$$\int \mathrm{tg}^8 x\,dx = \int \frac{1}{\sec^2 x}\,\mathrm{tg}^8 x\,\sec^2 x\,dx$$

$\sec^0 x$, 0 par

$u = \mathrm{tg}\,x$ $\quad du = \sec^2 x\,dx$ $\quad \sec^2 x = 1+\mathrm{tg}^2 x = 1+u^2$

$$= \int \frac{1}{1+u^2}\,u^8\,du = \int (u^6-u^4+u^2-1)\,du + \int \frac{1}{1+u^2}\,du$$

$$\begin{array}{l|l}
u^8 & u^2+1 \\ \cline{2-2}
-u^8-u^6 & u^6-u^4+u^2-1 \\ \cline{1-1}
\quad -u^6 & \\
\quad u^6+u^4 & \\ \cline{1-1}
\qquad u^4 & \\
\qquad -u^4-u^2 & \\ \cline{1-1}
\qquad\quad -u^2 & \\
\qquad\quad u^2+1 & \\ \cline{1-1}
\qquad\qquad 1 &
\end{array}$$

$$u^8 = (u^6-u^4+u^2-1)(u^2+1)+1$$

$$= \frac{u^7}{7} - \frac{u^5}{5} + \frac{u^3}{3} - u + \mathrm{arctg}\,u + C, \; C\in\mathbb{R}$$

$$= \frac{\mathrm{tg}^7 x}{7} - \frac{\mathrm{tg}^5 x}{5} + \frac{\mathrm{tg}^3 x}{3} - \mathrm{tg}\,x + x + C, \; C\in\mathbb{R}$$

$$\int tg^9 x\,dx = \int \frac{1}{\sec x}\, tg^8 x\; tg x \sec x\,dx$$

9 impar

$$\boxed{\begin{array}{l} u = \sec x \qquad du = \sec x\, tg x\, dx \\ u^2 - 1 = \sec^2 x - 1 = tg^2 x \end{array}}$$

2~ 1 2 1

3~ 1 3 3 1

4~ 1 4 6 4 1

$$= \int \frac{1}{u} (u^2-1)^4\, du$$

$$\boxed{\begin{array}{l} (u^2-1)^4 = (u^2)^4 \cdot (-1)^0 + 4(u^2)^3(-1) + 6(u^2)^2(-1)^2 + 4(u^2)^1(-1)^3 + (u^2)^0 \cdot (-1)^4 \\ = u^8 - 4u^6 + 6u^4 - 4u^2 + 1 \end{array}}$$

$$= \int u^7 - 4u^5 + 6u^3 - 4u + \frac{1}{u}\, du$$

$$= \frac{u^8}{8} - \frac{4}{6}u^6 + \frac{6}{4}u^4 - \frac{4}{2}u^2 + \ln|u| + c, \; c \in \mathbb{R}$$

$$= \frac{\sec^8 x}{8} - \frac{2}{3}\sec^6 x + \frac{3}{2}\sec^4 x - 2\sec^2 x + \ln|\sec x| + c, \; c \in \mathbb{R}$$

$\int \sec^8 x \, dx = \int \sec^6 x \sec^2 x \, dx$

8 par

$u = tg\,x \quad du = \sec^2 x \, dx$

$\sec^2 x = 1 + tg^2 x = 1 + u^2$

$\sec^6 x = (1+u^2)^3$

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$= \int (1+u^2)^3 \, du = \int 1.1^0(u^2)^3 + 3.1^1(u^2)^2 + 3.1^2.(u^2)^1 + 1.1^3(u^2)^0 \, du$

$= \int u^6 + 3u^4 + 3u^2 + 1 \, du$

$= \frac{u^7}{7} + \frac{3}{5}u^5 + \frac{3}{3}u^3 + u + C, \; C \in \mathbb{R}$

$= \frac{tg^7 x}{7} + \frac{3}{5} tg^5 x + tg^3 x + tg\,x + C, \; C \in \mathbb{R}$

$\int \sec^m x \, tg^n x \, dx$

m par $\Rightarrow u = tg\,x$

n impar $\Rightarrow u = \sec x$

Se m impar e n par então nem $u = tg\,x$ ou $u = \sec x$ funcionam.

$$\int \sec^3 x\,dx = \int \underbrace{\sec x}_{\int F} \cdot \underbrace{\sec^2 x}_{g}\,dx$$

(baixar 2 graus).

$$= \underbrace{\sec x}_{F}\,\underbrace{tg\,x}_{G} - \int \underbrace{\sec x\, tg\,x}_{\int f} \cdot \underbrace{tg\,x}_{G}\,dx \qquad 1+tg^2 x = \sec^2 x$$

$$= \sec x\, tg\,x - \int \sec x\,(\sec^2 x - 1)\,dx$$

$$= \sec x\, tg\,x - \int \sec^3 x - \sec x\,dx$$

$$= \sec x\, tg\,x - \int \sec^3 x + \int \sec x\,dx$$

$$\therefore\ 2\int \sec^3 x\,dx = \sec x\, tg\,x + \int \sec x\,dx$$

$$\therefore\ \int \sec^3 x\,dx = \frac{1}{2}\left(\sec x\, tg\,x + \ln|\sec x + tg\,x|\right) + C,\ C \in \mathbb{R}$$

$$\int \sec^{11} x\, tg^{10} x\,dx$$

(baixar de 21 para 19)

11 impar E 10 par

$$= \begin{cases} \int (\sec^{10} x\, tg^{9} x)(\sec x \cdot tg\,x)\,dx \\ \int (\sec^{9} x\, tg^{10} x)(\sec^2 x)\,dx \end{cases}$$

$$\int \sec^{11}x\, tg^{10}x\, dx = \int \underbrace{\sec^{10}x\, tg^{9}x}_{\int F}\,(\underbrace{\sec x\, tg x}_{g})\,dx$$

$$= \underbrace{\sec^{10}x\, tg^{9}x}_{F}\, \underbrace{\sec x}_{G} -$$

$$\int \underbrace{[10\sec^{9}x \cdot \sec x\, tg x\, tg^{9}x + \sec^{10}x \cdot 9\, tg^{8}x \sec^{2}x]}_{f'} \cdot \underbrace{\sec x}_{G}\, dx$$

$$= \sec^{11}x\, tg^{9}x - \int 10\sec^{11}x\, tg^{10}x + 9\sec^{11}x\, tg^{8}x\, \sec^{2}x\, dx$$

$$= \sec^{11}x\, tg^{9}x - \int 10\sec^{11}x\, tg^{10}x + 9\sec^{11}x\, tg^{8}x\,(1 + tg^{2}x)\,dx$$

$$= \sec^{11}x\, tg^{9}x - 10\int \sec^{11}x\, tg^{10}x\, dx$$

$$- 9\int \sec^{11}x\, tg^{8}x\, dx - 9\int \sec^{11}x\, tg^{10}x\, dx$$

$$\therefore 20\int \sec^{11}x\, tg^{10}x\, dx = \sec^{11}x\, tg^{9}x - 9\int \sec^{11}x\, tg^{8}x\, dx$$

$$\therefore \int \sec^{11}x\, tg^{10}x\, dx = \frac{\sec^{11}x\, tg^{9}x}{20} - \frac{9}{20}\int \sec^{11}x\, tg^{8}x\, dx$$

$11 + 8 = 19$

$\int \sec^{11}x \, tg^{10}x \, dx$

$= \int \sec^9 x \, tg^{10}x \, \sec^2 x \, dx$

$= \sec^9 x \, tg^{10}x \, tgx$

$\int [9\sec^8 x \sec x \, tgx \, tg^{10}x + \sec^9 x . 10 . tg^9 x \sec^2 x] \, tgx \, dx$

$= \sec^9 x \, tg^{11}x -$ $\qquad tg^2 x = \sec^2 x - 1$

$\int 9\sec^9 x \, tg^{12}x + 10\sec^{11}x \, tg^{10}x \, dx$

$= \sec^9 x \, tg^{11}x - 9\int \sec^{11}x \, tg^{10}x - \sec^9 x \, tg^{10}x \, dx$

$-10\int \sec^{11}x \, tg^{10}x \, dx$

$\therefore 20\int \sec^{11}x \, tg^{10}x \, dx = \sec^9 x \, tg^{11}x + 9\int \sec^9 x \, tg^{10}x \, dx$

$\therefore \int \sec^{11}x \, tg^{10}x \, dx = \dfrac{\sec^9 x \, tg^{11}x}{20} + \dfrac{9}{20}\int \sec^9 x \, tg^{10}x \, dx$

$9+10=19$