$$\int \underbrace{x}_{\int f} \underbrace{\ln(x^2+1)}_{G}\,dx = \underbrace{\frac{x^2}{2}}_{F}\underbrace{\ln(x^2+1)}_{G} - \underbrace{\int \frac{x^2}{2}\cdot\frac{1}{x^2+1}\,2x\,dx}_{-\int Fg}$$

$$= \frac{x^2}{2}\ln(x^2+1) - \frac{x^2}{2} + \frac{1}{2}\ln(1+x^2) + C,\quad C\in\mathbb{R}$$

$$= \frac{(x^2+1)}{2}\ln(x^2+1) - \frac{x^2}{2} + C,\ C\in\mathbb{R}.$$

$$\int \frac{x^3}{x^2+1}\,dx = \int x\,dx - \int \frac{x}{x^2+1}\,dx = \frac{x^2}{2} - \int \frac{1}{u}\,\frac{1}{2}\,du$$

$u = x^2+1$ $du = 2x\,dx$

$\therefore \boxed{\frac{1}{2}du = x\,dx}$

$$\begin{array}{r|l} x^3+0x^2+0x+0 & x^2+1 \\ \hline -x^3 \qquad\qquad -x & x \\ \hline -x & \end{array}$$

$\therefore x^3 = (x^2+1)\cdot x - x$

$\therefore \dfrac{x^3}{x^2+1} = x - \dfrac{x}{x^2+1}$

$$\therefore \int \frac{x^3}{x^2+1}\,dx = \frac{x^2}{2} - \frac{1}{2}\ln|u| + c$$

$$= \boxed{\frac{x^2}{2} - \frac{1}{2}\ln|x^2+1| + c,\ c\in\mathbb{R}}$$

$$\int \text{sen}^3 x \cos^7 x \, dx = \int \text{sen}^2 x \boxed{\cos^7 x} \boxed{\text{sen}\, x \, dx}$$

3 impar

2 par

$\boxed{u = \cos x}$ $\quad du = -\text{sen}\, x \, dx$

$\boxed{-du = \text{sen}\, x \, dx}$

$\text{sen}^2 x = 1 - \cos^2 x = 1 - u^2$

$$= \int (1-u^2) \boxed{u}^7 (-1) \, du$$

$$= \int u^9 - u^7 \, du = \frac{u^{10}}{10} - \frac{u^8}{8} + C = \frac{\cos^{10} x}{10} - \frac{\cos^8 x}{8} + C,$$

$C \in \mathbb{R}$.

$$\int \text{sen}^8 x \cos^3 x \, dx = \int \text{sen}^8 x \boxed{\cos^2 x} \cos x \, dx$$

3 impar

$u = \text{sen}\, x$ $\quad du = \cos x \, dx$

$$= \int u^8 \boxed{(1-u^2)} \, du = \int u^8 - u^{10} \, du$$

$$= \frac{u^9}{9} - \frac{u^{11}}{11} + C = \frac{\text{sen}^9 x}{9} - \frac{\text{sen}^{11} x}{11} + C, \ C \in \mathbb{R}$$

$$\int \sec x\,dx = \int \frac{1}{\cos x}\,dx = \int \frac{1}{\cos^2 x}\cos x\,dx$$

1é impar

$$\boxed{\begin{aligned} u &= \sin x \\ du &= \cos x\,dx \end{aligned}}$$

$$\cos^2 x = 1 - u^2$$

$$\overset{\star}{=} \int \frac{1}{1-u^2}\,du = \frac{1}{2}\int \frac{1}{1-u}\,du + \frac{1}{2}\int \frac{1}{1+u}\,du$$

$$\star \quad \frac{1}{1-u^2} = \frac{a}{1-u} + \frac{b}{1+u} = \frac{a(1+u)+b(1-u)}{1-u^2}$$

$$\therefore \; a+b+(a-b)u = 1$$

$$\Leftrightarrow \begin{cases} a+b=1 \\ a-b=0 \end{cases} \Leftrightarrow \begin{cases} a=b \\ 2a=1 \end{cases} \Leftrightarrow a=b=\frac{1}{2}$$

$$= -\frac{1}{2}\ln|1-u| + \frac{1}{2}\ln|1+u| + C$$

$$= \ln\left|\frac{1+u}{1-u}\right|^{\frac{1}{2}} + C = \ln\left|\frac{(1+u)^2}{1-u^2}\right|^{\frac{1}{2}} + C$$

$$= \ln\left|\frac{(1+\sin x)^2}{1-\sin^2 x}\right|^{1/2} + C = \ln\left|\frac{1+\sin x}{\cos x}\right| + C$$

$$= \ln|\sec x + \operatorname{tg} x| + C, \; C \in \mathbb{R}$$

abaixar grau de 22 para 20.

$$\int \operatorname{sen}^{10} x \cos^{12} x \, dx = \int \cos x \left(\operatorname{sen}^{10} x \cos^{11} x \right) dx$$

10 par   12 par   separa p/ partes $\int f$ G

$$= \operatorname{sen} x \left(\operatorname{sen}^{10} x \cos^{11} x \right) - \int \operatorname{sen} x \left[10 \operatorname{sen}^{9} x \cos x \cos^{11} x + \operatorname{sen}^{10} x \left(11 \cos^{10} x (-\operatorname{sen} x) \right) \right] dx$$

F G $-\int F$ g

$$= \operatorname{sen}^{11} x \cos^{11} x - \int 10 \operatorname{sen}^{10} x \cos^{12} x \, dx + \int 11 \operatorname{sen}^{12} x \cos^{10} x \, dx$$

10 12 ok   * 12 10 mexer

$$= \operatorname{sen}^{11} x \cos^{11} x - 10 \int \operatorname{sen}^{10} x \cos^{12} x \, dx + 11 \int \operatorname{sen}^{10} x \cos^{10} x \, dx - 11 \int \operatorname{sen}^{10} x \cos^{12} x \, dx$$

$$\boxed{\begin{aligned} * \int 11 \operatorname{sen}^{12} x \cos^{10} x \, dx &= \int 11 \left((1 - \cos^2 x) \operatorname{sen}^{10} x \cos^{10} x \right) dx \\ &= 11 \int \operatorname{sen}^{10} x \cos^{10} x \, dx - 11 \int \operatorname{sen}^{10} x \cos^{12} x \, dx \end{aligned}}$$

$$\therefore \int \operatorname{sen}^{10} x \cos^{12} x \, dx = \operatorname{sen}^{11} x \cos^{11} x + 11 \int \operatorname{sen}^{10} x \cos^{10} x \, dx - 21 \int \operatorname{sen}^{10} x \cos^{12} x \, dx$$

$$\therefore 22 \int \operatorname{sen}^{10} x \cos^{12} x \, dx = \operatorname{sen}^{11} x \cos^{11} x + 11 \int \operatorname{sen}^{10} x \cos^{10} x \, dx$$

$$\therefore \int \operatorname{sen}^{10} x \cos^{12} x \, dx = \frac{1}{22} \left[\operatorname{sen}^{11} x \cos^{11} x + 11 \int \operatorname{sen}^{10} x \cos^{10} x \, dx \right]$$