Gráfico de $f(x) = 9x^4 - 28x^3 + 12x^2 + 9$

$$f'(x) = 36x^3 - 28.3x^2 + 12.2x$$
$$= 12x(3x^2 - 7x + 2)$$

$$f''(x) = 36.3x^2 - 28.6x + 12.2$$
$$= 12(9x^2 - 14x + 2)$$

$f'(x) = 0 \Leftrightarrow x = 0$ ou $3x^2 - 7x + 2 = 0$

$$\Leftrightarrow x = 0 \quad x = \frac{7 \pm \sqrt{49-24}}{6} = \frac{7 \pm 5}{6} = 2 \text{ ou } \frac{1}{3}$$

$$f''(x) = 0 \Leftrightarrow x = \frac{14 \pm \sqrt{(-14)^2 - 4.9.2}}{18}$$
$$= \frac{14 \pm \sqrt{4[(-7)^2 - 18]}}{18}$$
$$= \frac{14 \pm 2\sqrt{49-18}}{18} = \frac{7 \pm \sqrt{31}}{9}$$

$$0, 2, \frac{1}{3}, \frac{7+\sqrt{31}}{9}, \frac{7-\sqrt{31}}{9}$$

$0 < \dfrac{7-\sqrt{31}}{9}$ | $\dfrac{7-\sqrt{31}}{9} < \dfrac{1}{3} \Leftrightarrow 7 - \sqrt{31} < 3$

$\Leftrightarrow 4 < \sqrt{31} \Leftrightarrow 16 < 31$

$$\frac{7+\sqrt{31}}{9} > \frac{1}{3} \Leftrightarrow 7+\sqrt{31} > 3 \Leftrightarrow 4+\sqrt{31} > 0$$

$$\frac{7+\sqrt{31}}{9} < 2 \Leftrightarrow 7+\sqrt{31} < 18 \Leftrightarrow \sqrt{31} < 11$$

assim

$$0 < \frac{7-\sqrt{31}}{9} < \frac{1}{3} < \frac{7+\sqrt{31}}{9} < 2$$

$$\frac{7+\sqrt{31}}{9} < 2 \Leftrightarrow 7+\sqrt{31} < 18$$

$$\Leftrightarrow \sqrt{31} < 11$$

$\therefore$

$$0 < \frac{7-\sqrt{31}}{9} < \frac{1}{3} < \frac{7+\sqrt{3}}{9} < 2$$

		0		$\frac{7-\sqrt{31}}{9}$		$\frac{1}{3}$		$\frac{7+\sqrt{31}}{9}$		2	
x	$-$	0	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$
$3x^2-7x+2$	$+$	$+$	$+$	$+$	$+$	0	$-$	$-$	$-$	0	$+$
$f'(x)$	$-$	0	$+$	$+$	$+$	0	$-$	$-$	$-$	0	$+$
c/dec f	$\searrow$	$\rightarrow$	$\nearrow$	$\nearrow$	$\nearrow$	$\rightarrow$	$\searrow$	$\searrow$	$\searrow$	$\rightarrow$	$\nearrow$
$9x^2-14x+2$	$+$	$+$	$+$	0	$-$	$-$	$-$	0	$+$	$+$	$+$
$f''(x)$	$+$	$+$	$+$	0	$-$	$-$	$-$	0	$+$	$+$	$+$
conc f	$\cup$	$\cup$	$\cup$	P.I	$\cap$	$\cap$	$\cap$	P.I	$\cup$	$\cup$	$\cup$

$$f(0)=9(0)^4-28(0)^3+12(0)^2+9=9$$

$$f\left(\frac{1}{3}\right)=\frac{9}{81}-\frac{28}{27}+\frac{12}{9}+9=$$

$$=\frac{3}{27}-\frac{28}{27}+\frac{36}{27}+9=\frac{11}{27}+9$$

$$f(2)=9.2^4-28.2^3+12.2^2+9=$$

$$=9.2^4-14.2^4+3.2^4+9=$$

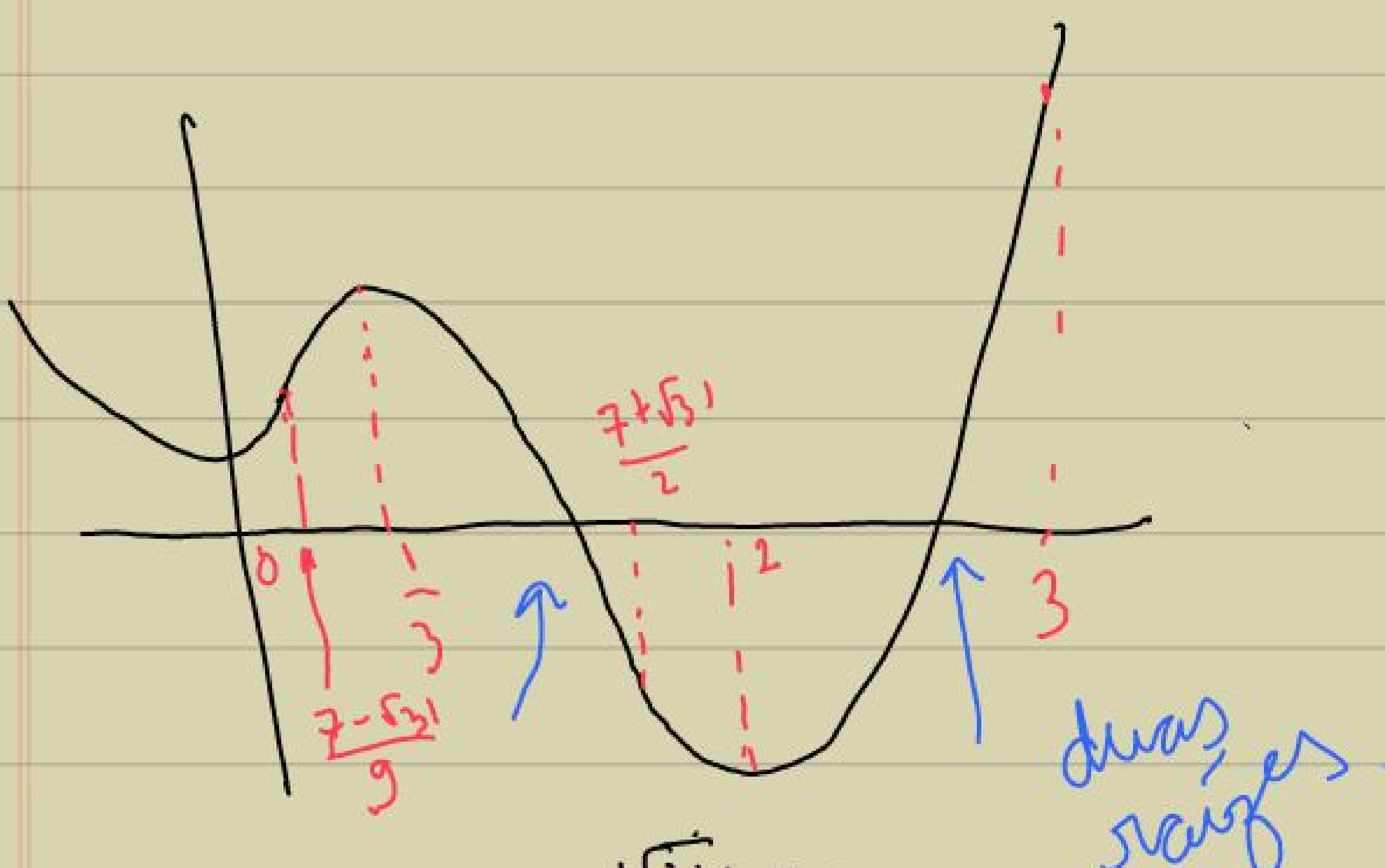
$$=-2.2^4+9=-32+9=-23$$

$$\lim_{x\to+\infty} 9x^4-28x^3+12x^2+9=$$

$$=\lim_{x\to+\infty} \underbrace{x^4}_{\to+\infty}\underbrace{\left(9-\frac{28}{x}+\frac{12}{x^2}+\frac{9}{x^4}\right)}_{\to 9}=+\infty$$

$$\lim_{x\to-\infty} 9x^4-28x^3+12x^2+9$$

$$=\lim_{x\to-\infty} \underbrace{x^4}_{\to+\infty}\underbrace{\left(9-\frac{28}{x}+\frac{12}{x^2}+\frac{9}{x^4}\right)}_{\to 9}=+\infty$$



$$f\left(\frac{7+\sqrt{31}}{9}\right)<0$$

$$f(3)=9.3^4-28.3^3+12.3^2+9$$

$$=9[3^4-28.3+12+1]$$

$$=9[3(27-28)+13]$$

$$=9[-3+13]=90$$

$$\frac{12}{9} < \frac{7+\sqrt{31}}{9} < \frac{13}{9}$$

$$12 < 7+\sqrt{31} < 13$$

$$5 < \sqrt{31} < 6$$

$$9\left(\frac{12}{9}\right)^4 - 28\left(\frac{12}{9}\right)^3 + 12\left(\frac{12}{9}\right)^2 + 9$$

$$= 9\left(\frac{4}{3}\right)^4 - 28\cdot\left(\frac{4}{3}\right)^3 + 12\left(\frac{4}{3}\right)^2 + 9$$

$$= \frac{4^4}{27} - 28\cdot\frac{4^3}{27} + 36\cdot\frac{4^2}{27} + \frac{9\cdot 27}{27}$$

$$= \frac{1}{27}\left(4^3(4-28+9) + 9\cdot 27\right)$$

$$= \frac{1}{27}\left(4^3(-15) + 9\cdot 27\right) = \frac{1}{9}\left(4^3\cdot(-5) + 3\cdot 27\right)$$

$$= \frac{1}{9}(-320+81) < 0$$