

PSI 2434 – MICROONDAS I

CAPÍTULO III

CAVIDADES RESSONANTES

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CAVIDADES RESSONANTES :

RETANGULARES E CILÍNDRICAS

FREQÜÊNCIAS DE RESSONÂNCIA

ÍNDICES DE MÉRITO

Direitos Autorais Reservados. Reprodução Proibida
Departamento de Engenharia de Sistemas Eletrônicos - PUC-Rio

I. INTRODUÇÃO -

APLICAÇÕES DAS CAVIDADES RESSONANTES

- FILTROS SINTONIZADOS

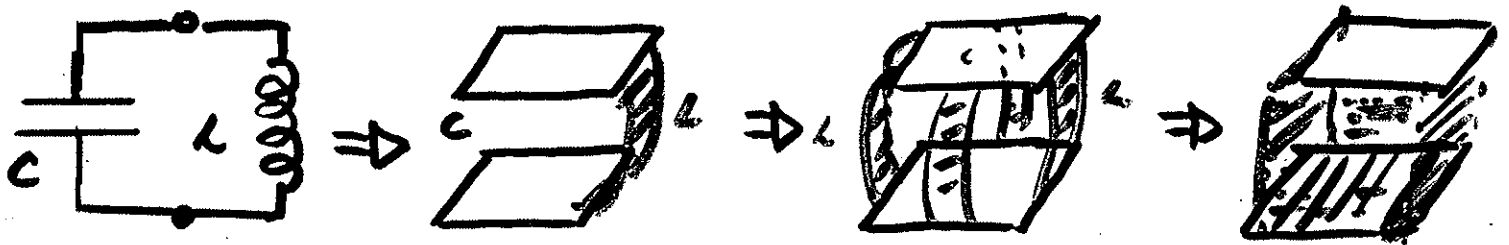
- OSCILADORES - FORNOS DE MICROONDAS

- AMPLIFICADORES SINTONIZADOS

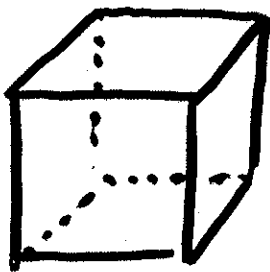
PSI-2434

Prof. KUEBER

CIRCUITOS RESSONANTES



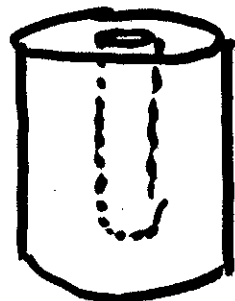
$\frac{1}{2\pi\sqrt{LC}}$ = frequência: $f_0 = \frac{1}{2\pi\sqrt{LC}}$



CAVIDADE
RETANGULAR



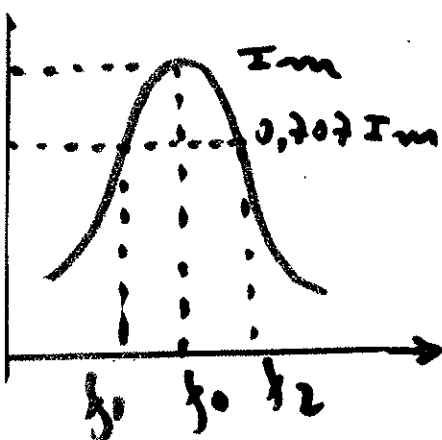
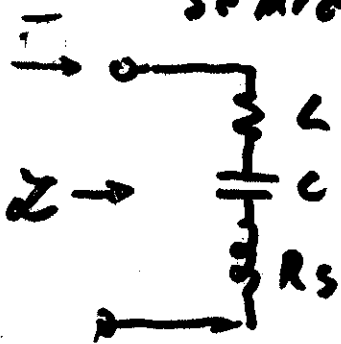
CAVIDADE
CILÍNDRICA



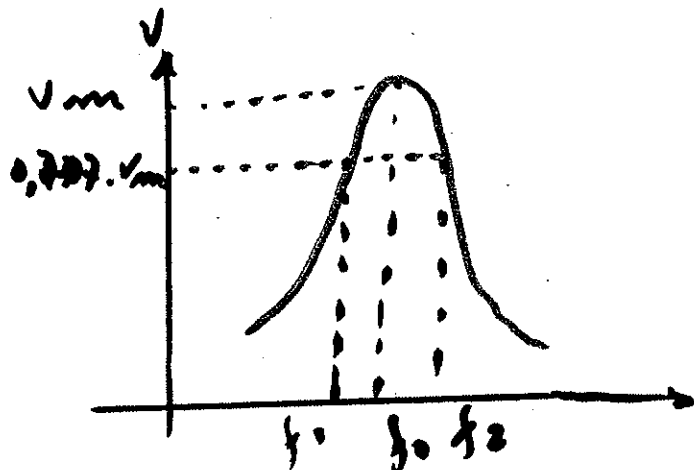
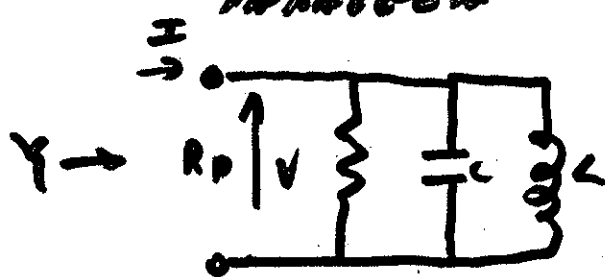
CAVIDADE
COAXIAL

CIRCUITOS EQUIVALENTES

RESONÂNCIA
SÉRIE



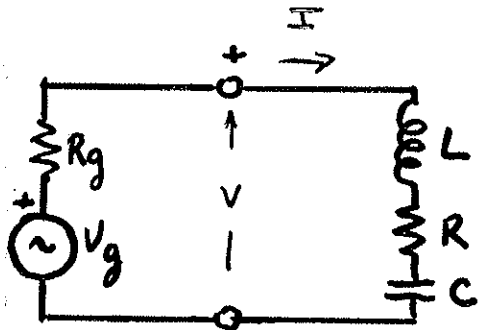
RESONÂNCIA
PARALELA



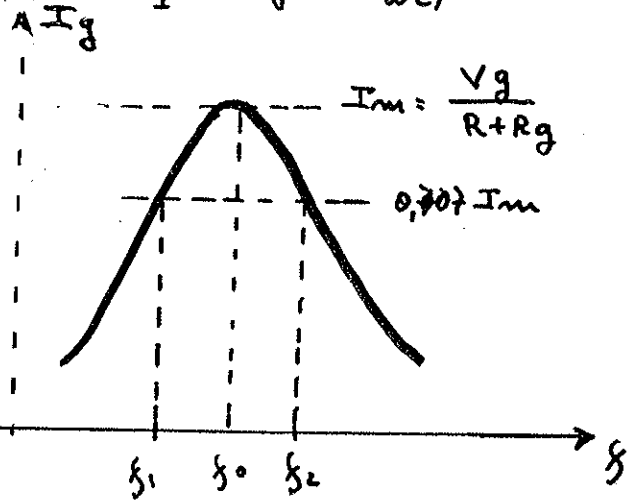
II. CIRCUITOS RESONANTES - NOÇÕES

As cavidades ressonantes podem ser representadas por dois tipos de circuitos equivalentes ressonantes:

CIRCUITO SÉRIE

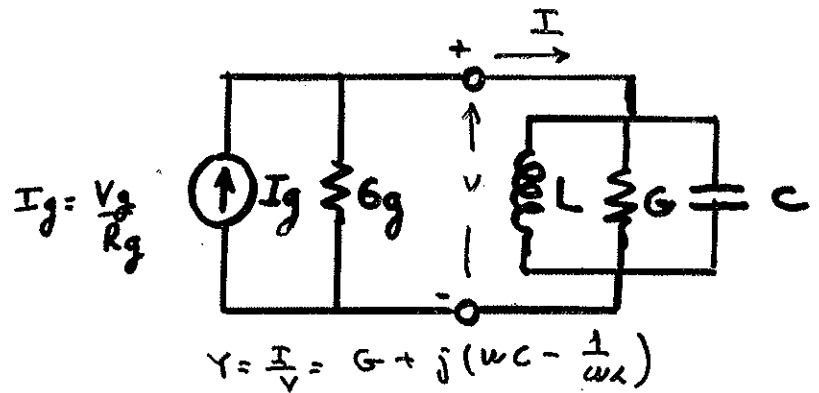


$$Z = \frac{V}{I} = R + j(\omega L - \frac{1}{\omega C})$$



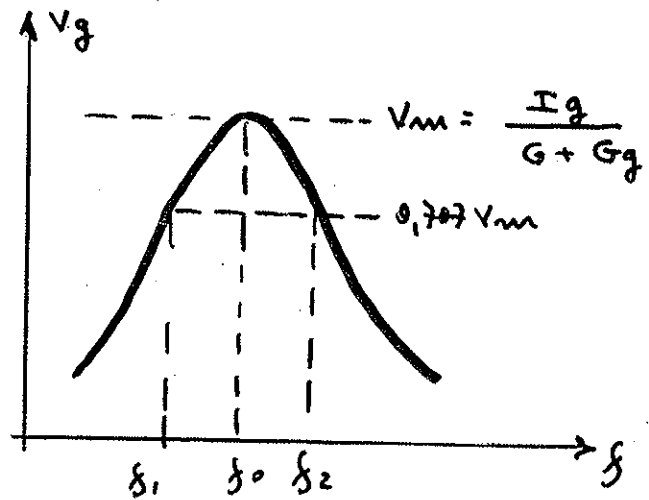
Frequência de ressonância

CIRCUITO PARALELO



$$I_g = \frac{V_g}{R_g}$$

$$Y = \frac{I}{V} = G + j(\omega C - \frac{1}{\omega L})$$



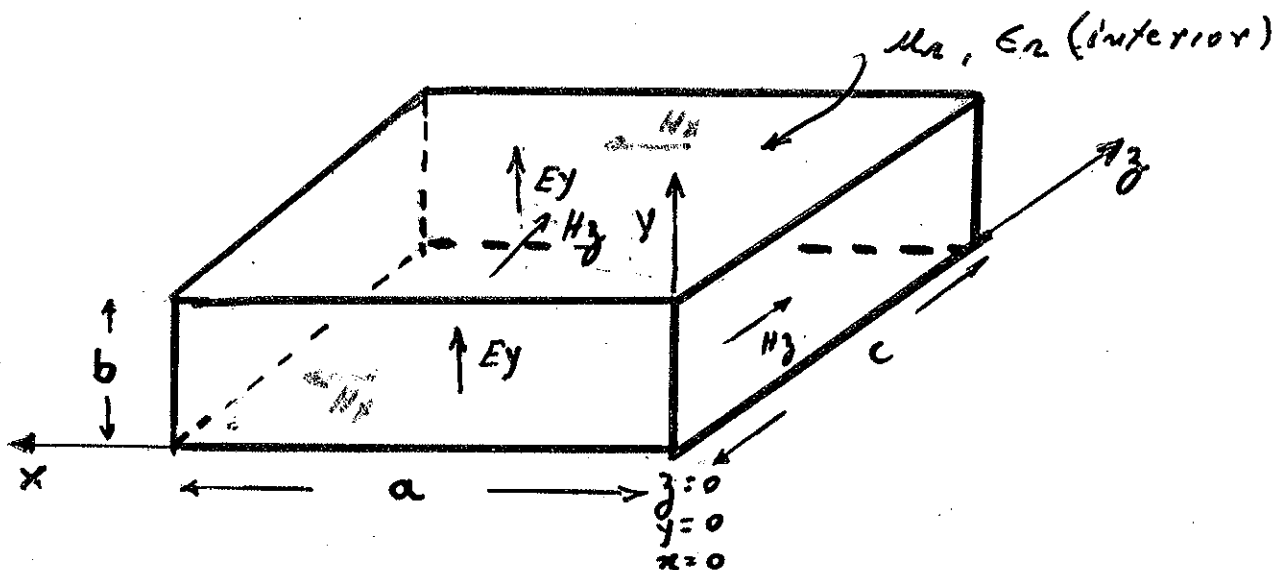
$$f_0 = f_2 = \frac{1}{2\pi\sqrt{LC}}$$

III - CAVIDADES RETANGULARES

III.1 - INTRODUÇÃO -

As cavidades retangulares podem ser consideradas como guias retangulares cujas extremidades estão em curto

CAVIDADE RETANGULAR - coordenadas cilíndricas - $a \times b \times c$



Para uma melhor compreensão dos vários modos presentes, imaginemos o mais simples: modo fundamental se propagando no sentido z
 expressão para o campo elétrico E_y no modo fundamental TE₁₀

$$E_y(x, y, z) = E_{y0+} \cdot \underbrace{\text{sen}\left(\frac{\pi}{a}x\right)}_{\vec{x}} e^{-\gamma z} + E_{y0-} \cdot \underbrace{\text{sen}\left(\frac{\pi}{a}x\right)}_{\vec{z}} e^{\gamma z}$$

Imaginando-se condutor e dielétrico perfeito $\gamma = j\beta$

$$E_y(x, y, z) = E_{y0+} \cdot \text{sen}\left(\frac{\pi}{a}x\right) e^{-j\beta z} + E_{y0-} \cdot \text{sen}\left(\frac{\pi}{a}x\right) e^{j\beta z}$$

$$= \text{sen}\left(\frac{\pi}{a}x\right) (E_{y0+} \cdot e^{-j\beta z} + E_{y0-} \cdot e^{j\beta z})$$

CONDICÕES DE CONTORNO - 1ª parede - $z=0$ - x -qual $E_{y0+} = -E_{y0-}$

pois $E_y(x, y, 0) = 0$

2ª parede - $z=c$ - x -qual $E_y(x, y, c) = 0$

CONDIÇÕES DE CONTORNO - CONTINUAÇÃO -
 SEPARAÇÃO DA CAVIDADE $z=c$

$$EY(x, y, z) = (E_{y_{0t}} \cdot e^{-j\beta z} - E_{y_{0t}} \cdot e^{j\beta z}) \text{sen}\left(\frac{\pi}{a}x\right)$$

$$EY(x, y, z=c) = E_{y_{0t}} \cdot (e^{-j\beta c} - e^{j\beta c}) \text{sen}\left(\frac{\pi}{a}x\right) = 0$$

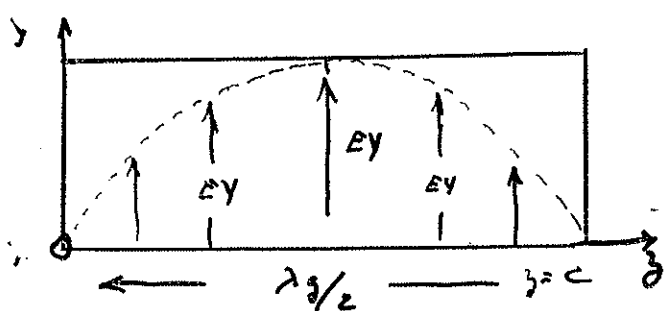
$$= -2j E_{y_{0t}} \cdot \text{sen}(\beta c) \cdot \text{sen}\left(\frac{\pi}{a}x\right) = 0$$

Soluções: $\beta c = p\pi$ ($p=1, 2, 3, \dots$) $p \neq 0$

EXPRESSIONO GERAL

$$EY(x, y, z) = -2j E_{y_{0t}} \cdot \text{sen}\left(\frac{\pi}{a}x\right) \cdot \text{sen}\left(\frac{p\pi}{c}z\right)$$

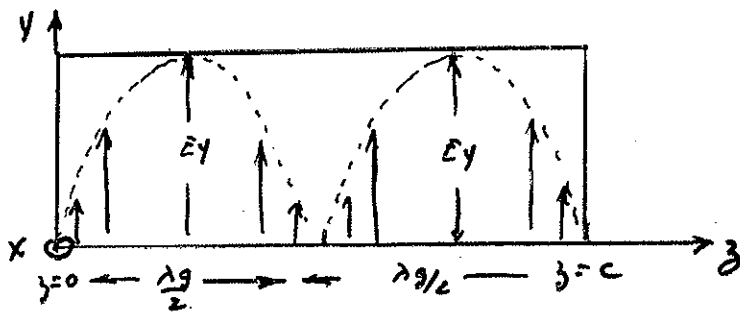
SOLUÇÕES - corte longitudinal - direção z .



1ª RESONÂNCIA

TE₁₀₁

$$\frac{\lambda_g}{2} = c$$



MODO TE_{10p}

2ª RESONÂNCIA -

TE₁₀₂

$$\lambda_g = c$$

é conhecido dos guias retangulares que $\lambda_g = \frac{\lambda_0}{[1 - (f_c/f)^2]^{1/2}}$
 $f_0 \rightarrow$ frequência de ressonância = f_n : $f_c/f_n = \lambda_n/\lambda_c$

$$\lambda_g = \frac{\lambda_n}{[1 - (f_c/f)^2]^{1/2}} = 2c \text{ (modo TE}_{101}\text{)} = \frac{1}{\left[\left(\frac{1}{\lambda_n}\right)^2 - \frac{1}{\lambda_c^2} \cdot \left(\frac{\lambda_n}{\lambda_c}\right)^2\right]^{1/2}}$$

Trabalhando-se a expressão, temos:

$$\left(\frac{1}{\lambda_n}\right)^2 = \left(\frac{1}{2c}\right)^2 + \left(\frac{1}{\lambda_c}\right)^2 \quad \text{- para o modo TE}_{10}\text{- } \lambda_c = 2a$$

$$\left(\frac{1}{\lambda_n}\right)^2 = \left(\frac{1}{2c}\right)^2 + \left(\frac{1}{2a}\right)^2 \quad \text{- sabemos que } v = f_n \cdot \lambda_n = \frac{c}{\sqrt{\mu_1 \epsilon_2}}$$

FREQUÊNCIA DE RESONÂNCIA - MODO TE₁₀₁

Apartir da expressão anterior $\left(\frac{1}{\lambda_n}\right)^2 = \left(\frac{1}{2c}\right)^2 + \left(\frac{1}{2a}\right)^2$

e $f_n \cdot \lambda_n = \frac{c}{\sqrt{\mu_r \epsilon_r}} = \frac{(\text{vel. da luz})}{\sqrt{\mu_r \epsilon_r}}$ teremos

$$\left(f_n\right)_{TE_{101}}^2 = \frac{c^2}{\mu_r \cdot \epsilon_r} \cdot \left[\left(\frac{1}{2a}\right)^2 + \left(\frac{1}{2c}\right)^2 \right] \quad m=1 \quad n=0 \quad p=1$$

FORMULA GERAL:

$$\left(f_n\right)_{TE_{m,n,p}}^2 = \frac{c^2}{\mu_r \epsilon_r} \left[\left(\frac{m}{2a}\right)^2 + \left(\frac{n}{2b}\right)^2 + \left(\frac{p}{2c}\right)^2 \right]$$

Para seguir vamos deduzir o caso geral - MODOS TE_{m,n,p} e TM_{m,n,p}
 ue tem como base as EQUAÇÕES DE ONDA

$$\nabla^2 \vec{E} + k^2 \vec{E} = 0$$

$$k^2 = \omega^2 \mu \epsilon$$

$$\nabla^2 \vec{H} + k^2 \vec{H} = 0$$

III-2. CAVIDADES RETANGULARES-

EXPRESSIONES GERAIS- MODOS TE e TM

Apartir das equações de onda, obtemos em coordenadas retangulares:

$$\frac{\partial^2 \vec{E}}{\partial x^2} + \frac{\partial^2 \vec{E}}{\partial y^2} + \frac{\partial^2 \vec{E}}{\partial z^2} + k^2 \vec{E} = 0 \quad (1)$$

$$\frac{\partial^2 \vec{H}}{\partial x^2} + \frac{\partial^2 \vec{H}}{\partial y^2} + \frac{\partial^2 \vec{H}}{\partial z^2} + k^2 \vec{H} = 0 \quad (2)$$

MODOS TE_{m,n,p} - Imposição $E_z = 0$ e $H_z \neq 0$

Utilizamos a equação de onda (2) para a variável $H_z \neq 0$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \frac{\partial^2 H_z}{\partial z^2} + k^2 H_z = 0 \quad (3)$$

Com o auxílio da técnica de separação de variáveis

$$H_z = X(x) \cdot Y(y) \cdot Z(z) = X \cdot Y \cdot Z \neq 0$$

Obtemos de (3)

$$\frac{1}{X} \cdot \frac{\partial^2 X}{\partial x^2} + \frac{1}{Y} \cdot \frac{\partial^2 Y}{\partial y^2} + \frac{1}{Z} \cdot \frac{\partial^2 Z}{\partial z^2} + k^2 = 0$$

$$-k_x^2 - k_y^2 - k_z^2 + k^2 = 0$$

Logo $k_x^2 + k_y^2 + k_z^2 = k^2 = \omega^2 \mu \epsilon$

SOLUÇÕES:

4ª solução $\Rightarrow \frac{1}{X} \cdot \frac{\partial^2 X}{\partial x^2} = -k_x^2 \quad \therefore X(x) = A \cdot \cos(k_x x) + B \cdot \sin(k_x x)$

Condições de contorno- 6 PAREDES DA CAVIDADE

1) e $x = a \quad \therefore \partial H_z / \partial x = k_x Y = 0 \quad \therefore B = 0$ e $k_x = \frac{m\pi}{a}$

2) e $y = b \quad \therefore \partial H_z / \partial y = k_y X = 0 \quad \therefore D = 0$ e $k_y = \frac{n\pi}{b}$

3) e $z = c \quad \therefore H_z = 0$

EXPRESSION GERAL. H_z - MODO $TE_{m,n,p}$

Como sabemos sempre $H_z = X \cdot Y \cdot Z \neq 0$

e pelas condições de contorno nas 6 paredes da

caixa de

$$\left. \begin{array}{l} B=0 \quad k_x = \frac{m\pi}{a} \\ D=0 \quad k_y = \frac{n\pi}{b} \\ \quad \quad k_z = \frac{p\pi}{c} \end{array} \right\} \begin{array}{l} m=0,1,2,\dots \\ n=0,1,2,\dots \\ p=1,2,3,\dots \quad p \neq 0 \end{array} \quad m+n \neq 0$$

Obtemos para $H_z = H_0 \cdot \cos\left(\frac{m\pi}{a}x\right) \cdot \cos\left(\frac{n\pi}{b}y\right) \cdot \rho \sin\left(\frac{p\pi}{c}z\right)$

Para a obtenção das frequências de ressonância, retomamos a expressão anterior.

$$-k_x^2 - k_y^2 - k_z^2 + k^2 = 0 \quad \therefore k_x^2 + k_y^2 + k_z^2 = k^2 = \omega^2 \mu \epsilon$$

$$\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{c}\right)^2 = (2\pi f_r)^2 \cdot \mu \epsilon$$

$$\left(f_r\right)_{TE_{m,n,p}} = \frac{c}{2\sqrt{\mu_r \epsilon_r}} \left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2 \right]^{1/2}$$

Durante as deduções nos guias retangulares, havíamos adotado $k_x^2 + k_y^2 = k_c^2 \quad \therefore k_c^2 + k_z^2 = k^2$ (caixas)

Continuando a adotar esse mesmo critério é possível exprimir todas as componentes do campo elétrico e magnético $[E_x, E_y, H_x, H_y]$ a partir das equações de Maxwell.

$$E_x = -\frac{j\omega\mu}{kc^2} \cdot \frac{\delta H_z}{\delta y} \quad \text{e} \quad E_y = \frac{j\omega\mu}{kc^2} \cdot \frac{\delta H_z}{\delta x}$$

$$H_x = -\frac{k_z}{kc^2} \cdot \frac{\delta H_z}{\delta x} \quad \text{e} \quad H_y = -\frac{k_z}{kc^2} \cdot \frac{\delta H_z}{\delta y}$$

MODOS TE_{m,n,p} - EXPRESÕES GERAIS

$$H_z = H_0 \cdot \cos\left(\frac{m\pi}{a}x\right) \cdot \cos\left(\frac{n\pi}{b}y\right) \cdot \text{sen}\left(\frac{p\pi}{c}z\right)$$

$$E_x = j \frac{\omega \mu_0 \cdot k_x}{k_c^2} \cdot H_0 \cdot \cos\left(\frac{m\pi}{a}x\right) \cdot \text{sen}\left(\frac{n\pi}{b}y\right) \cdot \text{sen}\left(\frac{p\pi}{c}z\right)$$

$$E_y = j \frac{\omega \mu_0 \cdot k_y}{k_c^2} \cdot H_0 \cdot \text{sen}\left(\frac{m\pi}{a}x\right) \cdot \cos\left(\frac{n\pi}{b}y\right) \cdot \text{sen}\left(\frac{p\pi}{c}z\right)$$

$$H_x = -\frac{k_z \cdot k_x}{k_c^2} \cdot H_0 \cdot \text{sen}\left(\frac{m\pi}{a}x\right) \cdot \cos\left(\frac{n\pi}{b}y\right) \cdot \cos\left(\frac{p\pi}{c}z\right)$$

$$H_y = -\frac{k_z \cdot k_y}{k_c^2} \cdot H_0 \cdot \cos\left(\frac{m\pi}{a}x\right) \cdot \text{sen}\left(\frac{n\pi}{b}y\right) \cdot \cos\left(\frac{p\pi}{c}z\right)$$

Apartir dessas equações torna-se possível desenharmos as configurações de cada campo (elétrico ou magnético) para cada modo, observando $\Rightarrow m=0,1,2,\dots$ $n=0,1,2,\dots$ $p=0,1,2,\dots$

MODOS TM_{m,n,p} - $E_z \neq 0$ e $H_z = 0$

de maneira análoga aos TE_{m,n,p} podemos resolver as equações de onda:

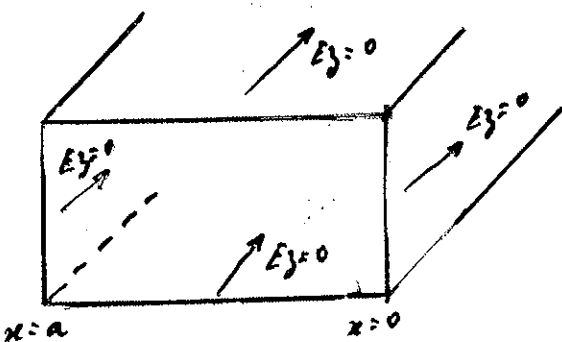
$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} + k^2 E_z = 0$$

com as condições de contorno apropriadas para as 6 paredes internas

$$E_z(x,y,z) = E_0 \cdot \text{sen}\left(\frac{m\pi}{a}x\right) \cdot \text{sen}\left(\frac{n\pi}{b}y\right) \cdot \cos\left(\frac{p\pi}{c}z\right)$$

$$m, n \neq 0$$

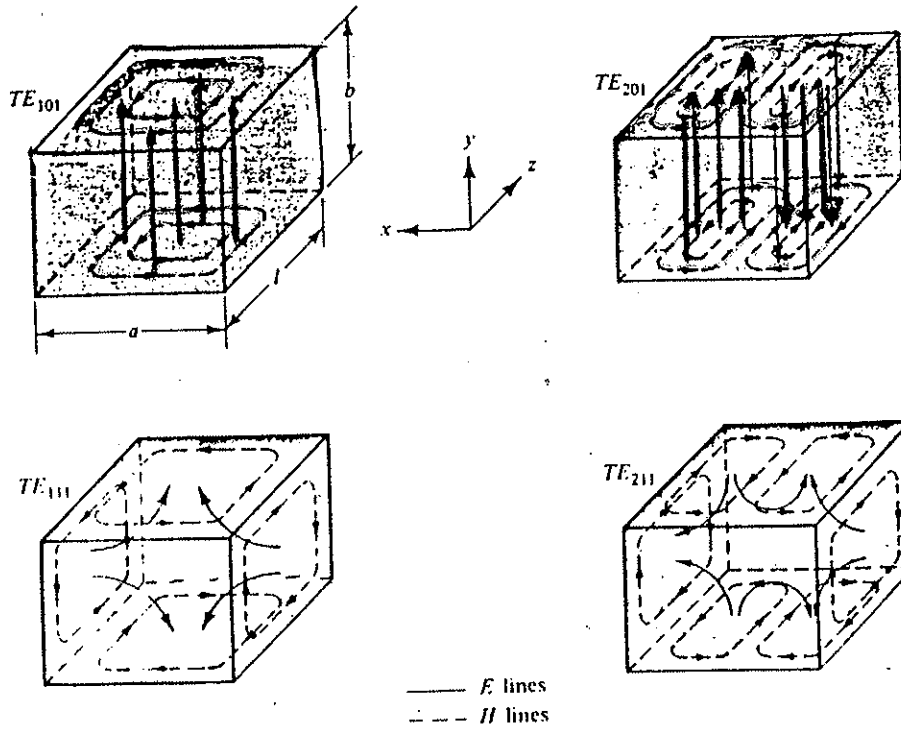
$$p = 0, 1, 2, 3, \dots$$



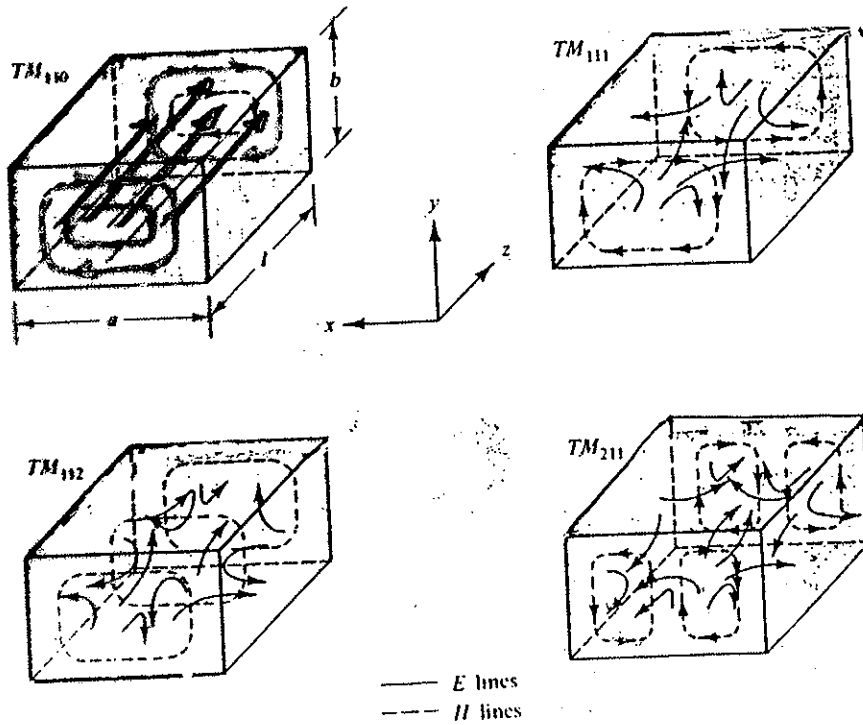
Parede da frente $z=0$

" de trás $z=c$

$$\frac{\partial E_z}{\partial z} = 0$$

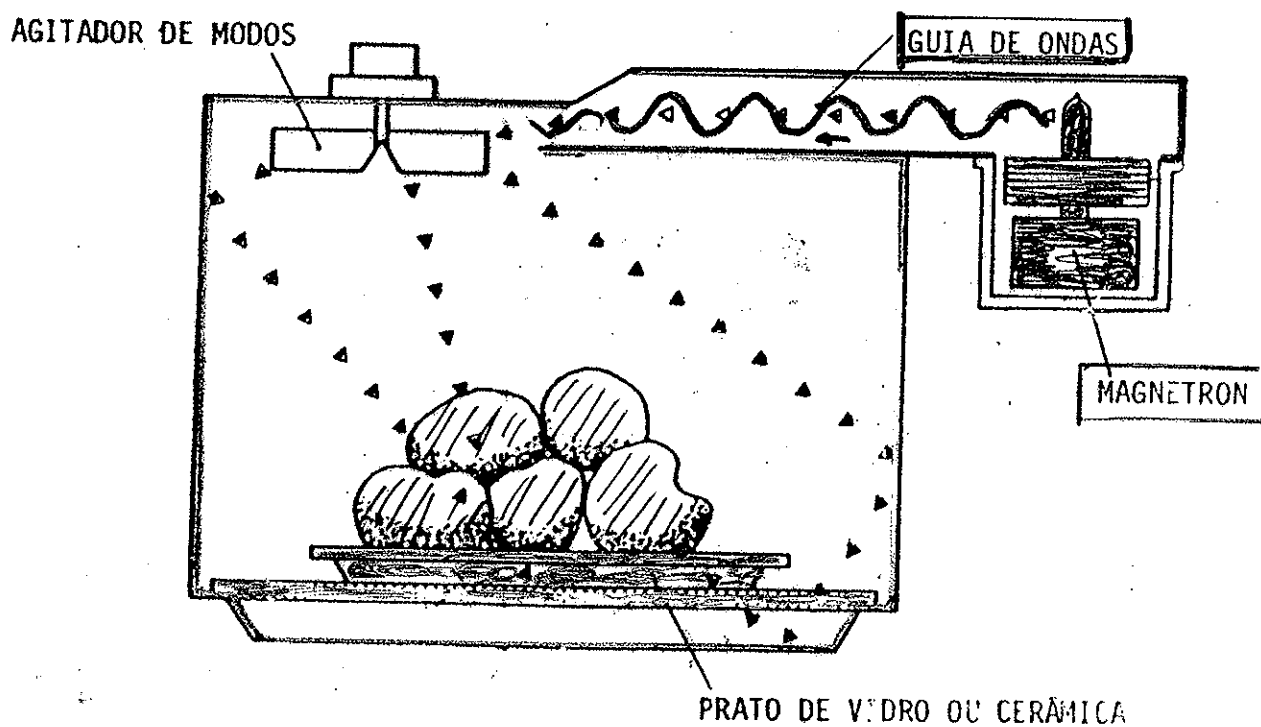
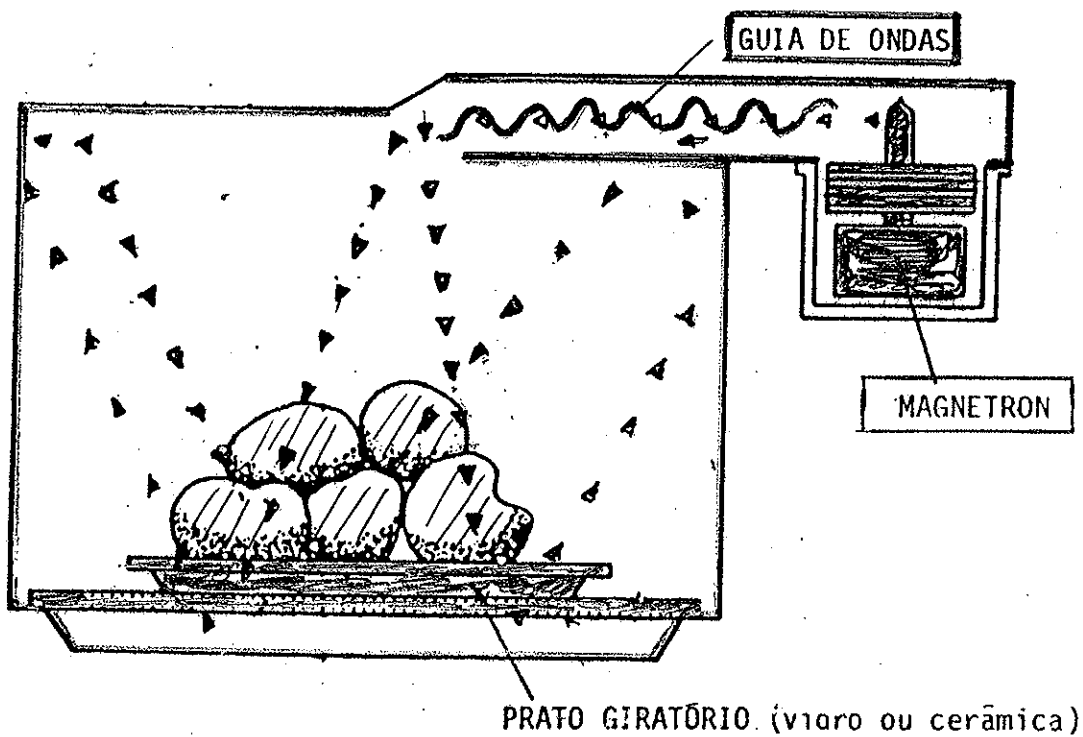


Field patterns for some TE_{mnp} modes in a rectangular cavity.
 (Note: The E and H fields are 90° out-of-phase.)



Field patterns for some TM_{mnp} modes in a rectangular cavity.
 (Note: The E and H fields are 90° out-of-phase.)

FORNO DE MICROONDAS - ESQUEMA GERAL



FORNO DE MICROONDAS

EXEMPLO - CUBO $a=b=c=43,2 \text{ cm}$

Volume: $80.000 \text{ cm}^3 = 80 \text{ L}$

$$f_0 = 15 \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2} = \frac{15}{a} \sqrt{(m)^2 + (n)^2 + (p)^2}$$

para o intervalo $2,4 \text{ GHz} < f_0 < 2,5$

$$\left(\frac{2,4 \cdot 43,2}{15}\right)^2 \leq m^2 + n^2 + p^2 \leq \left(\frac{2,5 \cdot 43,2}{15}\right)^2$$

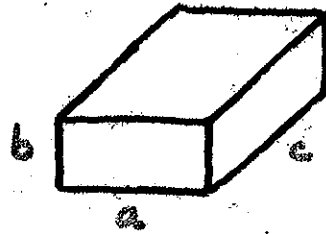
$$47,7 \leq m^2 + n^2 + p^2 \leq 51,84$$

m =	2	1	2	1	3	5	1	1	5	4
n =	4	3	3	4	4	0	0	1	1	6
p =	5	6	6	4	5	5	7	7	5	0
$m^2+n^2+p^2 =$	45	46	49	48	50	50	50	51	51	52

m =	3	3	6	6	2	3	4	3	5	5	5
n =	2	6	3	2	6	4	3	5	4	3	0
p =	6	2	2	3	3	5	5	4	3	4	5
$m^2+n^2+p^2 =$	49	49	49	49	49	50	50	50	50	50	50

CAVIDADES RETANGULARES

FREQUÊNCIAS DE Ressonância



$$b \leq a \leq c$$

\uparrow \uparrow \uparrow
 m m p

1º caso.

$a = b = c$ - CUBO

$$TE_{101} \quad (f_r)_{101} = \frac{v}{2} \left[\left(\frac{1}{a}\right)^2 + \left(\frac{1}{c}\right)^2 \right]^{1/2} = \frac{v}{2} \left[2 \cdot \left(\frac{1}{a}\right)^2 \right]^{1/2} = \frac{v}{2a} \cdot \sqrt{2}$$

$$TE_{011} \quad (f_r)_{011} = \frac{v}{2} \left[\left(\frac{1}{b}\right)^2 + \left(\frac{1}{c}\right)^2 \right]^{1/2} = \frac{v}{2} \left[\left(\frac{1}{a}\right)^2 + \left(\frac{1}{a}\right)^2 \right]^{1/2} = \frac{v}{2a} \sqrt{2}$$

$$TM_{110} \quad (f_r)_{110} = \frac{v}{2} \left[\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2 \right]^{1/2} = \frac{v}{2} \left[\left(\frac{1}{a}\right)^2 + \left(\frac{1}{a}\right)^2 \right]^{1/2} = \frac{v}{2a} \sqrt{2}$$

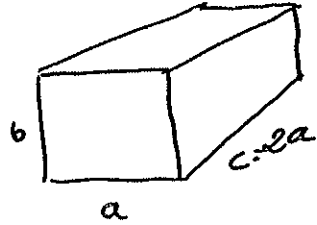
TE_{111} TM_{111}

$$(f_r)_{111} = \frac{v}{2} \left[\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2 + \left(\frac{1}{c}\right)^2 \right]^{1/2} = \frac{v}{2a} \sqrt{3} = 1,22(f_r)_{110} = 1,22(b)_{101}$$

	TE_{101}	TE_{011}	TM_{110}	$\frac{TM_{111}}{TE_{111}}$	TE_{102}
$(f_r)_{min}$	1	1	1	1,22	1,58
$(f_r)_{sem}$					

FREQUÊNCIAS DE RESONÂNCIA - CAVIDADE RETANGULAR

2.º caso - $a = b$ e $c = 2a$

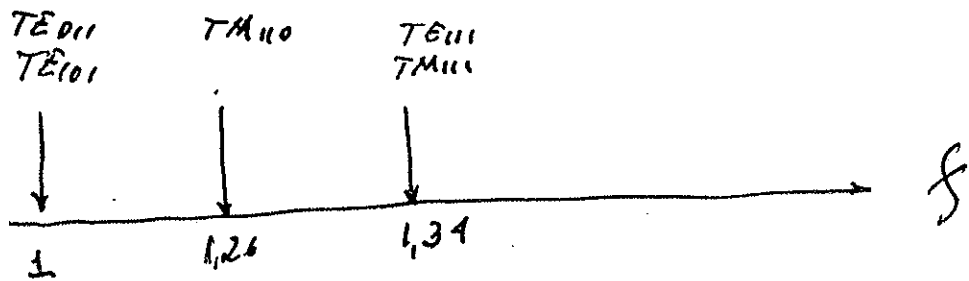


$$(f_1)_{TE_{101}} = \frac{v}{2} \left[\left(\frac{1}{a}\right)^2 + \left(\frac{0}{b}\right)^2 + \left(\frac{1}{c}\right)^2 \right]^{1/2} = \frac{v}{2} \left[\left(\frac{1}{a}\right)^2 + \left(\frac{1}{2a}\right)^2 \right]$$

$$(f_1)_{TE_{101}} = \frac{v}{2a} \sqrt{\frac{5}{4}} = (f_1)_{TE_{011}} = \frac{v}{2a} \times 1,118$$

$$(f_2)_{TM_{110}} = \frac{v}{2a} \left[\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2 + 0 \right]^{1/2} = \frac{v}{2a} \sqrt{2} = \frac{v}{2a} \times 1,414$$

$$(f_3)_{TE_{111}} = (f_3)_{TM_{111}} = \frac{v}{2} \left[\left(\frac{1}{a}\right)^2 + \left(\frac{1}{a}\right)^2 + \left(\frac{1}{2a}\right)^2 \right]^{1/2} = \frac{v}{2a} \cdot \sqrt{\frac{9}{4}} = \frac{v}{2a} \times 1,50$$



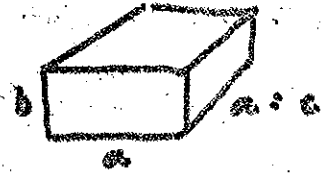
CANALDES RETANGULARES

FREQUENCIAS DE RERONANCIA

3º CASO

$a = 2b$

$c = a$



$$TE_{101} (f_n)_{101} = \frac{v}{2} \left[\left(\frac{1}{a}\right)^2 + \left(\frac{1}{c}\right)^2 \right]^{1/2} = \frac{v}{2} \left[\left(\frac{1}{a}\right)^2 + \left(\frac{1}{a}\right)^2 \right]^{1/2} = \frac{v}{2a} \sqrt{2}$$

$$TE_{011} (f_n)_{011} = \frac{v}{2} \left[\left(\frac{1}{b}\right)^2 + \left(\frac{1}{c}\right)^2 \right]^{1/2} = \frac{v}{2} \left[\left(\frac{2}{a}\right)^2 + \left(\frac{1}{a}\right)^2 \right]^{1/2} = \frac{v}{2a} [4+1]^{1/2}$$




$$TM_{110} (f_n)_{110} = \frac{v}{2} \left[\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2 \right]^{1/2} = \frac{v}{2a} [1+4]^{1/2}$$

$$TE_{111} (f_n)_{111} = \frac{v}{2} \left[\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2 + \left(\frac{1}{c}\right)^2 \right]^{1/2} = \frac{v}{2a} \left[\left(\frac{1}{a}\right)^2 + \left(\frac{2}{a}\right)^2 + \left(\frac{1}{a}\right)^2 \right]^{1/2} = \frac{v}{2a} \sqrt{6}$$

	TE_{101}	TE_{011}	TM_{110}	TE_{111} TM_{111}	TE_{102}
$(f_n)_{101}$	1	1,58	1,58	1,73	1,58
$(f_n)_{min}$					

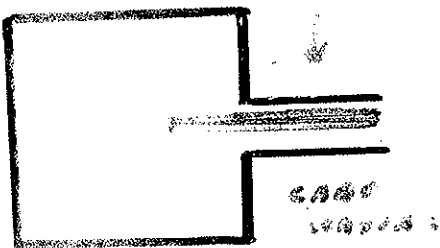
ORDÉM DE OCORRÊNCIA DAS FREQÜÊNCIAS DE RESSONÂNCIA

FREQÜÊNCIA
→

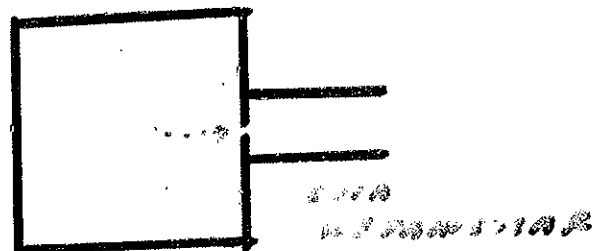
	f_r	f_r	f_r
<p>QUADRADO $c = a$</p> 	<p>TE₁₀₁ TE₀₁₁ TM₁₁₀</p> <p>$f_r = \frac{v}{2a} \sqrt{2}$</p>	<p>TE₁₁₁ TM₁₁₁</p> <p>$f_r = \frac{v}{2a} \sqrt{3}$</p>	
<p>RETO $c = 2a$</p> 	<p>TE₁₀₁-TE₀₁₁</p> <p>$f_r = \frac{v}{2a} \sqrt{\frac{5}{4}}$</p>	<p>TM₁₁₀</p> <p>$f_r = \frac{v}{2a} \sqrt{2}$</p>	<p>TE₁₁₁ TM₁₁₁</p> <p>$f_r = 1,34 f_{r101}$</p>
<p>RETO $a = 2b$ $c = 2a$</p> 	<p>TE₁₀₁</p> <p>$f_r = \frac{v}{2a} \sqrt{2}$</p>	<p>TE₀₁₁</p> <p>$f_r = \frac{v}{2a} \sqrt{5}$</p>	<p>TM₁₁₀</p> <p>$f_r = \frac{v}{2a} \sqrt{6}$</p>

MODOS DE EXCITAÇÃO DAS CAVIDADES

CAVIDADE COAXIAL



IRIS



CAVIDADE RECTANGULAR.

MODO TE101

$$\frac{E_{y_1}}{H_{x_0}} = -Z_{TE} \quad + \quad \frac{E_{y_2}}{H_{x_1}} = Z_{TE}$$

$$H_{x_1} = H_{x_0} + H_{x_2}$$

$$Z_{TE} = j \frac{\omega \mu}{\beta} = \frac{\omega \mu}{\beta}$$

$$H_{x_2} = -\frac{E_{y_1}}{Z_{TE}} + \frac{E_{y_2}}{Z_{TE}} = -\frac{1}{Z_{TE}} [E_{y_1} - E_{y_2}]$$

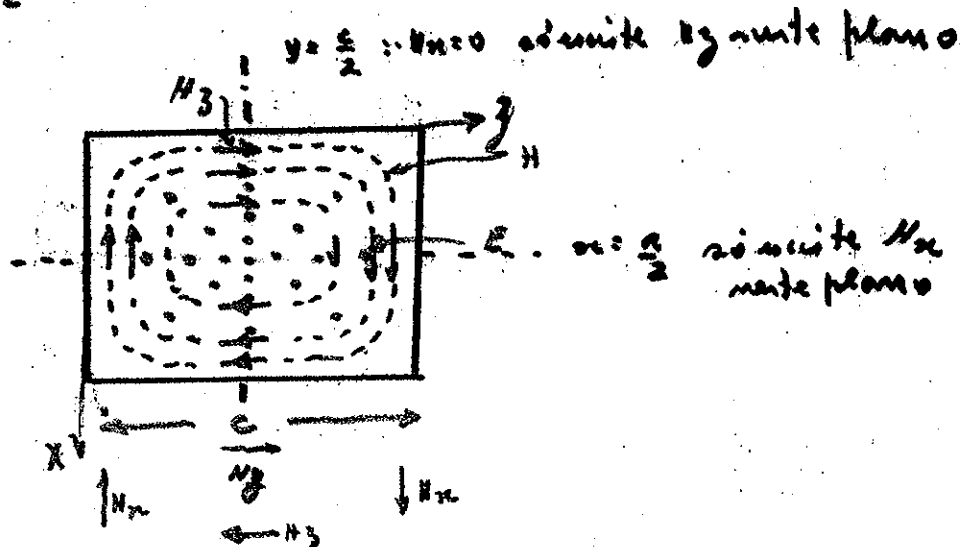
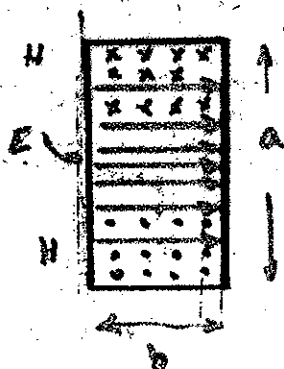
$$H_{x_2} = -\frac{1}{Z_{TE}} [E_{y_0_1} \cdot e^{-j\beta z} - E_{y_0_2} \cdot e^{j\beta z}] \cdot \text{sen}\left(\frac{\beta}{a} x\right)$$

$$H_{x_2} = -\frac{1}{Z_{TE}} E_{y_0_1} [e^{-j\beta z} - e^{j\beta z}] \cdot \text{sen}\left(\frac{\beta}{a} x\right)$$

$$= -\frac{2 E_{y_0_1}}{Z_{TE}} \cdot \cos(\beta z) \cdot \text{sen}\left(\frac{\beta}{a} x\right) = -\frac{2 E_0}{Z_{TE}} \cdot \cos\left(\frac{\beta}{a} z\right) \cdot \text{sen}\left(\frac{\beta}{a} x\right)$$

$$H_{x_2} = -\frac{2 E_0}{\omega \mu} \beta \cdot \text{sen}\left(\frac{\beta}{a} x\right) \cdot \cos\left(\frac{\beta}{a} z\right) \quad \beta = \pi/c$$

$$H_{x_2} = -\frac{2 E_0 \cdot a}{\eta \sqrt{a^2 + c^2}} \cdot \text{sen}\left(\frac{\beta}{a} x\right) \cdot \cos\left(\frac{\beta}{a} z\right)$$



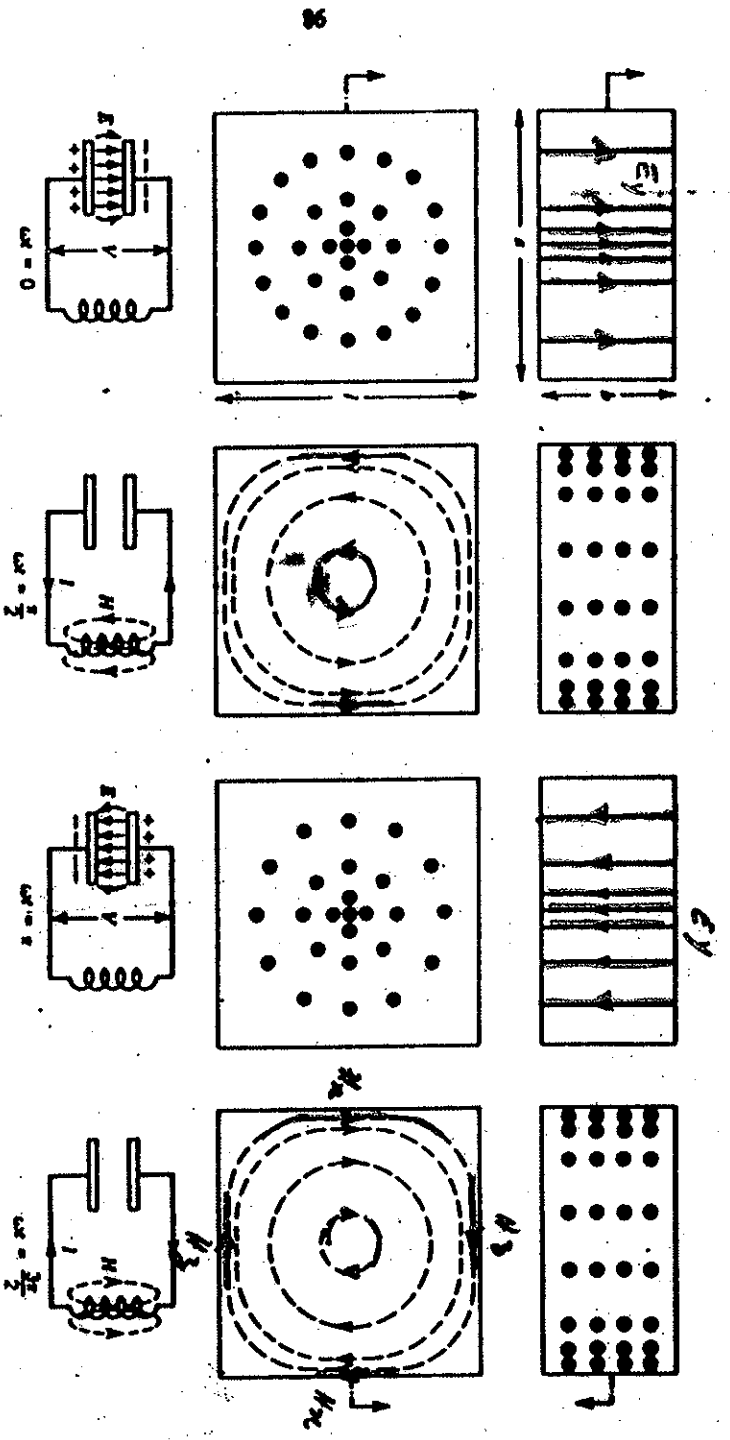


Figure 6.10. Elevation and plan view of E and H fields in rectangular waveguide cavity and the corresponding voltage and current in LC circuit.

FORNO DE MICROONDAS

EXEMPLO DE MODOS

$$X = 29,7 \text{ cm}$$

$$Y = 36,6 \text{ cm}$$

$$Z = 30,2 \text{ cm}$$

$$f_0 = 15 \left[\left(\frac{m}{29,5} \right)^2 + \left(\frac{n}{36,6} \right)^2 + \left(\frac{p}{30,2} \right)^2 \right]^{1/2}$$

TE_{m,n,p}

m	1	1	1	2	2	2	2	2	3
n	1	2	2	1	2	2	3	6	1
p	1	1	2	1	1	2	2	2	3
f ₀ (MHz)	0,818	1,08	1,38	1,19	1,39	1,63	1,87	2,8	2,68

TE_{m,n,p}

m	2	2	3	3	3	3	3	3
n	4	5	1	3	1	3	5	5
p	3	2	2	3	5	2	2	2
f ₀ (MHz)	2,434	2,491	2,443	2,455	3,3	2,18	2,7	

TM_{m,n,p}

m	0	0	0	0	0	0	0	0
n	1	2	3	4	5	6	7	1
p	0	0	0	0	0	0	0	1
f ₀ (MHz)	0,90	0,80	1,2	1,6	2,01	2,459	2,86	0,6

DICE DE MÉRITO - FATOR DE QUALIDADE - DEFINIÇÕES

$$\text{DE MÉRITO} = Q_U = \omega R \cdot \frac{\text{Energia armazenada no circuito ressonante}}{\text{Potência média dissipada no cir. ressonante}}$$

$\omega = \omega_n$

MODELO SÉRIE

$$Q_U = \frac{\omega R \cdot L}{R}$$

MODELO PARALELO

$$Q_U = \frac{\omega R \cdot C}{G}$$

$$\text{DE MÉRITO} = Q_E = \omega R \cdot \frac{\text{Energia armazenada no circuito ressonante}}{\text{Potência média dissipada no circuito externo}}$$

$\omega = \omega_n$

MODELO SÉRIE

$$Q_E = \frac{\omega R \cdot L}{R_g}$$

MODELO PARALELO

$$Q_E = \frac{\omega R \cdot C}{G_g}$$

$$\text{DE MÉRITO} = Q_L = \omega R \cdot \frac{\text{Energia armazenada no circuito ressonante}}{\text{Potência média total dissipada}}$$

$\omega = \omega_n$

Este índice depende do ^{gan}acoplamento entre o circuito externo e o circuito ressonante.

MODELO SÉRIE

$$Q_L = \frac{\omega R \cdot L}{R + R_g}$$

MODELO PARALELO

$$Q_L = \frac{\omega R \cdot C}{G + G_g}$$

ACÕES ENTRE OS ÍNDICES DE MÉRITO

$$\frac{1}{Q_L} = \frac{1}{Q_E} + \frac{1}{Q_U} \quad \therefore \quad Q_L = \frac{f_2}{f_2 - f_1} = \frac{f_2}{\Delta f} = \frac{f_{\text{freq. ressonância}}}{\text{Banda-3dB}}$$

f_2 - frequências onde a tensão ou a corrente corresponde a 0,707 de seu valor máximo = Potência cai pela metade

ÍNDICE DE MÉRITO DE CAVIDADES RETANGULARES

Como exemplo, iniciamos os cálculos com as ondas TE₁₀

Usando as fórmulas do índice de mérito-desacoplado

$$Q = \frac{W_r \cdot \text{Energia armazenada no circuito ressonante}}{\text{Potência média dissipada no circuito ressonante}}$$

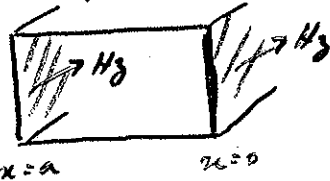
As paredes não são mais condutoras perfeitas, assim sendo a potência média dissipada nas paredes de resistência superficial R_s

$$P_d = \frac{R_s}{2} \int_{\text{paredes}} |H_t|^2 \cdot ds$$

Para cada uma das 6 paredes (2 a 2) é calculado P_d .

PAREDES LATERAIS

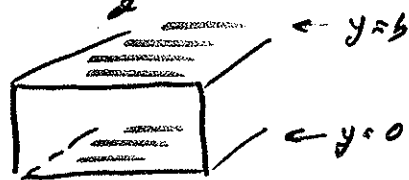
$$H_x = H_z$$



$$P_d = \frac{1}{2} R_s \int |H_z|^2 \cdot ds = \frac{1}{2} R_s \int_0^b \int_0^c H_0^2 \cdot \sin^2\left(\frac{\pi}{c} z\right) \cdot dy dz = \frac{R_s H_0^2 \cdot b \cdot c}{2}$$

Para as duas paredes idênticas (laterais) $P_d = \frac{R_s H_0^2 \cdot b \cdot c}{2}$

PAREDES SUPERIOR E INFERIOR



$$|H_t|^2 = |H_x|^2 + |H_z|^2$$

$$P_d = R_s H_0^2 \left(\frac{R_s}{k_n}\right)^2 \int_0^a \int_0^c \sin^2\left(\frac{\pi}{a} x\right) \cdot \cos^2\left(\frac{\pi}{c} z\right) dx dz + R_s H_0^2 \int_0^a \int_0^c \cos^2\left(\frac{\pi}{a} x\right) \cdot \sin^2\left(\frac{\pi}{c} z\right) dx dz$$

$$= \frac{H_0^2 R_s \cdot a^2}{2} \left[\frac{1}{2} \left(\frac{a}{c}\right) + \frac{1}{2} \left(\frac{c}{a}\right) \right]$$

PAREDES ANTERIOR E POSTERIOR

$$P_d = \frac{R_s a^2}{2} H_0^2 \left[\frac{a \cdot b}{c^2} \right]$$



POTENCIA TOTAL MEDIA DISSIPADA NAS 6 PAREDES

$$P_d = \frac{R_s}{2} \cdot a^2 \cdot H_0^2 \left[\frac{a \cdot b}{c^2} + \frac{b \cdot c}{a^2} + \frac{1}{2} \left(\frac{a}{c} + \frac{c}{a} \right) \right]$$

ENERGIA ARMAZENADA - campo elétrico

$$W_E = \frac{\epsilon}{2} \int_{\text{volume}} |E|^2 \cdot dV = \frac{\epsilon}{2} \int_V |E_y|^2 \cdot dV \quad \text{com } E_y = H_0 \text{ per } (1)$$

$$W_E = \frac{H_0^2 \cdot \epsilon}{2} \left(\frac{\eta \cdot f_2}{f_c} \right) \cdot \frac{a \cdot b \cdot c}{4}$$

ÍNDICE DE MÉRITO DE CARREGADO QV - TE101

$$Q_V = \frac{W_E \cdot W_E}{P_d} = \frac{(2\pi f_2)^2 \left[\frac{H_0^2 \epsilon}{2} \left(\frac{\eta \cdot f_2}{f_c} \right) \frac{a \cdot b \cdot c}{4} \right]}{\frac{R_s}{2} \cdot a^2 \cdot H_0^2 \left[\frac{a \cdot b}{c^2} + \frac{b \cdot c}{a^2} + \frac{1}{2} \left(\frac{a}{c} + \frac{c}{a} \right) \right]}$$

$$Q_V = \frac{\pi \eta}{4 R_s} \left[\frac{2b (a^2 + c^2)^{3/2}}{ac(a^2 + c^2) + 2b(a^3 + c^3)} \right] = \frac{2\pi \eta}{4 R_s} \left[\frac{\text{volume}}{\text{Área da superfície}} \right]$$

EXEMPLO (1) CAVIDADE CUBICA - $a=b=c$

$$Q_V = \frac{\pi \eta \sqrt{2}}{R_s} \quad \text{ar e cobre } \eta = 5,8 \times 10^{-7} \text{ } \Omega \cdot \text{m}$$

$$R_s = 2,6 \times 10^{-2} \text{ } \Omega$$

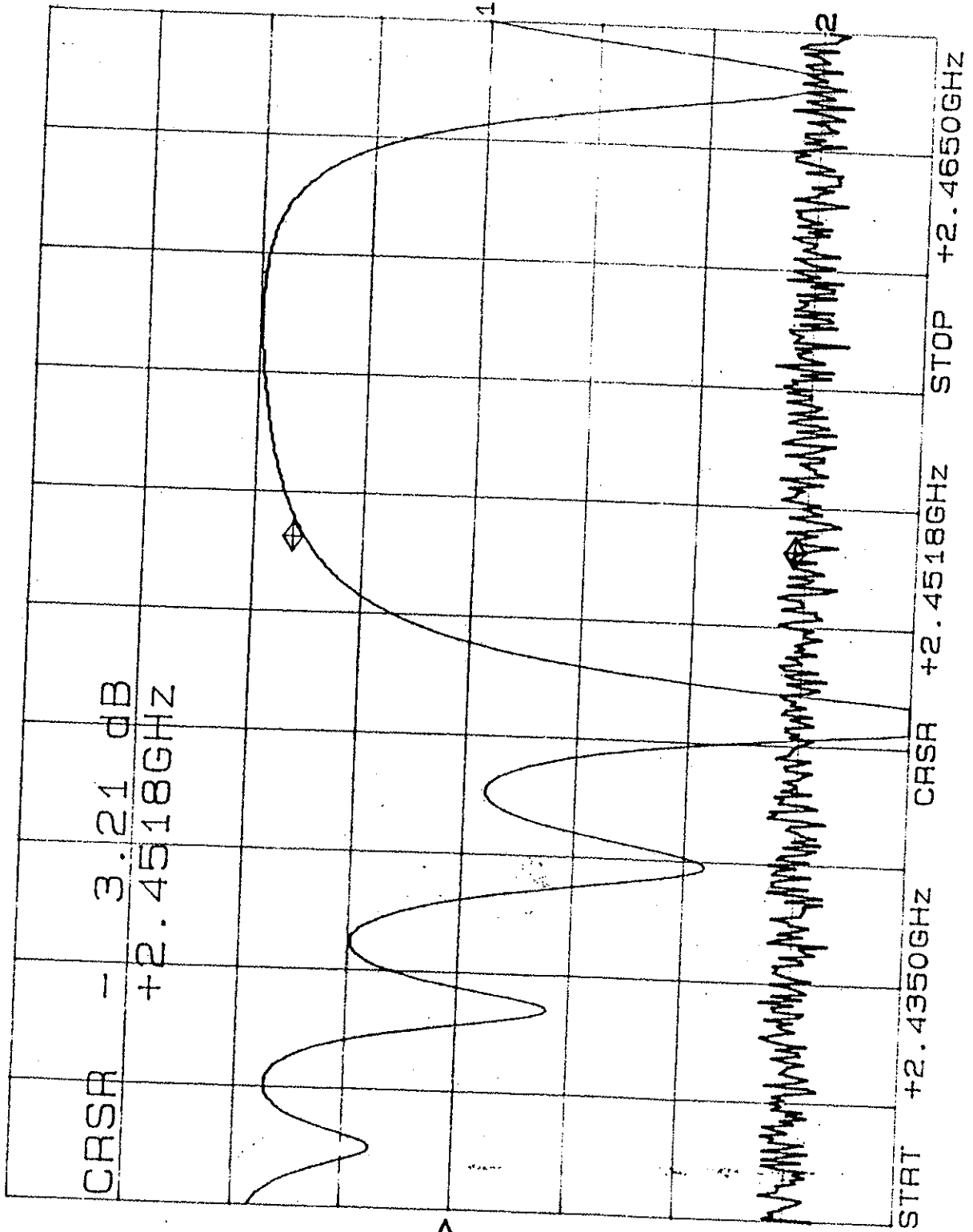
$$f = 10 \text{ GHz} \therefore Q = 10.737$$

(2) CAVIDADE RETANGULAR - $a=2b$ e $a=c$
COBRE - $f=10 \text{ GHz}$ TE101

$$Q_V = \frac{\pi \eta \sqrt{1 + \beta^2}}{2 R_s \left(2 + \frac{a}{b} \right)} = \underline{\underline{8.052}}$$

CH1: A -M REF = 1.98 dB
2.0 dB/ REF = 5.06 dB

CH2: B -M REF = 59.88 dB
20.0 dB/ REF = .00 dB



12 >

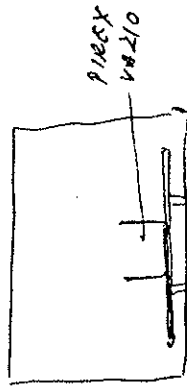
III-22

(7)

83/8/95

FORNO C/SUPORE/ PHOTO

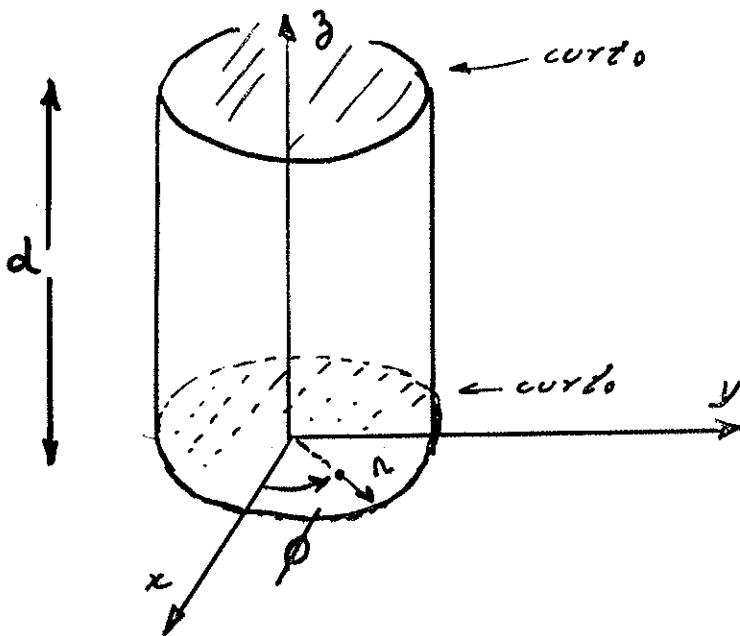
SO FRADCO APPAREX



CAVIDADE CILÍNDRICA

MODOS $TE_{m,l,p}$ - $H_z \neq 0$ e $E_z = 0$

cavidade cilíndrica é formada por um guia
cilíndrico, de raio $a = r$, com curto-circuitos nas
as extremidades (Pontos $z=0$ e $z=d$).



modo idêntico às cavidades retangulares, parte-se da
leis para $H_z(r, \phi)$ proveniente dos guias cilíndricos
e propagando segundo z

$$H_z(r, \phi) = H_0 \cdot J_m(k_c r) \cos(m\phi) \quad \text{com } k_c = \frac{\beta'_{m,l}}{a}$$

al se propagando segundo z (meio sem perdas) $m = 0, 1, 2, \dots$
 $l = 1, 2, 3, \dots$

$$H_z(r, \phi, z) = H_0 \cdot J_m(k_c r) \cdot \cos(m\phi) [H_{0+} \cdot e^{-j\beta z} + H_{0-} \cdot e^{j\beta z}]$$

colocar nos tampas metálicas (curto-circuitos) nos
nos $z=0$ e $z=d$. estabelecer as seguintes
condições de contorno.

II.2 CAVIDADE CILÍNDRICA - CASO GERAL - TEM. 1, p

EQUAÇÃO -

$$\frac{R}{R} \left[\frac{\delta R}{\delta r} + r \frac{\delta^2 R}{\delta r^2} \right] + r^2 k_c^2 - \eta^2 = 0 \quad \times \frac{R}{r^2}$$

$$\frac{1}{r} \frac{\delta R}{\delta r} + \frac{\delta^2 R}{\delta r^2} + R k_c^2 - \frac{\eta^2 R}{r^2} = 0$$

$$\frac{\delta^2 R}{\delta r^2} + \frac{1}{r} \frac{\delta R}{\delta r} + \left(k_c^2 - \frac{\eta^2}{r^2} \right) R = 0 \quad \text{- solução e'}$$

EQUAÇÃO DE BÉSSÉL - soluções.

$$R = F \cdot J_n(k_c r) + G \cdot Y_n(k_c r)$$

EQUAÇÃO - $\frac{\delta^2 \Phi}{\delta \phi^2} + \eta^2 \Phi = 0$

Solução $\Phi = C \cdot \cos(\eta \phi) + D \cdot \sin(\eta \phi)$

CONDIÇÕES DE CONTORNO

1) Para o centro da cavidade $r=0$ H_z - finito
então termo $Y_n(k_c r) \rightarrow \infty \therefore G=0$

$$R = F \cdot J_n(k_c r)$$

2) $\Phi = C \cdot \cos(\eta \phi) + D \cdot \sin(\eta \phi)$. deve ser sempre a mesma
função ϕ / cada rotacao, podemos

escrever $\Phi = C' \cdot \cos(\eta \phi)$

Assim sendo

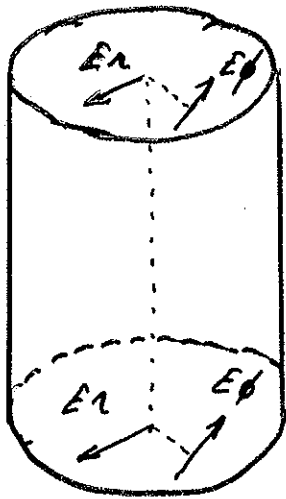
$$H_z = C' \cdot \cos(\eta \phi) \cdot F \cdot J_n(k_c r) \cdot [A \cdot \cos(k_z z) + B \cdot \sin(k_z z)]$$

Para $z=0 \rightarrow H_z=0$ logo $A=0$

$z=d \rightarrow H_z=0$ logo $k_z = \frac{p\pi}{d} \therefore p=1, 2, \dots$

$$H_z(n, p, z) = H_0 \cdot J_n(k_c r) \cdot \cos(\eta \phi) \cdot \sin(k_z z) \quad \text{III-24}$$

1A CILÍNDRICO - MODO TEM_{l,p}.
INDICÕES DE CONTORNO.



← PLANO $z = d$ → $E_\phi = 0$ mas $E_r = -\frac{j\omega\mu}{k_c^2} \frac{\delta H_z}{\delta r} = 0$
 $E_r = 0$ mas $E_\phi = -\frac{j\omega\mu}{k_c^2} \frac{\delta H_z}{\delta \phi} = 0$

← PLANO $z = 0$ → $E_\phi = 0$ contínuo $\therefore -\frac{\delta H_z}{\delta r} = 0$
 $E_r = 0$ " $\therefore -\frac{1}{r} \frac{\delta H_z}{\delta \phi} = 0$

INDICÃO DE CONTORNO - PLANO INFERIOR - $z = 0$

$H_z = 0$! $H_z = \text{cte} = 0 \therefore H_z(r, \phi, 0) = H_0 \cdot J_m(k_c r) \cdot \cos(m\phi) \cdot [H_{0+} e^{-j\beta z} + H_{0-} e^{j\beta z}]$
 $H_{0+} = H_{0-}$

$$H_z(r, \phi, z) = H_0 \cdot J_m(k_c r) \cdot \cos(m\phi) [H_{0+} e^{-j\beta z} - H_{0+} e^{j\beta z}]$$

INDICÃO DE CONTORNO - PLANO SUPERIOR - $z = d$

$$H_z(r, \phi, z=d) = H_0 \cdot J_m(k_c r) \cdot \cos(m\phi) [H_{0+} e^{-j\beta d} - H_{0+} e^{j\beta d}]$$

$$H_z(r, \phi, d) = H_0 \cdot J_m(k_c r) \cdot \cos(m\phi) \cdot H_{0+} (-2j \sin \beta d)$$

logo $\beta d = p\pi \therefore \beta = \frac{p\pi}{d}$ $p = 1, 2, \dots$
 $p \neq 0$

REISSÃO GERAL - TEM_{l,p}

$$H_z(r, \phi, z) = H_0 \cdot J_m(k_c r) \cdot \cos(m\phi) \cdot \sin\left(\frac{p\pi}{d} z\right)$$

$m = 0, 1, 2, \dots$ $l = 1, 2, 3, \dots$ $p = 1, 2, 3, \dots$

• TEM_{l,p} → $k_c = \frac{p\pi}{a} = \frac{1,841}{a}$

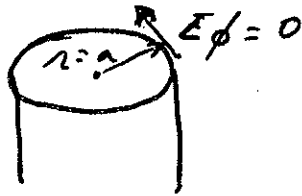
CAVIDADE CILINDRICA - MODOS TEM, l, p

1.3. FREQUÊNCIAS DE RESSONÂNCIA -

Das equações anteriores pode-se tirar a seguinte relação entre as constantes k_c, k e k_z

$$k_c^2 + k_z^2 = k^2 = \omega^2 \mu \epsilon$$

As condições para as guias cilíndricas $k_c = \frac{p'_{m,l}}{a}$ e foi retirada da importância



$$E_\phi = 0 \text{ mas } E_\phi = \frac{\delta H_z}{\delta r} = 0 \therefore J'_m(k_c r) = \frac{J'_m}{J_m} = 0$$

$$J'_m(k_c a) = 0 \therefore p'_{m,l} = k_c a$$

↑
ponto de 0

CONDICION DE RESSONÂNCIA -

$$k_c^2 + k_z^2 = k^2 = \omega^2 \mu \epsilon$$

$$\left(\frac{p'_{m,l}}{a}\right)^2 + \left(\frac{p}{d}\right)^2 = \omega^2 \mu \epsilon = (2\pi f_n)^2 \mu \epsilon$$

$$f_n^2 = \frac{1}{\mu \epsilon} \left[\left(\frac{p'_{m,l}}{2\pi a}\right)^2 + \left(\frac{p}{2\pi d}\right)^2 \right]$$

$$\left(f_n\right)_{TE_{m,l,p}} = \frac{c}{\sqrt{\mu_r \epsilon_r}} \left[\left(\frac{p'_{m,l}}{\pi D}\right)^2 + \left(\frac{p}{2d}\right)^2 \right]^{1/2}$$

$$m = 0, 1, 2, \dots \quad l = 1, 2, 3, \dots \quad p = 1, 2, \dots$$

ATIVIDADE CILINDRICA.

CASO GERAL - MODOS TEM_{l,p}

$$H_z \neq 0 \quad E_z = 0$$

EQUAÇÕES DE ONDA - $\nabla^2 \vec{E} + k^2 \vec{E} = 0$
 $\nabla^2 \vec{H} + k^2 \vec{H} = 0$

MODOS TEM_{l,p} - $H_z \neq 0 \quad E_z = 0$

$$\nabla^2 \vec{H} + k^2 \vec{H} = 0$$

Em coordenadas cilíndricas - direção z.

$$\frac{\partial}{\partial n} \left(n \frac{\partial H_z}{\partial n} \right) + \frac{1}{n^2} \frac{\partial^2 H_z}{\partial \phi^2} + \frac{\partial^2 H_z}{\partial z^2} + k^2 H_z = 0$$

separação de variáveis $H_z = R \cdot \Phi \cdot Z \neq 0$

$$\frac{1}{R} \frac{\partial}{\partial n} \left(n \frac{\partial R}{\partial n} \right) + \frac{1}{n^2 \Phi} \frac{\partial^2 \Phi}{\partial \phi^2} + \frac{1}{Z} \frac{\partial^2 Z}{\partial z^2} + k^2 = 0$$

$$\downarrow -k_c^2$$

$$-k_z^2 + k^2 = 0$$

$$k_c^2 + k_z^2 = k^2$$

EQUAÇÃO - $\frac{1}{Z} \frac{\partial^2 Z}{\partial z^2} = -k_z^2 \therefore Z = A \cdot \cos(k_z z) + B \cdot \sin(k_z z)$

EQUAÇÃO -

$$\frac{1}{R} \frac{\partial}{\partial n} \left(n \frac{\partial R}{\partial n} \right) + \frac{1}{n^2 \Phi} \frac{\partial^2 \Phi}{\partial \phi^2} + k^2 = 0 \quad \times n^2$$

$$\frac{\partial}{\partial n} \left(n \frac{\partial R}{\partial n} \right) + \frac{1}{\Phi} \frac{\partial^2 \Phi}{\partial \phi^2} + n^2 k^2 = 0 \quad \nearrow \frac{n}{R} \frac{\partial}{\partial n} \left(n \frac{\partial R}{\partial n} \right) + n^2 k_c^2 =$$

$$\downarrow = -\frac{1}{\Phi} \frac{\partial^2 \Phi}{\partial \phi^2} = \eta^2$$

CAVIDADES CILINDRICAS

FREQUÊNCIAS DE Ressonância

Para construir uma tabela da frequência de ressonância de cada modo, vamos pegar a frequência de ressonância do modo dominante. Tomemos como parâmetro d/a

1º caso. $\frac{d}{a} = 0,5 :: \frac{a}{d} = 2$

ou $\frac{2a}{d} = 4$



1.1 Modo dominante TM_{010} ($n=0, l=1, p=0$)

$p_{n,l} = p_{0,1} = 2,40$

$$\begin{aligned}
 (f_r)_{TM_{010}} &= \frac{c}{2\pi a} \left[(2,40)^2 + \left(\frac{p_{n,l} \cdot a}{d} \right)^2 \right]^{1/2} \\
 &= \frac{c}{2\pi a} [(2,40)^2] = \frac{c}{2\pi a} \cdot 2,40
 \end{aligned}$$

1.2. ~~TM~~ TE_{111} ($n=1, l=1, p=1$)

$p'_{n,l} = p'_{1,1} = 1,84$

$$\begin{aligned}
 (f_r)_{TE_{111}} &= \frac{c}{2\pi a} \left[(1,84)^2 + \left(\frac{1,84 \cdot a}{d} \right)^2 \right]^{1/2} \\
 &= \frac{c}{2\pi a} \left[(1,84)^2 + (2,2)^2 \right]^{1/2} = \frac{c}{2\pi a} \cdot 6,55
 \end{aligned}$$

$(f_r)_{TE_{111}} = 2,72 (f_r)_{TM_{010}}$

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FREQUÊNCIAS DE RESONÂNCIA

1.3- modo TM_{110} ($n=1, l=1, p=0$)

$p_{n,l} = p_{0,1} = 3,83$

$$(f_n)_{TM_{110}} = \frac{c}{2D_a} \left[(3,83)^2 + \left(\frac{0,2}{2} \cdot 0\right)^2 \right]^{1/2}$$

$$= \frac{c}{2D_a} \cdot 3,83$$

$$(f_n)_{TM_{110}} = 1,59 (f_n)_{TM_{010}}$$

1.4- modo TM_{011} ($n=0, l=1, p=1$)

$p_{n,l} = p_{0,1} = 2,40$

$$(f_n)_{TM_{011}} = \frac{c}{2D_a} \left[(2,40)^2 + (0,2)^2 \right]^{1/2}$$

$$= \frac{c}{2D_a} \cdot 2,43$$

$$(f_n)_{TM_{011}} = 2,8 (f_n)_{TM_{010}}$$

SEQUÊNCIA PARA $d/a = 0,5 \therefore \frac{a}{d} = 2$

TM_{010}	TM_{110}	TE_{111}	TM_{011}	TE_{211}		
2,00	1,59	2,72	2,80	2,90		

CAVIDADES CILINDRICAS

FREQUENCIAS DE RESONANCIA

1º Caso -

$$\frac{d}{a} = 1$$

$$\frac{a}{d} = 1 \therefore \frac{d^2}{a} = 2$$



2.1- modo dominante

$$TM_{010} \quad (m=0, l=1 \text{ e } p=0)$$

$$p_{m,l} = p_{0,1} = 2,4$$

$$\begin{aligned} (k^2)_{TM_{010}} &= \sum_{m,l,p} \left[(2,40)^2 + \left(\frac{d}{a}\right)^2 \right]^{1/2} \\ &= \sum_{m,l,p} \cdot 2,40 \end{aligned}$$

2.2- modo TE_{111}

$$\begin{aligned} (k^2)_{TE_{111}} &= \sum_{m,l,p} \left[(1,84)^2 + (1)^2 \right]^{1/2} \\ &= \sum_{m,l,p} \cdot 3,69 \end{aligned}$$

$$(k^2)_{TE_{111}} = 1,51 (k^2)_{TM_{010}}$$

2.3- modo TM_{110}

$$(k^2)_{TM_{110}} = \sum_{m,l,p} \cdot 3,83 = 1,59 (k^2)_{TM_{010}}$$

TM_{010}	TE_{111}	TM_{110}	TM_{011}	TE_{211}
1	1,50	1,59	1,63	1,80

CARACTERES CROMÁTICOS

FUNCIÓNES DE REFINANCIA

3º caso. $\frac{d}{a} = d \therefore \frac{a}{a} = \frac{1}{a} \therefore \frac{da}{a} = 1$

3.1. MODA DOMINANTE

$$(f_1)_{TM010} = \frac{c}{2da} [2,40]$$

3.2. MODA TERN

$$\begin{aligned} (f_1)_{TE111} &= \frac{c}{2da} \left[(1,34)^2 + \left(\frac{1,34}{a} \cdot a\right)^2 \right]^{1/2} \\ &= \frac{c}{2da} \left[(1,34)^2 + \left(\frac{c}{a}\right)^2 \right]^{1/2} \\ &= \frac{c}{2da} \cdot [2,40] \end{aligned}$$

$(f_1)_{TE111} = (f_1)_{TM010}$: 2 Modos Dominant

3.3. MODA TMO11

$$\begin{aligned} (f_1)_{TM011} &= \frac{c}{2da} \left[(2,40)^2 + \left(\frac{c}{a}\right)^2 \right]^{1/2} \\ &= \frac{c}{2da} \cdot [2,37] \end{aligned}$$

$(f_1)_{TM011} = 1,19 (f_1)_{TM010} = 1,19 (f_1)_{TE111}$

TM010	TE111	TMO11	TE211	TE112
1,07	1,00	1,19	1,42	1,50

CAVIDADES CILINDRICAS

FREQUÊNCIAS DE RESONÂNCIA

9.º CASO

$$\frac{d}{a} = 3$$

$$\frac{a}{d} = \frac{1}{3}$$

$$\frac{2a}{d} = \frac{2}{3}$$

$$\left(\frac{2a}{d}\right)^2 = 0,44$$



9.1- MODO TM010

$$(f_n)_{TM010} = \frac{c}{2\pi a} \cdot [2,40]$$

9.2- MODO TE111

$$(f_n)_{TE111} = \frac{c}{2\pi a} \left[(1,24)^2 + \left(\frac{2a}{d}\right)^2 \right]^{1/2}$$

$$= \frac{c}{2\pi a} \left[(1,24)^2 + \left(\frac{2}{3}\right)^2 \right]^{1/2}$$

$$= \frac{c}{2\pi a} [2,12]$$

$(f_n)_{TE111} \rightarrow$ DOMINANTE

$$(f_n)_{TM010} = 1/3 (f_n)_{TE111}$$

TE111	TM010	TM011	TE112	TE113
1,00	1,13	1,24	1,32	1,41

CAVIDADE CILÍNDRICA-

4. FREQUÊNCIAS DE RESONÂNCIA-

20 $TE_{m,l,p}$

MODOS $TM_{m,l,p}$

$$f_r = \frac{c}{\sqrt{\mu_0 \epsilon_0}} \left[\left(\frac{p_{m,l,p}}{\pi D} \right)^2 + \left(\frac{p}{2d} \right)^2 \right]^{1/2}$$

$m = 0, 1, 2, \dots$

$l = 1, 2, 3, \dots$

$p = 1, 2, 3, \dots$

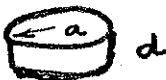



$m = 0, 1, 2, \dots$

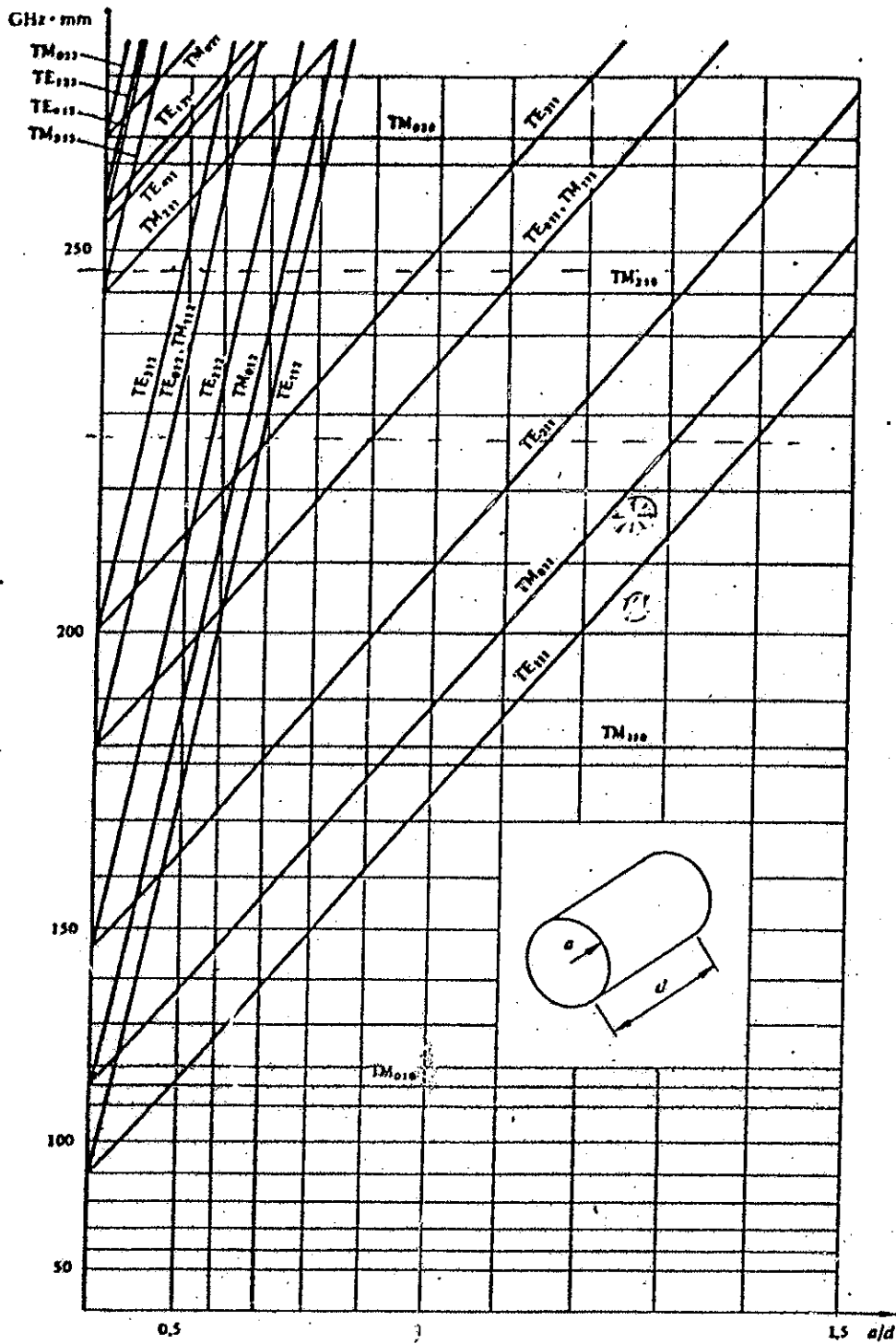
$l = 1, 2, 3, \dots$

$p = 0, 1, 2, 3, \dots$

COMPARAÇÃO DAS FREQUÊNCIAS DE RESONÂNCIA

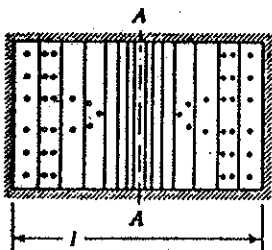
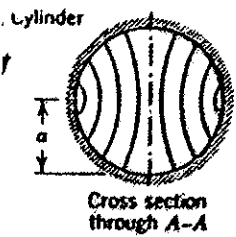
FREQUÊNCIA →

MOD	1º MODO	2º MODO	3º MODO
$\therefore \frac{a}{d} = 2$ 	TM_{010} $p_{0,1} = 2,40$ $f_r = \frac{c}{2\pi a} \cdot 2,40$	TM_{110} $p_{11} = 3,83$ $f_r = \frac{c}{2\pi a} \cdot 3,83$	TE_{111} $p'_{11} = 4,89$ $f_r = \frac{c}{2\pi a} \cdot 6,55$
$\therefore \frac{a}{d} = 1$ 	TM_{010} $f_1 = \frac{c}{2\pi a} \cdot 2,40$	TE_{111} $f_1 = \frac{c}{2\pi a} \cdot 3,64$	TM_{110} $f_1 = \frac{c}{2\pi a} \cdot 3,83$
$\therefore \frac{a}{d} = 0,5$ 	TM_{010} $f_2 = \frac{c}{2\pi a} \cdot 2,40$	TE_{111} $f_1 = \frac{c}{2\pi a} \cdot 2,40$	TM_{011} $f_2 = \frac{c}{2\pi a} \cdot 2,82$
$\therefore \frac{a}{d} = 0,33$ 	TE_{111} $f_r = \frac{c}{2\pi a} \cdot 2,12$	TM_{010} $f_1 = \frac{c}{2\pi a} \cdot 2,40$	TM_{011} $f_2 = \frac{c}{2\pi a} \cdot 2,82$

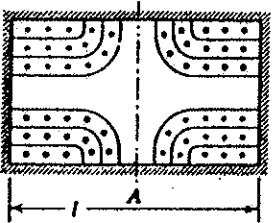
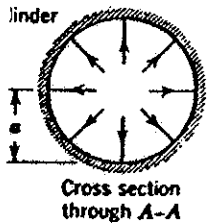


10.08 Circular Cylindrical Resonator

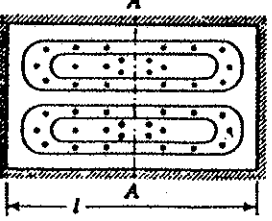
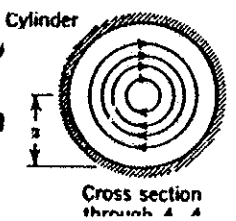
For a circular cylindrical resonator, Fig. 10.08, there are two modes analogous to that first studied for the rectangular box (vertical electric field has a maximum at the center and a minimum at the walls).



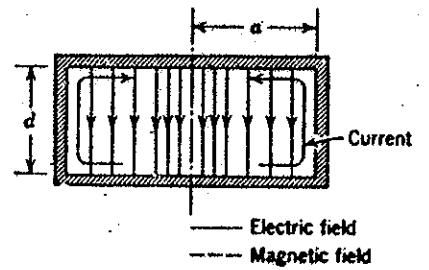
$$\lambda = \frac{2l}{\sqrt{1 + \left(\frac{2l}{3.41a}\right)^2}}$$



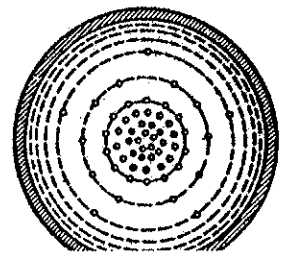
$$\lambda = \frac{2l}{\sqrt{1 + \left(\frac{2l}{2.61a}\right)^2}}$$



$$\lambda = \frac{2l}{\sqrt{1 + \left(\frac{2l}{1.84a}\right)^2}}$$



$$\begin{aligned}
 & TM_{010} \\
 & E_z = E_0 J_0(kr) \\
 & H_\phi = j \frac{E_0}{\eta} J_1(kr) \\
 & k = \frac{p_{01}}{a} = \frac{2.405}{a}
 \end{aligned}$$



CAVIDADES CILÍNDRICAS

FREQUÊNCIAS DE Ressonância

$$k_c^2 + k_z^2 = k^2 = \omega^2 \mu \epsilon$$

$$k_c = \frac{p'_{m,l}}{a} \quad \text{--- modos } TE_{m,l} \quad m=0,1,2,\dots \quad l=1,2,\dots$$

$$k_c = \frac{p_{m,l}}{a} \quad \text{--- modos } TM_{m,l} \quad m=0,1,2,\dots \quad l=1,2,3,\dots$$

$$k_c^2 + k_z^2 = \omega^2 \mu \epsilon$$

MODOS TE

$$\left(\frac{p_{m,l}}{a}\right)^2 + \left(\frac{p\pi}{d}\right)^2 = \omega^2 \mu \epsilon$$

$$p = 0, 1, 2, \dots$$

$$f_r^2 = \frac{1}{(2\pi)^2 \mu \epsilon} \left[\left(\frac{p_{m,l}}{a}\right)^2 + \left(\frac{p\pi}{d}\right)^2 \right]$$

$$f_r^2 = \frac{1}{(2\pi a)^2 \mu \epsilon} \left[(p_{m,l})^2 + \left(\frac{p\pi a}{d}\right)^2 \right]$$

$$f_r = \frac{1 \cdot v}{2\pi a} \left[(p_{m,l})^2 + \left(\frac{p\pi a}{d}\right)^2 \right]^{1/2}$$

MODOS TM

$$\left(\frac{p'_{m,l}}{a}\right)^2 + \left(\frac{p\pi}{d}\right)^2 = \omega^2 \mu \epsilon$$

$$p = 1, 2, 3, \dots$$

$$f_r^2 = \frac{1}{(2\pi)^2 \mu \epsilon} \left[\left(\frac{p'_{m,l}}{a}\right)^2 + \left(\frac{p\pi}{d}\right)^2 \right]$$

$$f_r^2 = \frac{1}{(2\pi a)^2 \mu \epsilon} \left[(p'_{m,l})^2 + \left(\frac{p\pi a}{d}\right)^2 \right]$$

$$f_r = \frac{v}{2\pi a} \left[(p'_{m,l})^2 + \left(\frac{p\pi a}{d}\right)^2 \right]^{1/2}$$

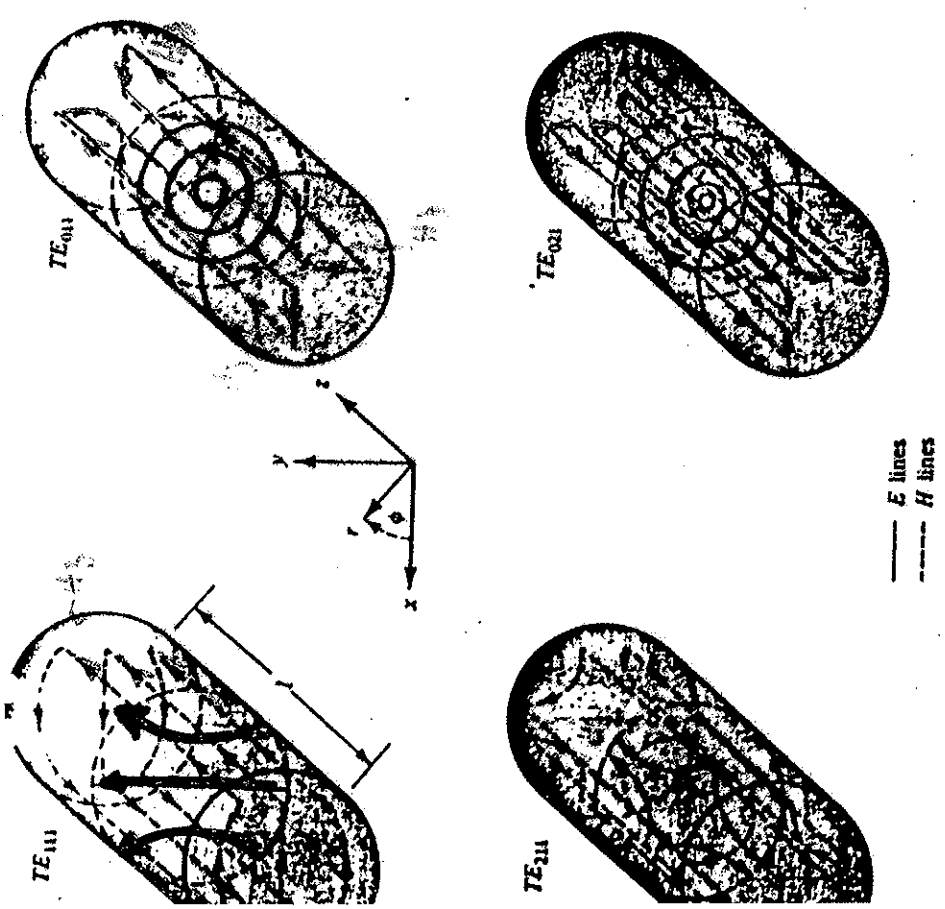


Figure 9-24 Field patterns for some TE_{mp} modes in a cylindrical cavity. (Note: The E and H fields are 90° out-of-phase.)

The field patterns for some of the TM cavity modes are shown in Fig. 9-25. Resonant frequencies and wavelengths are given by Eqs. (9-77) and (9-78) except $K_{mp} = \pi/q_m$. Values of K_{mp} for some of the TM modes are given in Table 9-1. In most cases, these values are different than those of the TE modes. As a result, TE and TM modes are not degenerate, an exception being the TE_{0mp} and TM_{0mp} modes. This follows from the fact that $J'_0(x) = -J_1(x)$. For $D/l \leq 0.99$, the primary mode in a cylindrical cavity is the TE_{111} ; otherwise it's the TM_{010} . The mode is analogous to the TE_{101} mode in a rectangular cavity. There are other similarities. For instance, the TM_{01p} modes are analogous to the $TM_{1,p}$ rectangular cavity modes.

Figure 9-17 illustrates the functional relationship between the resonant frequency and the D/l ratio of the cylindrical cavity. It is known as a mode chart and is shown in Fig. 9-2b. Note that for TM modes with $p = 0$, the resonant frequency is independent of cavity length. TE modes with $p = 0$ do not exist.

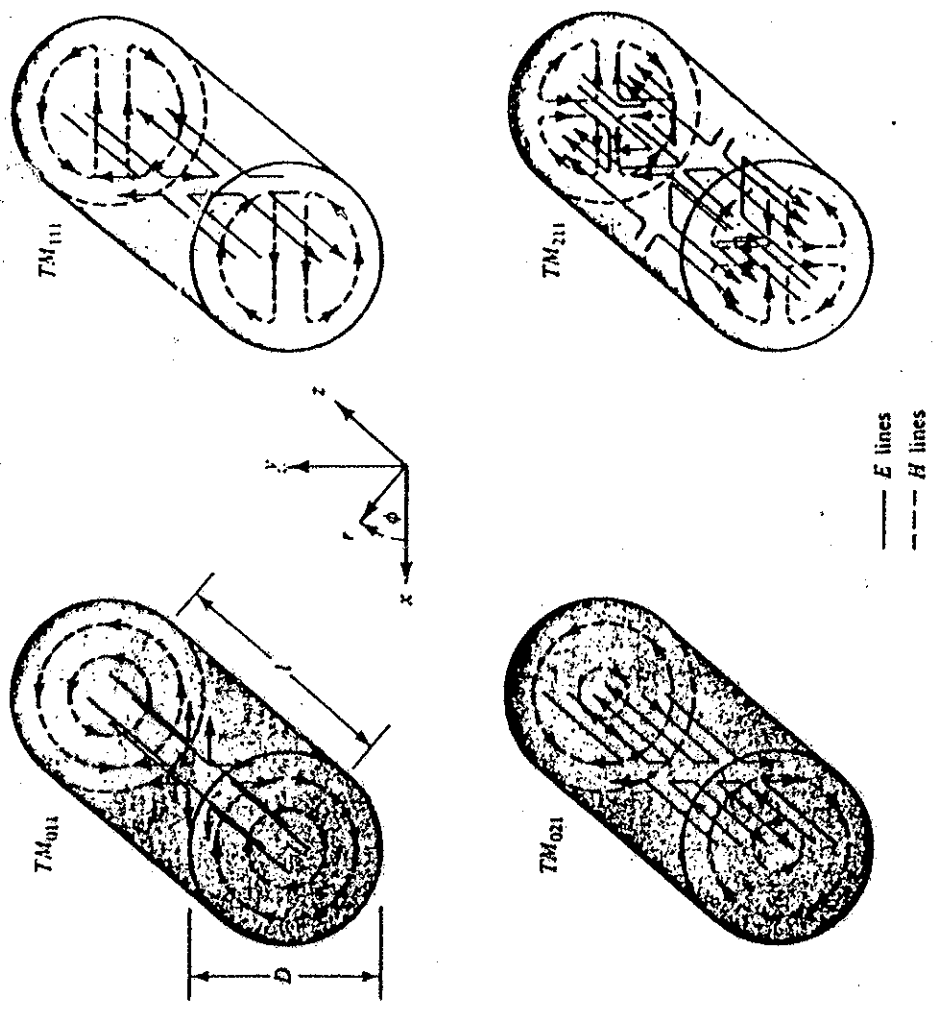


Figure 9-25 Field patterns for some TM_{mp} modes in a cylindrical cavity. (Note: The E and H fields are 90° out-of-phase.)

Unloaded Q of the $TE_{1,p}$ cavity modes. As discussed in Sec. 9-2b, the $TE_{1,p}$ modes in a cylindrical cavity are of considerable engineering interest because of their ease of tuning and high Q values. An expression for Q will now be derived using the field expressions in Eq. (9-79). The analysis proceeds in essentially the same manner as that used for the $TE_{1,p}$ rectangular cavity modes.

With $n = 0$ and $m = 1$, $E = E_0$ and its peak value is given by

$$E_0 = \sqrt{2} H_0 \frac{\omega_r \mu}{k_r} J_0'(k_r r) \sin \frac{p\pi}{l} z$$

where $k_r = 3.832/a$. Therefore,

$$U = \epsilon \left(\frac{\omega_r \mu}{k_r} \right)^2 H_0^2 \int_0^a \int_0^{2\pi} \int_0^l \left\{ J_0'(k_r r) \sin \frac{p\pi}{l} z \right\}^2 r \, dr \, dz \, d\phi$$

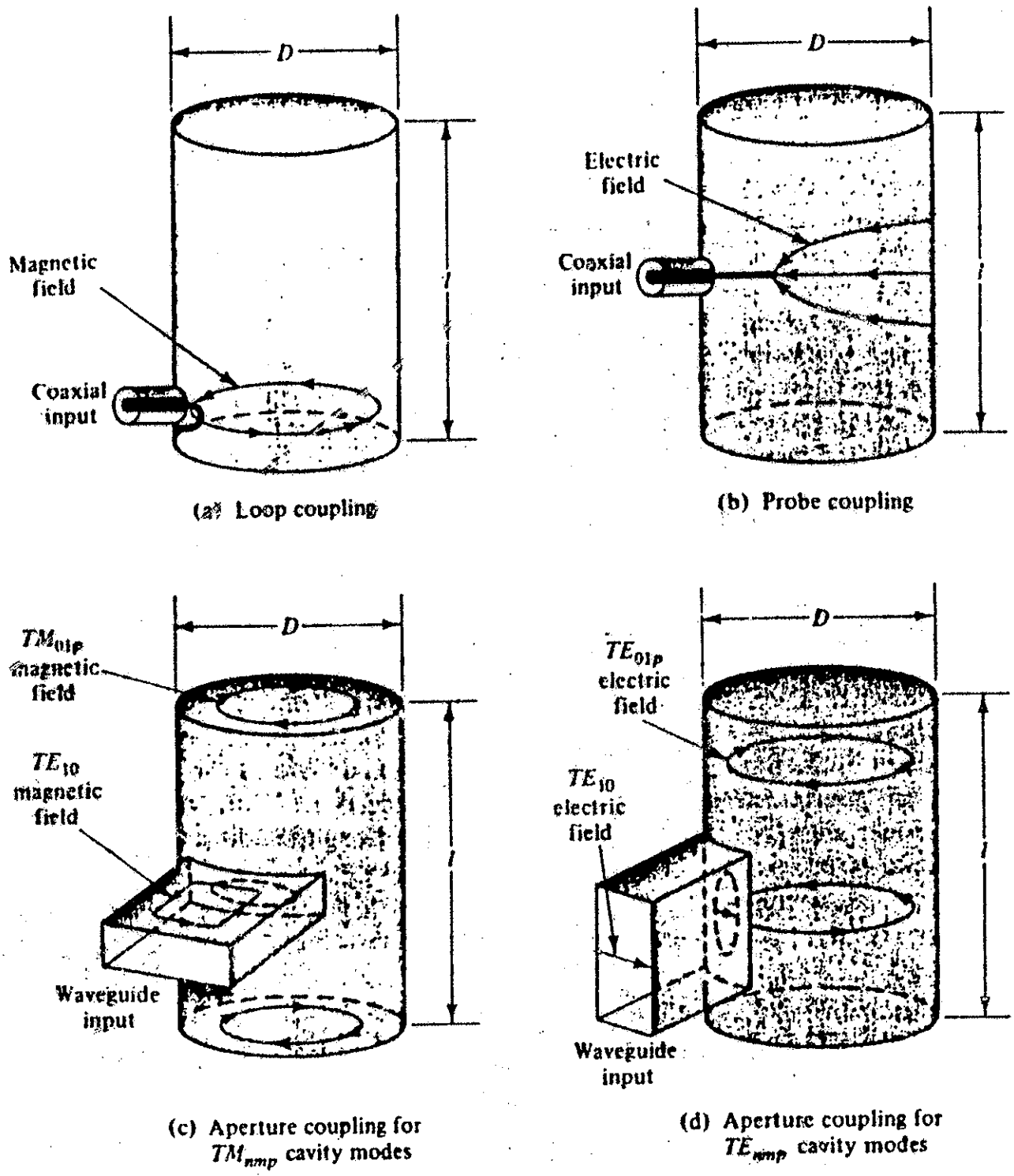


Figure 9-18 Four methods of coupling to a cavity resonator.

CIRCUITOS RESONANTES - Continuação

INDICE DE MÉRITO - FATOR DE QUALIDADE

$$Q_{\text{desenvolvido}} = Q_U = \frac{\omega_r \cdot \text{Energia Armazenada no Circuito Resonante}}{\text{Potência Média Dissipada no Circuito Resonante}}$$

Série

Paralelo

$$Q_U = \frac{\omega_r \cdot L}{R}$$

$$Q_U = \frac{\omega_r \cdot C}{G}$$

$$Q_E = \frac{\omega_r \cdot \text{Energia Armazenada no Circuito Externo}}{\text{Potência Média Dissipada no Circuito Externo}}$$

Série

Paralelo

$$Q_E = \frac{\omega_r \cdot L}{R_g}$$

$$Q_E = \frac{\omega_r \cdot C}{G_g}$$

$$Q_L = \frac{\omega_r \cdot \text{Energia Armazenada no Circuito Resonante}}{\text{Potência Média Total Dissipada}}$$

Por
relações:

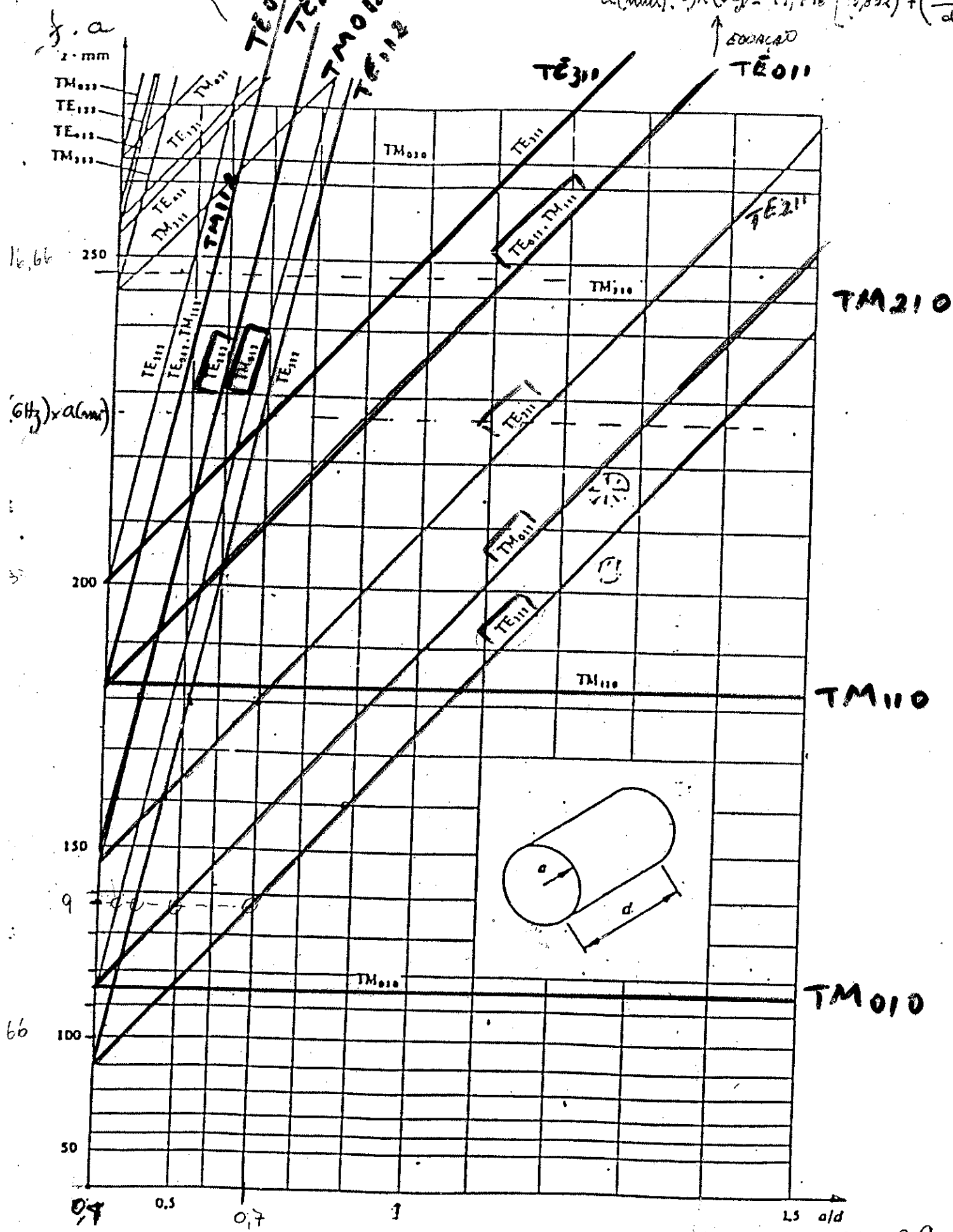
$$Q_L = \frac{\omega_r \cdot C}{G + G_g} = \frac{\omega_r \cdot C}{G + G_g}$$

$$\frac{1}{Q_L} = \frac{1}{Q_E} + \frac{1}{Q_U}$$

$$Q_L = \frac{f_2}{f_2 - f_1} = \frac{\text{Frequência de Ressonância}}{\text{Banda Passante (3dB)}}$$

$$a(\text{mm}) \sqrt{f(\text{GHz})} = 97,796 \left[(1,832)^2 + \left(\frac{1,77 a}{d} \right)^2 \right]^{1/2}$$

$$a(\text{mm}) \sqrt{f(\text{GHz})} = 97,796 \left[(1,832)^2 + \left(\frac{1,77 a}{d} \right)^2 \right]^{1/2}$$



Atenuação em Guias Cilíndricos

$$\alpha(TM_{nl}) = \frac{R_s}{a\eta} \frac{1}{\sqrt{1-(f_c/f)^2}} \text{ nepers/metro}$$

$$\alpha(TE_{nl}) = \frac{R_s}{a\eta} \frac{1}{\sqrt{1-(f_c/f)^2}} \left[(f_c/f)^2 + \frac{n^2}{(p'_{nl})^2 - n^2} \right] \text{ nepers/metro}$$

$$\gamma = \frac{\eta_0}{\sqrt{\epsilon_r}} \therefore \eta_0 = \sqrt{\frac{\eta_0}{\epsilon_0}} = 120\pi \text{ ohms}$$

$$v = \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{\sqrt{\epsilon_r}}$$

Índice de Mérito de Cavidades Retangulares

Modo: TE_{101}

$$Q = \frac{\pi\eta}{4R_s} \left[\frac{2b(a^2 + c^2)^{3/2}}{ac(a^2 + c^2) + 2b(a^3 + c^3)} \right]$$

Modo: TE_{10p}

$$Q = \frac{\pi\eta}{2R_s} \frac{\left\{ 1 + \left(p \frac{a}{c} \right)^2 \right\}^{3/2}}{2 \left\{ 1 + \left(\frac{a}{c} \right) \left(\frac{pa}{c} \right)^2 \right\} + \left(\frac{a}{b} \right) \left\{ 1 + \left(\frac{pa}{c} \right)^2 \right\}}$$

IV. CAVIDADE CILINDRICA

IV.5. INDICE DE MERITO

as cavidades cilíndricas os modos mais empregados são os TE_{01p}, pois dependem dos modos de menor perda nas condutores e maiores índices de merito.

Componentes TE_{01p} → TEM_{mp} $\begin{matrix} n=0 \\ m=1 \\ p=1,2,3,\dots \end{matrix}$

$$H_z = -2jH_0 \cdot J_0(kc r) \cdot \text{sen}\left(\frac{p\pi}{d} z\right)$$

$$H_r = -j \frac{p\pi}{k_c d} \cdot H_0 \cdot J_0'(k_c r) \cdot \text{sen}\left(\frac{p\pi}{d} z\right)$$

$$\begin{aligned} E_z &= k_c \cdot \text{sen}(u\phi) = 0 \\ H_\phi &= k' \cdot \text{sen}(u\phi) = 0 \end{aligned}$$

$$E_\phi = \frac{2\omega\mu}{k_c} \cdot H_0 \cdot J_0'(k_c r) \cdot \text{sen}\left(\frac{p\pi}{d} z\right)$$

$$k_c = 3,832/a$$

Energia armazenada na cavidade

$$W = \epsilon \left(\frac{\omega\mu}{k_c}\right)^2 H_0^2 \int_0^a \int_0^d \int_0^{2\pi} \left\{ J_0'(k_c r) \cdot \text{sen}\left(\frac{p\pi}{d} z\right) \right\}^2 r d\phi dz dr$$

$$W = \frac{\pi}{2} \frac{\eta}{\nu} \cdot H_0^2 \left(\frac{2\pi}{\lambda}\right)^2 \frac{a^2}{k_c^2} \cdot \left[-J_0'(k_c a) \cdot J_2(3,832) \right]$$

P_d = Potencia dissipada nas paredes

$$P_d = H_0^2 R_s \left[\int_0^d \int_0^{2\pi} \left[J_0'(k_c a) \cdot \text{sen}\left(\frac{p\pi}{d} z\right) \right]^2 a \cdot d\phi dz + 2 \int_0^a \int_0^{2\pi} \left[\frac{k_s}{k_c} J_0 \right]^2 r d\phi dr \right]$$

$$P_d = \pi H_0^2 R_s \left[a d J_0'^2(k_c a) + 2 a^2 \left(\frac{p\pi}{d}\right)^2 \frac{a}{3,832} \left[-J_0'(3,832) \cdot J_2(3,132) \right] \right]$$

$$Q_U = \frac{\omega W}{P_d} = \frac{\frac{\eta}{2R_s} \frac{a^2}{k_c^2} \left(\frac{2\pi}{\lambda}\right)^3}{a d + 2 a^2 \left(\frac{p\pi}{d}\right)^2 \frac{a}{3,832}}$$

$$Q_U = \frac{\eta}{2R_s} \frac{\left[(k_c a)^2 + \left(\frac{p\pi a}{d}\right)^2 \right]^{3/2}}{\left[(k_c a)^2 + \frac{2a}{d} \left(\frac{p\pi a}{d}\right)^2 \right]} \quad \therefore Q_U \text{ máximo p/ } d \approx 2a$$

CALCULO DO INDICE DE MERITO - EXEMPLO
CAVIDADE CILINDRICA.

1000 TEO11- $a = 2,875 \text{ cm}$
 $f_n = 7,07 \text{ GHz}$ $k_{ca} = 3,832$

$$VU = \frac{n_0}{2R_s} \left[\frac{(3,832)^2 + \left(\frac{\pi}{2}\right)^2}{(3,832)^2 + \left(\frac{\pi}{2}\right)^2} \right]^{3/2} = 39.500$$

$f_n = 10 \text{ GHz} = R_s = 26 \cdot 10^9$
 cavidade retangular $a = 3 \text{ cm}$ $b = 12 \text{ cm}$

OUTRA FORMA

FORMA ANTERIOR:

$$VU = \frac{n}{2R_s} \left[\frac{(k_{ca})^2 + \left(\frac{\pi a}{d}\right)^2}{(k_{ca})^2 + \frac{2a}{d} \left(\frac{\pi a}{d}\right)^2} \right]^{3/2}$$

$\frac{n}{R_s} = \frac{\sqrt{\mu}}{\epsilon} \Rightarrow \text{como } R_s = \frac{1}{\sigma} \therefore \delta = \frac{1}{\pi f \mu \sigma}$

$l = \frac{\sqrt{\mu}}{\epsilon} \cdot \sigma \cdot \delta = \frac{\sqrt{\mu}}{\epsilon} \cdot \sigma \cdot \delta^2 = \frac{\sqrt{\mu}}{\epsilon} \cdot \sigma$

$= \frac{1}{2 \delta f \sqrt{\mu \epsilon}} = \frac{v}{2 \delta f_n} = \frac{\lambda_n}{2 \delta \pi}$

$$VU = \frac{\lambda_n}{2 \delta \pi} \left[\frac{(k_{ca})^2 + \left(\frac{\pi a}{d}\right)^2}{(k_{ca})^2 + \frac{2a}{d} \left(\frac{\pi a}{d}\right)^2} \right]^{3/2}$$

TE_{0,11}/p
 $m=1$
 $p=(1,2,3)$
 III-42

INDICE DE MÉRITO DE CAVIDADE CILÍNDRICA

MODOS $TM_{m,l,p}$

$$Q_c = \frac{\lambda_n}{2\pi\delta_s} \frac{\left[(p_{m,l})^2 + \left(\frac{p\pi a}{d} \right)^2 \right]^{1/2}}{\left(1 + \frac{2a}{d} \right)} \quad p=0,1,2,\dots$$

$$p=0$$

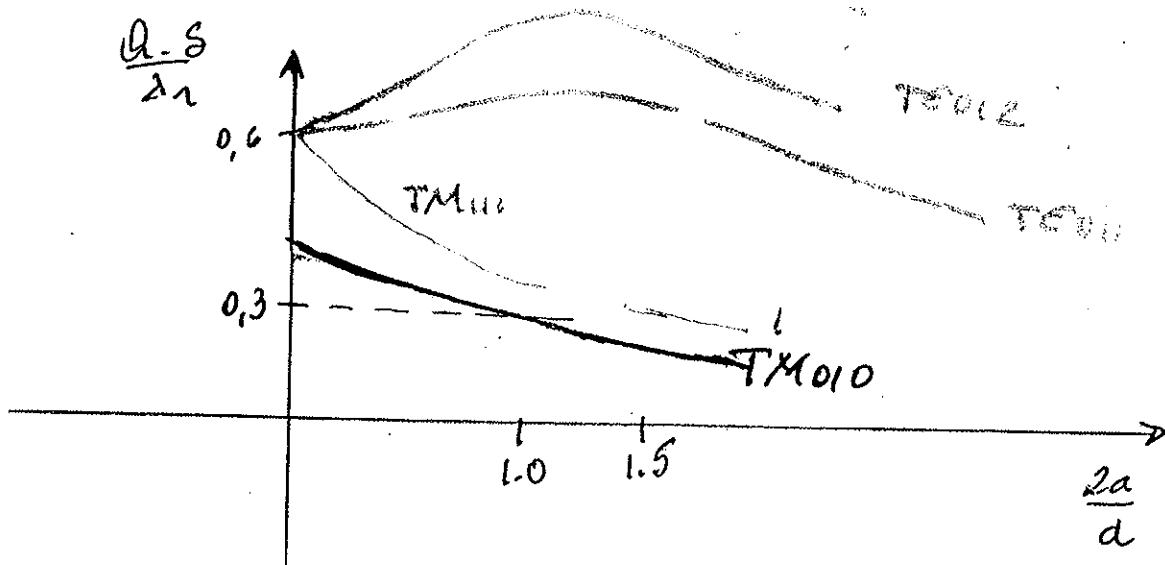
$$Q_c = \frac{\lambda_n}{2\pi\delta_s} \cdot \frac{p_{m,l}}{\left(1 + \frac{2a}{d} \right)}$$

$$\frac{Q_c \cdot \delta}{\lambda_n} = \frac{1}{2\pi} \cdot \frac{p_{m,l}}{\left(1 + \frac{2a}{d} \right)}$$

$$- TM_{010} - p_{m,l} = p_{0,1} = 2,41$$

$$\text{para } \frac{2a}{d} = 1,2$$

$$\frac{Q_c \cdot \delta}{\lambda_n} = \frac{2,41}{2\pi \cdot (1 + 1,2)} = 0,32$$



CAVIDADES CILINDRICAS.

ESTUDO COMPARATIVO ENTRE MODOS TE

Expressão geral para os modos TE_{np}

$$Q_c = \frac{\lambda_0}{2\pi\delta} \frac{\left[1 - \left(\frac{n}{p'm_e}\right)^2\right] \left[(p'm_e)^2 + \left(\frac{p\pi 2a}{2c}\right)^2 \right]^{3/2}}{\left[(p'm_e)^2 + \frac{2a}{c} \left(\frac{p\pi 2a}{2c}\right)^2 + \left(1 - \frac{2a}{c}\right) \left(\frac{p\pi 2a}{2cp'm_e}\right)^2 \right]}$$

2a = D - diâmetros

TE_{01p}

$$\begin{aligned} \frac{Q_c \delta}{\lambda_0} &= \frac{1}{2\pi} \frac{\left[1 - 0\right] \left[(p'o_e)^2 + \left(\frac{\pi D}{2c}\right)^2 \right]^{3/2}}{\left[(p'o_e)^2 + \frac{D}{c} \left(\frac{\pi D}{2c}\right)^2 + \left(1 - \frac{D}{c}\right) (0) \right]} \\ &= \frac{1}{2\pi} \frac{\left[(p'o_e)^2 + \left(\frac{\pi D}{2c}\right)^2 \right]^{3/2}}{\left[(p'o_e)^2 + \frac{D}{c} \left(\frac{\pi D}{2c}\right)^2 \right]} \end{aligned}$$

TE₀₁₁ — $\frac{D}{c} = 1,1$ ($Q_c \approx$ máximos)

$$\frac{Q_c \delta}{\lambda_0} = \frac{\left[(3,832)^2 + \left(\frac{1,1 \cdot \pi}{2}\right)^2 \right]^{3/2}}{2\pi \left[(3,832)^2 + 1,1 \left(\frac{1,1 \cdot \pi}{2}\right)^2 \right]} = 0,657$$

em $f = 7,07 \text{ GHz}$ - onde $Q_c = \frac{0,657 \cdot \lambda_0}{\delta}$

$Q_c = 35.928$

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{1}{\sqrt{\pi \cdot 7,07 \times 10^9 \cdot 4\pi \times 10^{-7} \cdot 58 \times 10^6}} = 7,89 \times 10^{-7} \text{ m}$$

" $0,789 \mu\text{m}$

$$\lambda = \frac{3 \times 10^8}{7,07 \times 10^9} = 4,2857 \times 10^{-2} \text{ m}$$

TE₁₁₁ - EXEMPLO - CAVIDADE CILINDRICA

$$\frac{D}{\lambda} = \frac{2a}{\lambda} = 1,1$$

$$\frac{Q_{c. \delta}}{\lambda_0} = \frac{\left[1 - \left(\frac{1}{1,191}\right)^2\right] \left[(1,841)^2 + \left(\frac{1,1 \cdot \pi}{2}\right)^2 \right]^{3/2}}{2\pi \left[(1,841)^2 + 1,1 \cdot \left(\frac{\pi \cdot 1,1}{2}\right)^2 + (1 - 1,1) \left(\frac{1,1}{2 \cdot 1,191}\right)^2 \right]}$$

$$= 0,275$$

Am 7 GHz - $\lambda_0 = 0,429 \text{ mm}$

$\delta = 7,9 \cdot 10^{-6} \text{ mm}$

$Q_c = 19.933$

TABELA DOS INDICES DE MERITO NORMALIZADOS $p / \frac{D}{\lambda} = 1,1$

p	TE _{11p}	TE _{01p}
	$\frac{Q_{c. \delta}}{\lambda_0}$	$\frac{Q_{c. \delta}}{\lambda_0}$
1	0,275	0,657
2	0,416	0,735
3	0,581	0,902
4	0,753	1,09

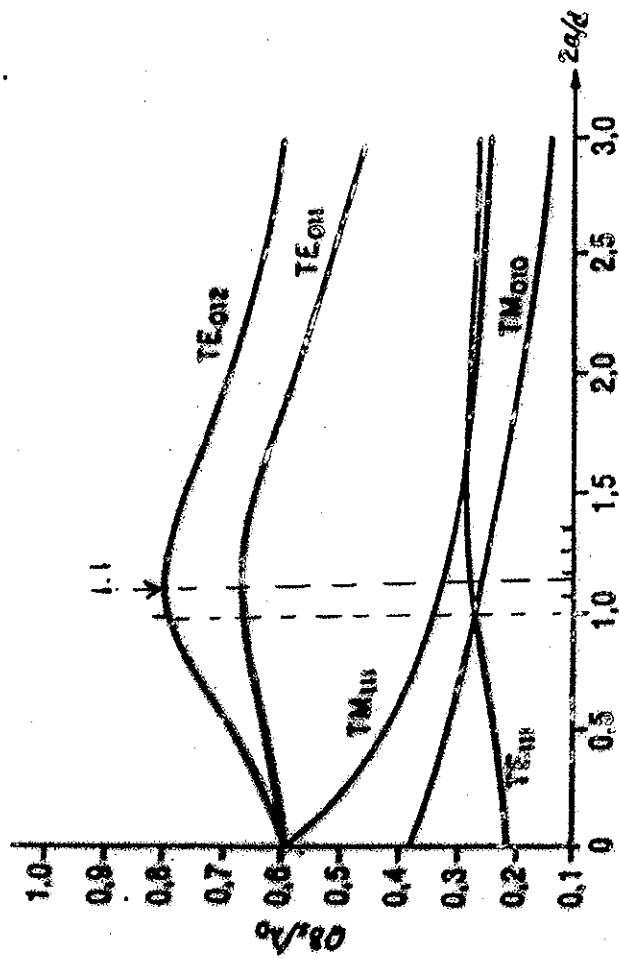


Fig. 7.12 Q de alguns modos da cavidade cilíndrica.

A Fig. 7.12 mostra um gráfico de $Q\delta_1/\lambda_0$ em função de $2a/d$ para vários modos. Note-se o valor consideravelmente mais alto do Q , obtido para o modo TE_{011} em relação ao modo TE_{111} . Um Q ótimo ocorre para $d \approx 2a$. Em $\lambda_0 = 3$ cm, $\delta_1/\lambda_0 = 2.2 \times 10^{-5}$, e assim sendo, da Fig. 7.12, verifica-se claramente que os valores típicos de Q são de 10.000 a 40.000 ou mais. Em $\lambda_0 = 10$ cm, os valores correspondentes de Q seriam $\sqrt{10/3}$ maiores.

7.4 Circuitos equivalentes para cavidades

