# Calculo lista 6

(7g)

$$\lim_{x\to+\infty}\left[x\ln 2-\ln(3^x+1)\right]$$

$$=\lim_{x\to+\infty}\left[\ln 2^x-\ln(3^x+1)\right]$$

$$=\lim_{x\to+\infty}\ln\left(\frac{2^x}{3^x+1}\right)=-\infty$$

$\frac{2^x}{3^x+1}\to 0^+$

$$\star\ \lim_{x\to+\infty}\frac{2^x}{3^x+1}=\lim_{x\to+\infty}\frac{2^x}{3^x}\left(\frac{1}{1+\frac{1}{3^x}}\right)$$

$$=\lim_{x\to+\infty}\left(\frac{2}{3}\right)^x\left(\frac{1}{1+\frac{1}{3^x}}\right)=0^+$$

$\frac{2}{3}<1$ $\to 0^+$ ; $\frac{1}{3^x}\to 0$ ; $\frac{1}{1+0}\to 1$

(8c)

$$\lim_{x\to+\infty}\left(\frac{x+2}{x+1}\right)^x = \lim_{x\to+\infty}\left(1+\frac{1}{x+1}\right)^x$$

$y=x+1,\ x=y-1$

$$= \lim_{y\to+\infty}\left(1+\frac{1}{y}\right)^{y-1} = \lim_{y\to+\infty}\left(1+\frac{1}{y}\right)^{y}\cdot\left(1+\frac{1}{y}\right)^{-1}$$

$\left(1+\frac{1}{y}\right)^{y} \to e$

$\left(1+\frac{1}{y}\right)^{-1} \to 1$

$$= e\cdot 1^{-1} = e$$

(9d)

$$\lim_{x\to 0} (1+2^x)^x$$

Vamos calcular $x \to 0^+$ primeiro

$$\lim_{x\to 0^+} (1+2^x)^x$$

$0 < x \leq 1$

$2^x$ é crescente

$1 \leq 1+2^x \leq 1+2^1 = 3$ como $x > 0$ temos

$1^x \leq (1+2^x)^x \leq 3^x$

$\downarrow \qquad \downarrow$ pelo confronto,

$1 \qquad 1$

$$\lim_{x\to 0^+} (1+2^x)^x = 1$$

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$$\lim_{x\to 0^-} (1+2^x)^x$$

$-1 \leq x < 0$

$1 \leq 1+2^x \leq 1+2^0 = 2$ ($2^x$ é crescente)

$1^x \geq (1+2^x)^x \geq 2^x$ ($x<0$)

pelo confronto $\lim_{x\to 0^-} (1+2^x)^x = 1$

como os limites laterais são iguais, segue que

$$\lim_{x \to 0} (1+2^x)^x = 1.$$

22 d

$$\left(\frac{x+\sqrt[4]{x}}{x^2+3}\right)' = \left(\frac{x+x^{\frac{1}{4}}}{x^2+3}\right)' =$$

$$\frac{(x+x^{\frac{1}{4}})'(x^2+3)-(x+x^{\frac{1}{4}})(x^2+3)'}{(x^2+3)^2} =$$

$$\frac{(1+\frac{1}{4}x^{-\frac{3}{4}})(x^2+3)-(x+x^{\frac{1}{4}})(2x)}{(x^2+3)^2} =$$ ← derivação já terminou

$$\frac{x^2+\frac{1}{4}x^{5/4}+3+\frac{3}{4}x^{-\frac{3}{4}}-2x^2-2x^{\frac{5}{4}}}{(x^2+3)^2} =$$

$$\frac{-x^2-\frac{7}{4}x^{5/4}+\frac{3}{4}x^{-\frac{3}{4}}+3}{(x^2+3)^2} =$$

$$= \frac{-x^2-\frac{7}{4}x^{5/4}+\frac{3}{4}\frac{1}{\sqrt[4]{x^3}}+3}{(x^2+3)^2}$$