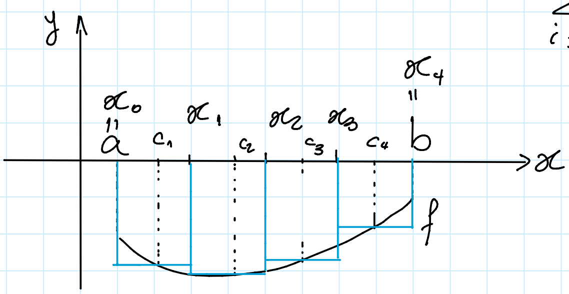


$$\sum_{i=1}^{4} f(c_i)\Delta x_i = f(c_1)\Delta x_1 + f(c_2)\Delta x_2 + f(c_3)\Delta x_3 + f(c_4)\Delta x_4 < 0$$

onde $\Delta x_i = x_i - x_{i-1} > 0$

Integral de Riemann

$$\int_a^b f(x)\,dx = \lim_{n \to +\infty} \sum_{i=1}^{n} f(c_i)\Delta x_i$$

1º Teorema Fundamental do Cálculo

Se f é integrável em $[a,b]$ e F é uma primitiva de f $(F'=f)$ então

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Notação: $\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$, F é uma primitiva de F $(F'=f)$

Exercícios: 1) $\int_1^2 \frac{x^7+x^2+1}{x^2}\,dx$

$$\int \frac{x^7+x^2+1}{x^2}\,dx = \int \left(\frac{x^7}{x^2} + \frac{x^2}{x^2} + \frac{1}{x^2}\right)dx = \int (x^5 + 1 + x^{-2})\,dx$$

$$= \frac{x^6}{6} + x + \frac{x^{-2+1}}{-2+1} + k = \frac{x^6}{6} + x - \frac{1}{x} + k$$

$$\int_1^2 \frac{x^7+x^2+1}{x^2}\,dx = \left(\frac{x^6}{6} + x - \frac{1}{x}\right)\Bigg|_1^2$$

$$= \left(\frac{2^6}{6} + 2 - \frac{1}{2}\right) - \left(\frac{1^6}{6} + 1 - \frac{1}{1}\right)$$

$$= \frac{2^5}{3} + \frac{3}{2} - \frac{1}{6} = \frac{2^6 + 9 - 1}{6} = \frac{72}{6} = 12$$

2) $\int_{-1}^{1} x^2 \sqrt{x^3+1}\,dx$

$$\int x^2\sqrt{x^3+1}\,dx \;,\quad u = x^3+1 \Rightarrow \frac{du}{dx} = u' = 3x^2 \Rightarrow \frac{du}{3} = x^2dx$$

$$\int x^2\sqrt{x^3+1}\,dx = \frac{1}{3}\int\sqrt{u}\,du = \frac{1}{3}\int u^{\frac{1}{2}}du = \frac{1}{3}\,\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + k$$

$$= \frac{1}{3}\cdot\frac{2}{3}\cdot u^{\frac{3}{2}} + k = \frac{2}{9}\sqrt{u^3} + k = \frac{2}{9}\sqrt{(x^3+1)^3} + k$$

$$\int_{-1}^{1} x^2\sqrt{x^3+1}\,dx = \frac{2}{9}\sqrt{(x^3+1)^3}\,\Big|_{-1}^{1} =$$

$$= \frac{2}{9}\sqrt{(1^3+1)^3} - \frac{2}{9}\sqrt{((-1)^3+1)^3} = \frac{2}{3}\sqrt{2^3} = \frac{4\sqrt{2}}{3}$$

3) $$\int_0^{\ln 3} x\,e^{-x}\,dx$$

$$\int x\,e^{-x}\,dx \qquad \boxed{\int u\,dv = uv - \int v\,du}$$

$$u = x \Rightarrow \frac{du}{dx} = 1 \Rightarrow du = dx$$
$$dv = e^{-x}dx \Rightarrow v = -e^{-x} \qquad (e^{-x})' = e^{-x}(-x)' = e^{-x}(-1) = -e^{-x}$$

$$\int x\,e^{-x}dx = -x\,e^{-x} + \int e^{-x}dx = -x\,e^{-x} - e^{-x} + k$$
$$= -(x+1)\,e^{-x} + k$$

$$\int_0^{\ln 3} x\,e^{-x}dx = -(x+1)\,e^{-x}\Big|_0^{\ln 3} = -(\ln 3+1)\,e^{-\ln 3} + (0+1)\,e^{-0}$$

$$= -(\ln 3+1)\,\frac{1}{e^{\ln 3}} + 1 = \frac{\ln 3+1}{3} + 1 = \frac{4+\ln 3}{3}$$

$$a^b = c \iff b = \log_a c$$
$$a^{\log_a c} = c\,, \quad b = \log_a a^b$$

$$e^{\ln 3} = e^{\log_e 3} = 3$$

2) Aplicações da Integral definida

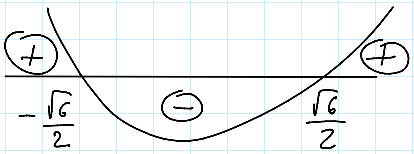
a) Calcule a área da região compreendida entre os gráficos de $f(x) = x^3 - 2x + 1$ e $g(x) = -x + 1$, com $-1 \leq x \leq 1$ (Resp. $\frac{1}{\ }$)

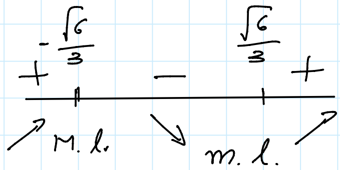
a) Calcule a área da região compreendida entre os gráficos de $f(x) = x^3 - 2x + 1$ e $g(x) = -x + 1$, com $-1 \leq x \leq 1$ (Resp. $\frac{1}{2}$)

Vamos esboçar o gráfico de $f(x) = x^3 - 2x + 1$

$D_f = \mathbb{R}$, $f'(x) = 3x^2 - 2$, $3x^2 - 2 = 0 \Rightarrow x^2 = \frac{2}{3} \Rightarrow x = \pm\sqrt{\frac{2}{3}} = \pm\frac{\sqrt{6}}{3}$

$\cong \pm 0{,}8$





$f''(x) = 6x$ — + (0) ; ∩ I ∪ $\qquad y = x^3 - 2x + 1$

$$\lim_{x\to+\infty} f(x) = \lim_{x\to} (x^3 - 2x + 1) = \lim_{x\to+\infty} x^3 \cdot \left(1 - \frac{2}{x^2} + \frac{1}{x^3}\right) = +\infty \quad (+\infty \cdot 1)$$

$x^3 \to +\infty$, $\frac{2}{x^2} \to 0$, $\frac{1}{x^3} \to 0$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} (x^3 - 2x - 1) = -\infty$$

$$f\left(-\frac{\sqrt{6}}{3}\right) = \left(-\frac{\sqrt{6}}{3}\right)^3 - 2\left(-\frac{\sqrt{6}}{3}\right) + 1 = -\frac{6\sqrt{6}}{3^3} + \frac{2\sqrt{6}}{3} + 1$$

$$= -\frac{2\sqrt{6}}{9} + \frac{2\sqrt{6}}{3} + 1 = \frac{-2\sqrt{6} + 6\sqrt{6} + 9}{9} = \frac{4\sqrt{6} + 9}{9} \cong 2{,}1$$

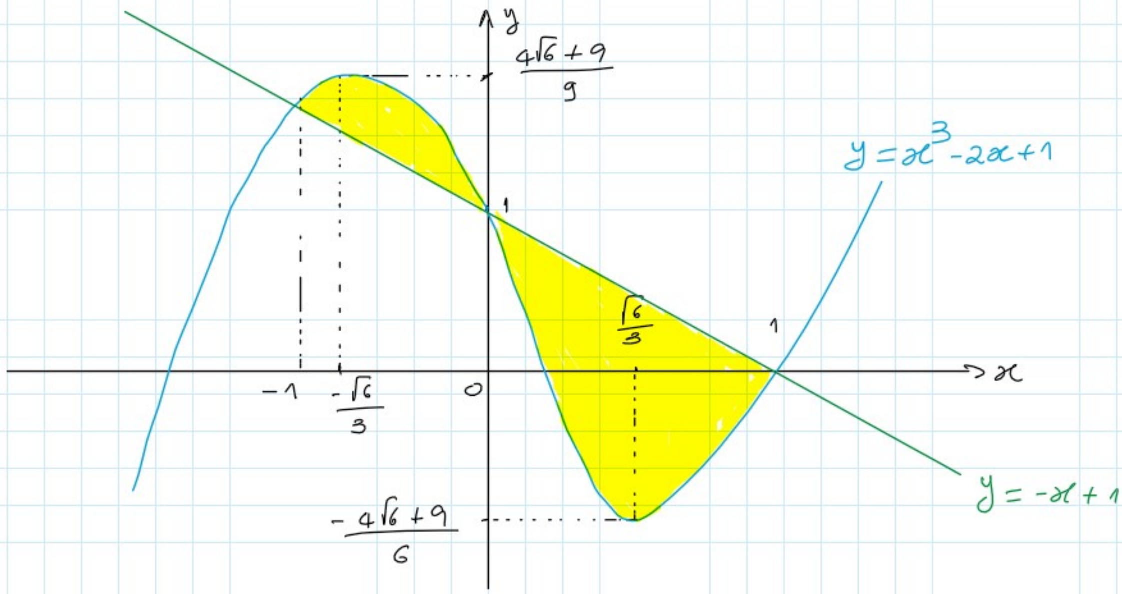
$$f\left(\frac{\sqrt{6}}{3}\right) = \frac{2\sqrt{6} - 6\sqrt{6} + 9}{9} = \frac{-4\sqrt{6} + 9}{9} \cong -0{,}8$$

$$f\left(\frac{\sqrt{6}}{3}\right) = \frac{2\sqrt{6} - 3}{9} \cong 0{,}2$$

$$x^3 - 2x + 1 = -x + 1 \Rightarrow x^3 - x = 0 \Rightarrow x(x^2 - 1) = 0$$

$$\Rightarrow x = 0 \text{ ou } x = \pm 1$$

$f(0) = 1$, $f(1) = 0$, $f(-1) = -1 + 2 + 1 = 2$



$$A = \int_{-1}^{0} \left[(x^3 - 2x + 1) - (-x+1) \right] dx + \int_{0}^{1} \left[(-x+1) - (x^3 - 2x + 1) \right] dx$$

$$= \int_{-1}^{0} (x^3 - 2x + 1 + x - 1)\,dx + \int_{0}^{1} (-x + 1 - x^3 + 2x - 1)\,dx$$

$$= \int_{-1}^{0} (x^3 - x)\,dx + \int_{0}^{1} (-x^3 + x)\,dx$$

$$= \left(\frac{x^4}{4} - \frac{x^2}{2} \right) \Big|_{-1}^{0} + \left(-\frac{x^4}{4} + \frac{x^2}{2} \right) \Big|_{0}^{1}$$

$$= \left(\frac{0^4}{4} - \frac{0^2}{2} \right) - \left(\frac{(-1)^4}{4} - \frac{(-1)^2}{2} \right) + \left(-\frac{1^4}{4} + \frac{1^2}{2} \right) - \left(-\frac{0^4}{4} + \frac{0^2}{2} \right)$$

$$= -\left(\frac{1}{4} - \frac{1}{2} \right) + \left(-\frac{1}{4} + \frac{1}{2} \right) = -\frac{1-2}{4} + \frac{-1+2}{4}$$

$$= \frac{1+1}{4} = \frac{1}{2} \quad \text{u. a. (unidades de área)}$$

PROPRIEDADES:

1) $\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

2) $\int_a^b (k f(x))\,dx = k \int_a^b f(x)\,dx$

3) $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx, \ \forall c \in [a,b]$

3) $\int_a f(x)\,dx = \int_a f(x)\,dx + \int_c f(x)\,dx$, $a < c < b$

4) $$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

5) $$\int_a^a f(x)\,dx = 0$$