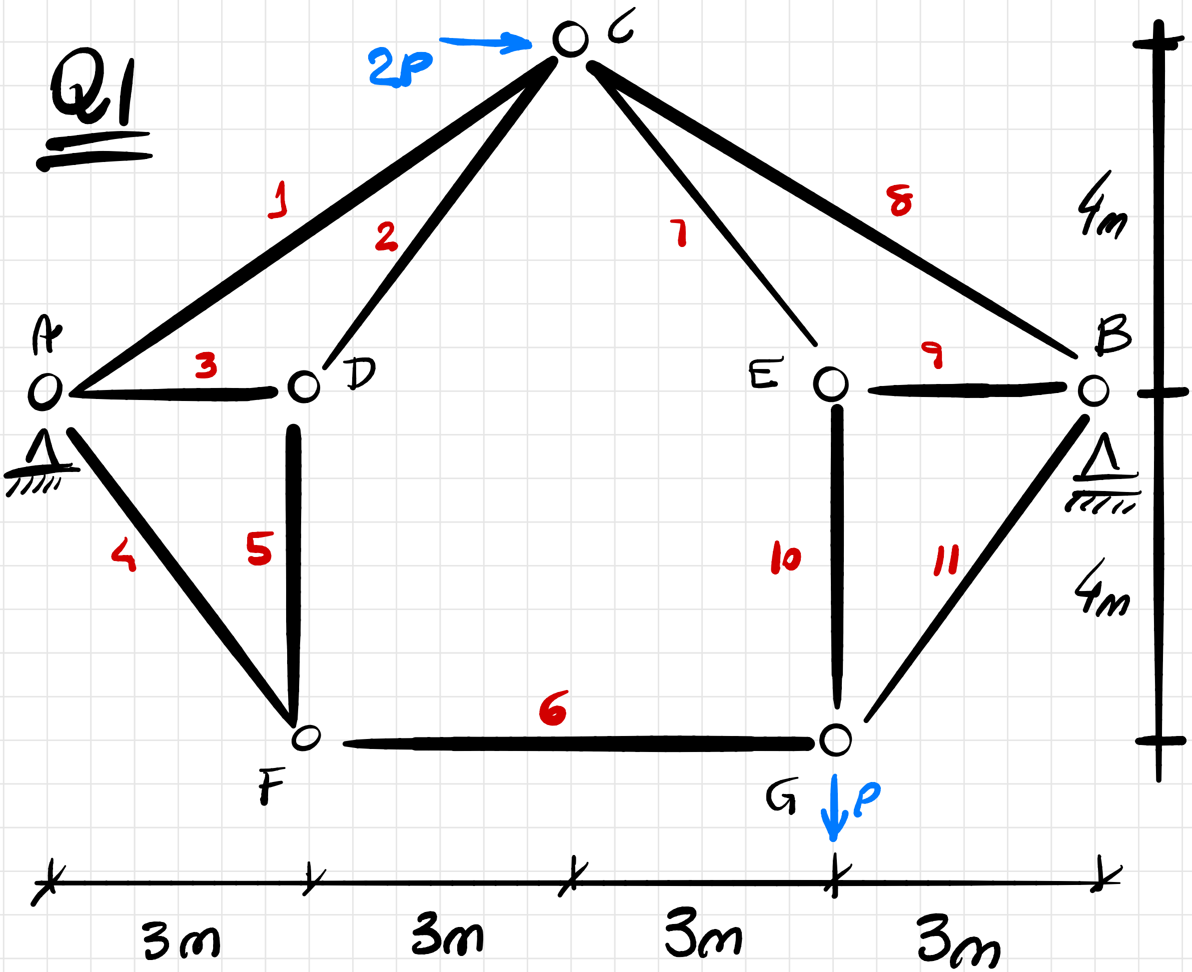
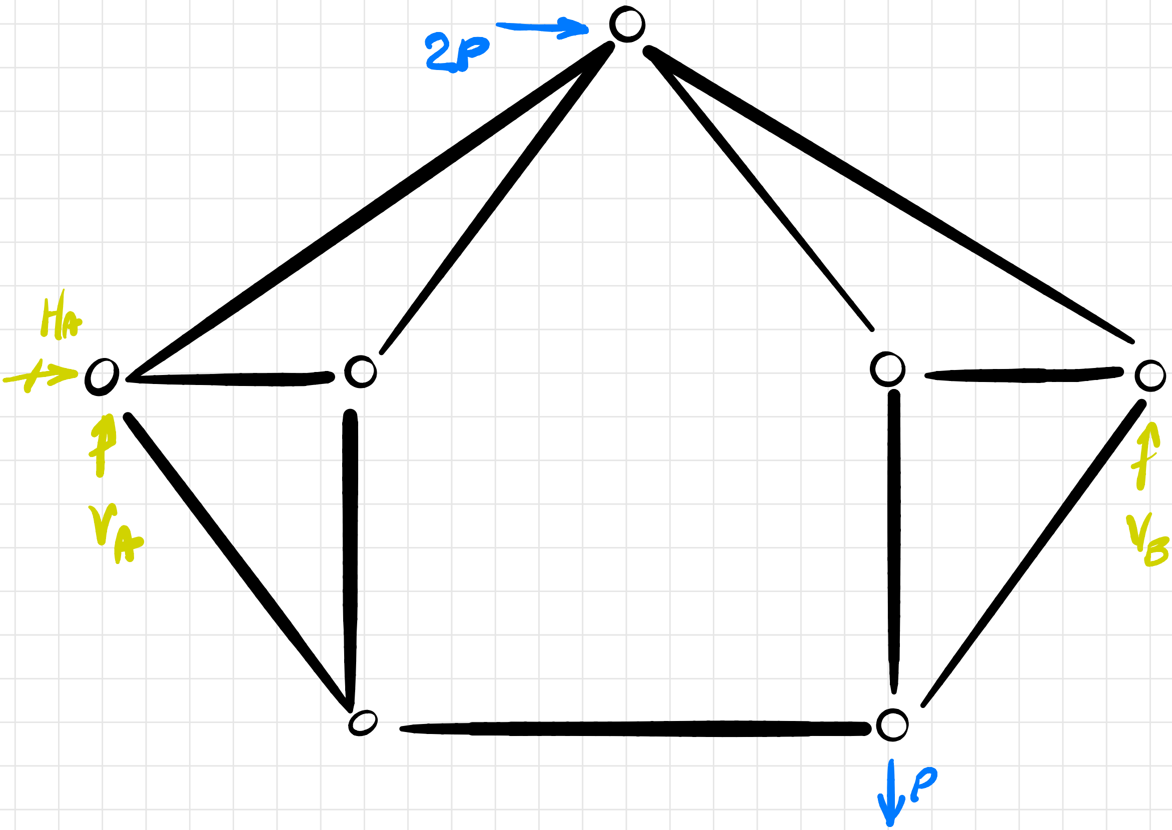
Q1



Determinar N_1, N_2 e N_6.

0,5 pontos por normal

total **1,5**



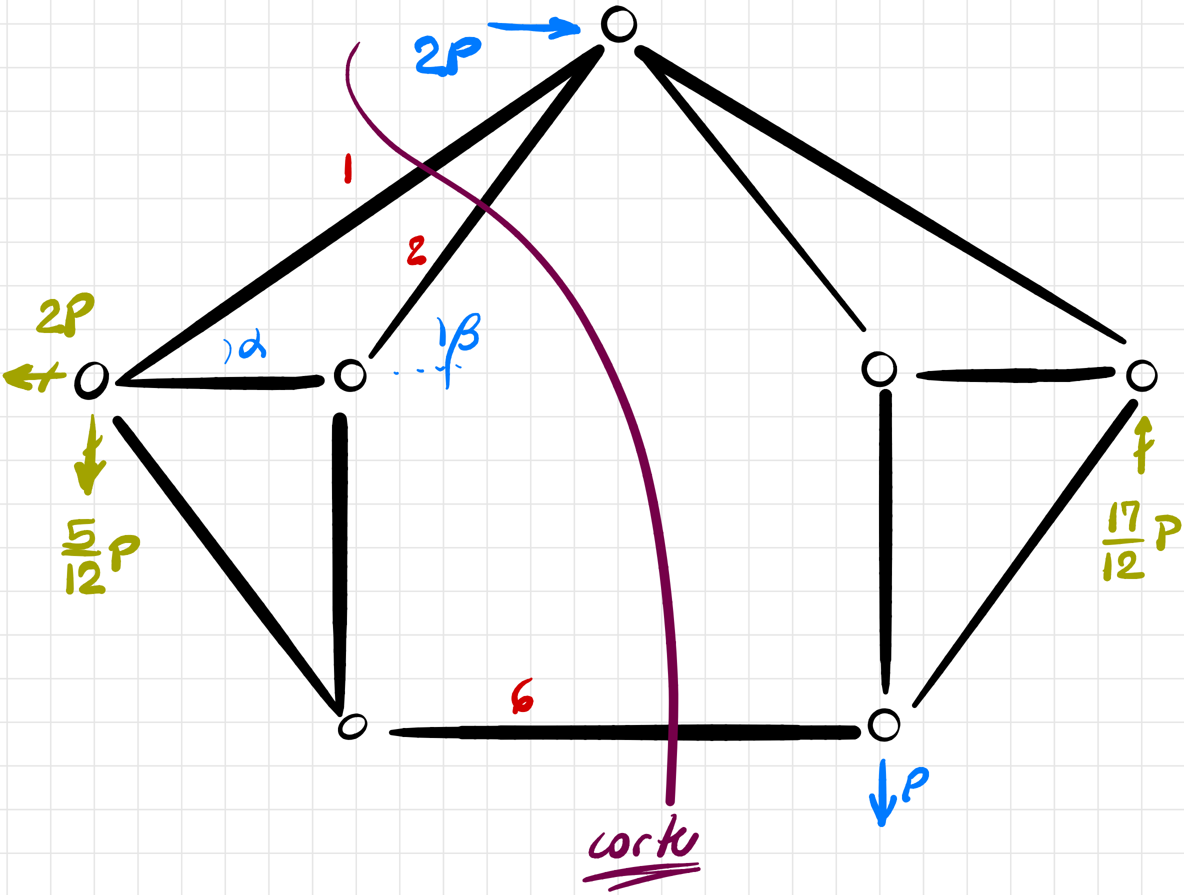
Equilíbrio:

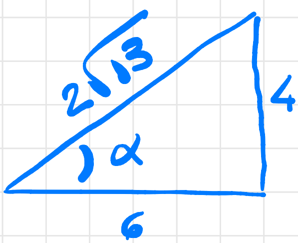
$$\sum F_H = 0: \; H_A + 2P = 0 \Rightarrow \boxed{H_A = -2P}$$

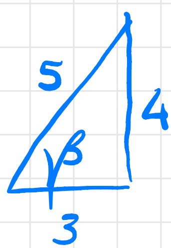
$$\sum F_V = 0: \; V_A + V_B - P = 0$$

$$(+) \sum M_A = 0: \; -2P \cdot 4 - P \cdot 9 + V_B \cdot 12 = 0$$

$$\boxed{V_B = \frac{17}{12}P} \Rightarrow \boxed{V_A = -\frac{5}{12}P}$$

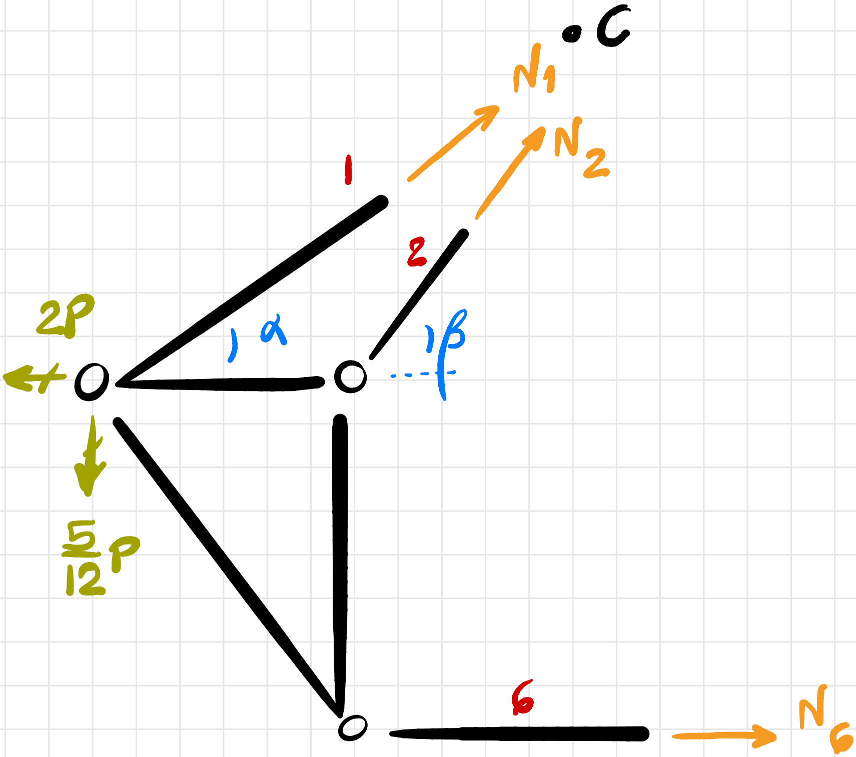






$$\text{sen}\,\alpha = \frac{2}{\sqrt{13}} \qquad \text{sen}\,\beta = 4/5$$

$$\cos\alpha = \frac{3}{\sqrt{13}} \qquad \cos\beta = 3/5$$



$$\sum F_H = 0: \; -2P + N_1\cos\alpha + N_2\cos\beta + N_6 = 0$$

$$\sum F_V = 0: \; -5/12\,P + N_1\sin\alpha + N_2\sin\beta = 0$$

$$(+)\sum M_C = 0: \; N_6 \cdot 8 + \frac{5}{12}P \cdot 6 - 2P \cdot 4 = 0$$

$$8 \cdot N_6 = \frac{11}{2}P \Rightarrow \boxed{N_6 = \frac{11}{16}P} \quad 0{,}6875P$$

$$\begin{cases} N_1 \cos\alpha + N_2 \cos\beta = 2P - {}^{11}/_{16}P = {}^{21}/_{16}P \\ N_1 \operatorname{sen}\alpha + N_2 \operatorname{sen}\beta = {}^{5}/_{12}P \end{cases}$$

$$\frac{3}{\sqrt{13}}N_1 + \frac{3}{5}N_2 = \frac{21}{16}P \Rightarrow \frac{N_1}{\sqrt{13}} + \frac{N_2}{5} = \frac{7}{16}P$$

$$\frac{2}{\sqrt{13}}N_1 + \frac{4}{5}N_2 = \frac{5}{12}P \Rightarrow \frac{N_1}{2\sqrt{13}} + \frac{N_2}{5} = \frac{5}{48}P$$

$$\frac{N_1}{\sqrt{13}} - \frac{N_1}{2\sqrt{13}} = \frac{7}{16}P - \frac{5}{48}P$$

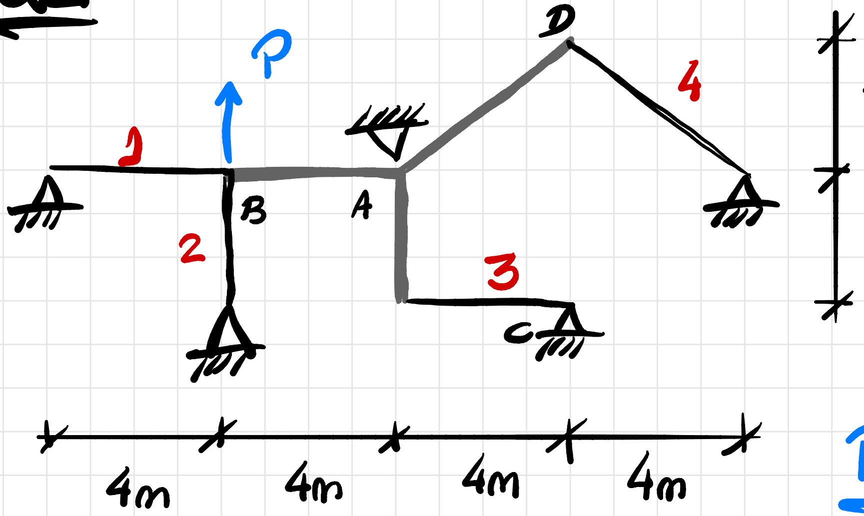
$$\frac{N_1}{2\sqrt{13}} = \frac{P}{3} \Rightarrow \boxed{N_1 = \frac{2\sqrt{13}}{3}P}$$ 2,404P

$$\frac{N_2}{5} = \frac{7}{16}P - \frac{1}{\sqrt{13}}\left(\frac{2\sqrt{13}}{3}P\right) = \frac{7}{16}P - \frac{2}{3}P = -\frac{11}{48}P$$

$$\boxed{N_2 = -\frac{55}{48}P}$$ $-1,146P$

Q2

3,5



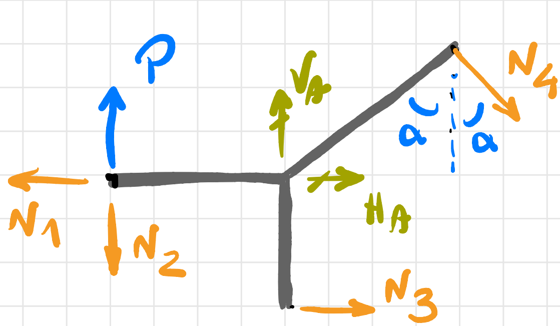
P em kN

Determinar a área mínima, considerando que a barra ABCD é rígida e:

$$\begin{cases} \bar{\sigma} = 160\,MPa \\ \bar{\varphi}_{AB} = 0{,}01\,rad \end{cases}$$

- considere que os fios são do mesmo material e tem a mesma área.

$E = 200\,GPa$



$\operatorname{sen}\alpha = 4/5$

$\cos\alpha = 3/5$

Equilíbrio:

$\sum F_H = 0: \; -N_1 + H_A + N_3 + N_4 \operatorname{sen}\alpha = 0$

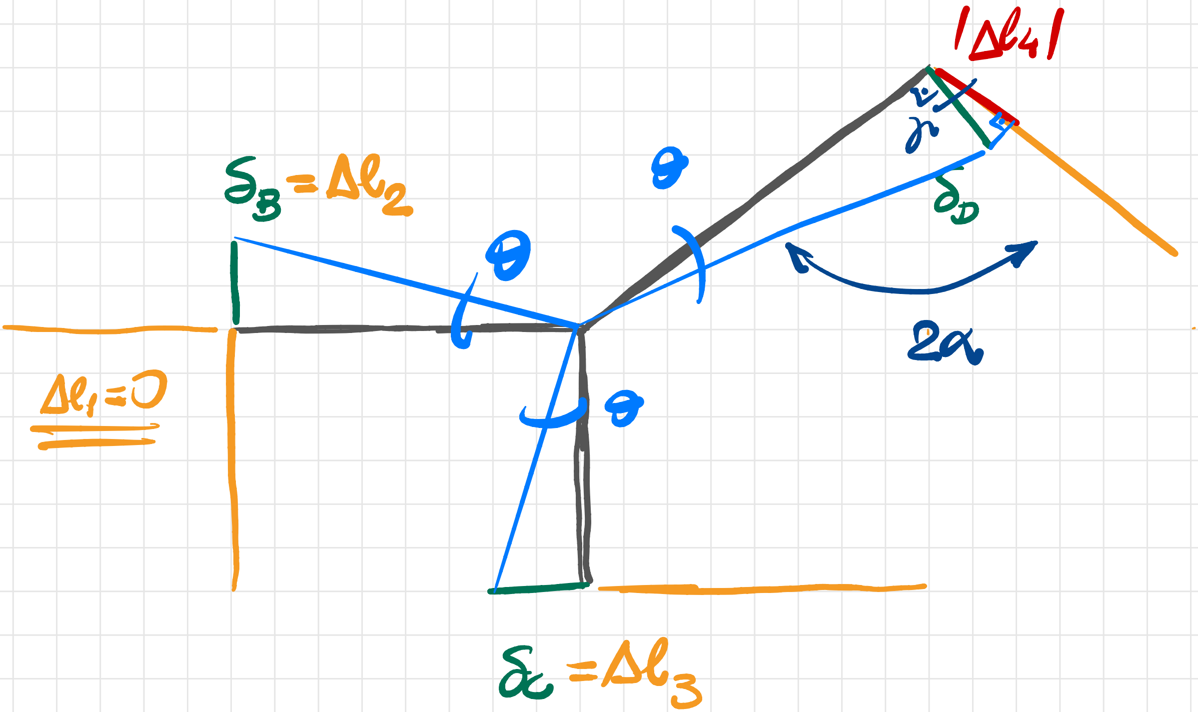
$\sum F_V = 0: \; -N_2 + P + V_A - N_4 \cos\alpha = 0$

$\circlearrowleft + \sum M_A = 0: \; -P.4 + N_2.4 + N_3.3 - N_4.\cos\alpha.4$
$- N_4 \operatorname{sen}\alpha.3 = 0$

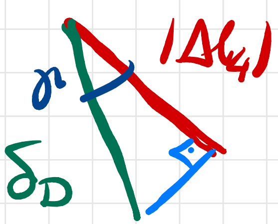
$4N_2 + 3N_3 - \frac{24}{5} N_4 = 4P$

$\boxed{20N_2 + 15N_3 - 24N_4 = 20P}$

Diagrama de Williot:



$$\frac{\pi}{2} + \gamma = 2\alpha \Rightarrow \gamma = 2\alpha - \pi/2$$



$$\cos\gamma = \frac{|\Delta l_4|}{\delta_D} \Rightarrow \delta_D = \frac{|\Delta l_4|}{\cos\gamma}$$

$$\cos\gamma = \cos(2\alpha - \pi/2) = \operatorname{sen} 2\alpha = 2\operatorname{sen}\alpha\cos\alpha = 24/25$$

Resumindo:

$\Delta l_1 = 0$

$\delta_B = \Delta l_2$

$\delta_C = \Delta l_3$

$\delta_D = \dfrac{|\Delta l_4|}{\cos \gamma}$

Mas:

$tg\theta = \dfrac{\delta_B}{4} = \dfrac{\delta_C}{3} = \dfrac{\delta_D}{5}$

E:

$\Delta l_i = \dfrac{N_i l_i}{EA} \rightarrow$ mesmos E e A

$$\frac{\Delta l_2}{4} = \frac{\Delta l_3}{3} \Rightarrow \frac{N_2 l_2}{4\cancel{EA}} = \frac{N_3 l_3}{3\cancel{EA}}$$

$$\frac{N_2 \cdot 3}{4} = \frac{N_3 \cdot 4}{3} \Rightarrow N_2 = \frac{16}{9} N_3$$

$$\frac{\Delta l_2}{4} = \frac{|\Delta l_4|}{5 \cdot \frac{24}{25}} \Rightarrow \frac{N_2 l_2}{4\cancel{EA}} = \frac{5|N_4| l_4}{24 \cdot \cancel{EA}}$$

$$\frac{N_2 \cdot 3}{4} = \frac{|N_4| \cdot 25}{24} \Rightarrow N_2 = \frac{100}{72}|N_4|$$

Assim:

$$N_1 = 0 \; ; \; N_3 = \frac{9}{16} N_2 \; ; \; N_4 = -\frac{18}{25} N_2$$

Voltando ao equilíbrio:

$$20N_2 + 15\left(\frac{9}{16}N_2\right) - 24\left(\frac{-18}{25}N_2\right) = 20P$$

$$\left(20 + \frac{135}{16} + \frac{432}{25}\right) N_2 = 20P$$

$$\frac{18287}{400} N_2 = 20P$$

$$\boxed{\begin{aligned} N_1 &= 0 \\ N_2 &= 0{,}437P \\ N_3 &= 0{,}246P \\ N_4 &= -0{,}315P \end{aligned}}$$

Dimensionando: ① tensão:

como $|N_2| > |N_3|$ e $|N_2| > |N_4|$

$\max(|\sigma_2|, |\sigma_3|, |\sigma_4|) = |\sigma_2|$

$|\sigma_2| \leq \bar{\sigma}$

$$\frac{N_2}{A} \leq \bar{\sigma} \Rightarrow A \geq \frac{N_2}{\bar{\sigma}} = \frac{\overbrace{0{,}437P.10^3}^{\text{em N}}}{\underbrace{160.10^6}_{\text{em Pa}}}$$

$A \geq 2{,}73.10^{-6}P$ [m^2]

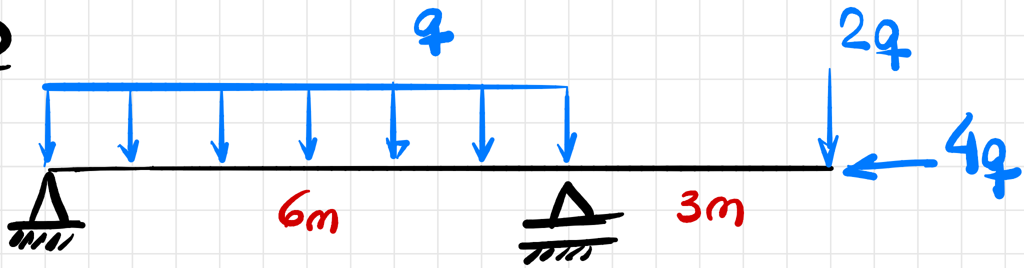
② giro máximo:

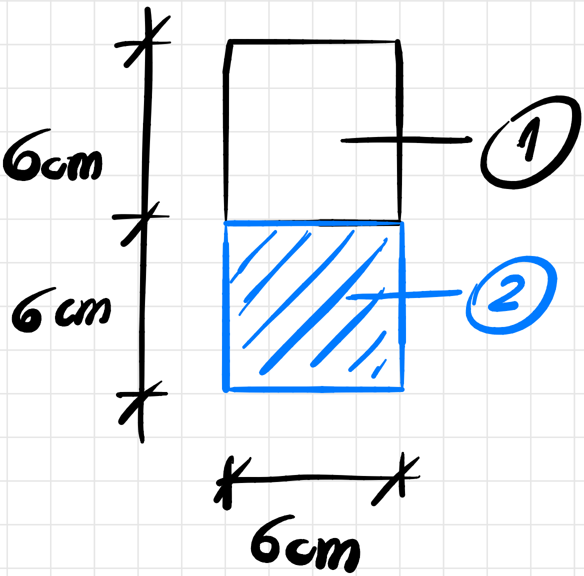
$$\varphi \simeq tg\theta \simeq \frac{\delta_B}{4} = \frac{N_2 \ell_2}{4EA} \leq \bar{\varphi}$$

$$A \geq \frac{N_2 \ell_2}{4E\bar{\varphi}} = \frac{0{,}437P.10^3.3}{4.200.10^9.0{,}01} = 1{,}63.10^{-7}P \text{ [}m^2\text{]}$$

Logo $A_{min} = 2{,}73.10^{-6}P$ [m^2]

Q3

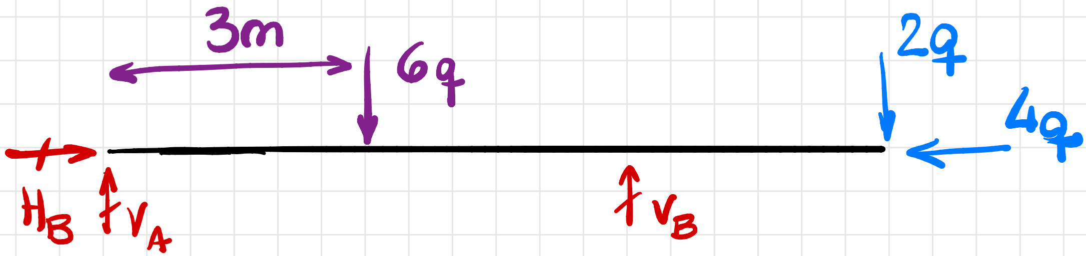




$E_1 = 210\,GPa$

$E_2 = 70\,GPa$

35



$$\sum F_H = 0: \quad H_B - 4q = 0 \Rightarrow \boxed{H_B = 4q}$$

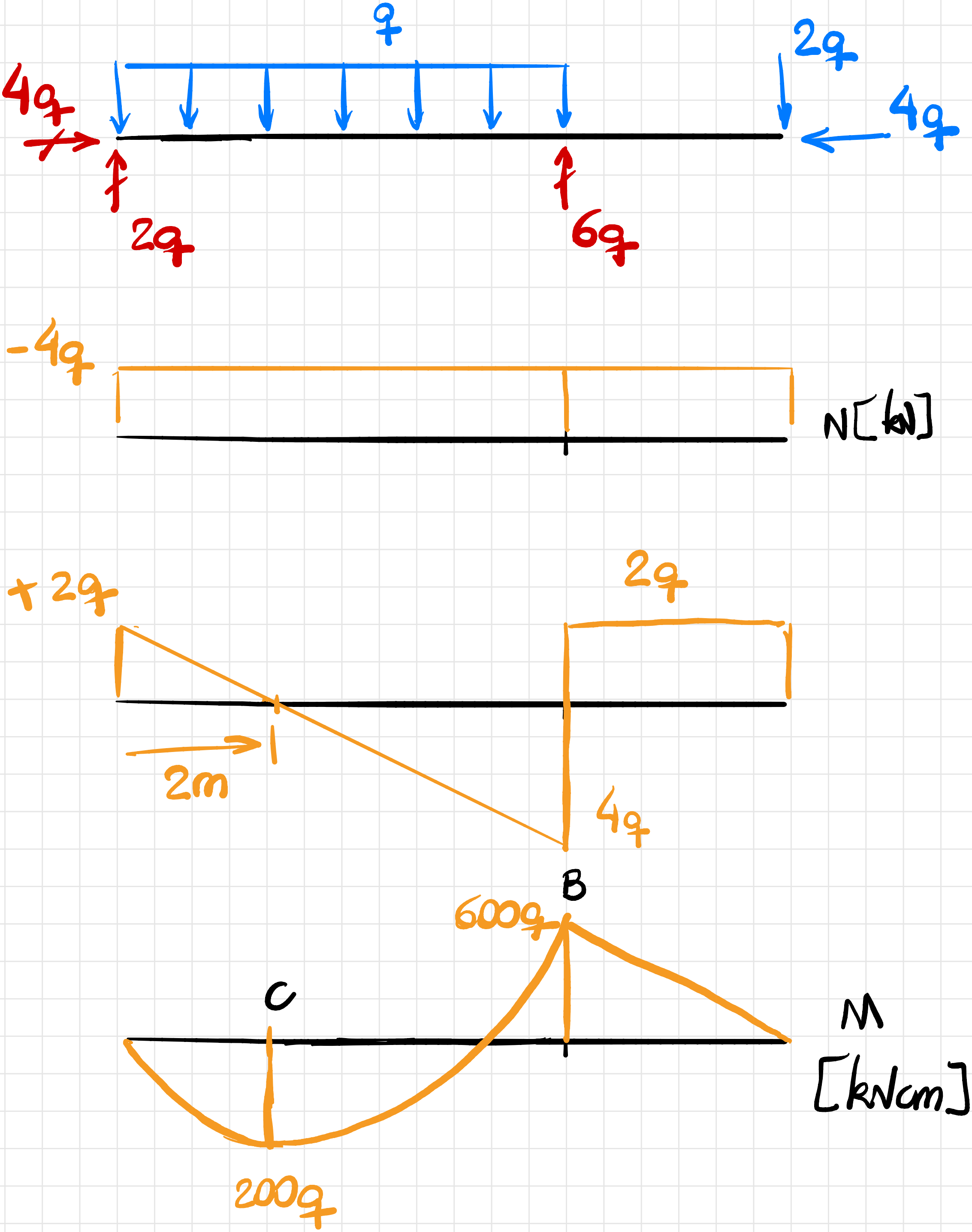
$$\sum F_V = 0: \quad V_A - 6q + V_B - 2q = 0 \Rightarrow V_A + V_B = 8q$$

$$(+)\ \sum M_A = 0: \quad -6q \cdot 3 + V_B \cdot 6 - 9 \cdot 2q = 0$$

$$6V_B = 18q + 18q$$

$$\boxed{V_B = 6q}$$

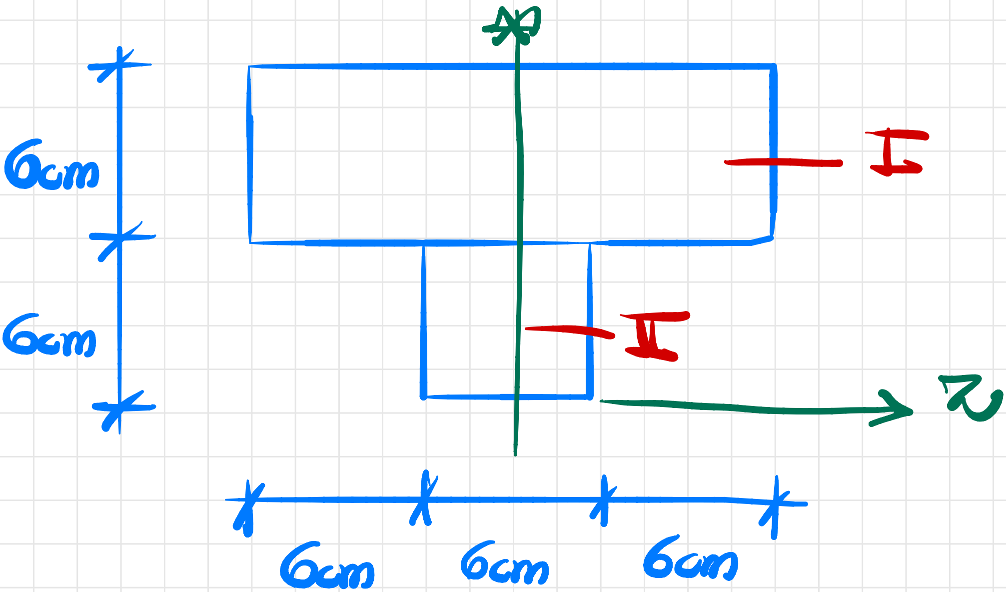
$$V_A = 8q - 6q \Rightarrow \boxed{V_A = 2q}$$


q
4q
2q
4q
2q
6q
-4q
N[kN]
+2q
2q
2m
4q
B
600q
C
M
[kNcm]
200q

Propriedades da seção transversal:

- escolhendo ② como base:

$$b_{eq} = \frac{E_1}{E_2} b = \frac{210}{70} b = 3.6 = 18cm$$



	y_G	A	d	d^2A	I_{z0}
I	9	108	1,5	243	324
II	3	36	4,5	729	108

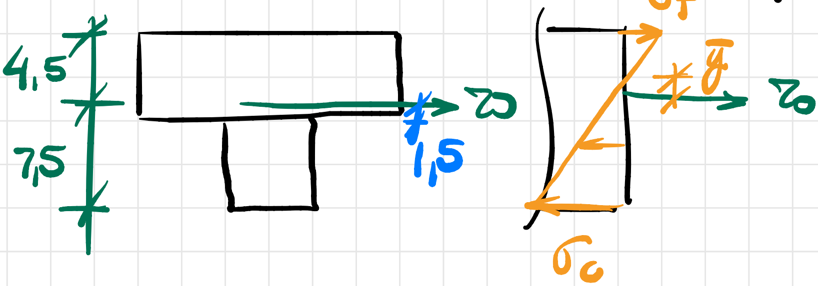
$$y_G = \frac{9.108+3.36}{108+36} = \frac{1080}{144} = 7{,}5\,cm$$

$$A = 144\ cm^2$$

$$I_{z_0} = (324+243)+(108+729)$$

$$I_{z_0} = 1404\,cm^4$$

Cálculo das tensões em B $\begin{cases} M = -600q \ [kNcm] \\ N = -4q \ [kN] \end{cases}$



$$\sigma = -\frac{4q}{144} - \frac{600q}{1404}\,y$$

y [cm]

σ [kN/cm²]

linha neutra: $\sigma = 0$:

$$-\frac{4q}{144} = \frac{600q}{1404}\,\bar{y} \Rightarrow \bar{y} = -0{,}065\,cm$$

$$\sigma_T = -\frac{4q}{144} - \frac{600q}{1404}\cdot(-4{,}5) = 1{,}895q$$

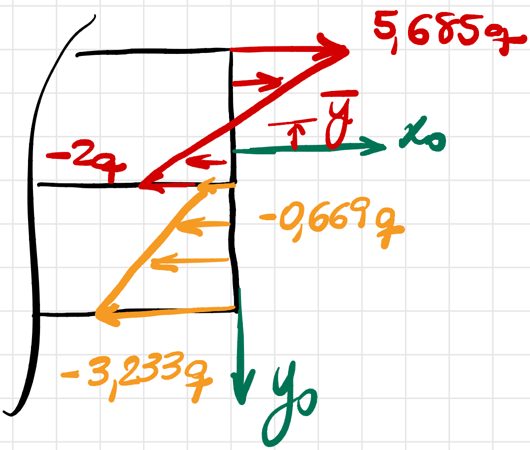
$$\sigma_C = -\frac{4q}{144} - \frac{600q}{1404}\cdot 7{,}5 = -3{,}233q$$

$$\sigma_I = -\frac{4q}{144} - \frac{600q}{1404}\cdot 1{,}5 = -0{,}669q$$

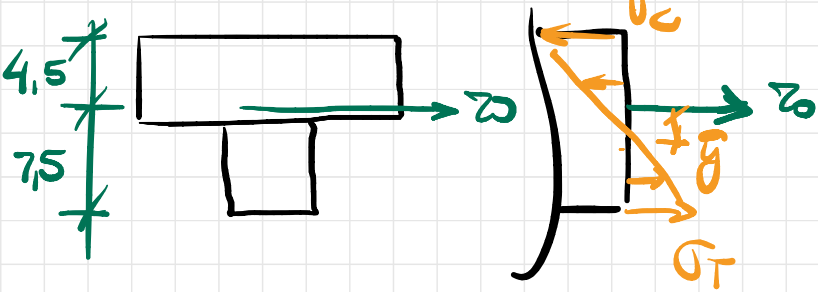
no material ①:

$$\sigma_T^{①} = 3\sigma_T = 5{,}685q$$

$$\sigma_I^{①} = 3\sigma_I = -2q \quad (\simeq 2{,}007q)$$



Cálculo das tensões em C $\begin{cases} M = 200q \\ N = -4q \end{cases}$



$$\sigma = -\frac{4q}{144} + \frac{200q}{1404} y$$

linha neutra: $\sigma = 0$

$$\frac{4q}{144} = \frac{200q}{1404}\bar{y} \Rightarrow \bar{y} = 0{,}195\,\text{cm}$$

$$\sigma_T = -\frac{4q}{144} + \frac{200q}{1404} \cdot 7{,}5 = 1{,}04\,q$$

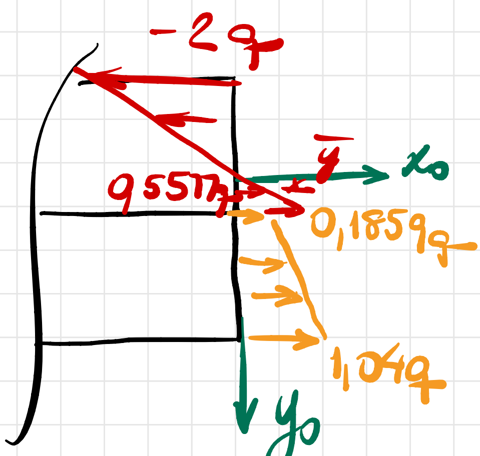
$$\sigma_c = -\frac{4q}{144} + \frac{200q}{1404} \cdot (-4{,}5) = -0{,}669\,q$$

$$\sigma_I = \frac{-4q}{144} + \frac{200q}{1404} \cdot 1{,}5 = 0{,}1859q$$

no material ①:

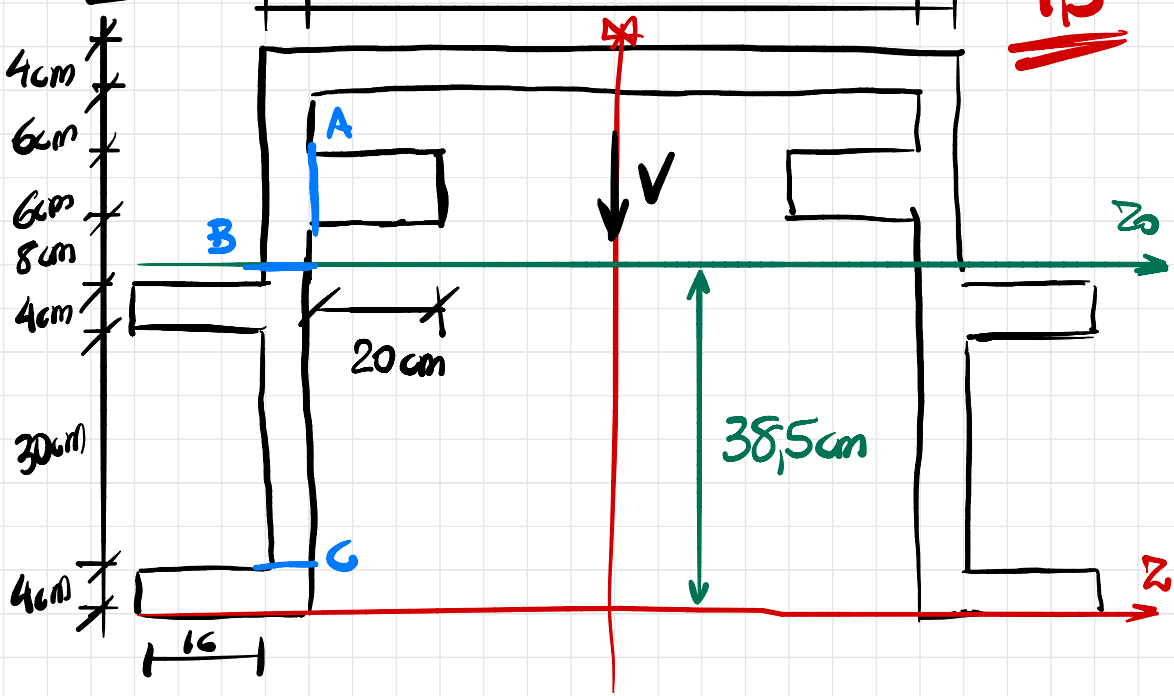
$$\sigma_T^{①} = 3\sigma_T = -2q \quad (\simeq -2{,}007q)$$

$$\sigma_I^{①} = 3\sigma_I = 0{,}5577q$$



Q4

1,5



Esboçar o diagrama de fluxo de cisalhamento e calcular as tensões em A, B e C.

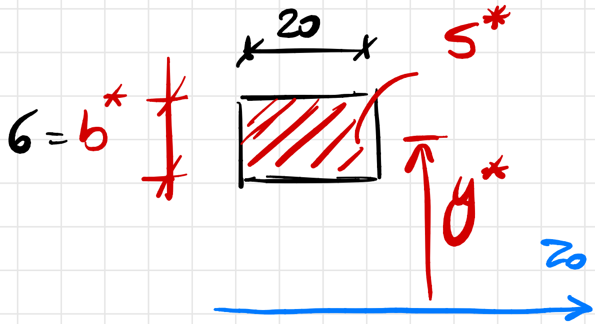
São dados: $y_G = 38{,}5\,cm$ (na figura) e $I_{z_G} = 5{,}2 \cdot 10^5\, cm^4$.

$V = 156P$ [kN]

0,3 por tensão

0,6 no esboço

Tensões em A:



$b^* = 6cm$

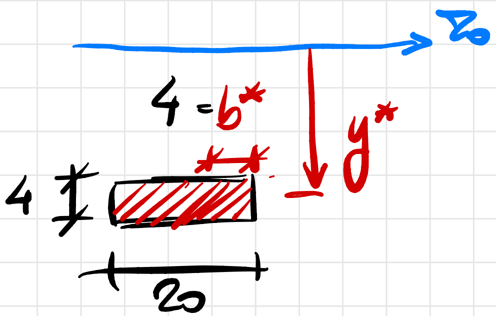
$S^* = 120\,cm$

$y^* = 49 - 38{,}5 = 10{,}5cm$

$M_S^* = 1260\,cm^3$

$$\tau_A = \frac{156P \cdot 1260}{6 \cdot 5{,}2 \cdot 10^5} = 0{,}063P\ kN/cm^2 = 0{,}63P\ MPa$$

Tensões em C:



$b^* = 4cm$

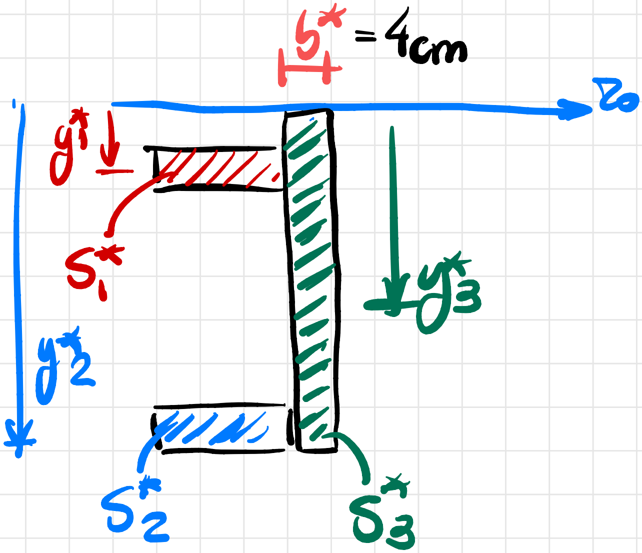
$S^* = 80cm^2$

$y^* = 38{,}5 - 2 = 36{,}5cm$

$M_S^* = 2920\,cm^3$

$$\tau_C = \frac{156P \cdot 2920}{4 \cdot 5{,}2 \cdot 10^5} = 0{,}219P\ kN/cm^2 = 2{,}19P\ MPa$$

Tensões em B:



$b^* = 4cm$

$y_1^* = 38{,}5 - 36 = 2{,}5cm$

$S_1^* = 4.16 = 64cm^2$

$y_2^* = 38{,}5 - 2 = 36{,}5cm$

$S_2^* = 4.16 = 64cm^2$

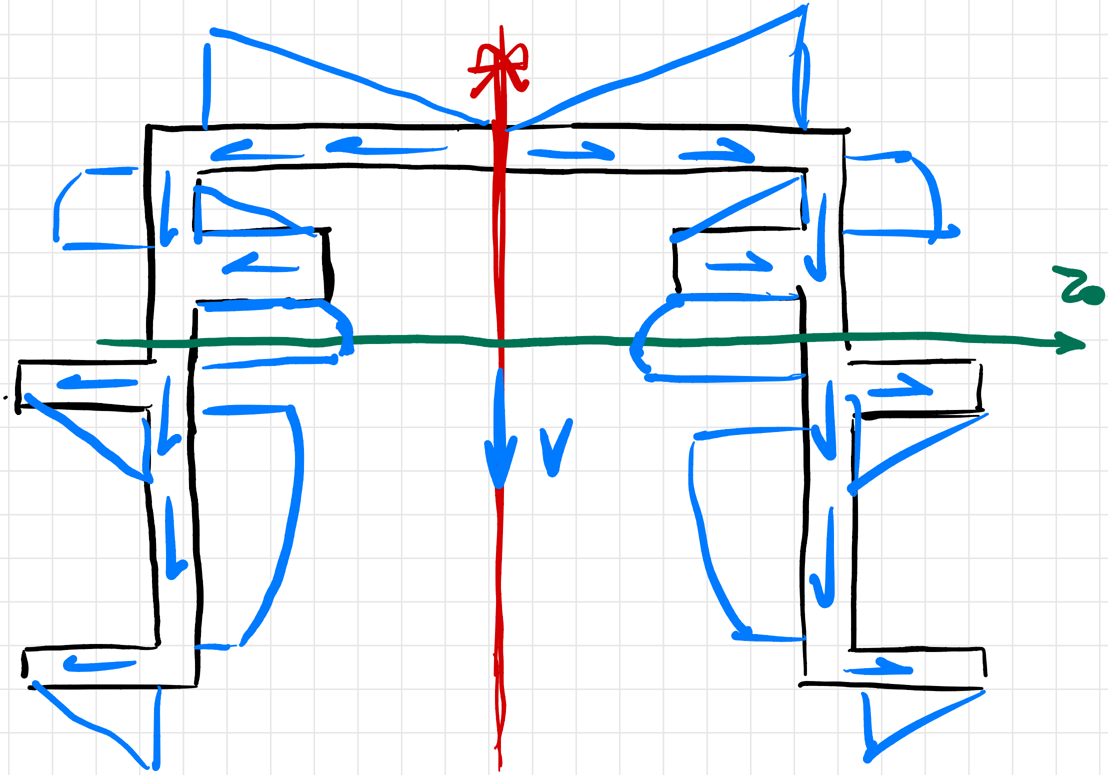
$y_3^* = 38{,}5/2 = 19{,}25cm$

$S_3^* = 154\,cm^2$

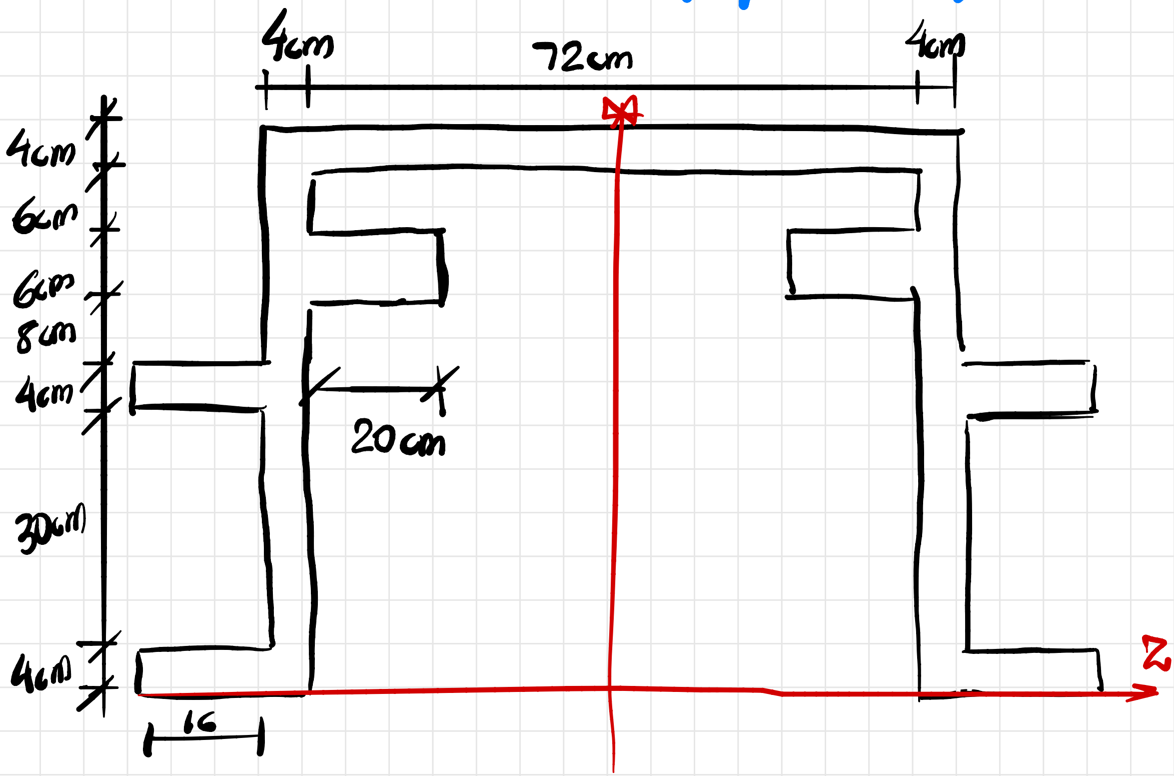
$$M_S^* = 2{,}5.64 + 36{,}5.64 + 19{,}25.154 = 5460{,}5\,cm^3$$

$$\tau_B = \frac{156\,P.5460{,}5}{4.5{,}2.10^5} = 0{,}41P\ kN/cm^2 = 4{,}1P\ MPa$$

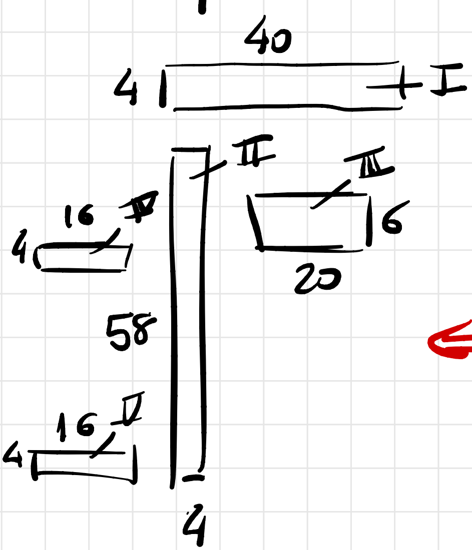
Esboço das tensões



Cálculos auxiliares (Propriedades)



Propriedades:



usando a simetria, tem-se que $I_{z_0} = 2\, I_{z_0}^{1/2}$

	y_G	A	d	d^2A	I_{Δ}	
I	60	160	21,5	73.960	640/3	
II	29	232	9,5	20.938	195112/3	
III	49	120	10,5	13.230	360	
IV	36	64	2,5	400	256/3	
V	2	64	36,5	85.264	256/3	

$$y_G = \frac{60.160 + 29.232 + 49.120 + 36.64 + 2.64}{160 + 232 + 120 + 64 + 64}$$

$$y_G = \frac{24640}{640} = \underline{\underline{38,5\,cm}}$$

$$I_{\Delta}^{1/2} = \sum \left(I_{\Delta}^{i} + d_i^2 A_i \right)$$

$$I_{\Delta}^{1/2} = 2,6.10^5\,cm^4$$

$$I_{\Delta} = 2\, I_{\Delta}^{1/2} = \underline{\underline{5,2.10^5\,cm^4}}$$