

PSI 2434 – MICROONDAS I

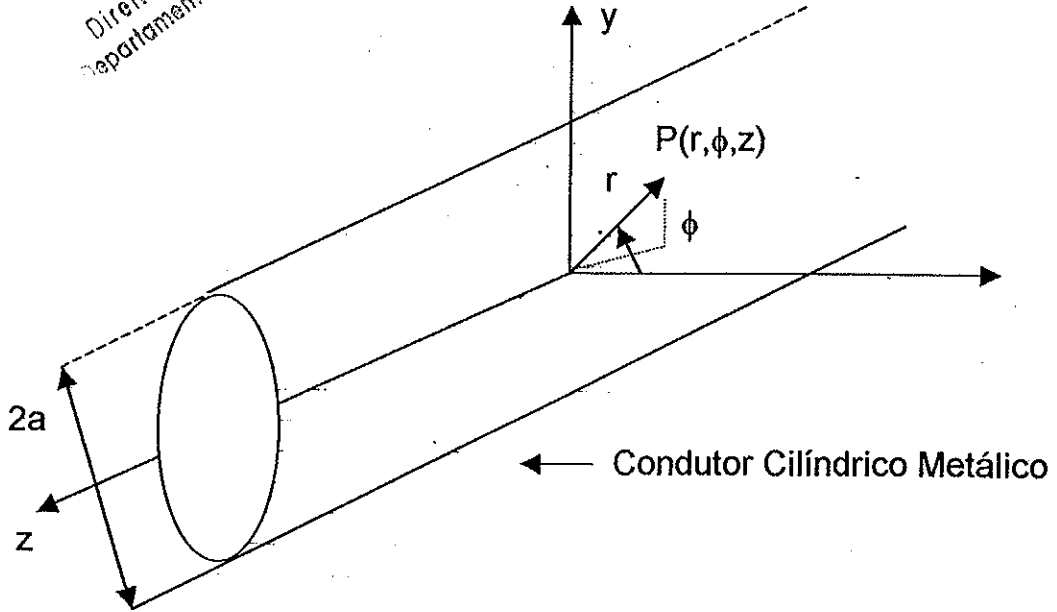
CAPÍTULO II
GUIAS DE ONDA CILÍNDRICO

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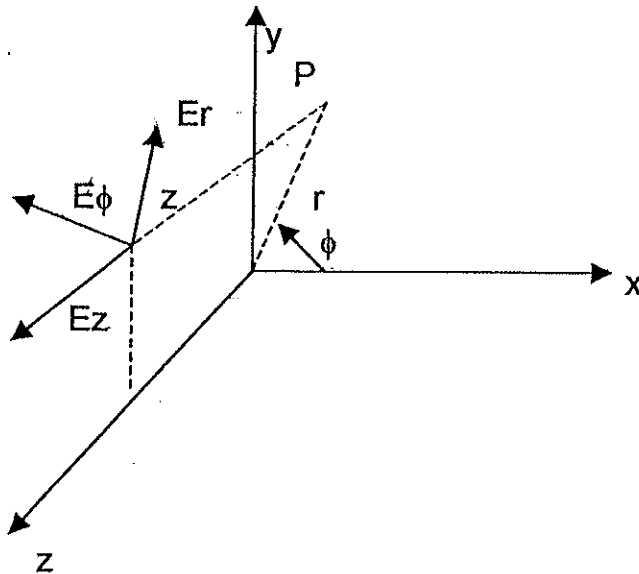
CAPÍTULO II

GUIAS DE ONDA CIRCULAR

COORDENADAS CILÍNDRICAS

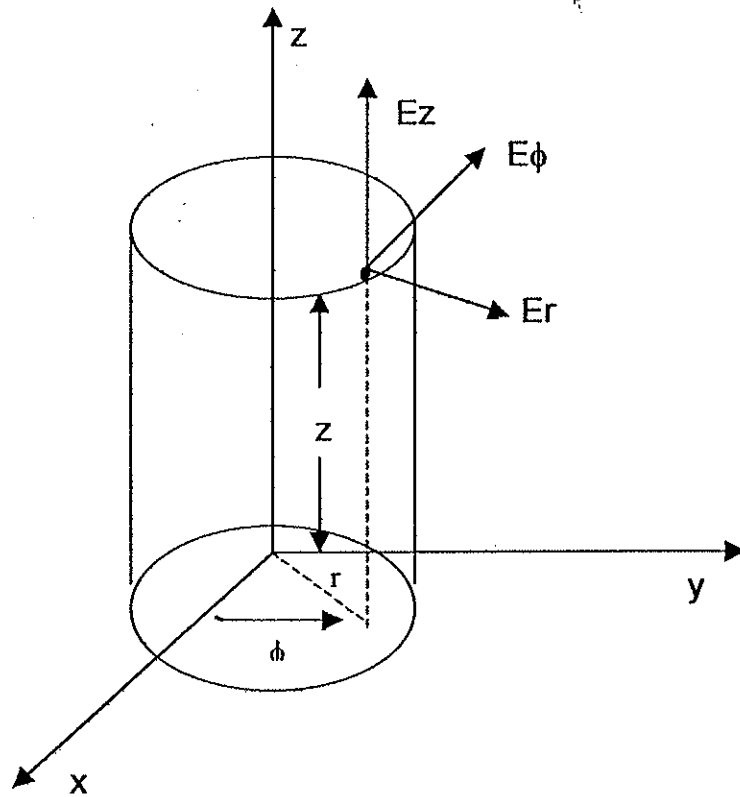


- Campo Elétrico em Coordenadas Cilíndricas



GUIAS CILÍNDRICOS

COORDENADAS CILÍNDRICAS



Equações de onda em coordenadas cilíndricas

$$\nabla^2 \bar{E} + k^2 \bar{E} = 0 \quad \text{e} \quad \nabla^2 \bar{H} + k^2 \bar{H} = 0 \quad \therefore k^2 = \omega^2 \mu \epsilon$$

Propagação segundo o eixo z.

$$\nabla^2 \bar{E} = \nabla_t^2 \bar{E} + \gamma^2 \bar{E} \quad \therefore \nabla_t^2 \text{ Laplaciano Transversal}$$

Equação de onda

$$\nabla^2 \bar{E} + \gamma^2 \bar{E} + k^2 \bar{E} = 0$$

$$\nabla_t^2 \bar{E} + (\gamma^2 + k^2) \bar{E} = 0$$

$$\nabla_t^2 \bar{E} + (\gamma^2 + k^2) \bar{E} = 0$$

EQUAÇÃO DE ONDA EM COORDENADAS CILÍNDRICAS

$$\nabla_t^2 \bar{E} + (\gamma^2 + k^2) \bar{E} = 0 = \nabla_t^2 \bar{E} + k_c^2 \bar{E} = 0$$

$$\nabla_t^2 \bar{H} + (\gamma^2 + k^2) \bar{H} = 0 = \nabla_t^2 \bar{H} + k_c^2 \bar{H} = 0$$

LAPLACIANO TRANSVERSAL EM COORDENADAS CILÍNDRICAS

$$\nabla_t^2 \bar{E} = \nabla_{r,\phi}^2 \bar{E} \quad \text{supondo propagação segundo } z$$

MODO TM

$$E_z \neq 0 \quad \text{e} \quad H_z = 0$$

$$\nabla_{r,\phi}^2 E_z = \frac{\delta^2 E_z}{\delta r^2} + \frac{1}{r} \cdot \frac{\delta E_z}{\delta r} + \frac{1}{r^2} \cdot \frac{\delta^2 E_z}{\delta \phi^2} = -k_c^2 E_z$$

MODO TE

$$H_z \neq 0 \quad \text{e} \quad E_z = 0$$

$$\nabla_{r,\phi}^2 H_z = \frac{\delta^2 H_z}{\delta r^2} + \frac{1}{r} \cdot \frac{\delta H_z}{\delta r} + \frac{1}{r^2} \cdot \frac{\delta^2 H_z}{\delta \phi^2} = -k_c^2 H_z$$

GUIAS CILÍNDRICOS OU CIRCULARES
RESOLUÇÃO DAS EQUAÇÕES DE ONDA

MODO TM $E_z \neq 0$ e $H_z = 0$

Façamos $E_z(r, \phi) = R(r) \cdot F(\phi)$

Lembrando que $E_z(r, \phi, z) = R(r) \cdot F(\phi) \left[G \cdot e^{-\gamma z} \right]$

Como $\nabla_{r,\phi}^2 E_z + k_c^2 E_z = 0$

Teremos:

$$\nabla_{r,\phi}^2 E_z = \frac{\delta^2 E_z}{\delta r^2} + \frac{1}{r} \cdot \frac{\delta E_z}{\delta r} + \frac{1}{r^2} \cdot \frac{\delta^2 E_z}{\delta \phi^2} = -k_c^2 E_z$$

$$= R''(r) \cdot F(\phi) + \frac{1}{r} \cdot R'(r) \cdot F(\phi) + \frac{1}{r^2} \cdot R(r) \cdot F''(\phi) = -k_c^2 E_z$$

$$= R'' \cdot F + \frac{1}{r} R' \cdot F + \frac{1}{r^2} R \cdot F'' = -k_c^2 R \cdot F$$

como $R \cdot F \neq 0$ pois $E_z \neq 0$

$$\frac{\nabla_{r,\phi} E_z}{R \cdot F} = \frac{R''}{R} + \frac{1}{r} \cdot \frac{R'}{R} + \frac{1}{r^2} \frac{F''}{F} = k_c^2 \quad \text{multiplicando-se por } r^2$$
$$= r^2 \cdot \frac{R''}{R} + r \cdot \frac{R'}{R} + \frac{F''}{F} = -k_c^2 r^2$$

ou ainda

$$\underbrace{r^2 \frac{R''}{R} + r \cdot \frac{R'}{R} + k_c^2 r^2}_{\substack{\text{constante} \\ = \eta^2}} = \underbrace{-\frac{F''}{F}}_{\substack{\text{constante} \\ = \eta^2}} = -\frac{\delta^2 E_z(r, \phi)}{E_z \delta \phi^2} \cdot r^2$$

GUIAS CILÍNDRICAS

RESOLUÇÕES DAS EQUAÇÕES DE ONDA

$$r^2 \cdot \frac{R''}{R} + r \cdot \frac{R'}{R} + k_c^2 r^2 = \eta^2 = - \frac{F''}{F}$$

η = constante de separação.

$$r^2 \cdot \frac{R''}{R} + r \cdot \frac{R'}{R} + k_c^2 r^2 = \eta^2 \quad \times \quad R \neq 0$$

$$r^2 R'' + r \cdot R' + k_c^2 r^2 R = \eta^2 R = n^2 R \quad \div \quad r^2$$

$$R'' + \frac{R'}{r} + R \left(k_c^2 - \frac{\eta^2}{r^2} \right) = 0 \quad \left[y'' + \left(1 - \frac{p^2}{x^2} \right) y = 0 \right] \quad \text{-----} \quad \frac{n^2}{k_c^2 r^2} = \frac{p^2}{x^2}$$

Equação Diferencial de Bessel Ordinária p/ $k_c > 0$

$$\text{Soluções} \begin{cases} J_n (J_0, J_1, J_2, \dots) & \rightarrow \text{primeira espécie} \\ N_n (N_0, N_1, N_2, \dots) & \rightarrow \text{segunda espécie} \end{cases}$$

Soluções Gerais

$$\begin{aligned} R(r) &= A \cdot J_n(k_c \cdot r) + B N_n(k_c \cdot r) \\ &= A \cdot J_n(n) + B \cdot N_n(x) \end{aligned}$$

A e B - constantes de integração que dependem das condições de contorno.

EQUAÇÕES LINEARES DE 2ª ORDEM

Equação Geral

$$y'' + p(x) y' + q(x) y = 0$$

$$y'' + p(x) y' + q(x) y = 0 \quad - \text{Equação Homogênea}$$

Solução $y = C_1 y_1 + C_2 y_2$

y_1 e y_2 - são duas soluções particulares linearmente independentes.

Caso seja, determina a solução y_1 , então:

$$y_2 = A y_1 \int \frac{e^{-\int p \cdot dx}}{y_1^2} \cdot dx \quad (A - \text{arbitrário})$$

Exemplo:

$$y'' + xy = 0 \quad \therefore y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots$$

Resolvendo-se, obtém-se

$$y = a_0 \left(1 - \frac{x^3}{2 \cdot 3} + \frac{x^6}{2 \cdot 3 \cdot 5 \cdot 6} - \dots \right) + a_1 \left(x - \frac{x^4}{3 \cdot 4} + \frac{x^7}{3 \cdot 4 \cdot 6} \right) + \dots$$

Equação de Bessel

$$x^2 y'' + xy' + (x^2 - \eta^2) y = 0 \quad (\eta - n^\circ \text{ inteiro})$$

Solução $y = x^\eta (a_0 + a_1 x + \dots)$

ou $y = C_1 J_n(x) + C_2 J_n(x)$

$$J_n(x) = \frac{x^n}{2^n \Gamma(n+1)} \cdot \left(1 - \frac{x^2}{2(2n+1)} + \frac{x^4}{2 \cdot 4 (2n+2)(2n+4)} \dots \right)$$

$$J_n(x) = (-1)^\eta J_n(x) = y_n(x) \quad (\text{Equação de Bessel de } 2^\circ \text{ espécie})$$

EQUAÇÃO DE BESSEL

$$x^2 y'' + x \cdot y' + (x^2 - p^2) y = 0$$

$$\div x^2 \neq 0$$

p - constante

$$y'' + \frac{y'}{x} + \left(1 - \frac{p^2}{x^2}\right) y = 0$$

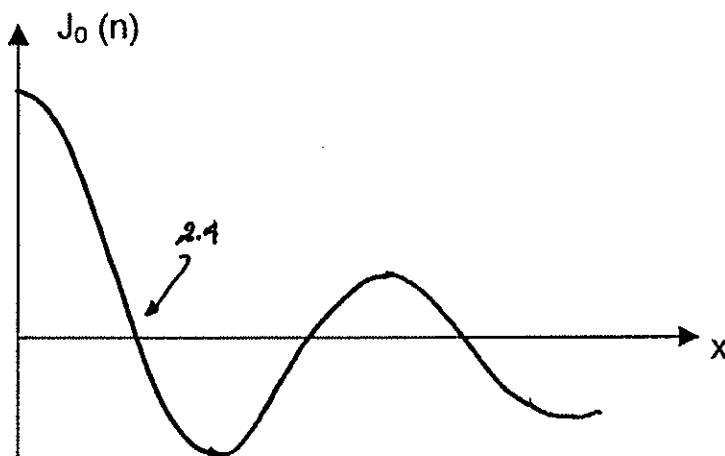
Soluções:

Forma geral:

$$J_p(x) = \sum_{m=0}^{\infty} \frac{(-1)^m \cdot (x)^{2m+p}}{m! (m+p)! 2^{2m+p}}$$

$$p=0 \therefore J_0(x) = \sum_{m=0}^{\infty} \frac{(-1)^m \cdot (x)^{2m}}{m! m! 2^{2m}}$$

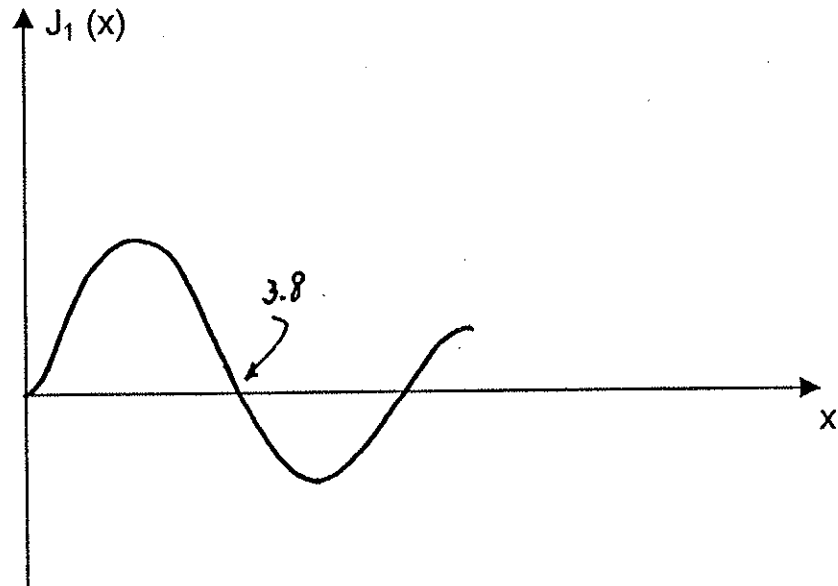
$$J_0(x) = 1 - \frac{x^2}{4} + \frac{x^4}{64} + \dots$$



$$p=1 \therefore J_1(x) = \sum_{m=0}^{\infty} \frac{(-1)^m \cdot (x)^{2m+1}}{m! (m+1)! 2^{2m+1}}$$

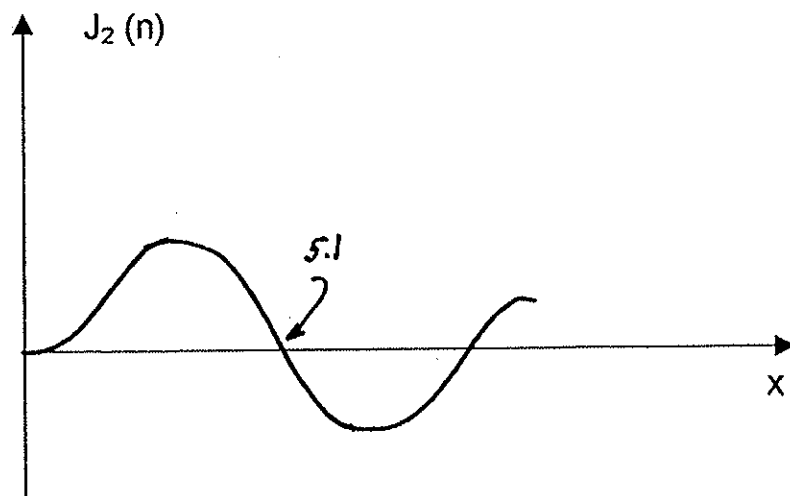
$$J_1(x) = \frac{x}{2} - \frac{x^3}{2 \cdot 2^3}$$

$$J_1(x) = \frac{x}{2} - \frac{x^3}{16} + \dots$$



$$p = 2 \quad J_2(x) = \sum_{m=0}^{\infty} \frac{(-1)^m \cdot (x)^{2m+2}}{m!(m+2)! 2^{2m+2}}$$

$$J_2(x) = \frac{x^2}{2! 4!} - \frac{x^4}{2! 2^4} \dots$$



CONDIÇÕES DE CONTORNO

$$R(r) = A \cdot J_n(k_c r) + B \cdot N_n(k_c r)$$

para $r = 0$

$$R(0) = A \cdot J_n(0) + B \cdot N_n(0)$$

mas $N_n(0) [N_0 \rightarrow \infty; N_1 \rightarrow \infty; N_2 \rightarrow \infty]$

$$N_n(0) \rightarrow \infty$$

Ora $N_n(0) \rightarrow \infty$ daria $R(0) \rightarrow \infty$ e $E_z \rightarrow \infty$, o que não é possível.

A única maneira de contornar esta situação é impor $B = 0$.

Assim sendo, a solução geral seria da forma:

$$R(r) = A \cdot J_n(k_c \cdot r)$$

Até o momento descobrimos o formato para a função $R(r)$. Vejamos agora a função:

$$F(\phi) = F$$

Como
$$\frac{-F''}{F} = \eta^2 \quad \therefore F'' + \eta^2 F = 0$$

$$F''(\phi) + \eta^2(\phi) = 0$$

As soluções são do tipo:

$$F''(\phi) = F\phi = C \cdot \cos(\eta\phi) + D \sin(\eta\phi)$$

SEM PERDA DE GENERALIDADE

$$F_\phi = F = C \cdot \cos(\eta\phi) + D \cdot \sin(\eta\phi) = F(\phi)$$

pode ser escrita na forma

$$F = \sqrt{C^2 + D^2} \cdot \cos(\eta\phi + \varphi)$$

$$F = C' \cdot \cos(n\phi + \varphi) = C' \cdot \cos(n\phi)$$

Como o guia é de secção circular, a função $F_\phi = F(\phi)$, deve se repetir a cada 2π radianos [$F(\delta) = F(\delta + 2\pi)$].

Assim sendo, η deve ser um número inteiro = $\eta = n$

SOLUÇÃO FINAL - MODO TM

$$E_z(r, \phi) = R(r) \cdot F(\phi) = A \cdot J_n(k_{c,r}) \cdot C' \cdot \cos(n\phi)$$

$$= E_0 \cdot J_n(k_{c,r}) \cdot \cos(n\phi) \quad n = 0, 1, 2, \dots$$

ou

$$= E_0 \cdot J_n(k_{c,r}) \cdot \sin(n\phi) \quad n \neq 0$$

ANALOGAMENTE - MODO TE

$$H_z(r, \phi) = A' \cdot J_n(k_{c,r}) \cdot C' \cdot \cos(n\phi)$$

$$= H_0 \cdot J_n(k_{c,r}) \cdot \cos(n\phi) \quad n = 0, 1, 2, \dots$$

SOLUÇÕES - GUIA CILÍNDRICO

MODO TM

$$E_z \neq 0 \quad \text{e} \quad H_z = 0$$

$$E_z(r, \phi) = E_0 \cdot J_n(k_c \cdot r) \cdot \cos(n\phi)$$

CONDIÇÕES DE CONTORNO

Paredes de guia = condutor perfeito

Para $r = a \quad \therefore \quad E_z = 0$

$$0 = E_0 \cdot \underbrace{J_n(k_{ca})}_{=0} \cdot \underbrace{\cos(n\phi)}_{\neq 0}$$

Raízes da Equação de Bessel (p_n, ℓ)

		ℓ			
		1	2	3	4
$J_0 \rightarrow$	0	TM ₀₁ $p_{0,1} = 2,405$	TM ₀₂ $p_{0,2} = 5,520$	TM ₀₃ $p_{0,3} = 8,654$	TM ₀₄ $p_{0,4} = 11,79$
$J_1 \rightarrow$	1	TM ₁₁ $p_{1,1} = 3,832$	TM ₁₂ $p_{1,2} = 7,016$	TM ₁₃ $p_{1,3} = 10,174$	TM ₁₄ $p_{1,4} = 13,32$
$J_2 \rightarrow$	2	TM ₂₁ $p_{2,1} = 5,135$	TM ₂₂ $p_{2,2} = 8,417$	TM ₂₃ $p_{2,3} = 11,62$	TM ₂₄ $p_{2,4} = 14,79$
$J_3 \rightarrow$	3				

FREQUÊNCIAS DE CORTE

MODO TM

$$1^{\circ} \text{ MODO TM}_{01} \quad \therefore \cdot (f_c)_{01} = \frac{2,405}{2\pi a} \cdot c = \frac{1,15}{a \text{ (cm)}} \text{ (GHz)}$$

$$2^{\circ} \text{ MODO TM}_{11} \quad \therefore \cdot p_{1,1} = 3,832$$

$$(f_c)_{11} = \frac{3,832}{2\pi a} \cdot c$$

$$(\lambda_c)_{11} = c \cdot \left(\frac{3,832}{2\pi a} \right)^{-1} = \underline{1,64a}$$

$$3^{\circ} \text{ MODO TM}_{21} \quad \therefore p_{21} = p_{21} = 5,135$$

TM21

GUIAS CIRCULARES

RELAÇÕES ENTRE AS VARIÁVEIS

As componentes dos campos, para os modos TM, poderão ser deduzidas de modo semelhante ao que foi feito nos guias retangulares.

$$\nabla \times \bar{E} = -j\omega \mu \bar{H}$$

Em coordenadas cilíndricas $\nabla \times \bar{E} = \frac{1}{r} \cdot \frac{\delta \bar{E}_z}{\delta \phi} + \gamma \cdot E \phi = -j\omega \mu H_r$

$$\frac{1}{r} \cdot \frac{\delta E_z}{\delta \phi} + \gamma \cdot E \phi = -j\omega \mu H_r \quad (1)$$

$$-\gamma \cdot E_r - \frac{\delta E_z}{\delta r} = -j\omega \mu H_\phi \quad (2)$$

$$\frac{1}{r} \cdot \frac{\delta}{\delta r} (r \cdot E \phi) - \frac{1}{r} \cdot \frac{\delta E_r}{\delta \phi} = -j\omega \mu H_z \quad (3)$$

$$\nabla \times \bar{H} = j\omega \varepsilon \bar{E}$$

$$\frac{1}{r} \cdot \frac{\delta H_z}{\delta \phi} + \gamma \cdot H \phi = j\omega \varepsilon \cdot E_r \quad (4)$$

$$-\gamma \cdot H_r - \frac{\delta H_z}{\delta r} = j\omega \varepsilon \cdot E \phi \quad (5)$$

$$\frac{1}{r} \cdot \frac{\delta}{\delta r} (r \cdot H \phi) - \frac{1}{r} \cdot \frac{\delta H_r}{\delta \phi} = j\omega \varepsilon \cdot E_z \quad (6)$$

com o emprego destas 6 equações, podemos relacionar:

$$H_r = f(E_z, H_z) \text{ ou } H_\phi = f(E_z) \text{ ou } H = f(E_z, H_z) \text{ etc....}$$

RELAÇÕES ENTRE AS COMPONENTES

GUIA CILÍNDRICO

$$H_r = \frac{1}{\gamma^2 + k^2} \cdot \left[j \frac{w\epsilon}{r} \cdot \frac{\delta E_z}{\delta \phi} - \gamma \cdot \frac{\delta H_z}{\delta r} \right] = \quad (7)$$

$$H_\phi = - \frac{1}{\gamma^2 + k^2} \left[jw\epsilon \cdot \frac{\delta E_z}{\delta r} + \frac{\gamma}{r} \cdot \frac{\delta H_z}{\delta \phi} \right] = \quad (8)$$

$$E_r = - \frac{1}{\gamma^2 + k^2} \left[\gamma \cdot \frac{\delta E_z}{\delta r} + j \frac{w\mu}{r} \cdot \frac{\delta H_z}{\delta \phi} \right] = \quad (9)$$

$$E_\phi = \frac{1}{\gamma^2 + k^2} = \left[- \frac{\gamma}{r} \cdot \frac{\delta E_z}{\delta \phi} + jw\mu \frac{\delta H_z}{\delta r} \right] = \quad (10)$$

MODO TM

$$E_z = E_0 \cdot J_n(k_c \cdot r) \cdot \cos(n\phi)$$

$$H_r = \frac{1}{k_c^2} \cdot j \frac{w\epsilon}{r} \cdot \frac{\delta E_z}{\delta \phi} = -j \cdot \frac{w\epsilon}{k_c \cdot k_c r} \cdot E_0 \cdot J_n(k_c r) \cdot n \cdot \text{sen}(n\phi)$$

$$= -j \cdot \frac{f \cdot n \cdot E_0}{k_c \cdot f_c \cdot \eta \cdot r} \cdot J_n(k_c r) \cdot \text{sen}(n\phi)$$

GUIAS CLÍNDRICAS - CIRCULARES

MODO TE

$$H_z \neq 0 \quad e \quad E_z = 0$$

Assim como no MODO TM, utilizamos a equação geral $\nabla_t^2 \bar{E} + (\gamma^2 + k^2) \bar{E} = 0$ e obtivemos $E_z = E_0 \cdot J_n(k_c r) \cdot \cos(n\phi)$, podemos agora empregar a equação geral:

$$\nabla_t^2 \bar{H} + (\gamma^2 + k^2) \bar{H} = 0$$

e obteremos

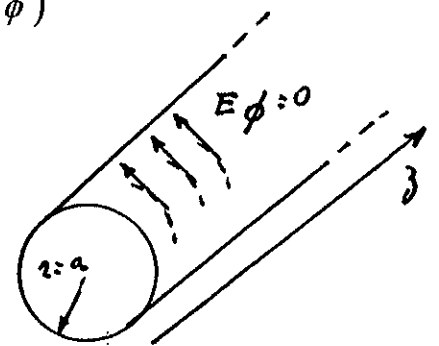
$$H_z(r, \phi) = H_0 \cdot J_n(k_c r) \cdot \cos(n\phi)$$

CONDIÇÃO DE CONTORNO

Nas paredes do guia cilíndrico, onde:

$$r = a$$

$E_\phi = 0$ (campo elétrico tangencial)



Pelas relações entre as componentes, temos (10),

$$E_\phi = \frac{1}{\gamma^2 \cdot k^2} \left[-\frac{\gamma}{r} \cdot \frac{\delta E_z}{\delta \phi} + j\omega\mu \cdot \frac{\delta H_z}{\delta r} \right] = \frac{1}{\gamma^2 + k^2} \cdot j\omega\mu \cdot \frac{\delta H_z}{\delta r}$$

$$E_\phi(r = a) = 0 = \frac{1}{k_c^2} \cdot j\omega\mu \cdot \frac{\delta H_z}{\delta r} = 0 \quad \therefore \frac{\delta H_z}{\delta r} = 0$$

Ora, $H_z(r, \phi) = H_0 \cdot J_n(k_c r) \cdot \cos(n\phi)$

$$\left. \frac{\delta H_z}{\delta r} \right|_{r=a} = 0 = H_0 \cdot \underbrace{J'_n(k_{ca})}_{=0} \cdot \underbrace{\cos(n\phi)}_{\neq 0}$$

conclui-se que $\rightarrow J'_n(k_{ca}) = 0$

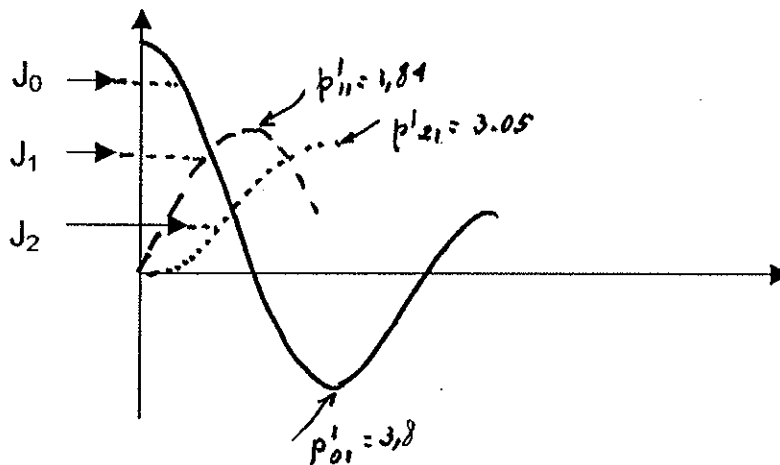
GUIAS CILÍNDRICOS = MODO TE

Nas paredes do guia $r = a \therefore J_n'(k_{ca}) = 0$ as raízes da função derivada são denominadas ou representadas por $p'_{n,\ell} \therefore k_c \cdot a = p'_{n,\ell}$

$$\frac{2\pi}{\lambda_c} \cdot a = p'_{n,\ell} \therefore \frac{2\pi}{v} \cdot f_c \cdot a = p'_{n,\ell} \therefore f_c = \frac{p'_{n,\ell} \cdot v}{2\pi a}$$

Frequências de Corte - Modo $TE_{M,\ell}$

$$(f_c)_{n,\ell}^{TE} = \frac{p'_{n,\ell} \cdot v}{2\pi a}$$



A menor raiz da função derivada será p'_{11} que corresponde ao 1º Modo TE_{11} é dominante, após este modo irá aparecer o TE_{21} pois $p'_{21} = 3,05$.

RAÍZES DA EQUAÇÃO DERIVADA – BESSEL

MODOS $TE_{m,l}$

	$n \backslash l$	1	2	3	4
$J'_0 \rightarrow$	0	TE ₀₁ $p'_{0,1} = 3,832$	TE ₀₂ $p'_{0,2} = 7,016$	TE ₀₃ $p'_{0,3} = 10,174$	TE ₀₄
$J'_1 \rightarrow$	1	TE ₁₁ $p'_{1,1} = 1,841$	TE ₁₂ $p'_{1,2} = 5,331$	TE ₁₃ $p'_{1,3} = 8,536$	TE ₁₄
$J'_2 \rightarrow$	2	TM ₂₁ $p'_{2,1} = 3,054$	TM ₂₂ $p'_{2,2} = 6,706$	TM ₂₃ $p'_{2,3} = 9,970$	TM ₂₄
$J'_3 \rightarrow$	3	TE ₃₁ $p'_{3,1} = 4,20$	TE ₃₂ $p'_{3,2} = 8,01$	TE ₃₃ $p'_{3,3} = 11,34$	TE ₃₄
$J'_4 \rightarrow$	4				

Modo Dominante $p'_{11} = 1,841$

$$(f_c)_{TE_{11}} = \frac{p'_{11} \cdot v}{2\mu a} \rightarrow \text{vácuo} = \frac{p'_{11} \cdot c}{2\pi a} = \frac{1,891c}{2\pi a}$$

$$(f_c)_{TE_{11}} = \frac{8,79}{a \text{ (cm)}} \text{ (GHz)}$$

$$(\lambda_c)_{TE_{11}} = \frac{2\pi \cdot a}{p'_{11}} = 3,41a$$

2º Modo - $p'_{21} = 3,054 \therefore (f_c)_{TE_{21}} = \frac{3,054 \cdot c}{2\pi a}$

$$\text{vácuo } (f_c)_{TE_{21}} = \frac{14,58}{a \text{ (cm)}} \text{ (GHz)}$$

$$(\lambda_c)_{TE_{21}} = 2,057a$$

GUIA CIRCULAR OU CILÍNDRICO

FREQUÊNCIAS DE CORTE

Pelas condições de contorno $E_z(r=a, \phi) = 0$ concluímos $k_c \cdot a = p_{n,\ell}$.

$$k_c \cdot a = p_{n,\ell} = \frac{2\pi}{\lambda} \cdot f_c \cdot a = p_{n,\ell}$$

$$(f_c)_{n,\ell} = \frac{p_{n,\ell}}{2\pi a \sqrt{\mu \epsilon}} = \frac{p_{n,\ell}}{2\pi a} \cdot v$$

No vácuo (guia sem material dielétrico) $v = c$

$$(f_c)_{n,\ell} = \frac{p_{n,\ell} \cdot c}{2\pi a}$$

MODO TM Dominante - menor valor para a raiz

$$p_{0,1} = 2,405$$

$$(f_c)_{01} = (f_c)_{TM_{01}} = \frac{2,405 \cdot c}{2\pi a} = \frac{c}{\lambda_c}$$

$$(\lambda_c)_{TM_{01}} = \frac{2\pi a}{p_{n,\ell}} = \frac{2\pi a}{2,405} = 2,61 a$$

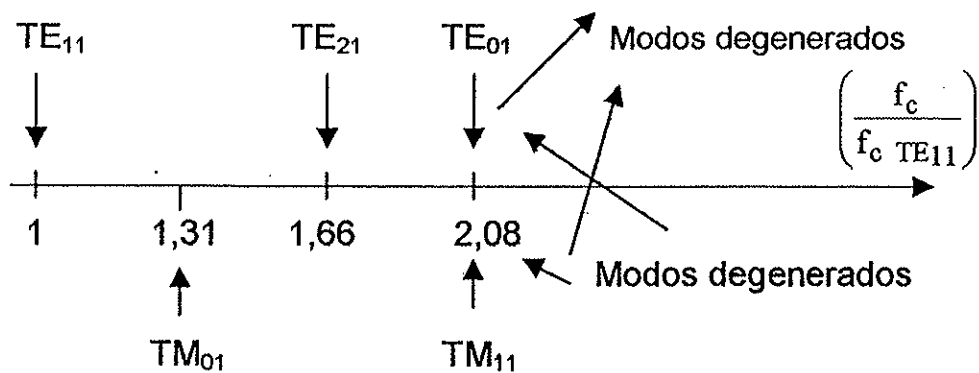
$$(f_c)_{01} = \frac{11,48}{a(\text{cm})} \text{ (GHz)}$$

Como é possível perceber se os guias de onda circulares apresentam uma faixa de frequência de operação muito pequena, na faixa em que apenas o modo dominante se propaga ($1 \rightarrow 1,31$).

A vantagem apresentada pelos guias cilíndricos é sua simetria, quando comparado aos guias retangulares e baixas perdas para os modos $TE_{0,\ell}$ (reduzem como o aumento de frequência).

GUIAS CILÍNDRICOS/ CIRCULARES. FREQUÊNCIAS DE CORTE

Modos $TE_{m, \ell}$ e $TM_{m, \ell}$	
Modo Dominante	$(f_c)_{TE_{11}} = \frac{1,84 c}{2 \pi a} = \frac{8,79}{a(\text{cm})} \text{ (GHz)}$
2º Modo	$(f_c)_{TM_{01}} = \frac{2,405 c}{2 \pi a} = \frac{11,48}{a(\text{cm})} \text{ (GHz)}$ $(f_c)_{TM_{01}} = 1,31 (f_c)_{TE_{11}}$
3º Modo	$(f_c)_{TE_{21}} = \frac{3,05 c}{2 \pi a} = \frac{14,58}{a(\text{cm})} \text{ (GHz)}$ $(f_c)_{TE_{21}} = 1,66 (f_c)_{TE_{11}}$
4º Modo	$(f_c)_{TE_{01}} = (f_c)_{TM_{11}} = \frac{3,832 c}{2 \pi a}$ $(f_c)_{TE_{01}} = (f_c)_{TM_{11}} = 2,08 (f_c)_{TE_{11}}$



RELAÇÕES ENTRE AS COMPONENTES

MODO TE

$H_z \neq 0$

Empregando-se as mesmas equações (7), (8), (9) anteriores

$$H_z(r, \phi) = H_0 \cdot J_n(k_c r) \cdot \cos(n\phi)$$

$$E_r(r, \phi) = \frac{-j\omega\mu}{k_c^2} \cdot \frac{1}{r} \cdot \frac{\delta H_z}{\delta \phi} = \frac{j\omega\mu \cdot n}{k_c^2 \cdot r} H_0 \cdot J_n(k_c r) \cdot \sin(n\phi)$$

Modos $TE_{0,\ell} \quad \therefore E_r = 0 \quad \text{e} \quad H_\phi = 0$

$$E_r(r, \phi) = \frac{jn \cdot \eta \cdot f}{k_c \cdot r \cdot f_c} \cdot H_0 \cdot J_n(k_c r) \cdot \sin(n\phi)$$

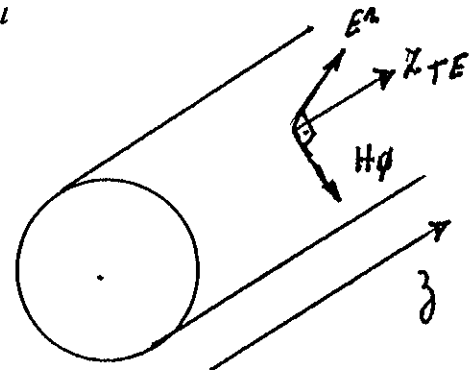
$$E_\phi(r, \phi) = \frac{j\omega\mu}{k_c} \cdot \frac{\delta H_z}{\delta r} = \frac{j\omega\mu}{k_c \cdot k_c} \cdot H_0 \cdot J_n'(k_c r) \cdot \cos(n\phi)$$

$$E_\phi(r, \phi) = \frac{j\eta \cdot f}{f_c} H_0 \cdot J_n'(k_c r) \cdot \cos(n\phi)$$

$$H_\phi(r, \phi) = -\frac{\gamma}{k_c^2} \cdot \frac{f}{r} \cdot \frac{\delta H_z}{\delta \phi} \quad \text{como} \quad \frac{\delta H_z}{\delta \phi} = \frac{-E_r \cdot k_c^2}{j\omega\mu}$$

Teremos
$$H_\phi = \frac{\gamma}{j\omega\mu} E_r = \frac{\beta}{\omega n} E_r$$

$$H_\phi = \frac{E_r}{Z_{TE}} \quad \therefore Z_{TE} = \frac{\omega\mu}{\beta} = \frac{\eta}{\left[1 - \left(\frac{f_c}{f}\right)^2\right]^{1/2}}$$



RELAÇÕES ENTRE AS COMPONENTES TM

$$H_r(r, \phi) = \frac{-j \cdot f \cdot n}{k_c \cdot f_c \cdot \eta \cdot r} \cdot E_0 \cdot J_n(k_c \cdot r) \cdot \text{sen}(\eta \phi)$$

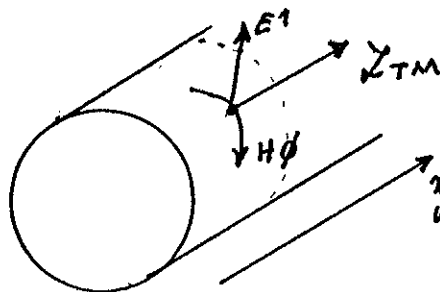
$$H_\phi(r, \phi) = \frac{-j \omega \varepsilon}{k_c^2} \cdot \frac{\delta E_z}{\delta r} = - \frac{j \cdot f \cdot E_0}{f_c \cdot k_c \cdot \eta} \cdot J'_n(k_c r) \cdot \cos(n \phi)$$

$$E_r = - \frac{\gamma}{k_c} \cdot \frac{\delta E_z}{\delta r} \quad \text{como} \quad \frac{\delta E_z}{\delta r} = - \frac{k_c^2}{j \omega \varepsilon} \cdot H_\phi$$

teremos:

$$\begin{aligned} E_r(r, \phi) &= \frac{-\gamma}{k_c^2} \left(\frac{-k_c^2}{j \omega \varepsilon} \right) \cdot H_\phi \\ &= \frac{\gamma}{j \omega \varepsilon} \cdot H_\phi = \frac{\beta}{\omega \varepsilon} \cdot H_\phi \end{aligned}$$

$$E_r = Z_{TM} \cdot H_\phi \quad Z_{TM} - \text{impedância de onda do Modo TM.}$$



Analogamente

$$E_\phi = \frac{-\gamma}{k_c^2 \cdot r} \cdot \frac{\delta E_z}{\delta \phi} \quad \therefore \quad E_\phi = - Z_{TM} \cdot H_r$$

RELAÇÕES ENTRE AS COMPONENTES - MODO TE

$$H_{\phi} = \frac{E_r}{Z_{TE}} \quad \therefore Z_{TE} = \frac{\eta}{\left[1 - \left(\frac{f_c}{f}\right)^2\right]^{1/2}}$$

$$Z_{TE} = \frac{\omega \mu}{\beta}$$

$$H_r = \frac{-\gamma}{k_c^2} \cdot \frac{\delta H_z}{\delta r} = -\frac{1}{Z_{TE}} \cdot E_{\phi}$$

Modo TM

$$E_r = Z_{TM} \cdot H_{\phi} \quad \therefore Z_{TM} = \eta \left[1 - \left(\frac{f_c}{f}\right)^2\right]^{1/2}$$

$$Z_{TM} = \frac{\beta}{\omega \epsilon}$$

TM

TE

$$E_z = E_0 \cdot J_n(k_c r) \cdot \cos(n\phi) \quad H_z = H_0 \cdot J_n(k_c r) \cdot \cos(n\phi)$$

$$E_r = -\frac{j\beta}{k_c} \cdot E_0 \cdot J'_n(k_c r) \cdot \cos(n\phi) \quad H_r = -\frac{j\beta}{k_c} \cdot H_0 \cdot J'_n(k_c r) \cdot \cos(n\phi)$$

$$E_{\phi} = \frac{j\beta n}{k_c^2 r} \cdot E_0 \cdot J_n(k_c r) \cdot \cos(n\phi) \quad H_{\phi} = \frac{j\beta n}{k_c^2} \cdot J_n(k_c r) \cdot \sin(n\phi)$$

$$H_r = \frac{-j\beta}{k_c^2} \cdot \frac{n \cdot E_0}{r} \cdot \frac{J_n(k_c r)}{Z_{TM}} \cdot \sin(n\phi) \quad E_r = \frac{j\beta}{k_c^2} \cdot \frac{n \cdot H_0}{r} \cdot Z_{TE}$$

$$H_{\phi} = \frac{-j\beta}{k_c} \cdot \frac{E_0}{Z_{TM}} \cdot J'_n(k_c r) \cdot \cos(n\phi) \quad E_{\phi} = \frac{j\beta}{k_c} \cdot Z_{TE} \cdot H_0 \cdot J'_n(k_c r) \cdot \cos(n\phi)$$

GUIAS CILÍNDRICOS OU CIRCULARES CONFIGURAÇÃO DOS CAMPOS - MODO TM_{01}

$$TM_{01} \quad m = 0 \quad \ell = 1$$

$$k_c r = \frac{p_n \cdot \ell \cdot r}{a} = \frac{p_{0,1} \cdot r}{a} = \frac{2,45 \cdot r}{a}$$

$$H_z = 0 \quad E_z(r, \phi) = E_0 \cdot J_n(k_c r) \cdot \cos(n\phi)$$

$$E_z(r, \phi) = E_0 \cdot J_0(k_c r) = E_0 \cdot J_0\left(\frac{2,45r}{a}\right)$$

Pontos importantes $r = 0 \rightarrow J_0 = 1 \therefore E_z = \text{máximo}$
 $r = a \rightarrow J_0 = 0 \therefore E_z = 0$

$$H_r = -\frac{j f \cdot n \cdot E_0}{k_c - f_c - \eta - r} \cdot J_n(k_c r) \cdot \text{sen}(n\phi)$$

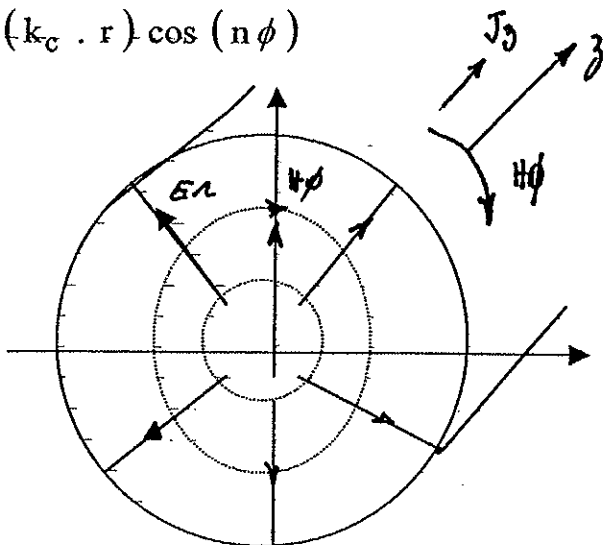
$$TM_{01} \therefore H_r = 0 \quad \therefore E_\phi = 0$$

(Campo elétrico radial $E_r \neq 0$).

$$H_\phi = -\frac{j \cdot f \cdot E_0}{f_c \cdot n} J_n'(k_c \cdot r) \cos(n\phi)$$

$$H_\phi = \frac{j \cdot f \cdot E_c}{f_c \cdot n} \cdot J_0'\left(\frac{2,45r}{a}\right) \therefore$$

$$r = 0 \therefore H_\phi = 0 \quad \text{e} \quad E_r = 0$$



GUIAS CILÍNDRICOS OU CIRCULARES CONFIGURAÇÃO DOS CAMPOS – MODO TM_{01}

$$TM_{01} \quad \eta = 0 \quad \text{e} \quad \ell = 1$$

$$k_c r = \frac{p_{n,\ell} \cdot r}{a} = \frac{p_{0,1} \cdot r}{a} = \frac{2,45 \cdot r}{a}$$

$$H_z = 0 \quad E_z(r, \phi) = E_0 \cdot J_n(k_c \cdot r) \cdot \cos(n\phi)$$

$$E_z(r, \phi) = E_0 \cdot J_0(k_c r) = E_0 \cdot J_0\left(\frac{2,45 r}{a}\right)$$

Pontos importantes: $r = 0 \rightarrow J_0 = 1 \therefore E_z = \text{máximo}$

$$r = a \rightarrow J_0 = 0 \therefore E_z = 0$$

$$H_r = \frac{j f \cdot \eta E_0}{k_c f_c \cdot \eta \cdot r} \cdot J_n(k_c r) \cdot \text{sen}(n\phi)$$

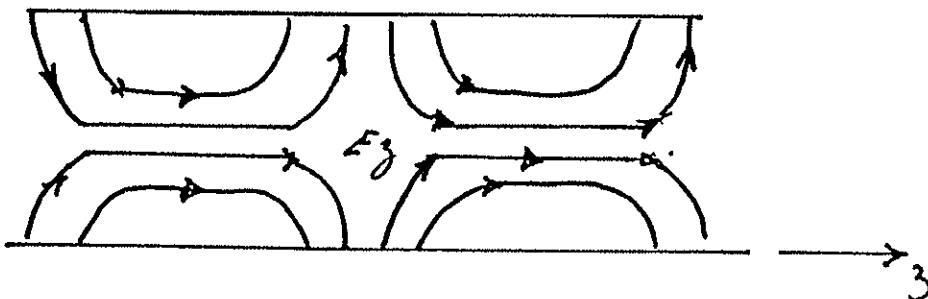
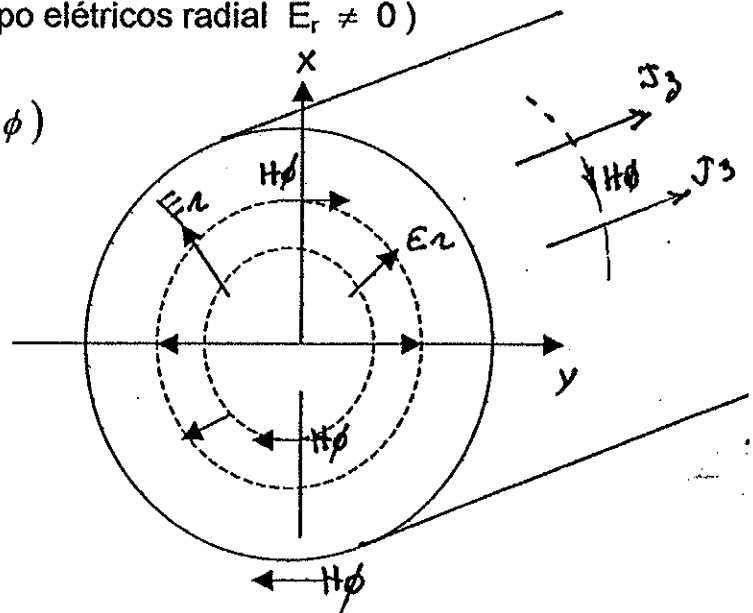
$$TM_{01} \quad \therefore \quad H_r = 0 \quad \therefore \quad E_\phi = 0$$

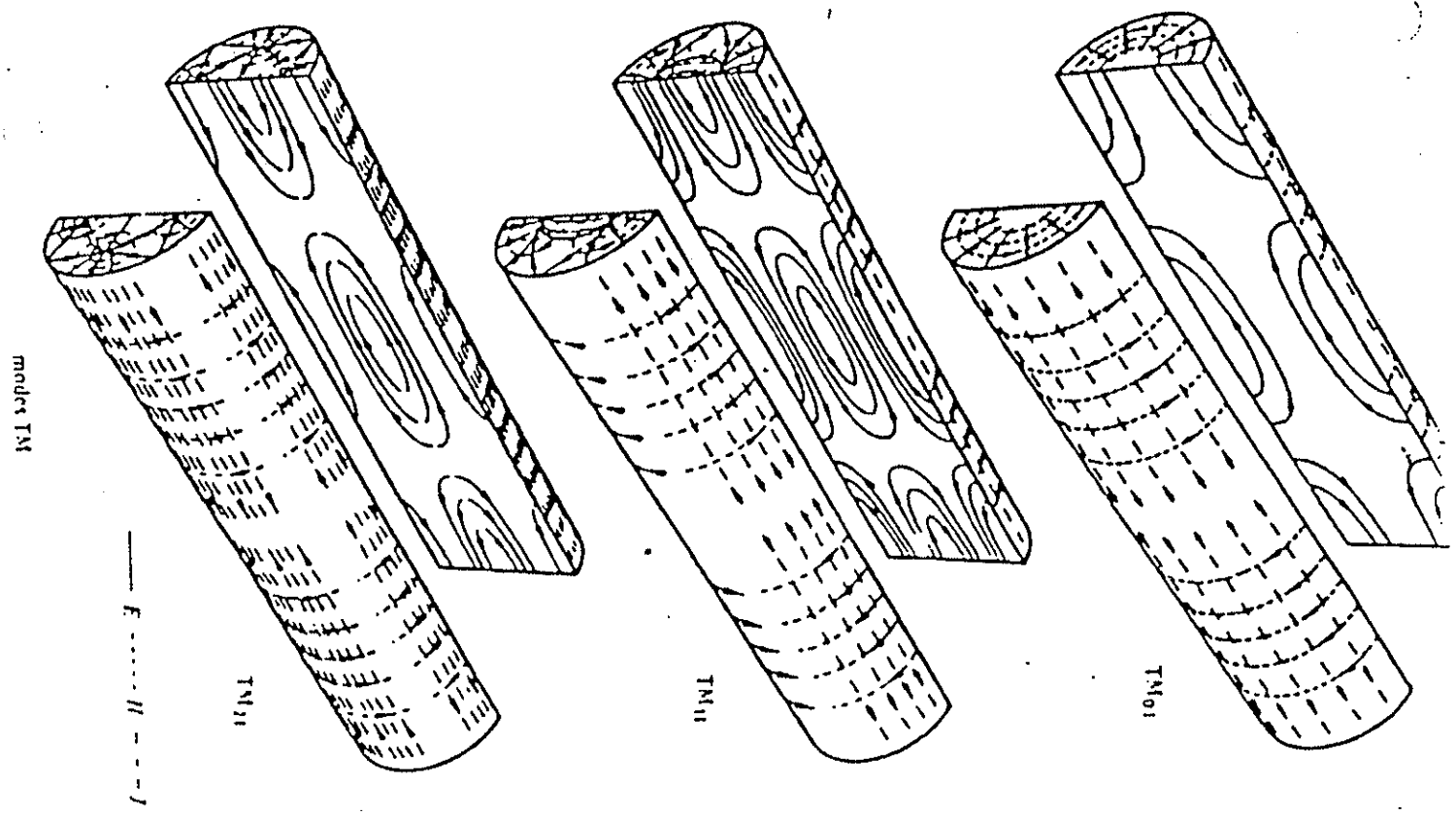
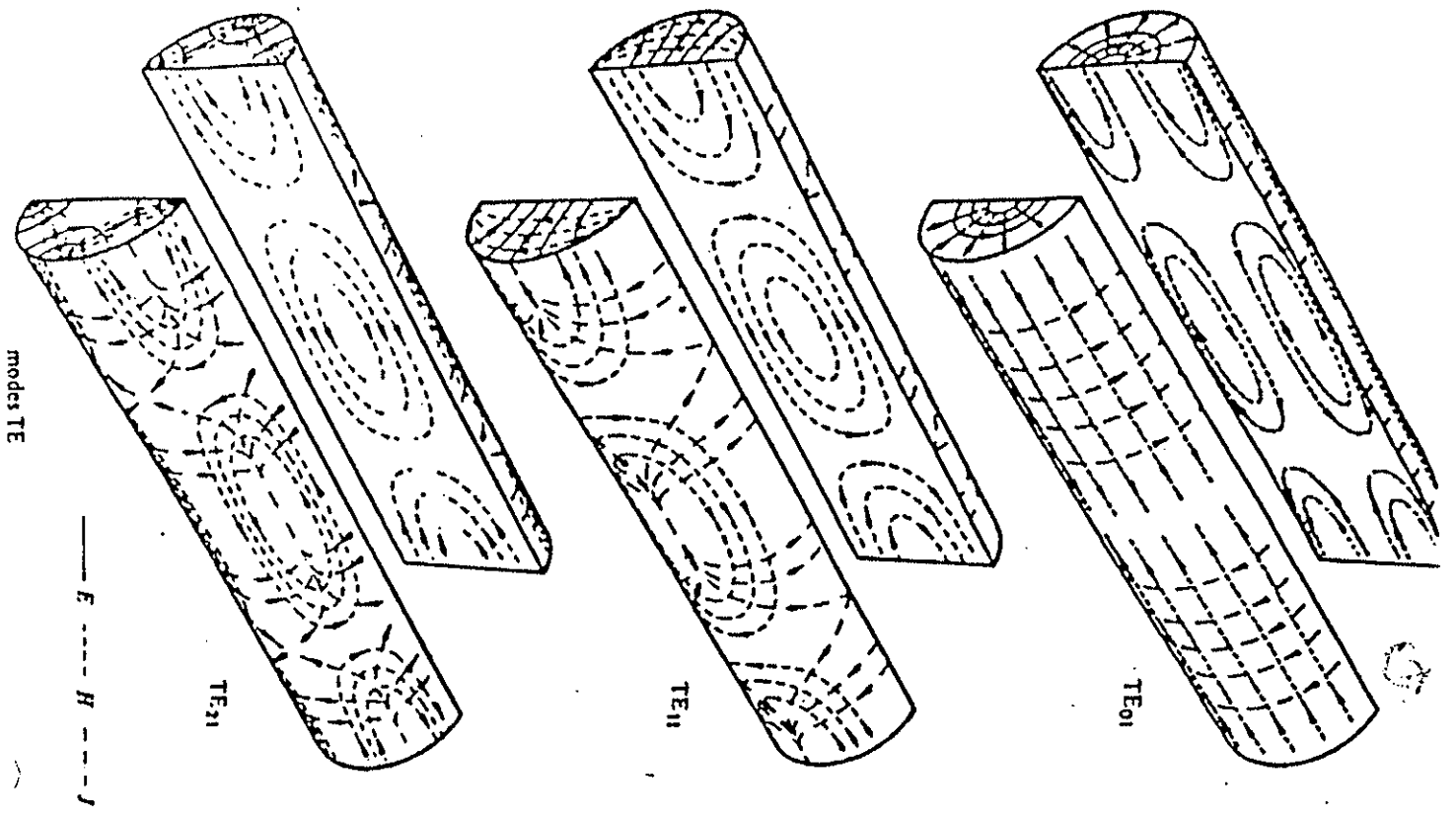
(campo elétrico radial $E_r \neq 0$)

$$H_\phi = -\frac{j \cdot f \cdot E_0}{f_c \cdot \eta} \cdot J_n'(k_c r) \cos(n\phi)$$

$$H_\phi = -\frac{j f \cdot E_0}{f_c \cdot \eta} \cdot J_0'\left(\frac{2,45 r}{a}\right) \therefore$$

$$r = 0 \therefore H_\phi = 0 \quad E_r = 0$$





ATENUAÇÃO EM GUIAS CIRCULARES

Da mesma forma que nos guias retangulares, a atenuação (ou perdas) nos guias circulares é devido:

1. Perdas no dielétrico que preenche o guia (α_d);
2. Propagação do sinal em freqüência abaixo da, freqüência de corte do modo considerado;
3. Perdas nas paredes do guia, devido à condutividade finita dos condutores que formam o guia (α_c).

As expressões para os dois primeiros tipo de perdas já foram reduzidas para as guias retangulares.

1. Perdas dielétricas $\rightarrow \alpha_d$.

$$\alpha_d \text{ (nepers/u.c)} = \frac{\omega \varepsilon''}{2 \varepsilon'} \cdot \sqrt{\mu \varepsilon'} \left[1 - \left(\frac{f_c}{f} \right)^2 \right]^{1/2}$$

$$= \frac{\pi f \operatorname{tg} \delta}{v} \cdot \left[1 - \left(\frac{f_c}{f} \right)^2 \right]^{1/2} \text{ para } f > f_c$$

2. Atenuação para freqüências abaixo de corte

$$\alpha = \frac{2\pi}{\lambda_c} \left[1 - \left(\frac{f}{f_c} \right)^2 \right]^{1/2} \text{ para } f < f_c$$

GUIAS CILÍNDRICOS – ATENUAÇÃO DEVIDO AO CONDUTOR

$$\alpha_c = \frac{P_\alpha}{2P_t} = \frac{\text{Potência dissipada nas paredes condutoras}}{2x \cdot \text{Potência Transmitida no interior do guia}}$$

$$P_t = \frac{1}{2Z_{TE}} \int_S |E_t|^2 \cdot ds = \frac{Z_{TE}}{2} \int_S |H_t|^2 \cdot ds \therefore Z_{TE} = \frac{\omega \mu}{\beta} = \frac{\eta}{\left[1 - \left(\frac{f_c}{f}\right)^2\right]^{1/2}}$$

$$P_t = \frac{Z_{TE}}{2} \int \left[|H_r|^2 + |H_\phi|^2 \right] ds = \frac{Z_{TE}}{2} \int \left[\left| -\frac{\gamma}{k_c^2} \frac{\delta H_z}{\delta r} \right|^2 + \left| -\frac{\gamma}{k_c^2} \frac{1}{r} \frac{\delta H_z}{\delta \phi} \right|^2 \right] ds$$

$$P_t = \frac{Z_{TE}}{2} \frac{|\gamma|^2}{k_c^2} \int \left[\left| \frac{\delta H_z}{\delta z} \right|^2 + \left| \frac{1}{z} \cdot \frac{\delta H_z}{\delta \phi} \right|^2 \right] ds$$

como $Z_{TE} = \frac{\omega \mu}{\gamma} = \frac{\eta}{\left[1 - \left(\frac{f_c}{f}\right)^2\right]^{1/2}}$ e $\lambda_c f_c = v = \frac{1}{\sqrt{\mu \epsilon}}$

teremos:

$$P_t = \frac{\eta}{2} \left(\frac{f}{f_c}\right)^2 \cdot \left[1 - \left(\frac{f_c}{f}\right)^2\right]^{1/2} \cdot \int_S |H_z|^2 \cdot dz$$

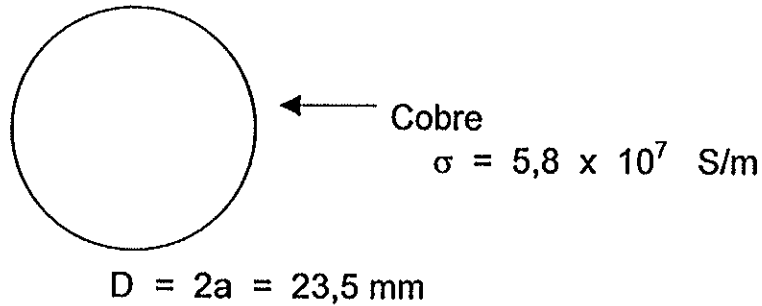
$$P_t = \frac{\pi \cdot a^2 \eta}{4} \cdot H_0^2 \cdot \left(\frac{f}{f_c}\right)^2 \cdot \left[1 - \left(\frac{f_c}{f}\right)^2\right]^{1/2} \cdot J_n^2(k_c a)$$

$$P_d = \frac{R_s}{2} \int_{\text{parede}} |H_z|^2 \cdot ds$$

$$\underline{P_d} = \frac{a \pi R_s \cdot H_0^2}{2} \cdot J_n^2(k_c a) \left[1 + \frac{n^2 \cdot \eta^2}{Z_{TE}^2 k_c^2 \cdot a^2} \cdot \left(\frac{f}{f_c} \right)^2 \right]$$

$$\alpha_c = \frac{P_d}{2P_t} = \frac{R_s}{a \eta \left[1 - \left(\frac{f_c}{f} \right)^2 \right]^{1/2}} \cdot \left[\left(\frac{f_c}{f} \right)^2 + \frac{n^2}{p'_{nl}{}^2 - n^2} \right]$$

EXEMPLO - ATENUAÇÃO EM GUIA CILÍNDRICO



$$(f_c)_{TE_{11}} = \frac{1,841 \cdot v}{2\pi a} = \frac{1,841 \cdot 30 \cdot 10^9}{\pi \cdot 23,5} = 7,48 \text{ GHz}$$

$$(f_c)_{TM_{01}} = \frac{2,405 \cdot v}{2\pi a} = \frac{2,405 \cdot 30 \cdot 10^9}{\pi \cdot 23,5} = 9,77 \text{ GHz}$$

$$(f_c)_{TE_{01}} = \frac{3,8 \cdot v}{2\pi a} = 15,57 \text{ GHz}$$

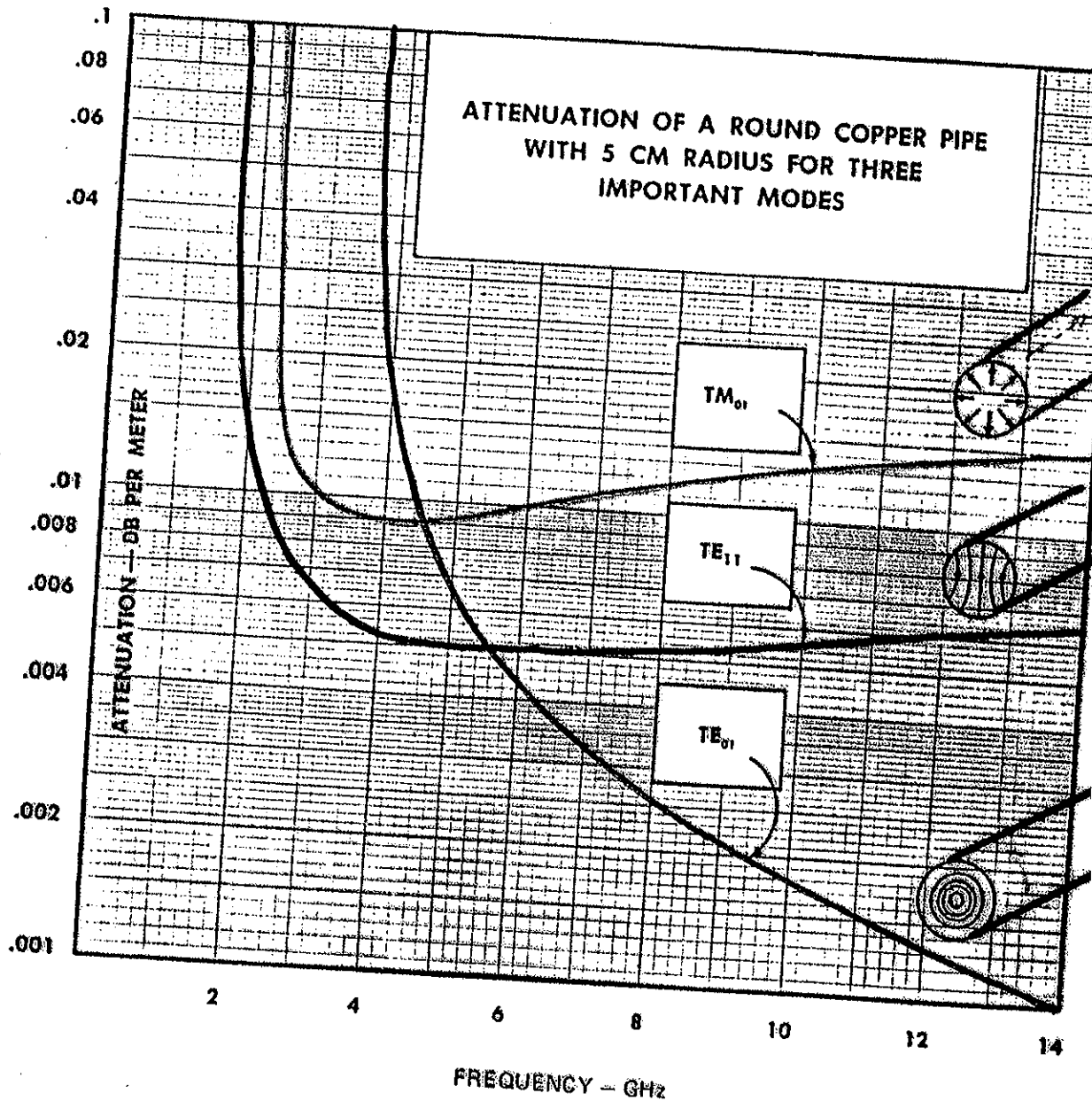
Determinar α_c - 10 GHz

$$R_s = \sqrt{\frac{\pi f \mu}{\sigma}} = 2,608 \times 10^{-2} \Omega = 0,026 \Omega$$

$$\alpha_{TE_{11}} \left(\frac{dB}{m} \right) = \frac{8,68 \cdot R_s}{377 \cdot a} \left[\left(\frac{f_c}{f} \right)^2 + \frac{1}{1,841^2 - 1} \right]$$

$$= 0,075 \text{ dB/m} = 7,5 \text{ dB/100 m}$$

$$\alpha_{TM_{01}} = 24,15 \text{ dB/100 m}$$



$$(f_c)_{TE_{11}} = \frac{p'_{11} \cdot v}{2\pi a} = \frac{1.841 \times 15}{\pi \cdot 5} = 1.75 \text{ GHz} \quad \odot$$

$$(f_c)_{TM_{01}} = \frac{p_{01} \cdot v}{2\pi a} = \frac{2.405 \times 15}{\pi \cdot 5} = 2.29 \text{ GHz} \quad \odot$$

From G. C. Southworth, Principles and Applications of Waveguide Transmission, copyright 1950 D. Van Nostrand Co., Inc. New York, N.Y.

$$(f_c)_{TE_{21}} = \frac{p'_{21} \cdot v}{2\pi a} = \frac{3.054 \times 15}{\pi \cdot 5} = 2.91 \text{ GHz}$$

$$(f_c)_{TE_{01}} = \frac{p'_{01} \cdot v}{2\pi a} = \frac{3.832 \times 15}{\pi \cdot 5} = 3.66 \text{ GHz} \quad \odot = (f_c)_{TM_{11}}$$

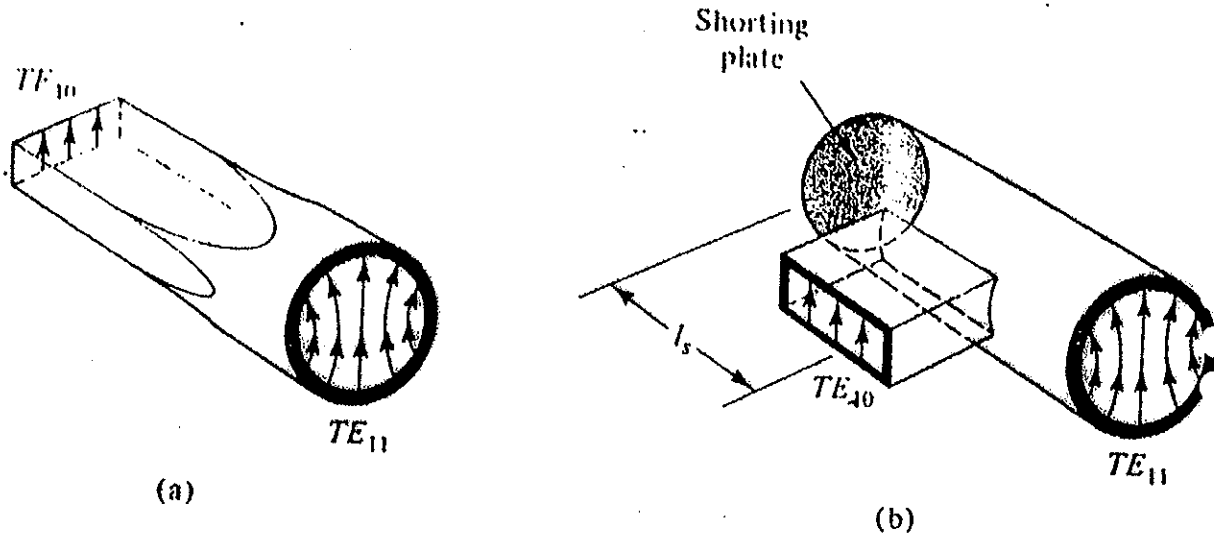


Figure 7-4 Two transitions between the TE_{10} mode in rectangular guide and the TE_{11} mode in circular guide.

TM_{01} mode in circular guide. The orientation of the waveguides encourages a smooth transition between the magnetic field patterns of the two modes. This is indicated in the figure. The diameter D of the circular guide must, of course, be sufficiently large to insure that the TM_{01} mode is above cutoff, but the next higher mode is not. Cutoff wavelengths for some circular guide modes are given in Fig. 5-27.

By proper choice of dimensions, the structure in Fig. 7-4b can also be used as a converter between the TE_{10} rectangular guide mode and the TE_{01} mode in circular guide. In this case, the circular guide diameter must be large enough to support TE_{01} mode propagation. The two-dimensional view in Fig. 7-6 describes the mode conversion process. The electric field pattern at the junction is shown on the left. As indicated in the figure, the distorted pattern may be viewed as mainly a combination of the TE_{11} and TE_{01} modes. With the guide sufficiently large to support the TE_{01} mode, it is also capable of propagating the TE_{11} mode. In order to insure TE_{01} mode purity in the circular guide, a mode filter (not shown) is usually inserted to suppress the unwanted TE_{11} mode. The TE_{10} to TM_{01} converter described in Fig. 7-5 also requires a TE_{11} mode suppressor to insure TM_{01} mode purity. In certain cases, the symmetrical nature of the transition prevents the generation of the dominant mode and hence a mode filter is not needed. One such case is the TEM to TM_{11} mode converter shown in Fig. 7-3.

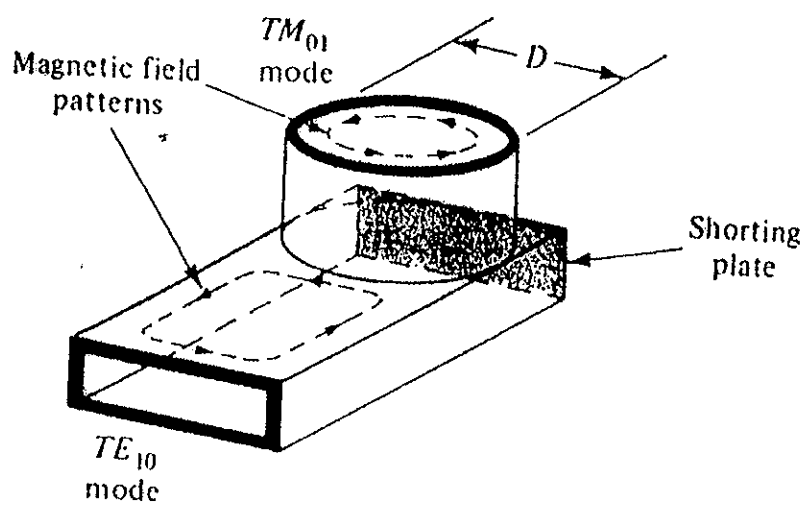


Figure 7-5 A transition between the TE_{10} mode in rectangular guide and the TM_{01} mode in circular guide.

... joint may be converted to a waveguide type by simply incorporating coax-to-waveguide transitions at each end. For example, using transition of the type shown in Fig. 7-2c, the coaxial unit in Fig. 7-32a becomes an waveguide rotary joint. A dashed outline of the transitions is shown in the figure. With this arrangement, the input and output guides have a common propagation axis and rotational motion takes place about the axis.

Waveguide rotary joints. Circular guide may be used to couple the halves of a waveguide rotary joint. The condition that the transmission characteristics remain constant during rotation requires that the circular waveguide mode be circularly symmetric. In most cases either the TM_{01} or TE_{01} mode is used. The joint shown in Fig. 7-33 makes use of the TM_{01} mode. It consists of two rectangular-to- TM_{01} circular mode transitions that are free to rotate about the propagation axis of the circular guide. The transitions are electrically and mechanically connected by a choke-type arrangement similar to that used in the coaxial rotary joint (Fig. 7-32a).

Since the TM_{01} is not the dominant mode in circular guide, TE_{11} propagation is possible. Therefore, it is critical that the transitions be designed with a high degree of TE_{11} mode suppression. One such transition is shown in Fig. 7-5. TE_{11} mode suppression is obtained by including mode filters of the type shown in Fig. 7-7b.

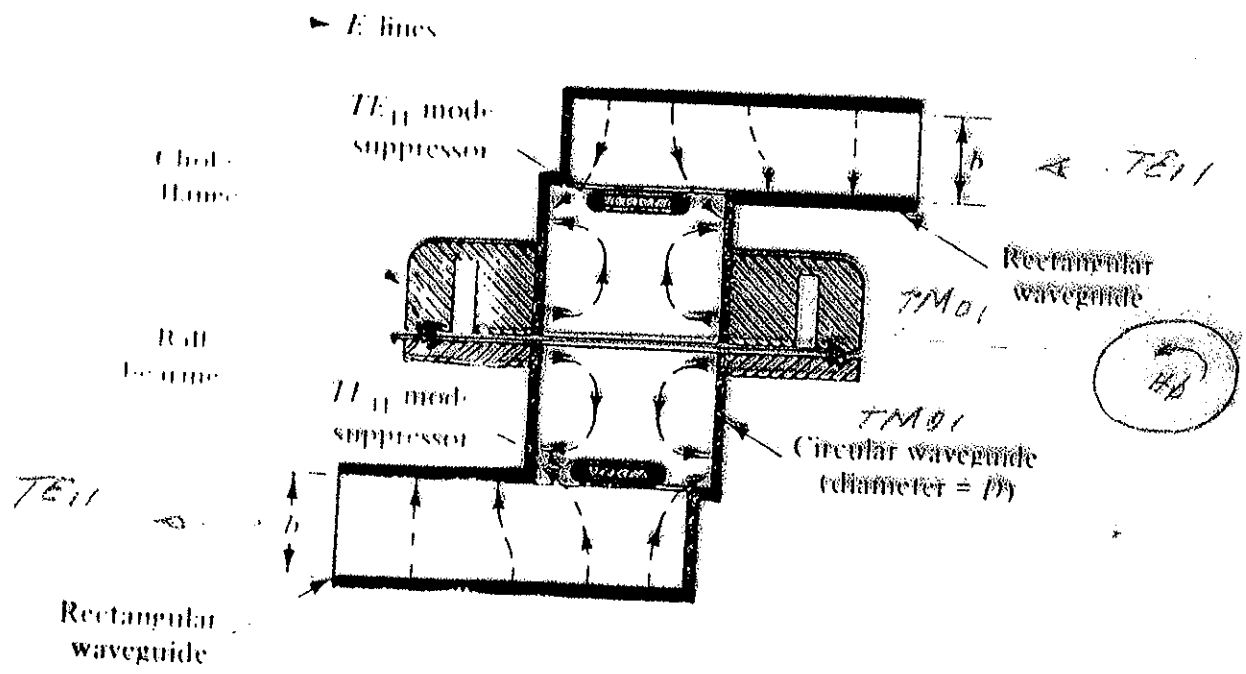
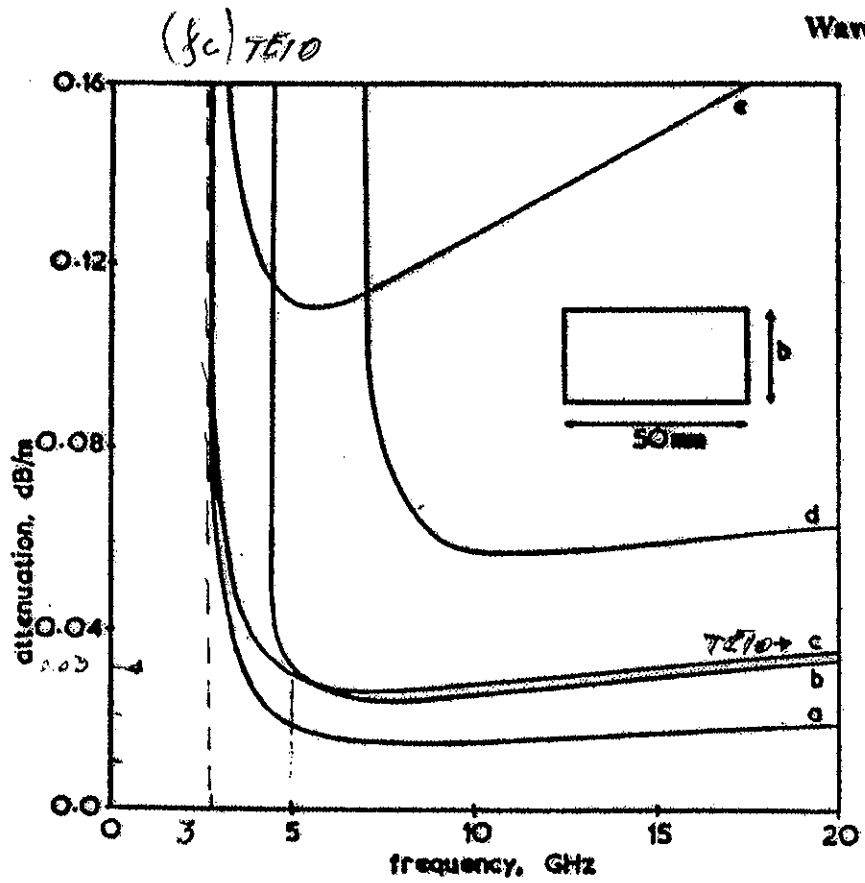
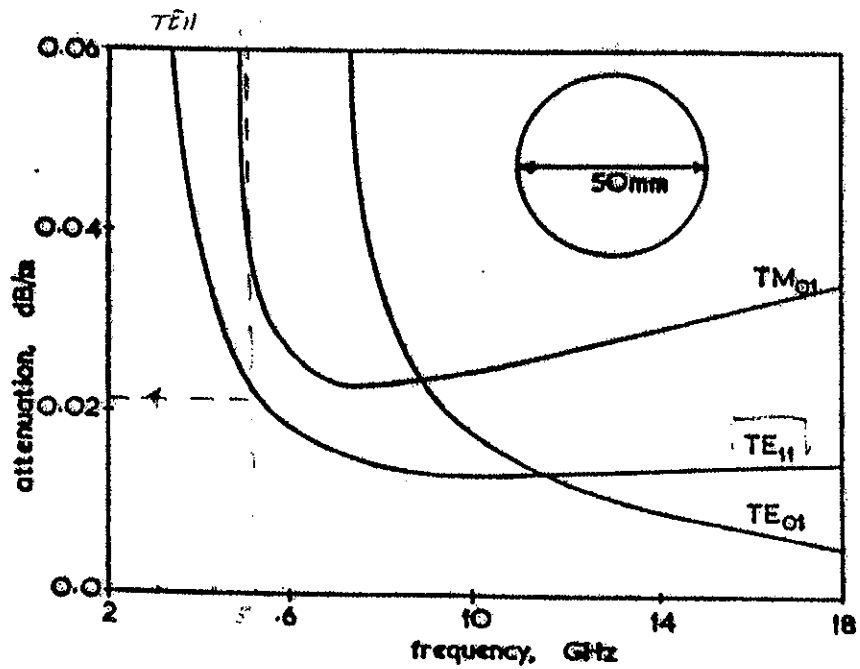


Figure 7-33 A waveguide rotary joint that utilizes the TM_{01} mode in circular waveguide.



Theoretical attenuation of a rectangular waveguide of fixed width

	mode	b/a
a	TE_{10}	1
b	TM_{11}	1
c	TE_{10}	1/2
d	TM_{11}	1/2
e	TE_{10}	1/10



Theoretical attenuation of a circular waveguide of fixed diameter