Chapter 2

Classical Control System Design

Overview

Ch. 2. Classical control system design

Introduction

- Steady-state errors
- Type *k* systems
- Integral control

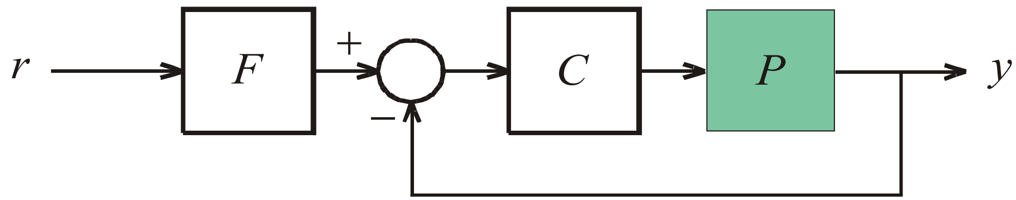
Frequency response plots

- Bode plots
- Nyquist plots
- *M*- and *N*-circles
- Nichols plots

Classical design techniques

- Classical design specifications
- Lead, lag, lead-lag compensation
- Guillemin-Truxal method
- Quantitative Feedback Theory
- Root locus

Steady-state errors-1



Tracking behavior: Assume $r(t) = \frac{t^n}{n!} 1(t)$ $\hat{r}(s) = \frac{1}{s^{n+1}}$

Response $\hat{y}(s) = \underbrace{\frac{L(s)}{1+L(s)} F(s)}_{H(s)} \hat{r}(s)$

Tracking error $\hat{\varepsilon}(s) = \hat{r}(s) - \hat{y}(s) = [1 - H(s)]\hat{r}(s)$

Steady-state errors-2

Steady-state tracking error

$$\varepsilon_\infty^{(n)} = \lim_{t\to\infty} \varepsilon(t) = \lim_{s\to 0} s\hat{\varepsilon}(s) = \lim_{s\to 0} \frac{1-H(s)}{s^n}$$

If $F(s)=1$ (no prefilter) then

$$1-H(s) = \frac{1}{1+L(s)}$$

$$\varepsilon_\infty^{(n)} = \lim_{s\to 0} \frac{1}{s^n[1+L(s)]}$$

Type *k* system

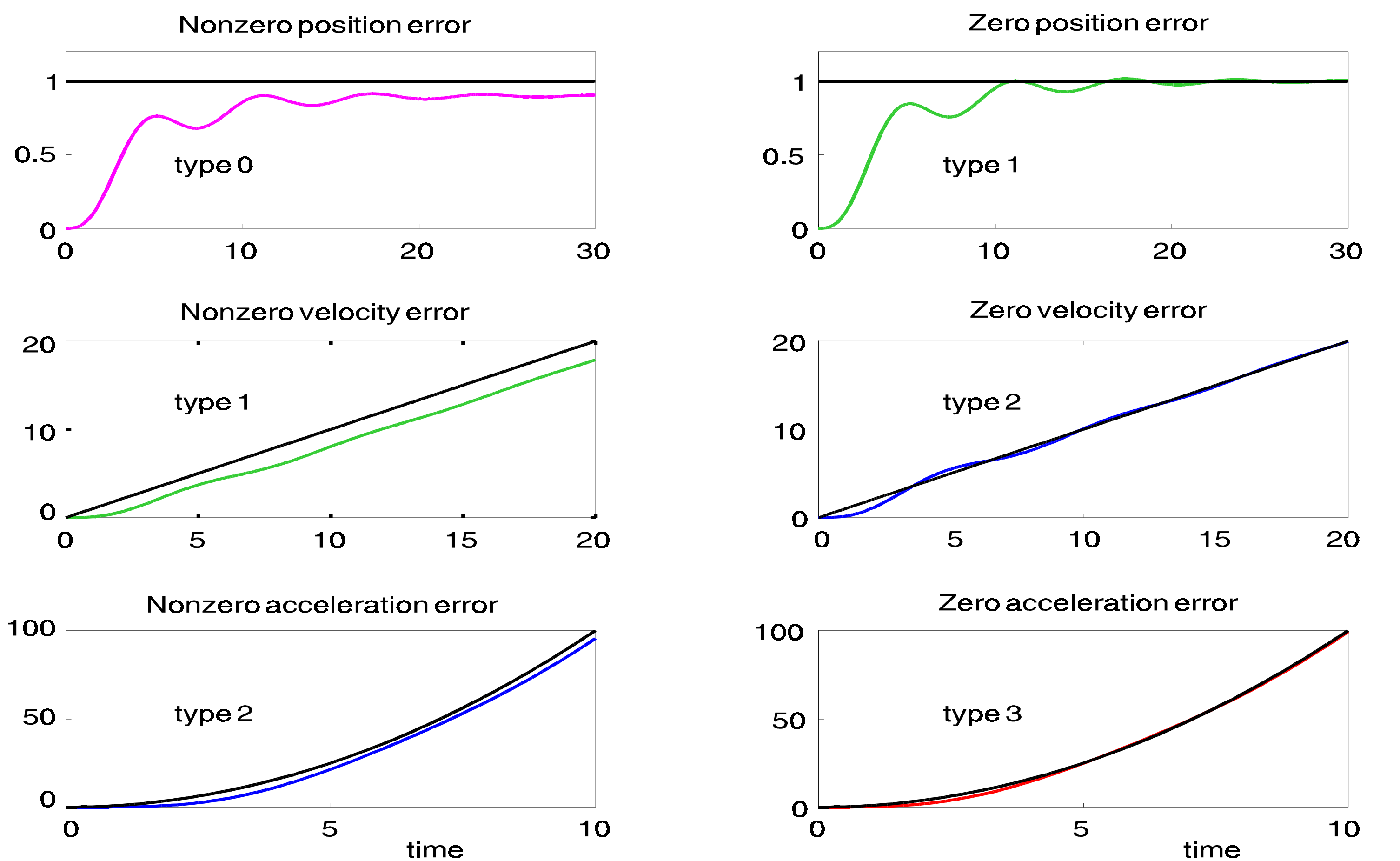
A feedback system is of type *k* if $L(s) = \frac{L_o(s)}{s^k}, \qquad L_o(0) \neq 0$

Then

$$\varepsilon_\infty^{(n)} = \lim_{s\to 0} \frac{1}{s^n[1+L(s)]}$$

$$= \lim_{s\to 0} \frac{s^{k-n}}{s^k + L_o(s)} = \begin{cases} 0 & \text{for } 0 \le n < k \\ 1/L_o(0) & \text{for } n = k \\ \infty & \text{for } n > k \end{cases}$$

Steady-state errors-3



Integral control-1

Integral control:

Design the closed-loop system such that $L(s) = \frac{L_o(s)}{s}$

Type *k* control: $L(s) = \frac{L_o(s)}{s^k}$

Results in good steady-state behavior

Also:

$$S(s) = \frac{1}{1 + L(s)} = \frac{s^k}{s^k + L_o(s)} = O(s^k) \quad \text{for} \quad s \to 0$$

Integral control-2

Type k control: $S(s) = \circ(s^k)$ for $s \to 0$

Hence if

$$v(t) = \frac{t^n}{n!} 1(t), \qquad \hat{v}(s) = \frac{1}{s^{n+1}}$$

then the steady-state error is zero if $n < k$ (*rejection*)

$k = 1$: *Integral control:* Rejection of constant disturbances

$k = 2$: *Type-2 control:* Rejection of ramp disturbances

Etc.

Integral control-3

Integral control:

$$L(s) = \frac{L_o(s)}{s^k} = P(s)C(s)$$

The loop has *integrating action* of order *k*

"Natural" integrating action is present if the plant transfer function has one or several poles at 0

If no natural integrating action exists then the compensator needs to provide it

Integral control-4

"Pure" integral control: $C(s) = \dfrac{1}{sT_i}$

PI control: $$C(s) = g\left(1 + \frac{1}{sT_i}\right)$$

PID control: $$C(s) = g\left(sT_d + 1 + \frac{1}{sT_i}\right)$$

Ziegler-Nichols tuning rules

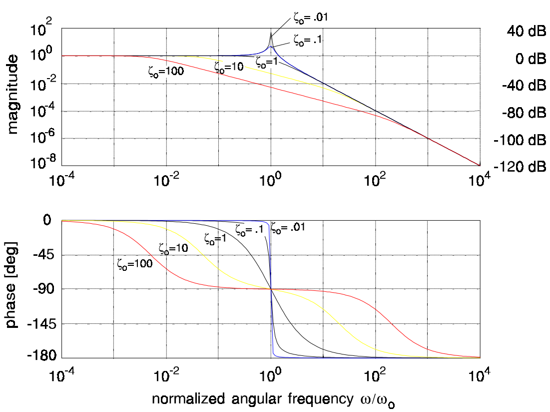
Internal model principle

Asymptotic tracking if model of disturbance is included in the compensator

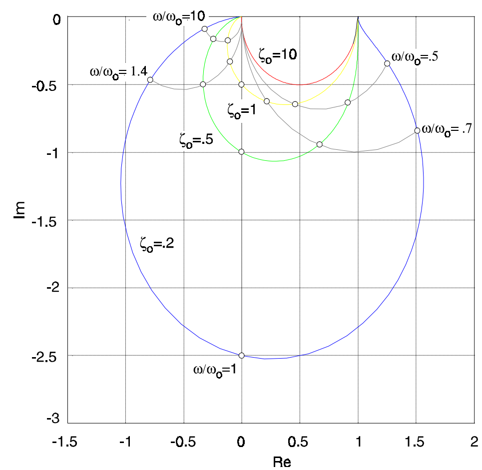
Francis, D.A. and Wonham, W.M., (1975) The internal model principle for linear multivariable regulators, Applied Mathematics and Optimization, vol 2, pp. 170-194

Frequency response plots

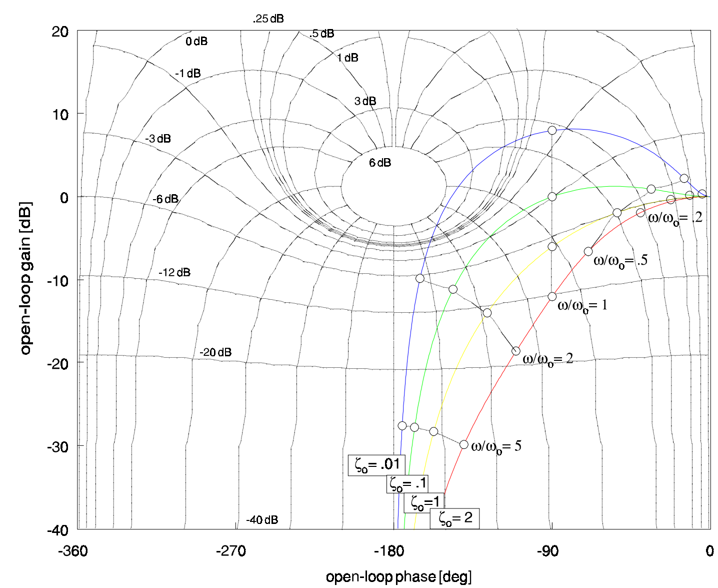
Bode plots



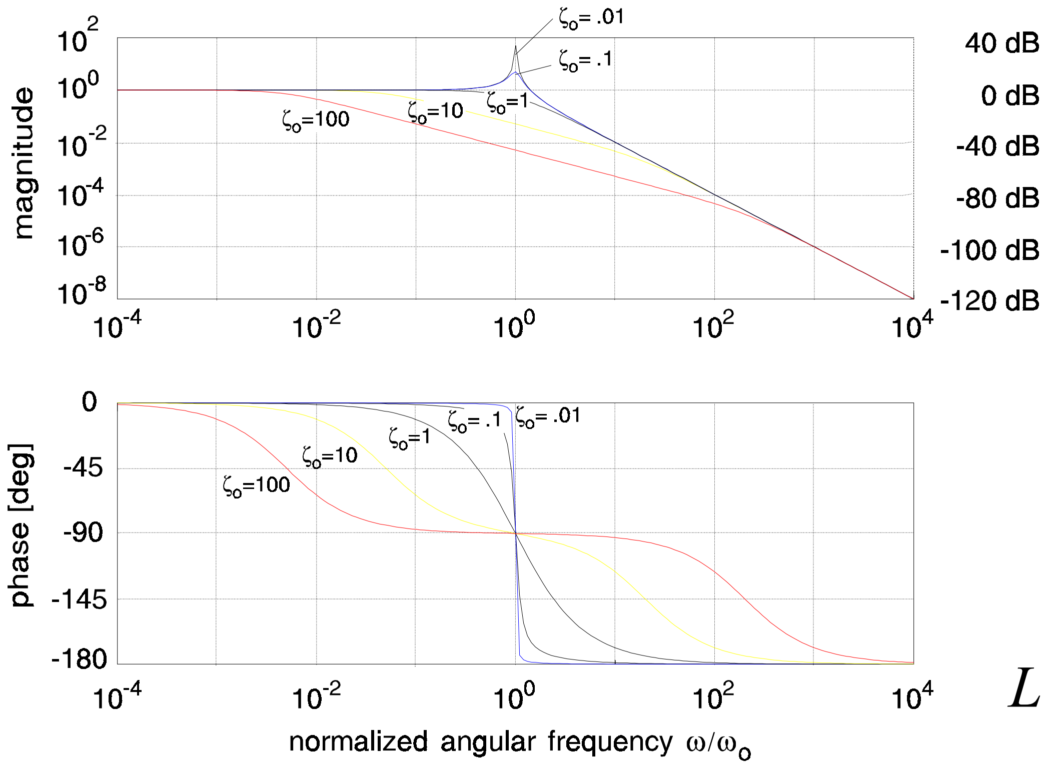
Nyquist plots



Nichols plots



Bode plots-1



Bode plot:

- doubly logarithmic plot of $|L(j\omega)|$ versus ω
- semi logarithmic plot of arg $L(j\omega)$ versus ω

$$L(j\omega) = \frac{\omega_o^2}{(j\omega)^2 + 2\zeta_o\omega_o(j\omega) + \omega_o^2}$$

Bode plots-2

Helpful technique:

By construction of the asymptotic Bode plots of elementary first- and second-order factors of the form

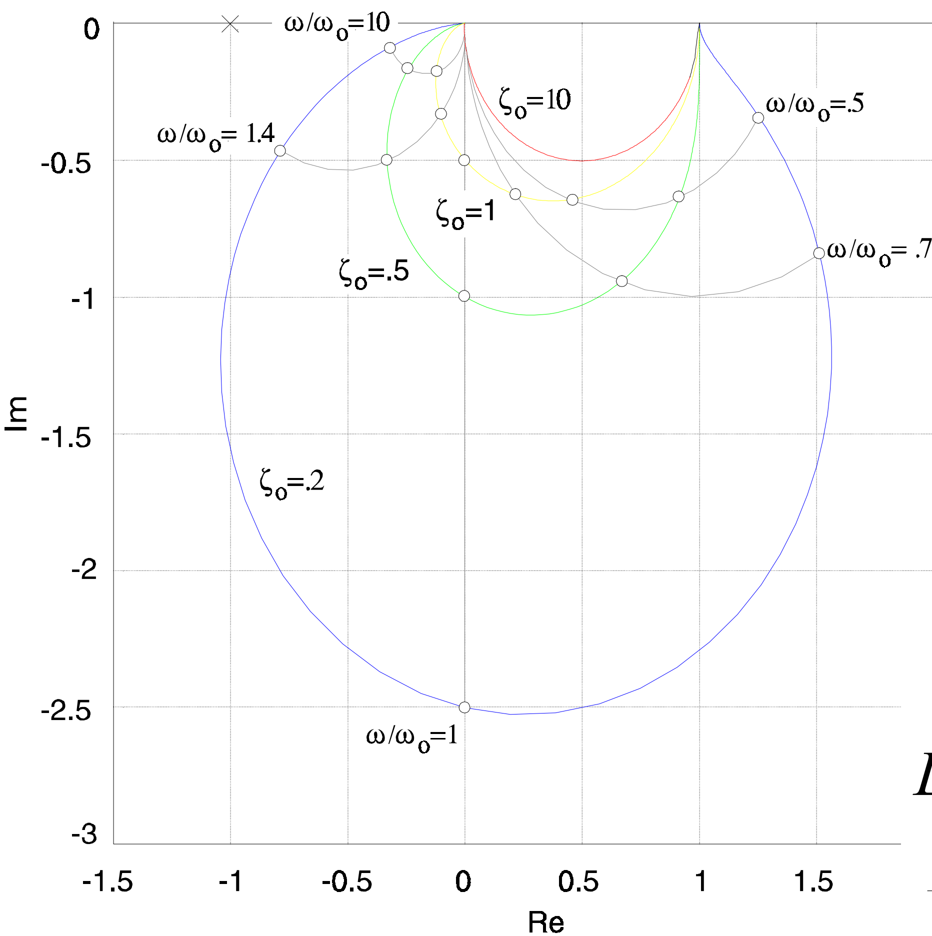
$$j\omega + \alpha \qquad \text{and} \qquad (j\omega)^2 + 2\zeta_0\omega_o^2(j\omega) + \omega_o^2$$

The shape of the Bode plot of

$$L(j\omega) = k\,\frac{(j\omega - z_1)(j\omega - z_2)\cdots(j\omega - z_m)}{(j\omega - p_1)(j\omega - p_2)\cdots(j\omega - p_m)}$$

may be sketched

Nyquist plots

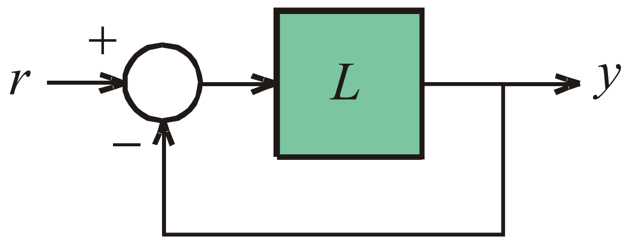


Nyquist plot: Locus of $L(j\omega)$ in the complex plane with ω as parameter

Contains less information than the Bode plot if ω is not marked along the locus

$$L(j\omega) = \frac{\omega_o^2}{(j\omega)^2 + 2\zeta_o\omega_o(j\omega) + \omega_o^2}$$

M- and *N*-circles-1



Closed-loop transfer function:

$$H = \frac{L}{1+L} = T$$

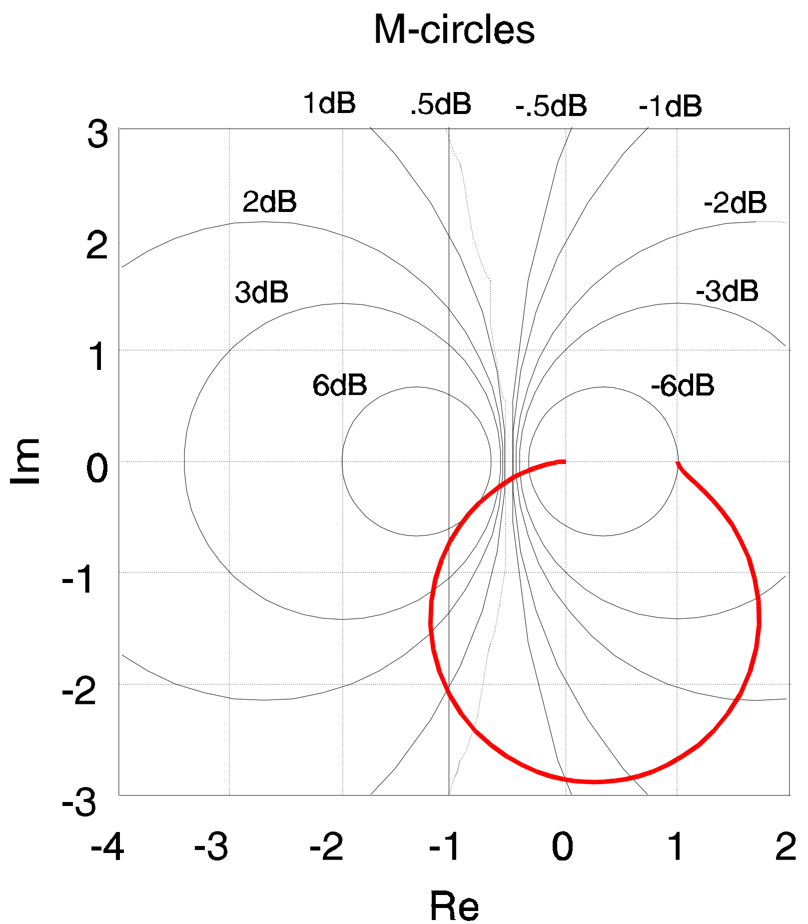
M-circle: Locus of points *z* in the complex plane where

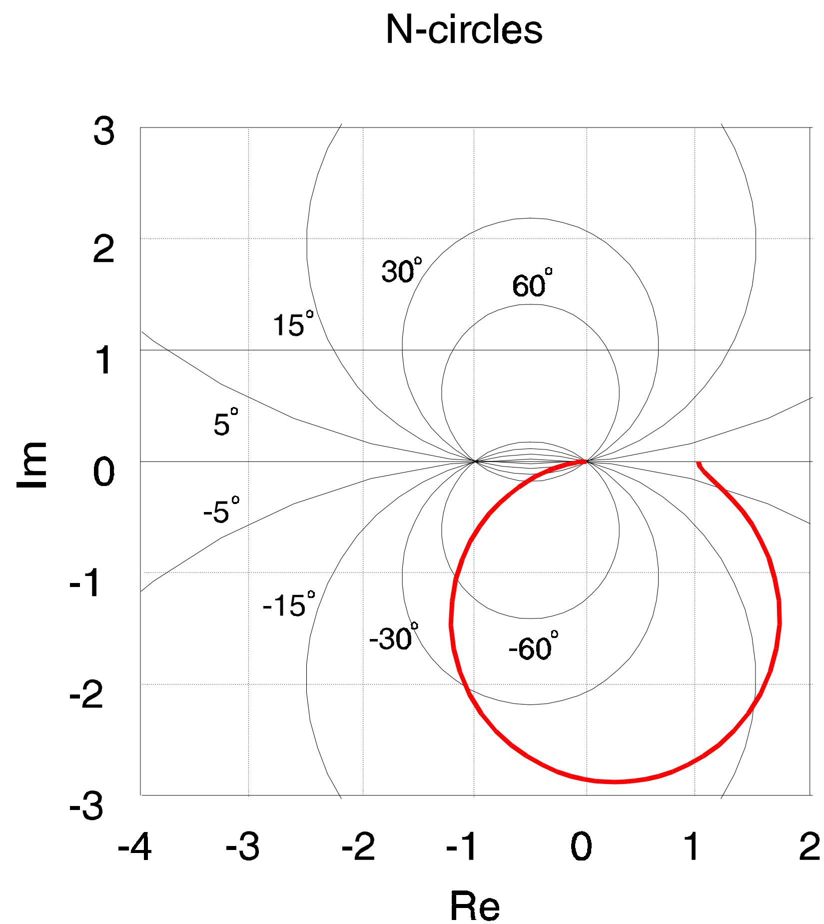
$$\left|\frac{z}{1+z}\right| = M$$

N-circle: Locus of points *z* in the complex plane where

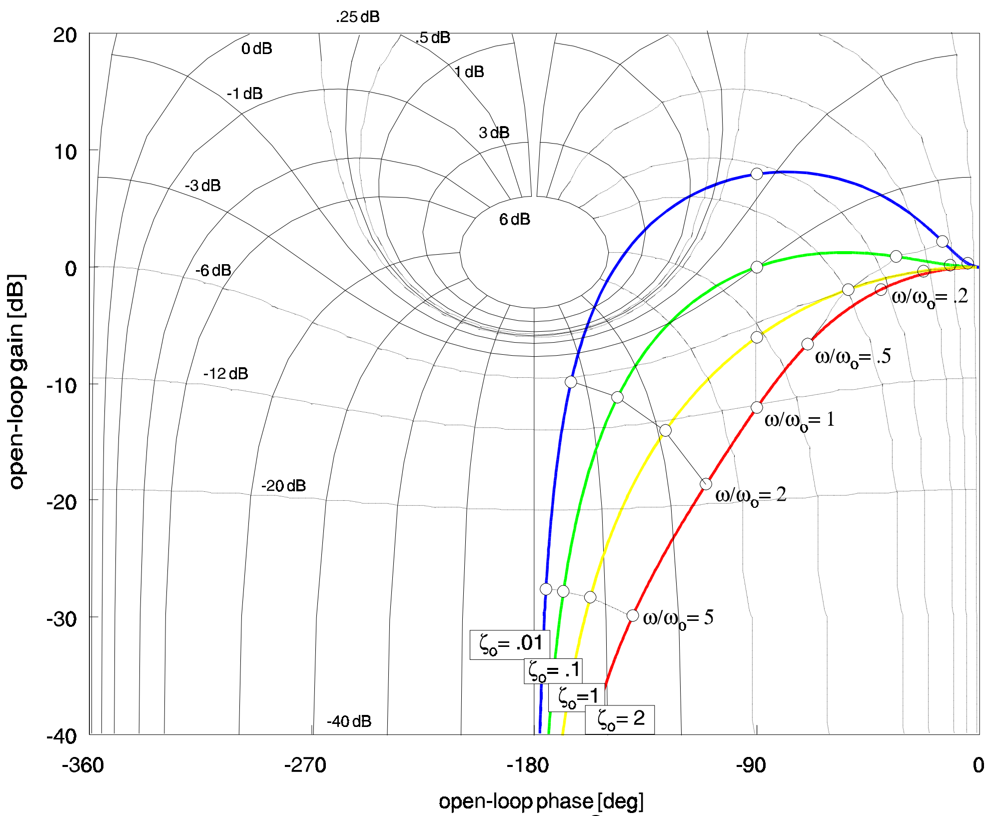
$$\arg\frac{z}{1+z} = N$$

M- and *N*-circles-2





Nichols plots



$$L(j\omega) = \frac{\omega_o^2}{(j\omega)^2 + 2\zeta_o\omega_o(j\omega) + \omega_o^2}$$

Nichols plot: Locus of $L(j\omega)$ with ω as parameter in the

log magnitude

versus

argument

plane

Nichols chart: Nichols plot with *M*- and *N*-loci included

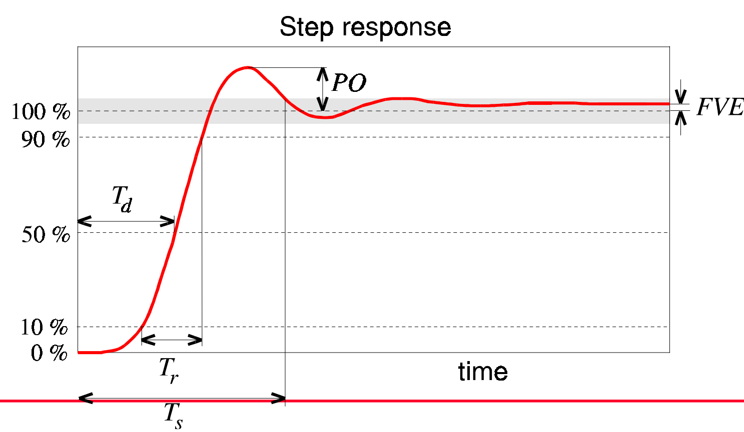
Classical design specifications

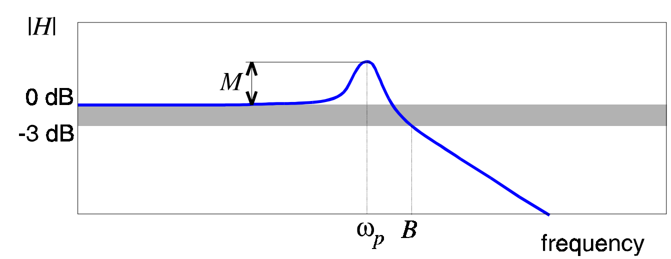
Time domain

- Rise time, delay time, overshoot, settling time, steady-state error of the response to step reference and disturbance inputs; error constants

Frequency domain

- Bandwidth, resonance peak, roll-on and roll-off of the closed-loop frequency response and sensitivity functions; stability margins





Classical design techniques

- Lead, lag, and lag-lead compensation (loopshaping)
- (Root locus approach)
- (Guillemin-Truxal design procedure)
- Quantitative feedback theory QFT (robust loopshaping)

Classical design techniques

Rules for loopshaping

- Change open-loop L(s) to achieve certain closed-loop specs
- first modify phase
- then correct gain

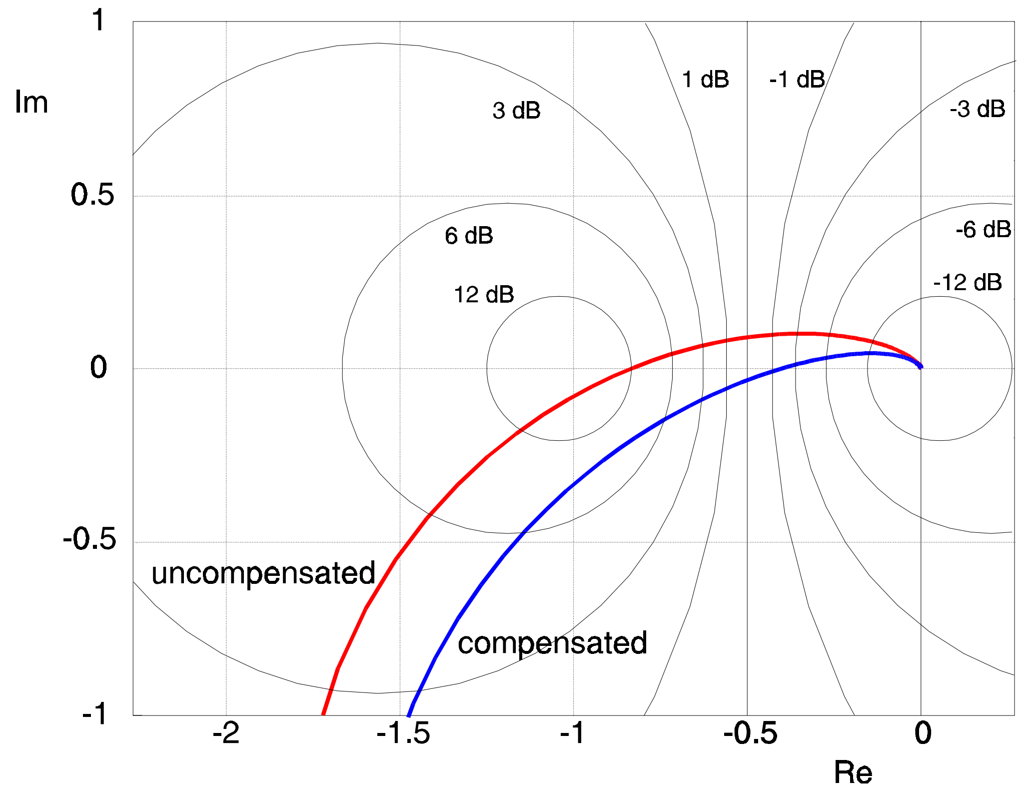
Lead compensation

Lead compensation:

Add extra phase in the cross-over region to improve the stability margins

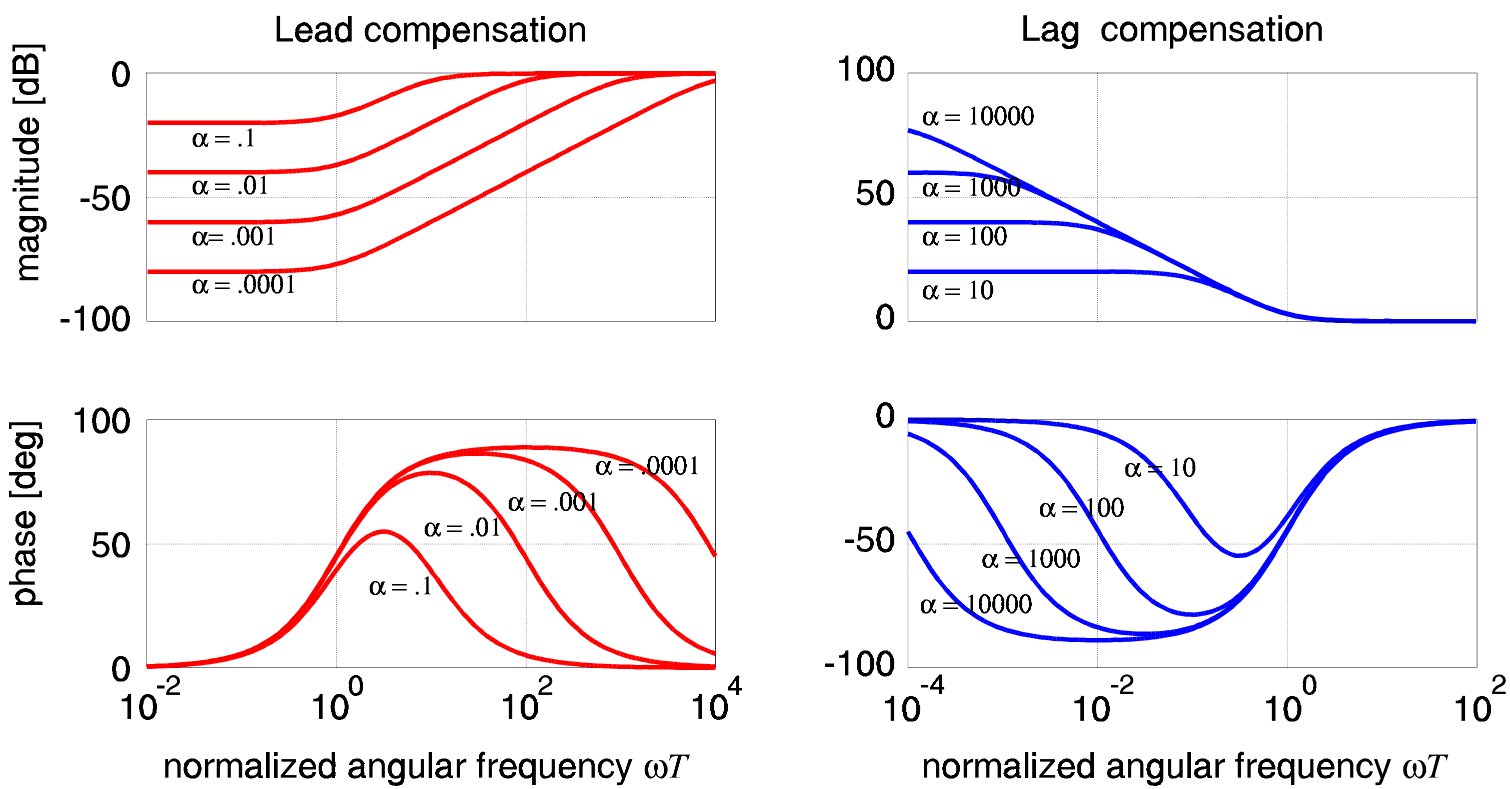
Typical compensator:

"Phase-advance network"



$$C(j\omega) = \alpha \frac{1 + j\omega T}{1 + j\omega\alpha T}, \quad 0 < \alpha < 1$$

Lead/lag compensator



$$C(j\omega) = \alpha \frac{1 + j\omega T}{1 + j\omega\alpha T}$$

Lag compensation

Lag compensation:

Increase the low frequency gain without affecting the phase in the cross-over region

Example: PI-control:

$$C(j\omega) = k\frac{1 + j\omega T}{j\omega T}$$

Lead-lag compensation

Lead-lag compensation: Joint use of

- lag compensation at low frequencies
- phase lead compensation at crossover

Lead, lag, and lead-lag compensation are always used in combination with gain adjustment

Notch compensation

(inverse) Notch filters:

- suppression of parasitic dynamics
- additional gain at specific frequencies

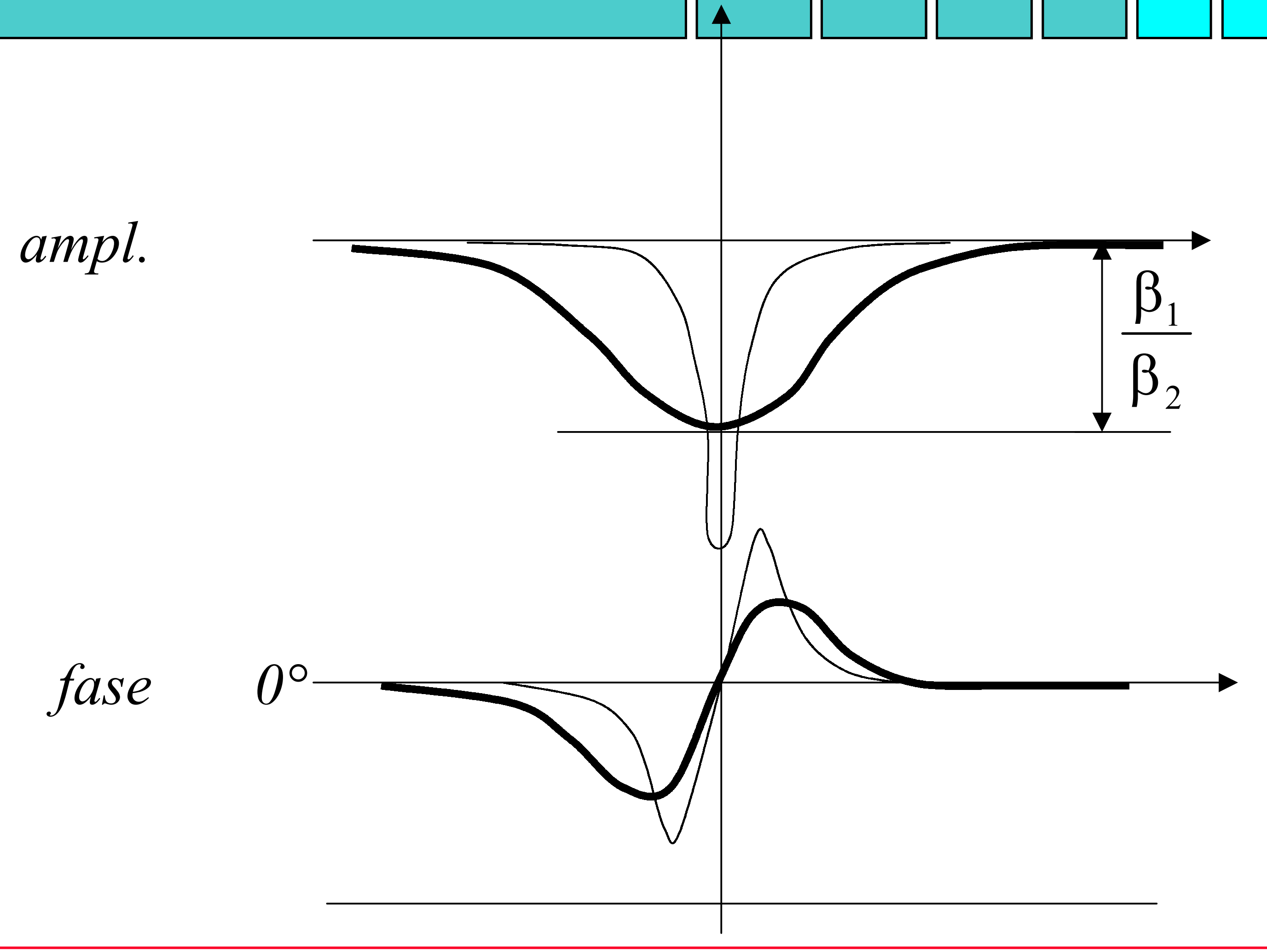
Special form of general second order filter

Notch compensation

$$H = \frac{u}{\varepsilon} = \frac{\dfrac{s^2}{\omega_1^2} + 2\beta_1 \dfrac{s}{\omega_1} + 1}{\dfrac{s^2}{\omega_2^2} + 2\beta_2 \dfrac{s}{\omega_2} + 1}$$

"Notch"-filter : $\omega_1 = \omega_2$

Notch compensation



Root locus method-1

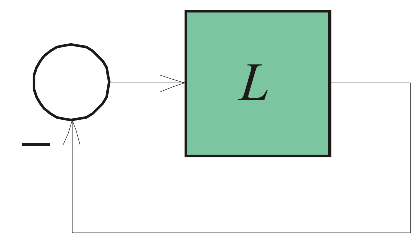
Important stage of many designs: Fine tuning of

- gain
- compensator pole and zero locations

Helpful approach: the root locus method (use rltool!)

Root locus method-2

$$L(s) = \frac{N(s)}{D(s)} = k\,\frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)}$$



Closed-loop characteristic polynomial

$$\begin{aligned}\chi(s) &= D(s) + N(s)\\ &= (s-p_1)(s-p_2)\cdots(s-p_n) + k(s-z_1)(s-z_2)\cdots(s-z_m)\end{aligned}$$

Root locus method: Determine the loci of the roots of χ as the gain k varies

Root locus method-3

$$\chi(s) = (s - p_1)(s - p_2)\cdots(s - p_n) + k(s - z_1)(s - z_2)\cdots(s - z_m)$$

Rules:

- For $k = 0$ the roots are the open-loop poles p_i
- For $k \rightarrow \propto$ a number m of the roots approach the open-loop zeros z_i. The remaining roots approach $\propto$
- The directions of the asymptotes of those roots that approach $\propto$ are given by the angles

$$\frac{2i+1}{n-m}\pi, \quad i = 0, 1, \cdots, n-m-1$$

Root locus method-4

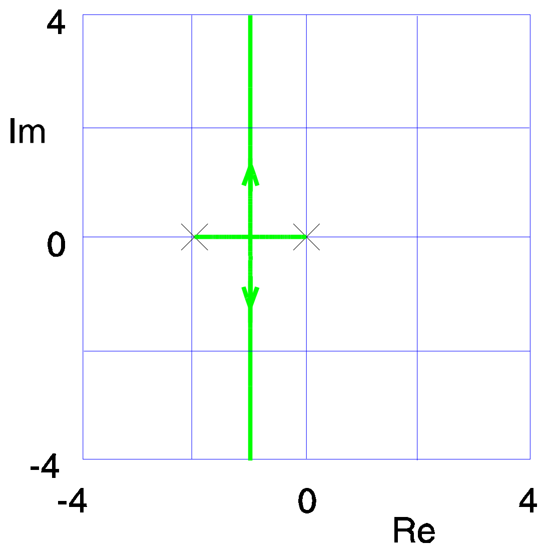
- The asymptotes intersect on the real axis in the point

$$\frac{(\text{sum of open-loop poles}) - (\text{sum of open-loop zeros})}{n-m}$$

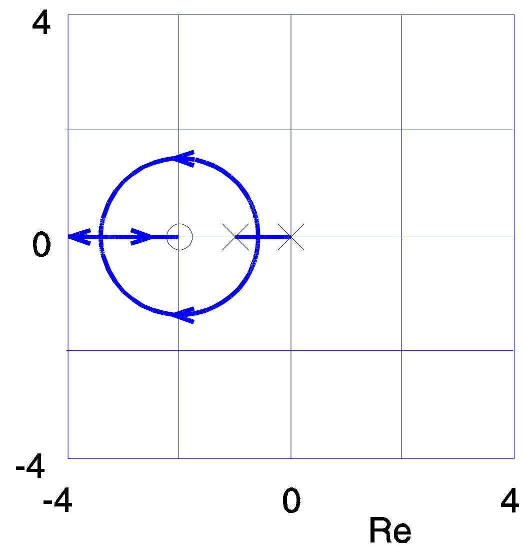
- Those sections of the real axis located to the left of an odd total number of open-loop poles and zeros on this axis belong to a locus
- The loci are symmetric with respect to the real axis
- ....

Root locus method-5

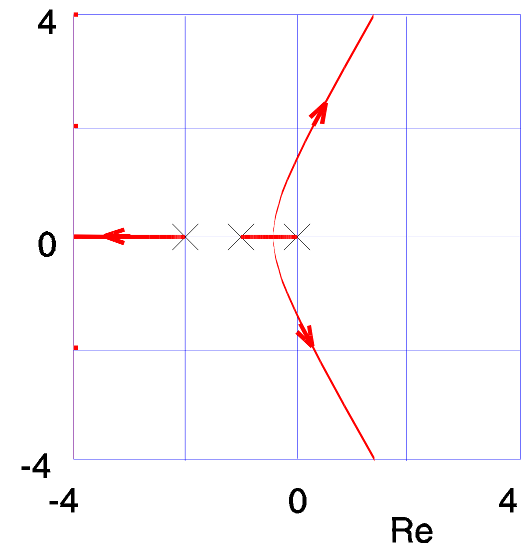
$$L(s) = \frac{k}{s(s+2)}$$



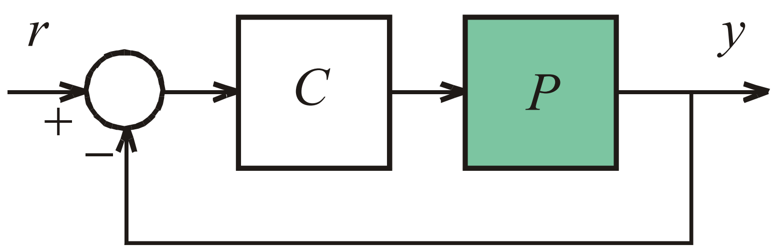
$$L(s) = \frac{k(s+2)}{s(s+1)}$$



$$L(s) = \frac{k}{s(s+1)(s+2)}$$



Guillemin-Truxal method-1



Closed-loop transfer function:

$$H = \frac{PC}{1+PC}$$

Procedure:

- Specify H
- Solve the compensator from $C = \frac{1}{P} \cdot \frac{H}{1-H}$

Guillemin-Truxal method-2

Example: Choose

$$H(s) = \frac{a_m s^m + a_{m-1} s^{m-1} + \cdots + a_0}{s^n + a_{n-1} s^{n-1} + \cdots + a_m s^m + a_{m-1} s^{m-1} + \cdots + a_0}$$

This guarantees the system to be of type $m+1$

How to choose the denominator polynomial?

Well-known options:

- Butterworth polynomials
- Optimal ITAE polynomials

Butterworth and ITAE polynomials

Butterworth polynomials

Choose the n left-half plane poles on the unit circle so that together with their right-half plane mirror images they are uniformly distributed along the unit circle

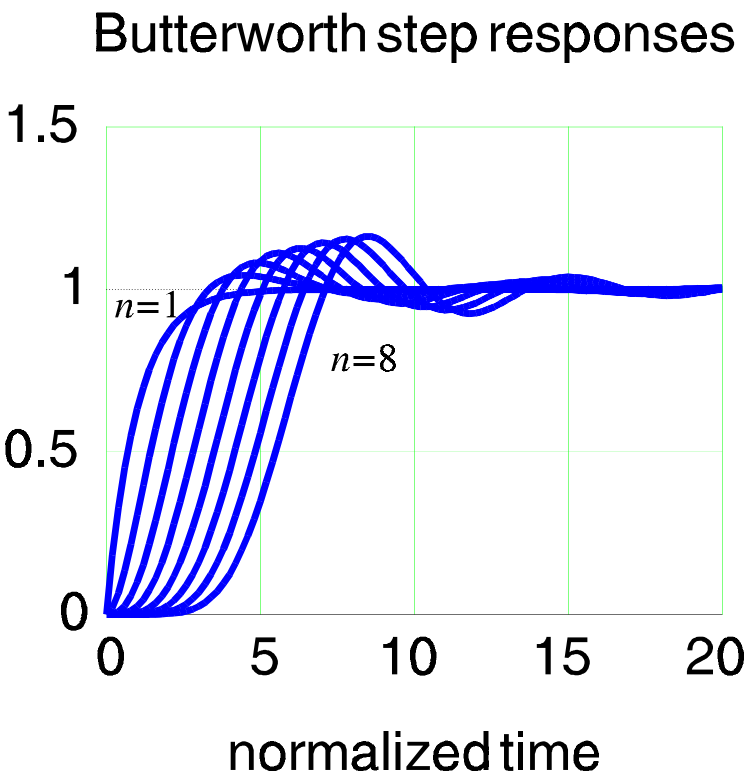
ITAE polynomials

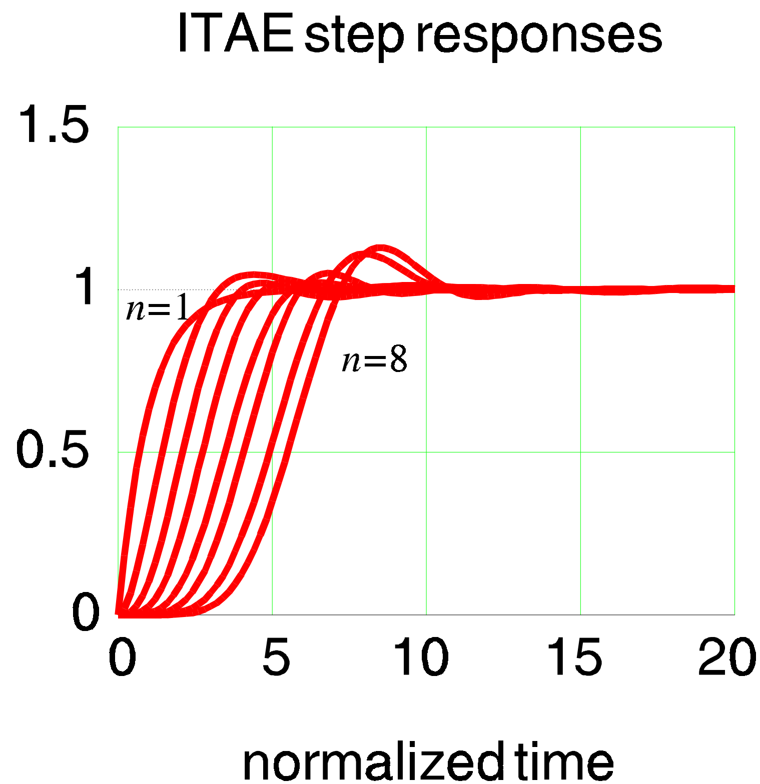
Place the poles so that

$$\int_0^\infty t\,|e(t)|\,dt$$

is minimal, where e is the tracking error for a step input

Butterworth and ITAE





$m = 0$

Guillemin-Truxal method-3

Disadvantages of the method:

- Difficult to translate the specs into an unambiguous choice of H. Often experimentation with other design methods is needed to establish what may be achieved. In any case preparatory analysis is required to determine the order of the compensator and to make sure that it is proper
- The method often results in undesired pole-zero cancellation between the plant and the compensator

Quantitative feedback theory QFT-1

Ingredients of QFT:

- For a number of selected frequencies, represent the uncertainty regions of the plant frequency response in the Nichols chart
- Specify tolerance bounds on the magnitude of T
- Shape the loop gain so that the tolerance bounds are never violated

QFT-2

Example: Plant $$P(s) = \frac{g}{s^2(1+s\theta)}$$

Nominal parameter values: $g = 1, \quad \theta = 0$

Parameter uncertainties: $0.5 \leq g \leq 2, \quad 0 \leq \theta \leq 0.2$

Tentative compensator:

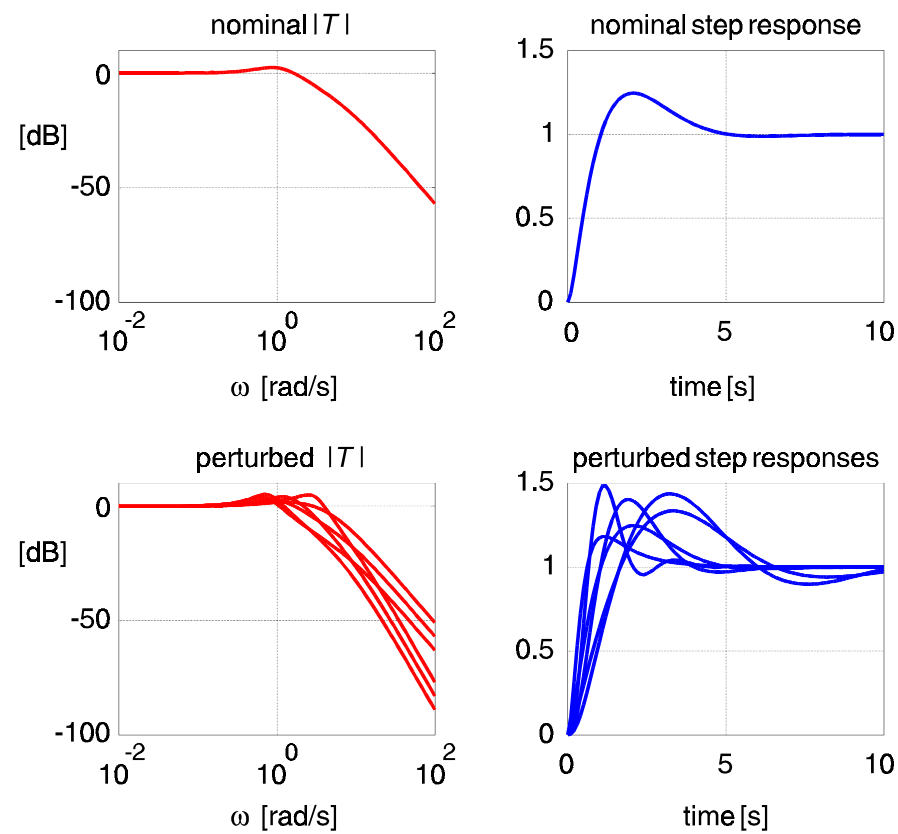
$$C(s) = \frac{k + sT_d}{1 + sT_o}, \quad k = 1, T_d = 1.414, T_o = 0.1$$

QFT-3

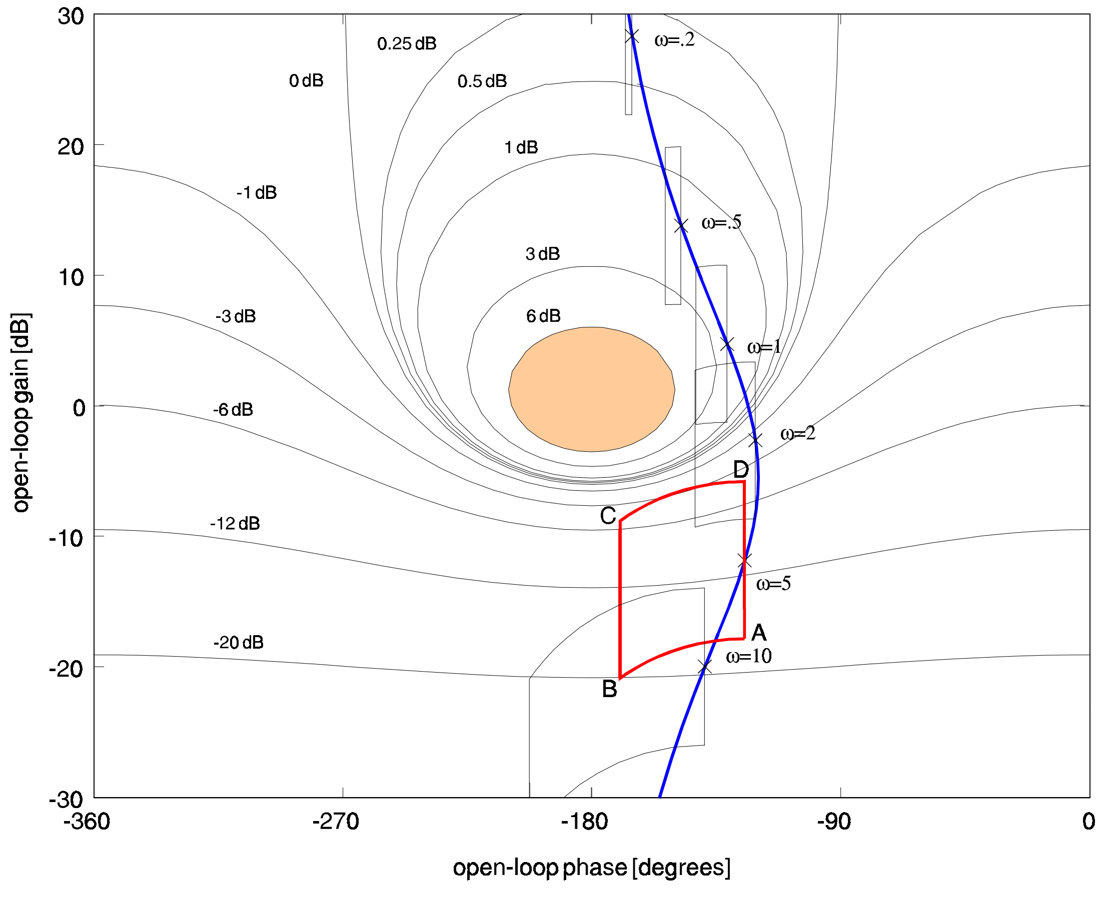
Responses of the nominal design

Specs on $|T|$

Frequency [rad/s]	**Tolerance band** [dB]
0.2	0.5
1	2
2	5
5	10
10	20



Uncertainty regions



Uncertainty regions for the nominal design

The specs are not satisfied

Additional requirement:

The critical area may not be entered

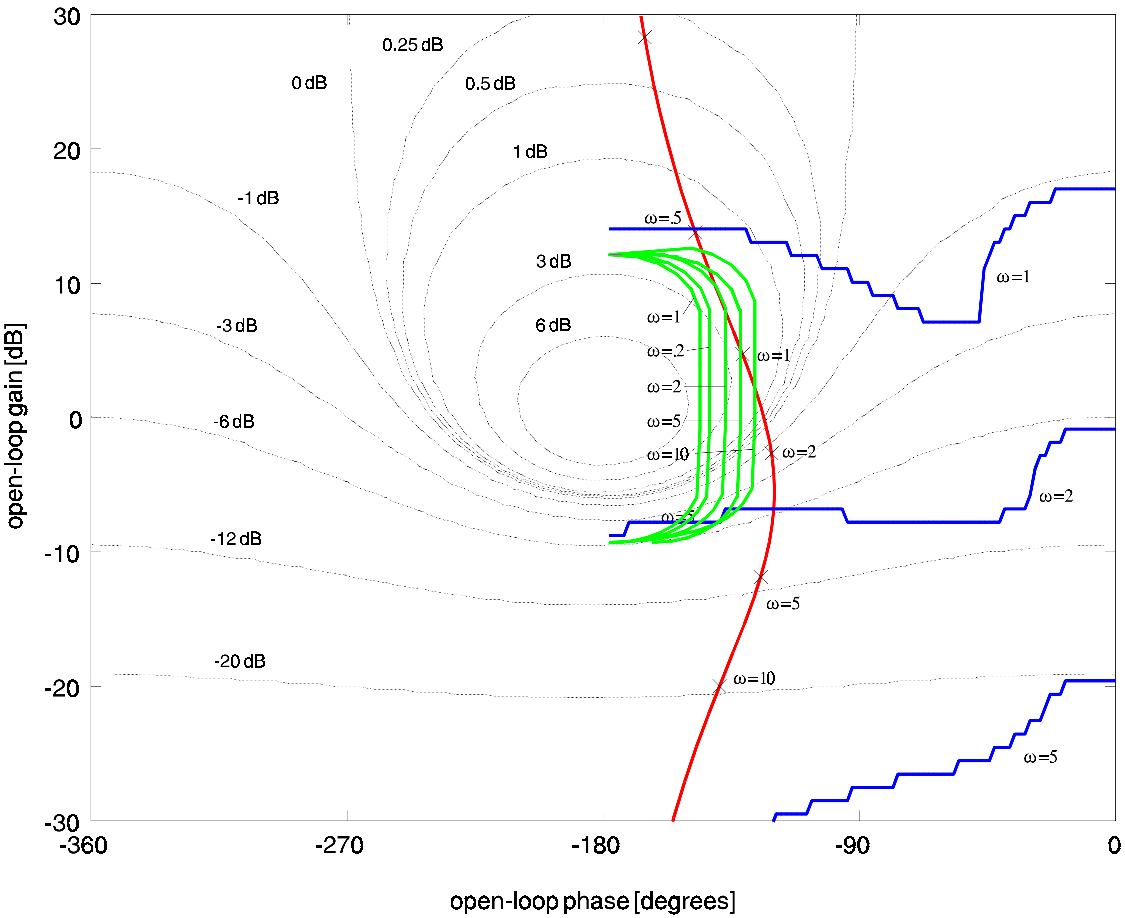
QFT-4

Design method: Manipulate the compensator frequency reponse so that the loop gain

- satisfies the tolerance bounds
- avoids the critical region

- Preparatory step 1: For each selected frequency, determine the performance boundary
- Preparatory step 2: For each selectedfrequency, determine the robustness boundary

Performance and robustness boundaries



Nominal plant frequency response

Robustness boundaries

Performance boundaries

QFT-5

Design step: Modify the loop gain such that for each selected frequency the corresponding point on the loop gain plot lies above and to the right of the corresponding boundary

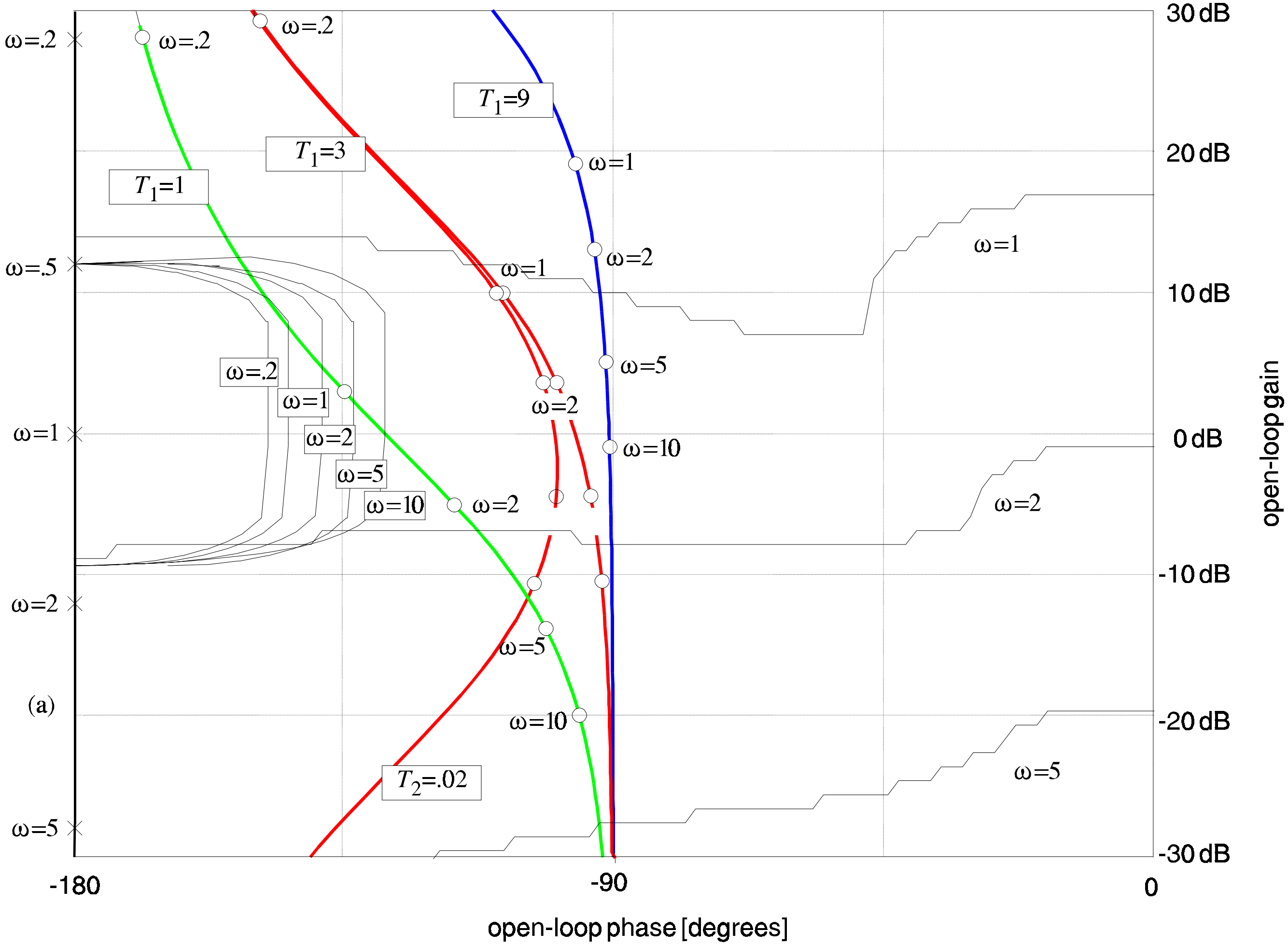
For the case at hand this may be accomplished by a lead compensator of the form

$$C(s) = \frac{1 + sT_1}{1 + sT_2}$$

Step 1: Set $T_2 = 0$, vary T_1

Step 2: Keep T_1 fixed, vary T_2

QFT-6



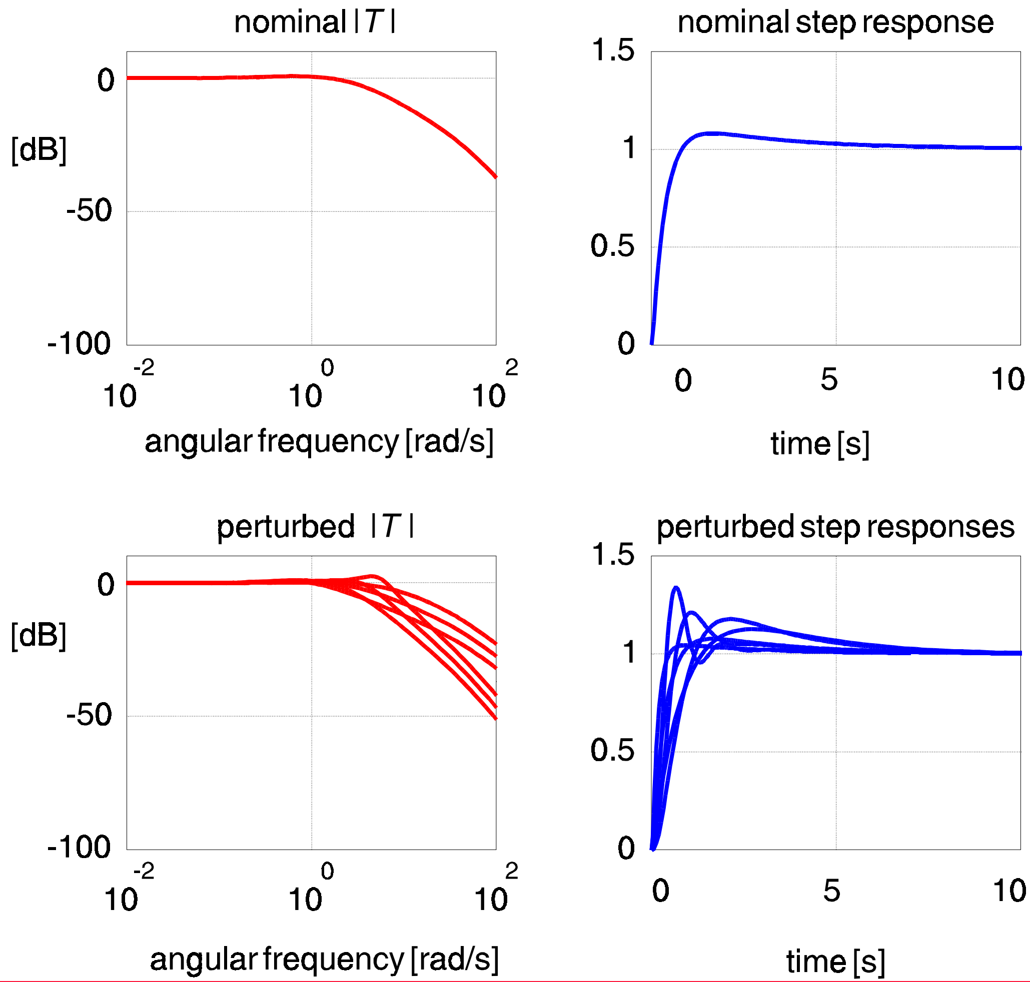
Eventual design:

$$T_1 = 3$$

$$T_2 = 0.02$$

QFT-7

Responses of the redesigned system

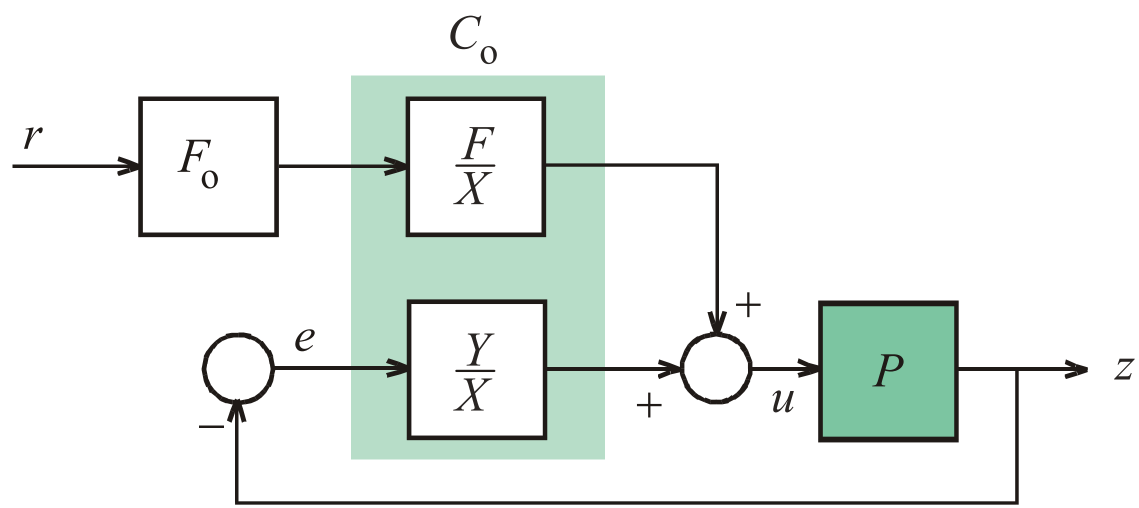


Prefilter design-1

2½-degree-of-freedom configuration

Closed-loop transfer function

$$H = \frac{NF}{D_{\rm cl}} F_o$$



For the present case:

$$D_{\rm cl}(s) = 0.02(s+0.3815)(s+2.7995)(s+46.8190)$$

$$N(s) = 1$$

Prefilter design-2

Use the polynomial F to cancel the (slow) pole at –0.3815, and let

$$F_o(s) = \frac{\omega_o^2}{s^2 + 2\zeta_o\omega_o s + \omega_o^2}, \qquad \omega_o = 1, \quad \zeta_o = \tfrac{1}{2}\sqrt{2}$$

Perturbed responses