \tag{4.8}$$

Solving (4.7) and (4.8) for t_1 and t_2, we have

$$t_1(u,v) - t_2(u,v) = L/[f(u,v) - v] - L/[f(u,-v) + v]. \tag{4.9}$$

But since L, the v-frame length of the train, does not depend on u, the train-frame speed of the balls, we can conclude from (4.9) and (4.1) that

$$1/[f(u,v) - v] - 1/[f(u,-v) + v] = g(v), \tag{4.10}$$

where the function $g(v)$ cannot depend on u.

Dividing by $2v$, we can rewrite (4.10) as

$$\frac{g(v)}{2v} = \frac{([f(u,-v) - f(u,v)]/2v) + 1}{[f(u,v) - v][f(u,-v) + v]}. \tag{4.11}$$

As v approaches zero the left side of (4.11) approaches some value K, and (4.11) gives

$$K = \frac{1 - [\partial f(u,v)/\partial v]|_{v=0}}{u^2}, \tag{4.12}$$

which [cf. (2.20)] again gives us the result

$$h'(u) = 1/(1 - Ku^2). \tag{4.13}$$

V. RELATIVITY WITHOUT LIGHT

Using the boundary condition (2.21) we can integrate (3.10) or (4.13) to get

$$h(z) = \frac{1}{2\sqrt{K}} \ln\left(\frac{1 + \sqrt{K}z}{1 - \sqrt{K}z}\right), \quad K > 0;$$

$$h(z) = \frac{1}{\sqrt{|K|}} \tan^{-1}(\sqrt{|K|}z), \quad K < 0. \tag{5.1}$$

Either of these expressions, when substituted into the general form (2.19) of the addition law, yields the result

$$w = (u + v)/(1 + Kuv). \tag{5.2}$$

The constant K cannot be negative, for if it were, the result of compounding two positive velocities both greater than $(-K)^{-1/2}$ would be a negative velocity. Positive velocities large enough to bring about this unsatisfactory state of affairs could always be attained by successive compoundings, since with K negative and w, u, and v all positive, w must *exceed* the sum of u and v.

If K is non-negative, however, it follows from (5.2) that $c(K) = K^{-1/2}$ is an invariant velocity: anything moving at this speed will have the same speed in all frames. The Galilean case is, of course, included in this range of possibilities, since $K = 0$ gives $w = u + v$, and $c(0) = \infty$. Historically, evidence for the nonzero value of K was provided by the growing body of evidence that the speed of light in empty space was in fact the invariant velocity.

It is not, however, necessary for there to be phenomena propagating at the invariant speed to reveal the value of K. One only requires sufficiently accurate measurements of the speed of any uniformly moving object from two different frames of reference. If the object C moves to the right in frame B, which itself moves to the right in frame A, then the addition law

$$v_{CA} = (v_{CB} + v_{BA})/(1 + Kv_{CB}v_{BA}) \tag{5.3}$$

can be used to express K in terms of the various velocities as

$$K = (1/v_{BA}v_{CA})[1 + x(1 - v_{CA}/v_{BA})], \tag{5.4}$$

where

$$x = v_{BA}/v_{CB} = -v_{AB}/v_{CB}. \tag{5.5}$$

Except for the quantity x, Eq. (5.4) expresses K entirely in terms of two velocities measured in the A frame. To determine the value of K, A-frame observers need therefore only measure the velocities v_{CA} and v_{BA} of the object C and the frame B, and collect from their associates in the B frame the value they have measured for the dimensionless ratio x. Since x does not depend on the time and distance units used by B-frame observers, it is not even necessary to crosscalibrate the B- and A-frame units. Equation (5.4) directly gives the inverse square of the invariant velocity in whatever units are employed in the A-frame.

Nor is light needed to establish in either frame a reliable method for measuring velocities. A and B could, for example, measure the velocities of each other and of C by setting up appropriate realizations of the Gedanken race of Sec. II, using the fact that in the rest frame of the race track Eq. (3.5) relates the measured fraction r to the speed s of the hare and speed u of the tortoise simply by

$$r = (s - u)/(s + u). \tag{5.6}$$

Thus A, by arranging a race between his own hare (the "A hare") and the object C, could measure r and determine the velocity u of C in terms of the speed s_A of the A hare. A similar determination could be made of the velocity of B and B's race track in units of s_A. B, on the other hand, would make measurements in terms of the B-hare velocity s_B. Because x is dimensionless, it would not be necessary to take care that B hares and A hares were prepared identically in their respective frames.

One can also dispense with the observers A and B. All that is really required are two superimposed parallel rigid race tracks, in uniform relative motion, each with its own hare, designed with a mechanism of propulsion that couples each hare to its track in the same way for the right-left and left-right parts of its race. By suitably lining everything up at the start and recording the fractional positions along each track where its hare first reencounters C and first reencounters the appropriate section of the other track, one can extract everything needed to give the value of K. No clock synchronization procedures—indeed, no clocks at all— are required.

Having demonstrated that there must be an invariant velocity, one can build up the rest of special relativity along more conventional lines. Alternatively, with the addition law at hand from the start, one can vary the development in any number of ways, avoiding, if one wishes, the use of anything that travels at the invariant speed.

A more interesting, instructive, and challenging exercise, however, is to try to refine those stages of the argument that I have already presented above. What, for example, is the simplest piece of apparatus necessary to extract K from the kinds of elementary counting procedures used in the Gedanken race or the comparison of commensurate Gedanken oscillators? What are the least assumptions about the nature of time, distance, and velocity necessary to give meaning to such procedures? How weak or natural can one make the smoothness assumptions?

There is, at a minimum, much good pedagogy to be had in the process of understanding the relativistic interconnections of space, time, and velocity, without the use of light. Working in the dark can be illuminating.

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[1]A. Einstein, Ann. Phys. **17**, 891 (1905).

[2]Another way to derive relativity without light is given by Y. P. Terletskii, *Paradoxes in the Theory of Relativity* (Plenum, New York, 1968), Sec. 7. Terletskii shows that a general linear transformation between two sets of space-time coordinates must be of the Lorentz form, with c^2 an undetermined constant, if the inverse and products of all transformations are to have the appropriate structure. (This argument stands on its own, without the appeals to the dependence of mass on velocity with which Terletskii precedes and concludes it.) My approach is conceptually simpler (though analytically somewhat more intricate) because I require no comparison of space and time measurements in different frames to establish the general form of the velocity addition law; my argument is based on *Gedankenexperimente* that make no use of clocks at all.

[3]Einstein begins Ref. 1 with the formulation of two postulates. The first, the principle of relativity, is that "dem Begriffe der absoluten Ruhe nicht nur in der Mechanik, sondern auch in der Elektrodynamik keine Eigenschaften der Erscheinungen entsprechen." (Not only in mechanics but also in electrodynamics the phenomena have no properties corresponding to the concept of absolute rest.) The second is that " sich das Licht im leeren Raume stets mit einer bestimmten, vom Bewegungszustande des emittierenden Körpers unabhängigen Geschwindigkeit V fortpflanze." (Light always propagates in empty space with a definite velocity c, independent of the state of motion of the emitting body.)

[4]In this paper I only establish the existence of a velocity that is invariant in frames of reference moving parallel to that velocity. The extension to the other two spatial dimensions can be achieved by a somewhat lengthy but straightforward modification of the usual arguments used to develop the relativistic behavior of length and time measurements. To avoid distracting the reader from the central part of the argument and to keep the paper to a reasonable length, I have omitted this exercise.

[5]One might object that such measurements make implicit use of light to establish a synchronization procedure for the clocks used to time the speeds. The method I describe for determining K in Sec. V, however, requries no clocks. (It should nevertheless be emphasized that there are methods for synchronizing distant clocks that do not rely on light. They can, for example, be synchronized by direct comparison at the point midway between their intended positions and then symmetrically transported to those positions.)

[6]N. David Mermin, Am. J. Phys. (in press). This article is an expanded version of homework problem 5, in N. David Mermin, *Space and Time in Special Relativity* (McGraw-Hill, New York, 1968), p.230.

[7]Equation (2.18) also follows directly from (2.17) and the condition $f(x,0) = x$.

[8]In Ref. 6 the hare is called a photon and its speed c is asserted to be the same in all frames of reference. Here there is nothing special about the speed s of the hare.

[9]At this point the argument of Ref. 6 is virtually complete. One need only note that when $v = 0$, Eq. (3.7) reduces the expression (3.5) for r to $r = (s - u)/(s + u)$. But if $s = s_1 = s_2 = c$, then setting the two expressions for r equal to one another gives an equation for the only remaining unknown w, whose solution is the relativistic addition law. Here, however, s_1 and s_2 are related to s through the very addition law we seek to determine, and we arrive not at the addition law, but at a nonlinear equation it must satisfy. Further analysis is required to solve that equation.

[10]The two balls can be regarded as the tortoise and hare of Sec. III, constrained now to have commensurate velocities, and required to keep running until they meet again at the original starting line.